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Averaging techniques in the time minimal orbital transfer using low propulsion

Jérémy Rouot

NONLINEAR CONTROL AND GEOMETRY
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Joint work with B. Bonnard, J.B. Caillau and J.B. Pomet.

Some historical points about optimal control and orbital transfer with low thrust

- $\approx$1970, electro-ionic propulsion where the thrust is very low (spiral transfer).
  Missions: Artemis-ESA (2001, GST), Smart-1-ESA (2006, GST to lunar orbit), BepiColombo-ESA (2016, Mercury).

- Epenoy-Geffroy (1997) use averaging techniques coupled with the Pontryagin Maximum Principle for low thrust transfer.

- Bonnard-Caillau (2007) computed explicitly the averaged Hamiltonian in the case of the energy cost.
  It defines a Riemannian metric which allows to compute extremal curves for coplanar transfers.

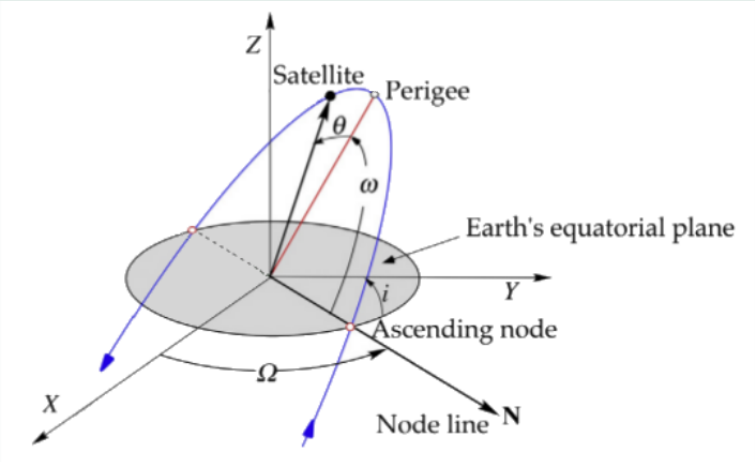
Time minimal orbital transfer problem

$q \in \mathbb{R}^3$: position vector of the satellite,

$$\ddot{q} + \nabla V(q) = 0, \quad V(q) = -\frac{1}{|q|}.$$

Elliptic orbits: restriction to $\mathcal{E} = \{(q, \dot{q}) \mid q \times \dot{q} \neq 0 \text{ and } \frac{1}{2}|\dot{q}|^2 - V(q) < 0\}$.

Kepler's orbital elements



$(a, e, i, \omega, \Omega)$: shape and orientation of the ellipse,
θ: angle locating the satellite on its orbit.
The uncontrolled motion can be described by $(I, \varphi = F(I, \theta))$

$$\begin{aligned} \dot{I} &= 0 \\ \dot{\varphi} &= \nu(I). \end{aligned}$$

$0 < \varepsilon \ll 1$, $(I(t), \varphi(t)) \in \mathbb{R}^5 \times S^1$: state variables of the satellite, $u(t) \in \mathbb{R}^3$, $F_i(I,\varphi,\varepsilon), G_i(I,\varphi,\varepsilon)$ $i = 1..3$ are **2π-periodic** vector fields with respect to φ.

Time minimal orbital transfer problem

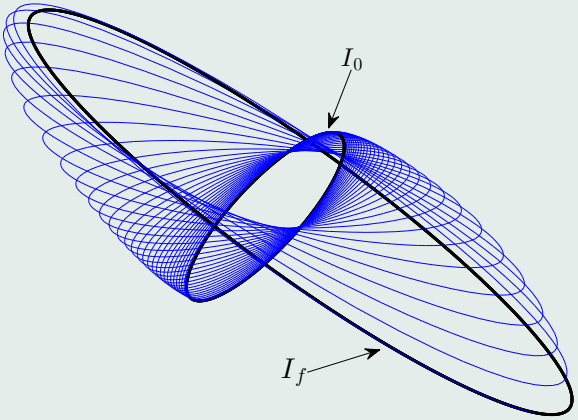
$$\min_{|u| \leqslant 1} \quad t_f(u)$$

$$\text{subject to} \quad \dot{I} = \varepsilon F_0(I,\varphi,\varepsilon) + \varepsilon \sum_{i=1}^{3} u_i F_i(I,\varphi,\varepsilon), \quad \dot{\varphi} = \nu(I) + \varepsilon \sum_{i=1}^{3} u_i G_i(I,\varphi,\varepsilon)$$

$$I(0) = I_0, \qquad I(t_f(u)) = I_f.$$

The vector field F_0 takes into account conservative perturbations such as

- lunar and solar attraction,
- J_2-effect.



Averaging principle for dynamical system

Let $f(X,\psi,\varepsilon), g(X,\psi,\varepsilon)$ be two smooth $\mathbf{2\pi}$-**periodic** vector fields with respect to $\psi \in S^1$.

Perturbed system

$$\begin{aligned} \dot{X} &= \varepsilon f(X,\psi,\varepsilon) \\ \dot{\psi} &= \nu(X) + \varepsilon g(X,\psi,\varepsilon). \end{aligned}$$

Averaged system

$$\dot{Y} = \varepsilon F(Y)$$

where $F(Y) = \dfrac{1}{2\pi}\int_0^{2\pi} f(Y,\psi,\varepsilon=0)\mathrm{d}\psi.$

Theorem 1. *Assume $|\nu(X)| > R > 0$ and let $\tau > 0$ fixed.*
If

- $(X_\varepsilon(.),\psi_\varepsilon(.))_\varepsilon$ *is a family of solutions of the* ***perturbed system*** *on* $[0,\frac{\tau}{\varepsilon}]$,
- $X_\varepsilon(0) \underset{\varepsilon\to 0}{\longrightarrow} X_0^0$.

Then $t \to X_\varepsilon(\varepsilon t)$ uniformly converges on $[0,\tau]$ to $t \to \overline{Y}(t)$ such that

- $\overline{Y}(.)$ *is a solution of the* ***averaged system***,
- $\overline{Y}(0) = X_0^0$.

Non averaged system

Applying the PMP, a minimizing trajectory is projection of an extremal $(I(.),\varphi(.),p_I(.),p_\varphi(.))$ solution of the Hamiltonian system defined by

$$H(I,\varphi,p_I,p_\varphi,\varepsilon) = p_\varphi \nu(I) + \varepsilon K(I,\varphi,p_I,p_\varphi,\varepsilon),$$

$$K := \sqrt{\sum_{i=1}^m H_i^2}, \quad H_i(I,\varphi,p_I,p_\varphi,\varepsilon) = p_I F_i(I,\varphi,\varepsilon) + p_\varphi G_i(I,\varphi,\varepsilon), \quad i=0,\ldots,m.$$

The extremal system is

$$\begin{aligned} \dot{I} = \varepsilon \frac{\partial K}{\partial p_I}, \quad \dot{p_I} = -p_\varphi \nu' - \varepsilon \frac{\partial K}{\partial I} \\ \dot{\varphi} = \nu + \varepsilon \frac{\partial K}{\partial p_\varphi}, \quad \dot{p_\varphi} = -\varepsilon \frac{\partial K}{\partial \varphi}. \end{aligned} \qquad (*)$$

The adjoint vector p_I might be not a slow variable ($\dot{p}_I = O(\varepsilon)$?).

Normalization of the non averaged system

We decide to normalize on the energy level $\boldsymbol{H(I,\varphi,p_I,p_\varphi,\varepsilon)=\varepsilon}$, then one can reduce our system by setting $p_\varphi = \varepsilon\frac{K-1}{\nu}$, and locally we can write

$$\boldsymbol{p_\varphi = -\varepsilon h(I,\varphi,p_I,\varepsilon).}$$

This yields to have p_I as a **slow variable**, so is I.
$\rightarrow$ The system can be averaged classically with respect to φ.

Boundary conditions and transversality conditions

$$I(0)=I_0, \quad p_\varphi(0)=0, \quad I(t_f(\varepsilon))=I_f, \quad p_\varphi(t_f(\varepsilon))=0, \quad H=\varepsilon.$$

Remark. *Parameterizing by φ, the non averaged extremal system (*) is hamiltonian*

$$\frac{\mathrm{d}I}{\mathrm{d}\varphi}=\varepsilon\frac{\partial h}{\partial p_I}, \quad \frac{\mathrm{d}p_I}{\mathrm{d}\varphi}=-\varepsilon\frac{\partial h}{\partial I}. \qquad \textit{(non averaged)}$$

Averaged of the orbital transfer problem

The averaged dynamic is defined by

$$\begin{aligned}\overline{h}(I,p_I) &:= \frac{1}{2\pi}\int_0^{2\pi} h(I,\varphi,p_I,\varepsilon=0)\mathrm{d}\varphi \\ &= \frac{1}{2\pi}\int_0^{2\pi} \frac{K-1}{\nu}(I,\varphi,p_I,p_\varphi=0,\varepsilon=0)\mathrm{d}\varphi.\end{aligned}$$

Parameterizing by $s := \varepsilon(\varphi-\varphi_0)$ leads to the **averaged boundary value problem**

$$\begin{aligned}&\frac{\mathrm{d}I}{\mathrm{d}s} = \frac{\partial\overline{h}}{\partial p_I}, \quad \frac{\mathrm{d}p_I}{\mathrm{d}s} = -\frac{\partial\overline{h}}{\partial I}, \\ &I(0) = I_0, \quad I(s_f) = I_f, \quad \overline{h} = 0.\end{aligned} \qquad \textit{(averaged)}$$

This system defines a distance on the set of elliptic Keplerian orbits

$$\mathrm{d}(I_0, I_f) = s_f.$$

Singularity

The Hamiltonian H is non smooth on the set

$$\mathcal{S}_\varepsilon = \{(I,\varphi,p_I,p_\varphi) | H(I,\varphi,p_I,p_\varphi) - \varepsilon = 0 \text{ and } H_i(I,\varphi,p_I,p_\varphi,\varepsilon) = 0,\ i = 1,2,3\}.$$

$\mathrm{codim}(\mathcal{S}_\varepsilon) = 3$ in $\{(I,\varphi,p_I,p_\varphi) | H(I,\varphi,p_I,p_\varphi) - \varepsilon = 0\}$.

We define

$$\mathcal{S}_0 = \{(I,\varphi,p_I,p_\varphi) | p_\varphi = 0 \text{ and } H_i(I,\varphi,p_I,p_\varphi = 0,\varepsilon = 0) = 0,\ i = 1,2,3\}.$$

Then, the averaged Hamiltonian is non smooth on

$$\overline{\mathcal{S}} = \{(I,p_I) | \exists \varphi, (I,\varphi,p_I,0) \in \mathcal{S}_0\}$$

which is of **codimension 2**.

Convergence theorem

Let V an open subset of the lifted Keplerian elliptic orbits set such that $\overline{\mathcal{S}} \cap V = \varnothing$.

Theorem 2. *If*

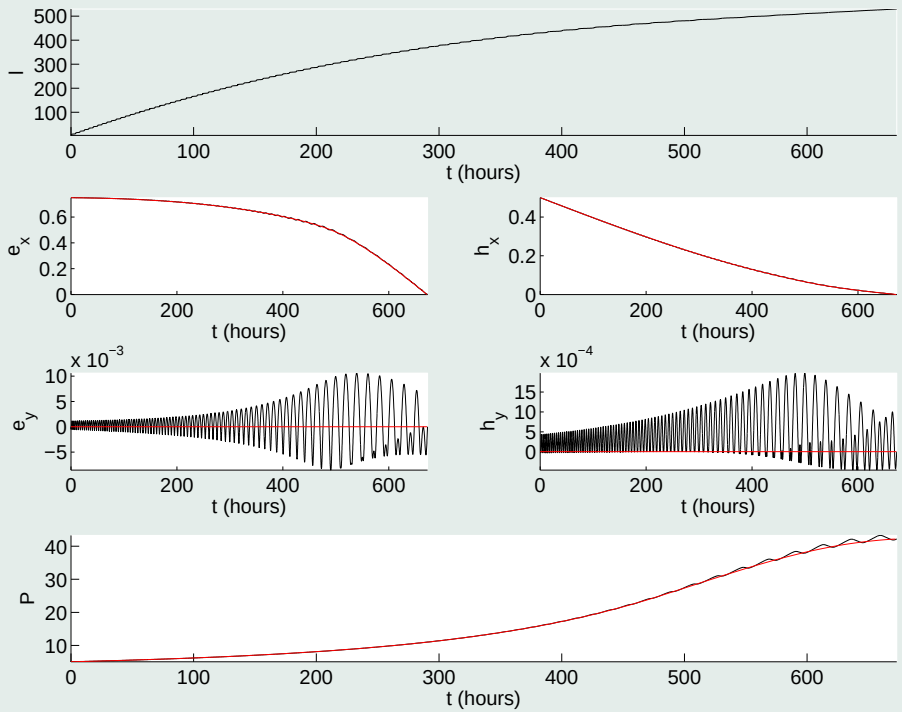
- $(I^\varepsilon(.), \varphi^\varepsilon(.), p_I{}^\varepsilon(.), p_\varphi^\varepsilon(.))_\varepsilon$ *is a family of extremals defined on* $[0, t_f(\varepsilon)]$ *of the non averaged boundary value problem,*
- $\boldsymbol{\forall t \in [0, t_f(\varepsilon)],\ (I^\varepsilon(t), p_I{}^\varepsilon(t)) \in K \subset V}$, K *compact, so that, for* ε *small enough* $(I^\varepsilon(t), \varphi^\varepsilon(t), p_I{}^\varepsilon(t), p_\varphi^\varepsilon(t)) \notin \mathcal{S}_\varepsilon$.

Then $t \to (I^\varepsilon(\varepsilon t), p_I{}^\varepsilon(\varepsilon t))$ *converges uniformly to* $t \to (\overline{I}(t), \overline{p}_I(t))$ *on* $[0, \overline{t_f}]$ *which is an extremal of the averaged boundary values problem with conditions*

$$\overline{I}(0) = I_0, \quad \overline{I}(\overline{t_f}) = I_f.$$

Dowód. Using the uniform convergence on $[0, T]$ of the previous averaging theorem and the equicontinuity of the family $(I^\varepsilon(.))_\varepsilon$, the uniform convergence and the boundary conditions are satisfied. $\square$

States : $I(t) = (P, e_x, e_y, h_x, h_y)(t)$ et $\varphi(t) = l(t)$



Minimizing trajectories - ***Thrust 3N***, $\varepsilon \approx 2\mathrm{e}-2$

black: non averaged, *red: averaged.*

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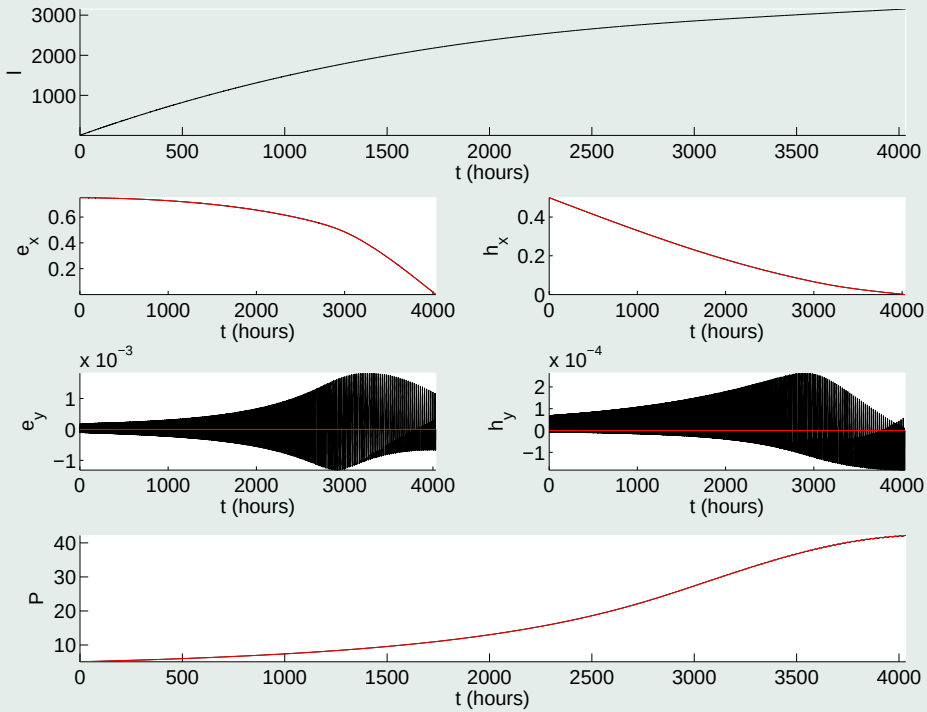
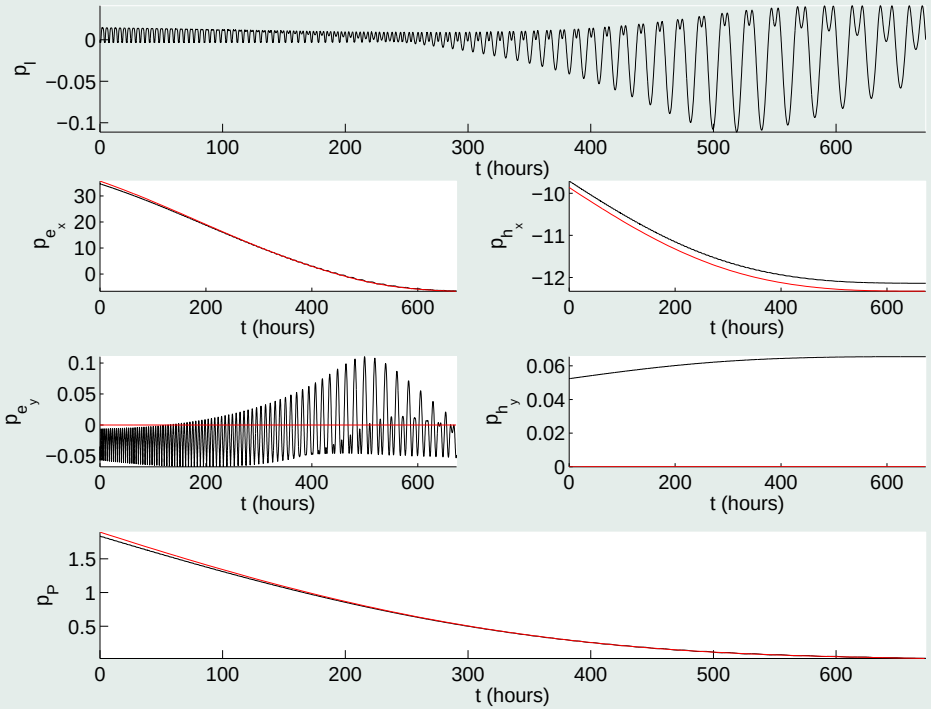


Minimizing trajectories - ***Thrust*** $\mathbf{0.5N}$, $\varepsilon \approx 4\mathrm{e}-3$

black: non averaged, *red: averaged.*

Adjoints $p_I(t) = (p_P, p_{e_x}, p_{e_y}, p_{h_x}, p_{h_y})(t)$ et $p_\varphi(t) = p_l(t)$



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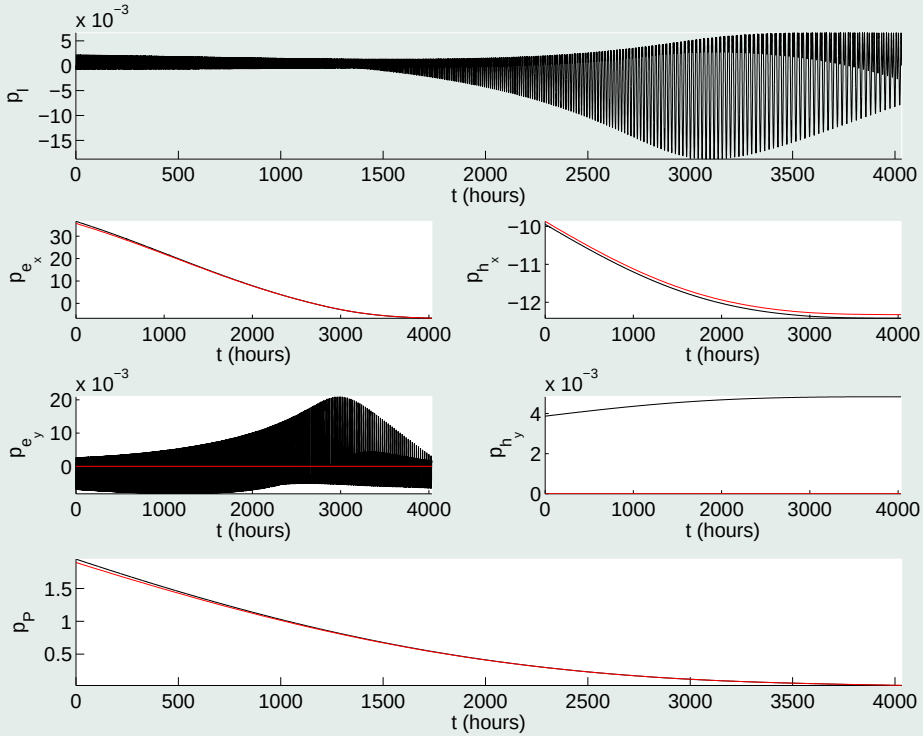
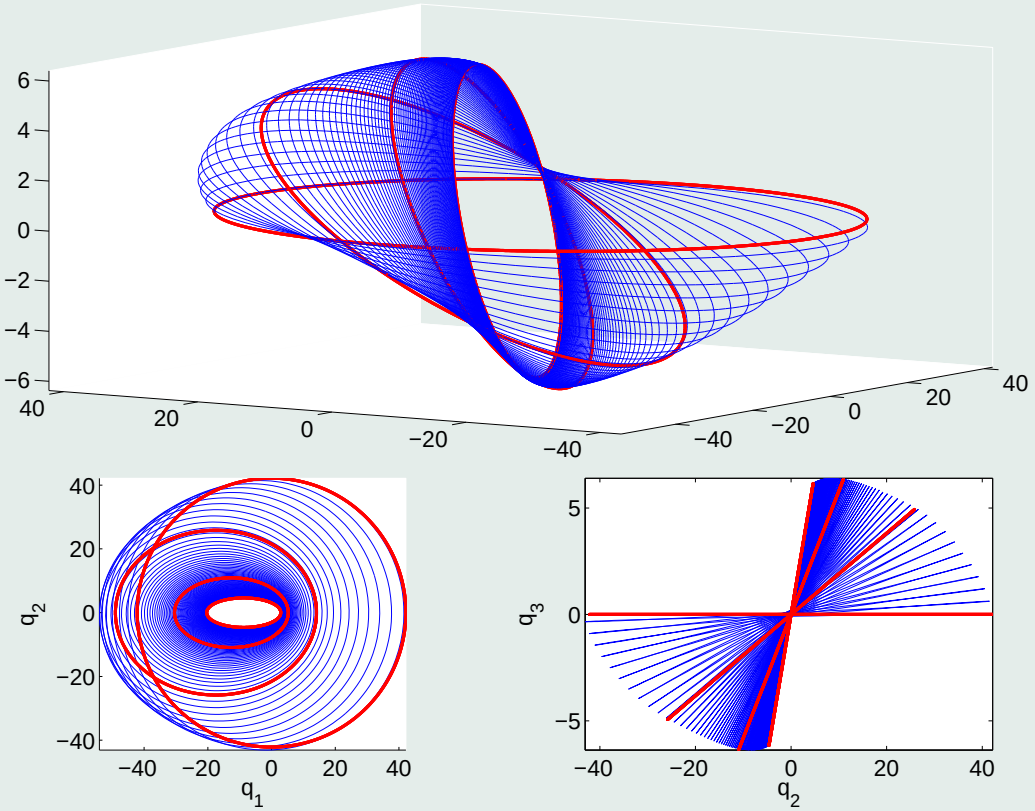


Minimizing trajectories - ***Thrust*** $\mathbf{0.5N}$, $\varepsilon \approx 4\mathrm{e}-3$

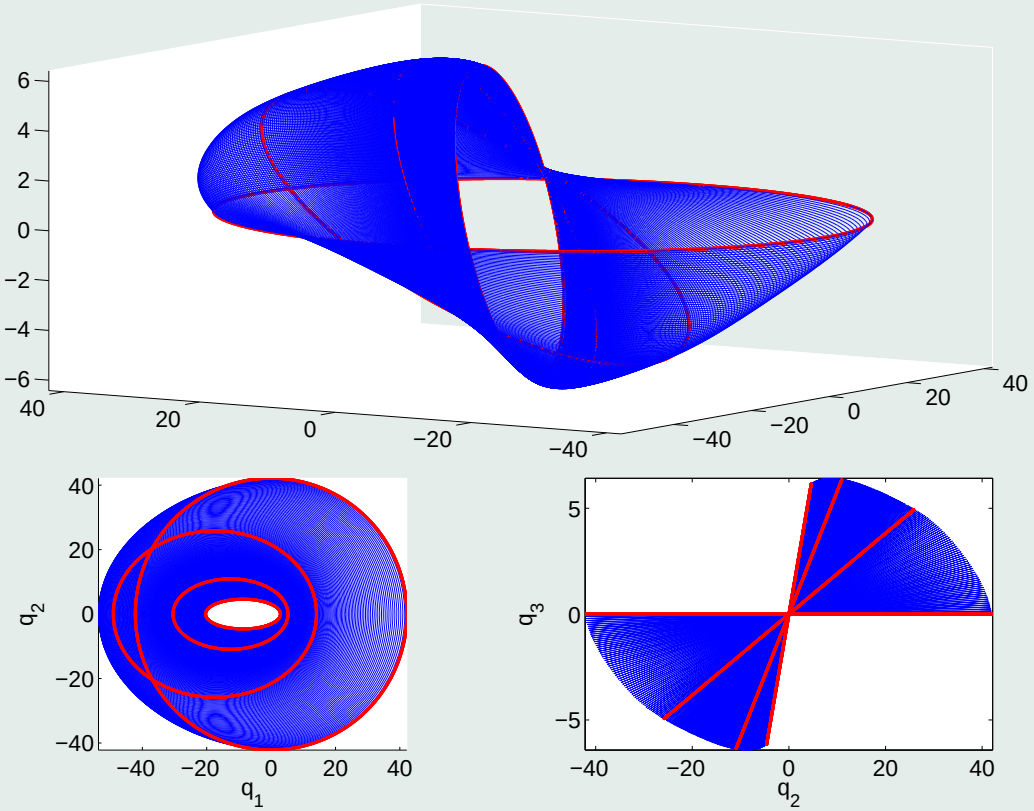
black: non averaged, *red: averaged.*

Cartesian coordinates

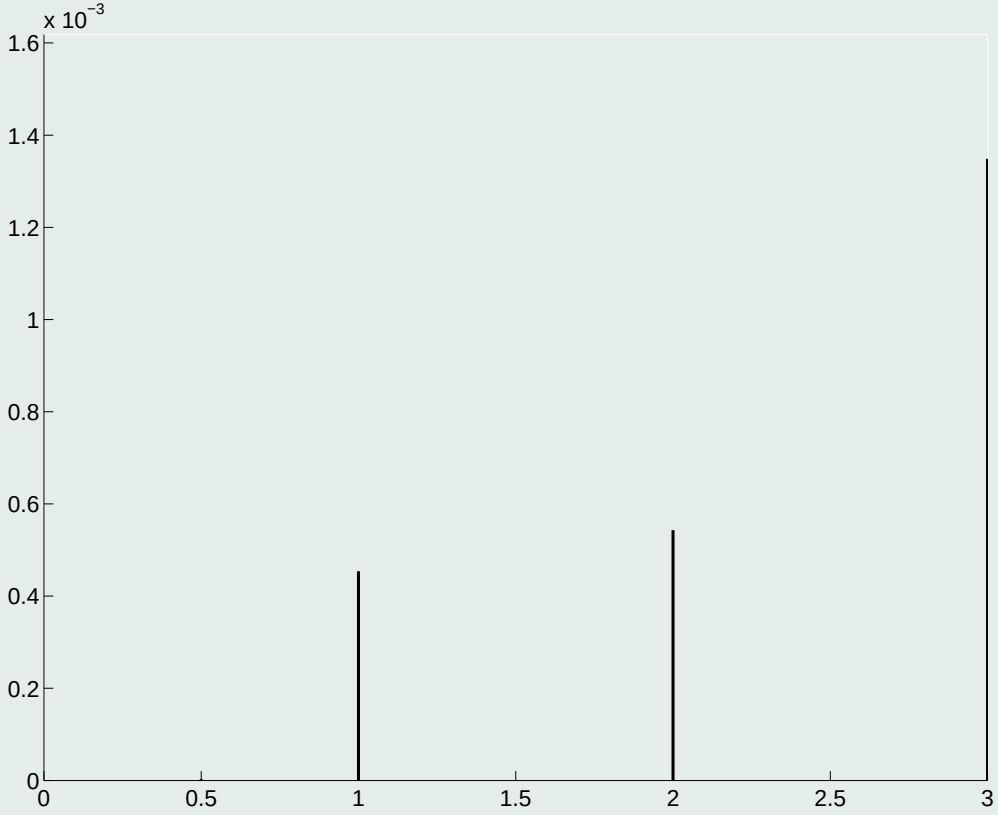


Thrust 3N - some orbits of the averaged trajectories are represented in red.

Cartesian coordinates



Thrust 0.5N - some orbits of the averaged trajectories are represented in red.



Relative error between $t_{f_{optimal}}$ (non averaged optimal time) et $t_{f_{moy}}$ (estimated from normalized optimal angle s_f of the averaged system).

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