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Freely orientable microstructures for designing deformable 3D prints

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1 INTRODUCTION

With the advent of additive manufacturing (AM), we can envision the design and fabrication of objects exhibiting a wide palette of deformation behaviors. The elastic response of a part can be significantly modified by carefully crafting a small scale architecture within its volume, a metamaterial. Yet, despite major advances, we are still relatively limited when it comes to obtaining large controlled deformations within an object, across different directions.

A promising approach is to exploit a contrast in the directional elastic response of a shape: being near non-stretchable along some...
directions while being comparatively very flexible along others. Under large deformations, the contrast induces internal curvature, resulting in highly non-homogeneous deformations that would be challenging to obtain otherwise. Recent examples include pneumatic plates [Siefert et al. 2019] and the design of flexures [Shaw et al. 2019]. However, the studied structures are tailored to a specific case and offer limited design freedom. In this work we propose a freely orientable metamaterial with a high directional rigidity contrast. It affords designers unprecedented freedom, allowing to mimic the behavior of flexible fibrous materials (iniquitous in animal/plant tissue), and to trigger highly anisotropic, large deformations within a volume.

Numerically simulating under such large deformations is extremely challenging. Notably due to thin walls, complex stochastic geometry, and the difficulty to model the effect of layered fabrication. For large deformations, we thus resort to physical measurements on actual fabricated samples. This has the advantage to take into account the actual final product as opposed to an idealization of it. This is especially important as we print walls at the smallest possible thickness. The downside is that we cannot explore the material space or undertake inverse problems in a numerical fashion. However, our structures provide a fiber analogy allowing an intuitive approach for modeling.

The designer directly controls the local fiber orientations and structure sizing through a painting interface and can fabricate designs at low cost, on widely available filament 3D printers.

This idea is illustrated in Figure 1 where the fiber-like microstructures — produced by our method — makes the top of the deformed cube wrap around the bar being pressed on it. The yellow dashed line in Figure 1 (right) outlines a curve along which our structure resists stretch and preserves the curvilinear distance, as a rigid fiber would. We call the resulting material *transverse-rigid* as its maximal rigidity is along the selected transverse (fiber) direction and is much stiffer than its average response in the orthogonal plane.

Our approach allows to freely orient and grade such an elastic directional response throughout the volume of an object. The structures afford for contrast gradation between the most rigid and most flexible directions (see Figure 1, bottom part of the cube). This allows to locally dampen or cancel the directional contrast, progressively transitioning towards a structure that has the same elastic response in all directions.

Defining periodic structures having a transverse-rigid behavior in a fixed reference frame is relatively straightforward. But without the ability to freely orient the transverse direction and grade the rigidity contrast, the design possibilities are severely restricted. Making such controls possible, however, requires synthesizing a geometry with specific spatially varying characteristics (Section 3 and Figures 2 and 3). This leads to the conflicting requirements of having changes in local orientation and scale while preserving an overall smooth, locally regular structure.

Besides inducing the target elastic response, a key challenge is to ensure that the synthesized geometry can be fabricated with AM. For the large deformations we target, internal voids and thin walls are paramount, as the combination of both will produce the least dense and most flexible objects while being less prone to fracture than open-cell (truss) structures. On most technologies, this leads to the major difficulty of avoiding any sort of support structure within the volume. 3D printing thin walls — at the minimal possible thickness — also implies enforcing a maximum overhang constraint: no wall should have an angle that exceeds a maximum with respect to the vertical direction of fabrication. Otherwise, the solidified curves forming the walls in each slice will not vertically bond to one another [Livesu et al. 2017].

The microstructure we synthesize is carefully crafted to achieve all the required properties. It prints reliably on even the most affordable AM process (filament printers), behaves like a transverse-rigid material, it can be freely spatially graded in terms of orientation and rigidity contrast, and it is efficient to synthesize, making it scalable (embarrassingly parallel algorithms with no global memory storage of the result).

To achieve this, we define the microstructures from a stochastic process introduced in the field of procedural texturing [Lagae et al. 2010a]. The stochastic component provides direct control over the average distribution of geometric features, which translates into control over the average mechanical response. However, this comes with a specific challenge: such processes generate structures that can have a large number of localized discontinuities that negatively impact the structural properties. We propose a specialized filtering method that removes most of these issues from the final results.

We describe the geometry synthesis process in Section 3 and Section 4. To verify that the geometry we synthesize indeed produces a transverse-rigid oriented response, we fabricate a variety of samples covering the parameter space and submit each to mechanical testing. We report results in Section 5. We also fabricate a number of designs with interesting deformations, shown in Section 6 as well as in the accompanying video.

**Contributions.** In summary, our contributions are:

1. The definition, from stochastic processes, of a microstructure with high transverse-rigidity, which orientation, scale, and rigidity contrast can be freely graded.
2. A filtering approach to strongly reduce the occurrence of defects in the synthesized structures.
3. An evaluation of the obtained results in terms of manufacturability and mechanical properties.

![Fig. 2. Left: Microstructure without gradations. The extruded diamond shape has a high rigidity in the direction of extrusion u and is comparatively very flexible in the two orthogonal directions v and w. Right: Illustration of orientation gradation along an arbitrary direction field. Conforming to changes in direction while maintaining a near-constant structure scale inevitably yields scattered singularities (red circles). Several types of singularities exist, the ones shown here are not problematic.](image)
There is a large body of work considering how microstructures, or small-scale architectures, modify the macro-scale mechanical behavior of an object. We describe here those most related to our approach, and in particular those studied in the context of AM.

Throughout this discussion, and in the remainder of the paper, it is important to recall that the mechanical behavior of microstructures is to be understood as the average behavior of a sufficiently large volume filled with the microstructure [Gibson and Ashby 1999].

Microstructures are synthesized inside an object following a variety of approaches. A traditional way is to repeat carefully optimized tiles within a periodic grid [Panetta et al. 2017, 2015; Schumacher et al. 2015; Zhu et al. 2017]. The periodicity affords for efficient simulation of material properties [Michel et al. 1999] and slicing [Brennan-Craddock 2011]. Gradation is achieved by choosing different tiles in different locations, ensuring their boundaries match, either by construction [Panetta et al. 2015], optimization [Schumacher et al. 2015] or parametric interpolation [Fryazinov et al. 2013; Li et al. 2015]. Microstructures can also be defined through subdividing geometries [Kuipers et al. 2019; Medeiros e Sá et al. 2015; Wu et al. 2016], in which case density is directly controlled by the subdivision level.

While tile-based and subdivision approaches may allow for large deformations (locally placing tiles with anisotropic response), the underlying grid makes it very difficult to follow arbitrary orientations and arbitrary control fields. Indeed, the pre-computed tile set can only practically include a small set of orientations, given that all possible transition tiles would also have to be included.

Another approach is to generate microstructures from stochastic processes [Martínez et al. 2016, 2018, 2017]. The study of parameterized stochastic microstructures [Pham et al. 2019; Tarantino et al. 2019] has been gaining research focus in the last few years due to certain advantages it offers compared to periodic microstructures. First, stochastic microstructures allow for very progressive gradation and increased design freedom, as no grid or subdivision rule is constraining the structures [Siddhant et al. 2020]. Namely, the spatial gradation of a stochastic process is intrinsically encoded by its parameters. This is particularly relevant for anisotropic 3D microstructures that are graded following an arbitrary 3D vector field. Recent works dealing with the 3D periodic case [Geoffroy-Donders et al. 2020] require additional processing to grade and conform the periodic geometry to a surface. By contrast, stochastic microstructures do not require any additional processing. Second, it has been reported that stochastic microstructures are more resilient to fabrication imperfections arising from symmetry-breaking issues, compared to periodic microstructures [Portela et al. 2020]. Finally, procedural stochastic microstructures afford for scalable and efficient slicing [Pasko et al. 2011; Vidimče et al. 2013].

When using microstructures, spatial gradations may be directly designed [Ion et al. 2016; nTopology 2019], or may be optimized for various objectives such as rigidity [Li et al. 2015; Zhu et al. 2017], target deformations [Panetta et al. 2015], or balance [Wu et al. 2016].

Microstructures dedicated to additive manufacturing typically take fabrication constraints into account. These include minimal geometry size, maximal overhang [Martínez et al. 2018; Wu et al. 2016], the existence of unsupported islands in slices [Panetta et al. 2015] as well as overlaps in deposition paths [Kuipers et al. 2019].

Freely orientable microstructures. Several works orient structures in a volume or along a surface, for instance, conformal truss structures [Wang et al. 2005], periodic structures following a texture mapping analogy [Chen 2007], or procedural synthesis with free orientation [Martínez et al. 2017]. It is also possible to directly optimize a microstructure within a part, performing high-resolution topology optimization [Aage et al. 2017; Wu et al. 2018] that allows complex oriented structures to emerge. A recent development consists in synthesizing microstructures following the output of a topology optimization process [Geoffroy-Donders et al. 2020; Groen et al. 2020; Pantz and Trabelsi 2008; Wu et al. 2019]. These techniques produce cellular (hexahedral) and lattice (truss) structures following an optimized direction field. This has direct links to the field of hex-meshing [Gao et al. 2017; Jakob et al. 2015; Ray et al. 2006].

To the best of our knowledge, none of the approaches we mentioned above allow for directional grading of a microstructure inducing large, inhomogeneous deformations within the object. Martínez et al. [2017] orient orthotropic microstructures within a volume. However, the open-cell beam structures can only be fabricated on SLA/SLS: the beams take arbitrary angles well beyond overhang constraints and beams print poorly due to the many small disconnected regions appearing in every slice. The structures also tend to be fragile under large deformations, as stress accumulates on joints: for the same density, closed-cell structures have in general higher fracture toughness than open-cell structures [Maiti et al. 1984]. The closed-cell structure introduced by Martínez et al. [2018] can be graded but is orientable only in the plane aligned with the printer bed, and is significantly less collapsible than our structures (see Section 5). The alternative closed-cell structure proposed by Kuipers et al. [2019] is very flexible and collapsible along the vertical axis; however, no gradations other than density are possible.
This is an angle in radians, from 0 to 2π, which controls how much the cells orientations locally deviate from \( \mathbf{u} \).

The fields are created by the designer using a basic painting tool that we discuss in Section 6. In practice, the user only controls \( \mathbf{u} \) and the scale of \( \mathbf{v} \) and \( \mathbf{w} \): their directions are obtained as \( \mathbf{v} = \mathbf{u} \times \mathbf{z} \) and then \( \mathbf{w} = \mathbf{v} \times \mathbf{u} \).

In terms of physical properties, we seek to design a structure such that the transverse rigidity is everywhere aligned with the \( \mathbf{u} \) axis defined by \( F \). The local density is inversely proportional to the scale of \( \mathbf{v}, \mathbf{w} \), and derived from the wall thickness. Note that ideally the solidified thickness in a 3D printed slice should adjust for the local slope of the wall, from a ratio of 1 (vertical) to \( \sqrt{2} \) (max overhang of 45 degrees). \( \Gamma \) changes the rigidity contrast from the maximum at the given density towards a more isotropic result. \( \Gamma \) has no significant impact on density. Please refer to Section 5.3 for measurements validating these relationships on actual samples.

### 3.1 Design challenges

Consider a very flexible periodic 2D pattern in the \( xy \)-plane, typically a bending dominated structure [Ashby 2006]. By extruding it vertically along the \( z \)-axis, we produce a 3D pattern that remains flexible in the \( xy \)-plane but is very rigid along the \( z \)-axis (as vertical walls strongly resist extension and compression).

Thus, we can expect that by sweeping such a 2D pattern along a direction \( \mathbf{u} \) in space, the alignment of the extruded walls with \( \mathbf{u} \) would locally trigger a similarly aligned local transversal rigidity. To control the rigidity contrast, we have to modulate the microstructure geometry from a pattern that has mainly walls along \( \mathbf{u} \), to a pattern that has a more uniform distribution of wall orientations.

We are now facing an especially difficult task, as we seek to have strongly oriented regular patterns in some locations (locally an extruded 2D pattern), and progressively grade towards evenly distributed spatial structures. Meanwhile, the walls have to obey everywhere the overhang constraints of AM processes, as they will be printed at minimal thickness. We address these challenges next.

### 3.2 Stochastic structures

Stochastic procedural texturing methods offer rich controls over the patterns that can be synthesized. These are usually focused on producing seemingly random continuous patterns that obey a specific spectral content in terms of frequency, direction and bandwidth [Lagae et al. 2010b]. We note that a perfectly regular stripe pattern can be described in terms of spectral content. For instance, a perfect sine wave is a Dirac in the spectral domain. As procedural methods with increasingly precise spectral content are developed [Goldberg et al. 2008; Lagae et al. 2009], these offer an interesting alternative to parameterization methods for generating stripe patterns (e.g. [Jakob et al. 2015; Knöppel et al. 2015]).

We define our microstructures from a phasor noise [Tricard et al. 2019]; a stochastic pattern generator that can produce sharp; contrasted oscillating patterns (see Figure 4). The oscillation’s frequency and orientation can be spatially graded at will, and can spatially evolve from a locally regular periodic oscillation towards a stochastic pattern having no clear preferred orientation.
The produced oscillations are less regular than those produced by
parametrization methods. However, this trade-off is compensated by
the ability to spatially grade the patterns, as well as the
computational efficiency of procedural methods: the approach scales
trivially, is supported by embarrassingly parallel algorithms, and
does not require meshing the domain. We therefore use this method-
ology to define our 3D microstructure geometry and later focus on
improving the quality of the synthesized oscillations (Section 4).

3.2.1 Phasor noise. A phasor noise is an instantaneous phase field \( \phi \)
which is defined from a complex version \( G \) of Gabor noise:

\[
\phi(x) = \text{Arg}(G(x))
\]

with:

\[
G(x) = \sum_j e^{-b||x-x_j||^2} e^{2\pi f_j d_j \cdot (x-x_j)+i\varphi_j}
\]

This is a sparse convolution process [Lewis 1989]. A finite set of
kernels located at random locations \( x_j \) in a domain combine into the
final signal. Each kernel is a product of two complex functions. The
first is a clamped (99% of stddev) Gaussian of bandwidth \( b \) centered
on the kernel, the second is an oscillator of frequency \( f_j \), phase \( \varphi_j \)
and direction \( d_j \). In our case, \( x, x_j \) and \( d_j \) are 3D vectors, thus \( G \)
and \( \phi \) define solid noises (see Figure 5, left).

Feeding the phase \( \text{Arg}(G) \) into a periodic function produces
oscillations having the profile of the function. The produced patterns
can be freely graded in orientation and frequency (see Figure 4).

3.2.2 Local parametrizations. To generate a cellular pattern such as
the one shown in Figure 6, we need to determine whether a
point belongs to a black or a white cell. A convenient approach is
to define a 2D parametrization in space (e.g. a set of texture
coordinates) from which we can associate every 3D point with a
2D point in a base profile. The profile defines the cross-sectional
shape of the cells, as illustrated in Figure 5. The choice of profile will
impact the final mechanical response as well as determine whether
the result is fabricable under overhang constraints. We discuss it in
Section 3.2.3.

![Image](https://via.placeholder.com/150)

Fig. 5. Overview of the microstructure geometry synthesis, without any spatial gradation for clarity. We define two orthogonal solid phasor noises \( p_v \) and \( p_w \) with a sawtooth profile. Each produces a cyclic linear ramp in \([0, 1]\). Combining both defines local \( u, v \) parametrizations in \([0, 1]^2\). These local parametrizations are then used to define diamond cells, from which the structure walls are extracted. AM toolpaths are directly generated from this structure definition.

![Image](https://via.placeholder.com/150)

Fig. 7. The axis of a diamond cell can be oriented within the full unit-sphere,
while being under a 45 degrees overhang constraint. **Left:** Square diamond shape extruded along its normal, \( Y \). All walls have angles of exactly 45 degrees with respect to the horizontal plane. **Right:** Rotation of the diamond cell around its horizontal diagonal (\( X \)). This further increases the vertical cross-section wall angles (red lines), which remain below the constraint.

Producing such a parameterization in our context is challenging. It is akin to computing a globally periodic parametrization (e.g. [Ray et al. 2006]) – with the additional challenge of providing the ability to grade the structure’s regularity based on the field \( \Gamma \). The method should also easily scale to large volumes and not require meshing the domain.

We propose to exploit the oscillation profile control of phasor noise. In particular, an oscillation with a sawtooth profile can be seen as a periodic local 1D parameterization which varies linearly from 0 to 1 in each period. By computing two orthogonal oscillating waves each with a sawtooth profile, we obtain local 2D parameterization patches (each wave provides one coordinate).

We orient two 3D phasor waves such that their directions are
locally orthogonal to \( u \) and respectively oscillate along \( v \) and \( w \). We
denote the waves respectively \( p_v \) and \( p_w \). At a given point \( x \), the phasor sawtooth evaluates as:

\[
p_v(x) = \frac{\text{Arg}(G_v(x))}{2\pi}
\]

where \( G_v \) has the form given in Equation 2, with the kernel direc-
tions \( d_j \) obtained as \( v(x_j) \) (and similarly for \( p_w \), using \( w(x_j) \)), see
Figure 5 (left). The phasor sawtooth waves depicted in the remainder
of the paper are planar slices of the 3D noises.
To generate cells with an increasing angular spread of wall directions we use $\Gamma$ to perturb the kernel orientations in $p_v$. We add a rotation around $w$ to the base orientation of the kernels, randomly sampling an angle in $[-\Gamma(x),\Gamma(x)]$. When $\Gamma(x) = \pi/2$ this results in oscillations in all directions (Figure 4 right), that still properly define parametric patches. Roughly speaking, in 3D the structures locally take the geometry of an hourglass around the $w$ axis.

For the sake of conciseness, we move implementation details of the Phasor noise — which follow the state of the art — to the supplemental material (Section 4 therein).

While very effective, defining the local parameterizations with phasor noise comes with its own challenges: the phasor fields have local defects that impact the structure and its mechanical properties. We discuss these in Section 3.2.4.

3.2.3 Diamonds and overhangs. To allow for free orientation, the transverse axis of the generated microstructures should be able to cover any direction in the full unit-sphere $\mathbb{S}^2$. However, the structure also has to always verify the overhang constraints for correct fabrication.

Under this objective, a square diamond extrusion profile has desirable properties. Let us consider a 2D diamond shape in the $x,z$ plane having one of its diagonal aligned with the vertical printing direction $z$. The extraction of the diamond shape along $y$ defines a cell with a diamond cross-section (see Figure 7, left). The wall angles with the horizontal plane are exactly 45 degrees, which typically enforces the maximum overhang angle constraint [Livese et al. 2017]. Any such diamond cell remains printable for arbitrary rotations around the $x$ then $z$ axis (see Figure 7, right), covering the full sphere of directions.

We however have to cancel any roll around $u$ from $v$ and $w$ in $F$, ensuring that $v$ remains orthogonal to $z$. While the user as complete freedom over $u$, we also have to constrain $\Gamma$ to avoid overhangs. Indeed, $\Gamma$ adds random rotations to the kernels around $w$, which has a similar effect as adding roll around the diamond axes. The range of values $\Gamma(x)$ can take is thus restricted based on $u(x)$ (see supplemental for details). The recommendation is thus to keep $u$ in the $x,y$ plane where a large $\Gamma$ is desired. In practice this is a minor inconvenience: a large $\Gamma$ is used to cancel the directional effects, and therefore the choice of $u$ is less important in such regions.

We obtain the black diamond cells (Figure 6) by thresholding the local parametrizations, using the following formula:

$$\left| \frac{1}{2} - p_v(x) \right| + \left| \frac{1}{2} - p_w(x) \right| < \frac{1}{2}$$

White cells are obtained by reversing the inequality.

3.2.4 Singularities. Using phasor noises as a basis to generate our microstructures comes with a specific challenge: the waves exhibit singularities where they diverge from an ideal regular oscillation. These defects occur where the complex Gabor field vanishes ($G(x) = 0$), but also where the phase field of the phasor noise varies abruptly, distort the oscillations. While Neyret and Heitz [2016] proposed an FFT-based iterative filter to reduce singularities in the context of Gabor noise, it is not applicable in the presence of spatial gradations.

Note that in a non trivial orientation field a number of singularities have to exist [Bommes et al. 2013]. However, these should ideally be points (in a slice) or curves (in 3D) where oscillations meet or split (see Figure 2, right). For the sake of clarity, since we show in figures 2D slices of 3D noises we will refer to the ideal case as point singularities.

Point singularities at the apex of branching oscillations (forks) do not endanger the overall properties of the structure: around the singular point, the produced shape still exhibits a diamond extrusion profile and prints reliably, with only a small hole at their base.

Unfortunately, we also observe curve singularities in the phasor noise (surface singularities in the 3D case); across such curves the local parameterization values abruptly change and oppose on both sides (see red circles in Figure 4). These are more problematic than point singularities as they induce a change from a black to a white diamond (and vice-versa) resulting in walls orthogonal to the local diamond extrusion direction — see red walls in the inset. Such walls violate the overhang constraint if the orientation is vertical, and make the structure stiffer overall.

As an illustration of the scope of the problem, a structure of fixed orientation is about four times stiffer in $v$, $w$ compared to $u$ when these singularities are not dealt with (see details in supplemental material, Figure 9). This results in a significantly lower rigidity contrast, which is detrimental to our use cases.

We focus in Section 4 on the problem of filtering curve singularities from phasor noises.

3.3 Toolpath extraction

For the sake of completeness, before we proceed to filtering let us briefly describe how we extract the final structure for 3D printing. We seek to produce a single, as thin as possible wall between diamonds. Given a slice, we first draw an image of the sliced structure by evaluating whether each pixel belongs to a black or white diamond cell, as defined in Figure 5. This results in a binary image, from which we extract a solidification trajectory along each boundary between black and white pixels.

This is a robust process, which only drawback is the image discretization error. However, we can use a higher image resolution — smaller than half of the minimal thickness — since memory consumption is low: we work in a single slice and evaluate the pixel ‘color’ using a GPU pixel shader. The shader evaluates the phasor
noises with a sawtooth profile directly from the kernels. The kernels themselves use little memory, and the memory allocated for a slice is freed immediately after trajectory extraction.

4 FILTERING SINGULARITIES

The main drawback of relying on phasor noises is that the oscillations exhibit many singularities even in simple cases ($F$ smooth overall and $\Gamma = 0$). One would instead expect a very regular result with smooth and progressive changes, and only a limited number of singularities where waves fork (see Figure 8).

We now describe how to strongly reduce the occurrence of singularities, while preserving the variability associated with changes of orientation, frequency, and angular spread.

Recall that our focus is on curve singularities, which have a detrimental effect on the synthesized geometry (see Section 3.2.4 and Figure 4). Such singularities most notably arise when two neighboring kernels with similar direction $d_j$ are out of phase — see Figure inset.

Our approach works at two fundamentally different levels. The stochastic process of the phasor noise cannot possibly produce phase coherency between kernels across long distances. Thus, we apply an iterated regularization process that operates on the phasor noise kernels and seeks to align the phases of neighboring kernels — removing inconsistencies — in order to produce longer coherent wavefronts (see Figure 8 left). It is described in Section 4.1.

A perfect phase alignment is generally impossible in the presence of variations in the frame field $F$. Therefore, even after phase alignment, some local singularities remain where neighboring kernels of different orientation or frequency combine. To further improve the result, we define a specialized filter that locally removes deviations from an idealized oscillation estimated in every sampling point. We describe the filter in Section 4.2.

4.1 Iterated phase alignment

To regularize the oscillations of the phasor noise, we first adjust the phase shifts of neighboring kernels, as illustrated in Figure 8. Interestingly, this problem appears in a related form in recent interactive quad meshing techniques [Evgeny and Harders 2019; Lichtenberg et al. 2018]. These operate on triangles and have one oriented oscillator (akin to a phasor kernel) per vertex. They attempt to globally align the phases of the per-vertex oscillators, so as to define continuous patterns across triangles by applying an iterative diffusion process. To achieve high performance, the phase at every vertex is iteratively updated, in parallel for all vertices, from the average of phases suggested by the neighbors. Note that the orientation and frequency of the oscillators remains fixed. Our iterated phase alignment for phasor noise bears some similarities but operates without a mesh on the initial kernels of the underlying (complex) Gabor noise.

At each iteration, the algorithm adjusts the phase of each kernel to be the phase of the average complex oscillator obtained from its neighbors. Aligning phases between kernels suppose they share a similar orientation. Thus, the average is weighted by the dot product of the orientation vectors of the kernel and its neighbor. The dot product result is clamped to zero. The algorithm for one iteration is given in Algorithm 1, where $\mathcal{N}(k_j)$ is the set of kernels neighboring $k_j$ (they belong to the same or neighboring cells in the Gabor noise evaluation grid [Lagae et al. 2009]).

We repeat Algorithm 1 for a fixed number of iterations (20 in our implementation), which we experimentally adjusted (please refer to supplemental material for details, Section 5). The algorithm trivially maps to an efficient GPU implementation, with one thread per kernel. The average time for one iteration on an NVIDIA GTX 1080 is 7ms for 551K kernels for a 100 mm $\times$ 100 mm $\times$ 100 mm cube.

4.2 Evaluation Filter

While phase alignment removes overall inconsistencies, the complex local interactions between disagreeing kernels still create local singularities. We propose a filter that regularizes the signal, filtering away discontinuities which locally distort the noise profile.

We start by examining the singularities and their properties in order to identify and classify them, and quantify their presence in the generated noise.

4.2.1 Identifying singularities. Let us first consider a simple case where the frequency and the orientations are constant in space, noted respectively $f$ and $\mathbf{d}$. In this case, we can extract the phasor field of the phasor noise. It is obtained at each point $\mathbf{x}$ by dividing the complex Gabor noise by a global oscillator $\omega_0$:

$$\omega_0(\mathbf{x}) = e^{2\pi i f \mathbf{d} \cdot \mathbf{x}} \quad (4)$$

The phasor field is thus obtained as:

$$\varphi(\mathbf{x}) = \frac{\mathcal{G}(\mathbf{x})}{\omega_0(\mathbf{x})} \quad (5)$$

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Fig. 10. Phasor noise filtering with increasing filter kernel sizes. **Left**: phasor field argument encoded as hue. **Right**: phasor sawtooth. **Inset**: filter. **Top to bottom**: increasing kernel size. Note how the phasor field becomes smoother and remaining singularities tend to become point singularities.

Fig. 11. Filtered phasor sawtooth (right) vs. phasor sawtooth with an equivalent noise bandwidth $b$ (left). Our filter formulation results in a pattern that matches the input fields more closely and is less prone to leading to undesired angular spread.

It represents the phase shift between a specific phasor noise pattern and a hypothetical perfectly regular global oscillator (i.e. a simple sine wave). Examples of phasor fields are shown in Figure 9.

Fig. 12. Phasor field (right) obtained by division of the input noise (left) by the corresponding global oscillator. Note how there are strong oscillations along a preferred direction in the field, which is unexpected from a phasor field such as in Figure 10.

Fig. 13. Non-attenuated filter (middle) vs. attenuated filter (bottom). Without attenuation, the filter privileges the direction given by the input orientation field and discards other orientations. This erases the angular spread (red), which is a superposition of kernels of spread-out directions. The attenuated filter does not exhibit this behavior (green), and thus better preserves the properties of the original noise (top).

**Remark 1.** The smoother the phasor field is, the less the pattern deviates from an ideal oscillator. Thus the phasor field variations characterize deviations from the ideal signal (the singularities).

Note that there is a notion of scale in this remark: deviations become problematic when they happen over a short distance compared to the oscillations wavelength. This wavelength is determined by the local frequency:

**Remark 2.** The frequency of the phasor noise defines the relative variation scale at which deviations can be considered as singularities.

Indeed, variations that are smooth with respect to the local frequency are not problematic, as the phasor noise smoothly and progressively shifts around the ideal oscillator. However, where the phasor field varies too abruptly, the noise profile is locally compressed, which results in discontinuities. This is coherent with the visual examination of the fields in Figure 9.

Remarks 1 and 2 open the possibility to filter singularities by directly smoothing the phasor field (Section 4.2.2). However, this only holds for a constant orientation and frequency over the whole surface. A primary objective of our work is to allow arbitrary spatial gradations, therefore a more general point of view is needed.

We note that under the assumption that the control fields are smooth, they can be locally considered constant. We start by rewriting the global oscillator for an arbitrary origin point $c$:

$$\omega_c(x) = e^{2\pi i f d \cdot (x-c)} \tag{6}$$
Fig. 14. Comparison of the contribution of the different parts of the method to the reduction of singularities. **Left**: from top to bottom: input orientation and scale, original phasor sawtooth wave, phasor sawtooth wave after iterated kernel phase alignment, phasor sawtooth wave after filtering, phasor sawtooth wave after iterated phase alignment and filtering. From left to right: various input fields and random seeds. **Right**: close-up of the effect of the iterated phase alignment, filtering, and both methods on the second example. Both approaches contribute to the overall improvement (see circled singularity).
The field \( \frac{G(x)}{\omega_c(x)} \) is the same as \( \varphi(x) \) with an additional constant phase shift \( e^{i2\pi f d} \). Thus, it has the same gradient, and gives the same result under Remark 1.

Then, substituting \( f \) and \( d \) for \( F(c) \) and \( D(c) \) — the approximated parameters around \( c \) — in \( \omega_c \), we obtain a local definition of the phasor field:

\[
\varphi_c(\delta) = \frac{G(c + \delta)}{\omega_c(c + \delta)}
\]

(7)

\( \varphi_c(\delta) \) provides a good approximation of the phase shift relative to an ideal oscillator at any offset \( \delta \) around \( c \), as long as \( \delta \) remains small compared to the spatial variations of the control fields.

This allows us to evaluate functions on the locally-defined phasor field in a neighborhood centered around \( c \). We use this property to define a measure for singularities and then express the evaluation filter.

4.2.2 Filter. From Equation 5 we see that the initial complex Gabor noise can be reconstructed from the phasor field, multiplying it back by the oscillator. This opens the possibility to directly manipulate the phasor field, and in particular, apply a smoothing filter that will remove abrupt variations. Following Remark 1, doing so will reduce the extent of singularities in the phasor noise.

4.3 Results of singularity reduction

Figure 14 shows a variety of qualitative examples using the sawtooth profile, and for each, the effect of both approaches separately, and then combined. In all cases, we observe a significant reduction in the number of singularities by the regularization, as well as a much more regular oscillation pattern after the evaluation filter is applied.

Applying only the filter results in a high number of curve discontinuities which cannot be solved without aligning phases, but applying only the iterated phase alignment results in remaining local distortions. Using both approaches gives the best results.
We now consider how well the synthesized geometries match the user-provided target fields. First, we measure how well the scale $v$ in $F$ correlates with the final structure density. Second, we verify that the structure orientation matches the user specifications $u$ in $F$.

For analysis, we convert the produced solidification trajectories (G-code) into a high resolution voxel model. We then analyze the data to extract local density and local orientation measures.
The results are shown in Table 1. The experiments reveal that our method obtains a higher contrast in transversal rigidity, which is reflected in a greater collapsibility in the transverse axis (see Figure 19). The extracted orientation also follows the input orientation closely as the scale, angular spread). Figure 19 shows the input scale and the obtained output density. The extracted orientation also follows the input orientation closely as the bottom histogram shows, with most structures within 20 degrees of the input direction.

Table 1. We compare our approach having $\gamma = 0$ with [Martínez et al. 2018] having $\mu = 0.85$ (see article for details of the effect of parameter $\mu$) to produce two comparable structures exhibiting transverse rigidity. The specimens are cubes of 30 mm of side with similar density. We perform compression tests to evaluate the directional Young’s modulus (MPa) in each orthogonal direction. Our method creates a structure that is more compliant on $x$ an $y$, stiffer along $z$, thus manifesting a greater rigidity contrast (see Figure 19).

<table>
<thead>
<tr>
<th>Axis</th>
<th>[Martínez et al. 2018]</th>
<th>Ours</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x$</td>
<td>0.76 MPa</td>
<td>0.12 MPa</td>
</tr>
<tr>
<td>$y$</td>
<td>0.60 MPa</td>
<td>0.08 MPa</td>
</tr>
<tr>
<td>$z$</td>
<td>1.175 MPa</td>
<td>2.78 MPa</td>
</tr>
</tbody>
</table>

User painting tool. The fields are stored and manipulated as low resolution 3D textures ($64^3$), which are directly painted by users through a dedicated interface shown in Figure 20. We describe here our simple approach for this, but of course many other options would be possible.

The user selects a brush radius, a scale factor (for $v,w$), a value for $\Gamma$, and paints onto the shape. The stroke direction is used as the vector $u$. The brush updates the volume field within a radius around its center. To paint inside the volume, the user can interactively move a cut plane through the part.

Direct visual feedback is obtained by rendering the phasor local parameterizations along the surface or cuts (without singularity correction). This gives an accurate indication of the structures that will be locally synthesized.

Usage guidelines. We found the analogy with long rigid fibers to be effective when designing the fields. Thanks to the very high rigidity contrast of our structures, a designer can reason in only two terms: directions that will be rigid (as if long rigid fibers were embedded in the print) and directions that will be flexible. Under deformation, the fibers resist stretch and pull/push other parts of the volume alongside them. These other parts then easily collapse along flexible directions. These effects cascade throughout the volume.

In addition to these base principles, some typical patterns exist that can be reused in many parts. For instance, the cylindrical collapse of Figure 1 generates a gripper/engulfing behavior also visible in Figures 26, 22. Radial and tangential patterns trigger (respectively) collapse or absorption effects as in Figure 24. Changes to $\Gamma$ cancel out the contrast achieving different effects along a shape, as in Figure 25. These behaviors can combine towards more complex effects such as in Figures 23, 29, 27, 28. We next detail each of these examples.

Examples. Our first example, shown in Figure 1, is a cube filled with a complex structure. The direction field rotates such that the transverse rigidity remains parallel to an axis, located in the middle of the top face. Some angular spread is added to the opposite (bottom) face, making it more rigid in all directions. Pressing on the top face collapses the cube while wrapping the top around the pressing object, an unexpected gripping behavior for the unsuspecting observer. Pressing on the bottom face produces nothing special, the cube reacts as if made of a homogeneous material. Such gripping behaviors are useful for soft robotics.

Fig. 17. Fidelity of the synthesized geometry to the control fields, for the 3D printed knee example (Figure 29). Although the output density suffers from filtering artifacts at the geometry boundary, the rest of the model correlates strongly with the input scaling field (Pearson correlation coefficient: 0.814). The extracted orientation also follows the input orientation closely as the bottom histogram shows, with most structures within 20 degrees of the input direction.

6 Printed Examples

In this section we design and fabricate several examples with our method, demonstrating the variety of deformation behaviors that can be achieved by manipulating the control fields (orientation, scale, angular spread).

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https://icles.loria.fr/
the radial arrangement of rigidity makes the entire plate bend along the $x$ axis. 

The Young’s modulus $E$ is in Mpa. The printing direction is along the $w$ axis. Please refer to the text for discussion.

Figure 24 shows a thick circular plate, where one face has rigidity oriented radially and the other in a tangential manner. This reveals how the direction field alone strongly influences the deformation. In Figure 24 (bottom) we see that pressing the center has no global effect since the structures flex and absorb the distortion. In Figure 24 (top) the radial arrangement of rigidity makes the entire plate bend under pressure. This is easily explained by the orientation of the structures that prevent distances to change within the volume along specific directions. Note that this very different behavior occurs on both sides of the same object.

Figure 23 is another circular plate where a tangential spiral displacement occurs under radial compression, thanks to the specific arrangement produced by the direction field. This example could also act as a shock-absorbing wheel for a robot or rover.

Figure 27 is a toy mechanism inspired by Figure 1 right. The fields are overall the same, but an internal geometry allows a column to raise from the opposite face when pressing on the sides. This can serve as an actuator or a lateral pressure sensor (measuring the displacement of the column).

Figure 26 is an example design of a simple chair. We oriented the structure to follow the seat surface and be less dense near it.

Figure 28 is a pair of pliers synthesized with our method. The jaws and the hand grips are denser whereas the joint part is softer and uses a radial field to guide the deformation.

Our last and example is an illustration of a knee-like ‘joint-less’ articulation, shown in Figure 29 and which principle is more clearly isolated in Figure 21. A flexible front plate is created using a low rigidity contrast. Behind, two main directions are used in a way that 1. allows the volume to collapse where we want the fold to occur and 2. resists lateral bending. We also added a region of low contrast to mimic the bone in the upper and lower leg. This inner core is easily felt when pressing on the sides and gives vertical rigidity to the overall part.

To the best of our knowledge, no other available approach can produce results with such deformation behaviors under large displacements. Freedom of gradation in orientation, density (through scale), and rigidity contrast (through $\Gamma$) interact to provide very effective control over the volume reshaping under external loads. Our examples are only scratching the surface of potential applications in soft robotics, soft mechanisms, prosthetics, and orthoses design, as well as shock or vibrations absorbing pads.

7 LIMITATIONS

Our structures are stiffer along directions $v + w$, $v - w$ while they would ideally be equivalently flexible for all directions in the $v, w$ plane. On less demanding fabrication technologies, using a hexagonal profile would reduce this effect.

Our current implementation does not adapt the deposited thickness with respect to the local wall slope, which implies that density varies with slope. While we did not observe detrimental effects in our results (where walls are often everywhere slanted), a varying width deposition would provide a more accurate result [Kuipers et al. 2020].

Finally, having the ability to perform simulation – even under crude approximations – would help explore more subtle designs and effects, and help the adoption of the approach for engineering applications. This is however an orthogonal, albeit important and difficult problem.

8 CONCLUSION

By allowing to design where and how a volume compresses or extends when manipulated, our method enables unprecedented control over 3D printed objects. We made careful trade-offs to obtain a balance between reliability of fabrication — including on the most...
Fig. 20. User interface for designing a frame field. From left to right: The original field with only one direction, The field with another direction painted along the red line, a section view of the field (section in green), the field with one more direction painted along the blue line on the green section.

Fig. 21. Principle of a knee-like flexure. A flexible front plate provides the overall strength. The V-shaped transverse-rigid structures behind resist lateral bending, while freely collapsing at the rear to let the front flex in the unique allowed direction (at comparable stress levels).

Fig. 22. A structured plate. The orientations produce a strong collapsing/gripping effect when pressing on a specific location.

Fig. 23. A wheel design with a spiraling direction field. Left: The structure at rest, with black lines drawn on it to better outline the deformation. Right: The wheel under radial compression. Note the spiral motion occurring around the center, thanks to the spiraling fiber structures.

Fig. 24. Top: Thick disc with radial fiber pattern. Pressing in the center lifts the sides up. Bottom: With a circular direction field, the central pressure is absorbed and has no effect on the sides.

Fig. 25. Plate with a progressive gradation of $\Gamma$ along its length. One end bends easily across its width while the other is overall more rigid. If pressed on a specific point it bends out of plane, due to how the deformations are absorbed on one side and transferred to the other.

affordable filament printers — and freedom of gradation of the mechanical properties. Experimental tests confirm that a high-contrast transverse rigidity is achieved over a wide range of deformations. The ability to print walls at the thinnest thickness with a single material allows to produce very flexible objects using off-the-shelf materials.

Our technique is defined from a stochastic approach. This affords for freedom of gradation of the rigid axis orientation, as well as spatial gradation in density and rigidity contrast. Thanks to its favorable computational properties, our technique allows rapid, efficient extraction of toolpaths and is highly scalable.
Freely orientable microstructures for designing deformable 3D prints

Fig. 26. Seat printed with our method, a lower frequency is used on the top of the structure to make it softer.

Fig. 27. Example of a soft mechanism created with our method. Lateral pressure on the sides of the object induces a deformation that makes the embedded cylinder move up, lifting a small object.

Fig. 28. Pliers printed with our method. The direction control induces the movement. Scaling control is used to rigidify the hand grips and the jaws, only allowing bending in the joint part.

In terms of applications, an important feature of our method is that it works on low-end 3D printers using inexpensive materials. This will allow enthusiasts, makers, and students in engineering to experiment with novel designs. We encourage this by releasing the technique in a freely available design/3D printing software.

Code availability. We provide the full implementation of a 2D version, available as a Shadertoy https://www.shadertoy.com/view/WtjfzW.

Fig. 29. Knee 3D printed using our diamond structures. (a) Input model: in blue the direction field $D$, and in green the angular spread field. (b) shows the generated toolpaths. (c) and (e) show the printed model at rest (d): Knee bended. (f): Stress is applied across the section. The less dense regions collapse, while the denser parts remain the same.

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