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RESEARCH PROBLEM

Problem 83. J.-C. Bermond and M. Paoli

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We present here, in an array, problems concerning the hamiltonian properties of the powers of a graph G depending on certain properties of G . We mention also the results known on these problems.

Let us recall some definitions. A graph G is n -hamiltonian (resp. n edge-hamiltonian) if after removal of at most n vertices (resp. edges) the resulting graph is hamiltonian. A graph G is n -mixed hamiltonian if after removal of any p vertices and any q edges with $p + q \leq n$, the resulting graph is hamiltonian. A graph G is strongly (p, q) -hamiltonian if for any set S of vertices of G with $|S| \leq p$ and any set T of edges of the complete graph built on $G - S$, so that $|T| \leq q$ and the edges of T form a collection of independent paths, the graph $(G - S) \cup T$ contains a Hamilton cycle including all the edges of T . The k th power of G , denoted G^k , is the graph obtained from G , by joining two vertices of G if and only if their distance in G is less than or equal to k .

The table must read as follows. The conjectures are followed by a question mark. The results that would be best possible, if true, are starred. It is understood that in all the rows, k will be at least 2, except for the rows G connected and G 2-edge connected, where k must be at least 3. It is also understood that the number of vertices is large enough to insure that the property has a meaning: For example, if we want G^k to be n -hamiltonian, then the number of vertices is at least $n + 3$.

Other similar properties of the graph G can be considered, for example G h -edge connected with $h \geq 3$. However, as no results are known to us, we did not include them in the table. The references include the last work on a case, although other important papers exist before.

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Table 1

G	G^k	n -hamiltonian	n -edge hamiltonian	n -mixed hamiltonian	Strongly (p, q) -hamiltonian
Connected ($k \geq 3$)		$n = k - 2^*$ [2, 4, 7, 13]	$n = k - 2^*$ [10]	$n = k - 2^* ?$	$p + q \leq k - 2^*$ [1]
h -Connected ($h \geq 2$)		$n = hk - 2^* ?$ except $h = k = 2$ [7, 17] known $n \geq h + k - 3$ [3, 8]	$n = hk - 2^* ?$ known $n \geq k - 1,$ for $h = 2$	$n = hk - 2^* ?$ except for $k = h = 2$	$p + q \leq k + h - 3^*$ [11] true for $h = 2$ [11] $p + q \leq kh - 2^* ?$ [11] expect for $k = h = 2$ [7, 11, 15]
Hamiltonian		$n = 2k - 2^*$ [5]	$n = 2k - 2^*$ [16]	$n = 2k - 2^* ?$ [16]	$p + q \leq 2k - 2^* ?$
h -Hamiltonian ($h \geq 1$)		$n = (h + 2) \cdot k - 2^* ?$ [11] for $h = 1, n = 3 \cdot k - 2^*$ [11] for $h = k = 2, n = 6^*$ [11]	$n = (h + 2) \cdot k - 2^* ?$	$n = (h + 2) \cdot k - 2^* ?$	$p + q \leq (h + 2) \cdot k - 2^* ?$ [11]
2-edge connected ($k \geq 3$)		for $k = 3, n = 2^*$ [14] $k > 3 ?$	for $k = 3, n \geq 2$ [12] $n = 3^*$ for $k = 3 ?$	for $k = 3, n = 2^*$ [12]	for $k = 3, p + q \leq 2^*$ [9, 12]

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