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## RESEARCH PROBLEM

**Problem 83.** J.-C. Bermond and M. Paoli

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We present here, in an array, problems concerning the hamiltonian properties of the powers of a graph  $G$  depending on certain properties of  $G$ . We mention also the results known on these problems.

Let us recall some definitions. A graph  $G$  is  $n$ -hamiltonian (resp.  $n$  edge-hamiltonian) if after removal of at most  $n$  vertices (resp. edges) the resulting graph is hamiltonian. A graph  $G$  is  $n$ -mixed hamiltonian if after removal of any  $p$  vertices and any  $q$  edges with  $p + q \leq n$ , the resulting graph is hamiltonian. A graph  $G$  is strongly  $(p, q)$ -hamiltonian if for any set  $S$  of vertices of  $G$  with  $|S| \leq p$  and any set  $T$  of edges of the complete graph built on  $G - S$ , so that  $|T| \leq q$  and the edges of  $T$  form a collection of independent paths, the graph  $(G - S) \cup T$  contains a Hamilton cycle including all the edges of  $T$ . The  $k$ th power of  $G$ , denoted  $G^k$ , is the graph obtained from  $G$ , by joining two vertices of  $G$  if and only if their distance in  $G$  is less than or equal to  $k$ .

The table must read as follows. The conjectures are followed by a question mark. The results that would be best possible, if true, are starred. It is understood that in all the rows,  $k$  will be at least 2, except for the rows  $G$  connected and  $G$  2-edge connected, where  $k$  must be at least 3. It is also understood that the number of vertices is large enough to insure that the property has a meaning: For example, if we want  $G^k$  to be  $n$ -hamiltonian, then the number of vertices is at least  $n + 3$ .

Other similar properties of the graph  $G$  can be considered, for example  $G$   $h$ -edge connected with  $h \geq 3$ . However, as no results are known to us, we did not include them in the table. The references include the last work on a case, although other important papers exist before.

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Table 1

$G$	$G^k$	$n$ -hamiltonian	$n$ -edge hamiltonian	$n$ -mixed hamiltonian	Strongly $(p, q)$ -hamiltonian
Connected ( $k \geq 3$ )		$n = k - 2^*$ [2, 4, 7, 13]	$n = k - 2^*$ [10]	$n = k - 2^* ?$	$p + q \leq k - 2^*$ [1]
$h$ -Connected ( $h \geq 2$ )		$n = hk - 2^* ?$ except $h = k = 2$ [7, 17] known $n \geq h + k - 3$ [3, 8]	$n = hk - 2^* ?$ known $n \geq k - 1,$ for $h = 2$	$n = hk - 2^* ?$ except for $k = h = 2$	$p + q \leq k + h - 3^*$ [11] true for $h = 2$ [11] $p + q \leq kh - 2^* ?$ [11] expect for $k = h = 2$ [7, 11, 15]
Hamiltonian		$n = 2k - 2^*$ [5]	$n = 2k - 2^*$ [16]	$n = 2k - 2^* ?$ [16]	$p + q \leq 2k - 2^* ?$
$h$ -Hamiltonian ( $h \geq 1$ )		$n = (h + 2) \cdot k - 2^* ?$ [11] for $h = 1, n = 3 \cdot k - 2^*$ [11] for $h = k = 2, n = 6^*$ [11]	$n = (h + 2) \cdot k - 2^* ?$	$n = (h + 2) \cdot k - 2^* ?$	$p + q \leq (h + 2) \cdot k - 2^* ?$ [11]
2-edge connected ( $k \geq 3$ )		for $k = 3, n = 2^*$ [14] $k > 3 ?$	for $k = 3, n \geq 2$ [12] $n = 3^*$ for $k = 3 ?$	for $k = 3, n = 2^*$ [12]	for $k = 3, p + q \leq 2^*$ [9, 12]

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