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Averaging techniques in the time minimal orbital transfer using low propulsion

Jérémy Rouot

NONLINEAR CONTROL AND GEOMETRY
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Joint work with B. Bonnard, J.B. Caillau and J.B. Pomet.

Some historical points about optimal control and orbital transfer with low thrust

- \approx 1970, electro-ionic propulsion where the thrust is very low (spiral transfer).
Missions: Artemis-ESA (2001, GST), Smart-1-ESA (2006, GST to lunar orbit), BepiColombo-ESA (2016, Mercury).
- Epenoy-Geffroy (1997) use averaging techniques coupled with the Pontryagin Maximum Principle for low thrust transfer.
- Bonnard-Caillau (2007) computed explicitly the averaged Hamiltonian in the case of the energy cost.
It defines a Riemannian metric which allows to compute extremal curves for coplanar transfers.

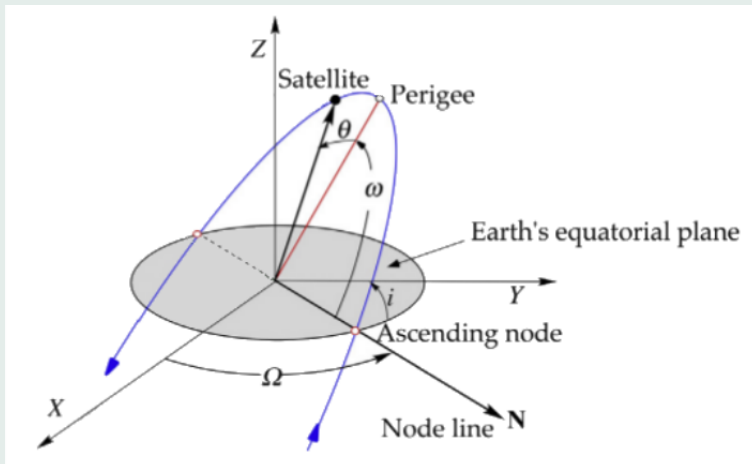
Time minimal orbital transfer problem

$q \in \mathbb{R}^3$: position vector of the satellite,

$$\ddot{q} + \nabla V(q) = 0, \quad V(q) = -\frac{1}{|q|}.$$

Elliptic orbits: restriction to $\mathcal{E} = \{(q, \dot{q}) \mid q \times \dot{q} \neq 0 \text{ and } \frac{1}{2}|\dot{q}|^2 - V(q) < 0\}$.

Kepler's orbital elements



$(a, e, i, \omega, \Omega)$: shape and orientation of the ellipse,

θ : angle locating the satellite on its orbit.

The uncontrolled motion can be described by $(I, \varphi = F(I, \theta))$

$$\dot{I} = 0$$

$$\dot{\varphi} = \nu(I).$$

$0 < \varepsilon \ll 1$, $(I(t), \varphi(t)) \in \mathbb{R}^5 \times S^1$: state variables of the satellite, $u(t) \in \mathbb{R}^3$, $F_i(I, \varphi, \varepsilon), G_i(I, \varphi, \varepsilon)$ $i = 1..3$ are **2π -periodic** vector fields with respect to φ .

Time minimal orbital transfer problem

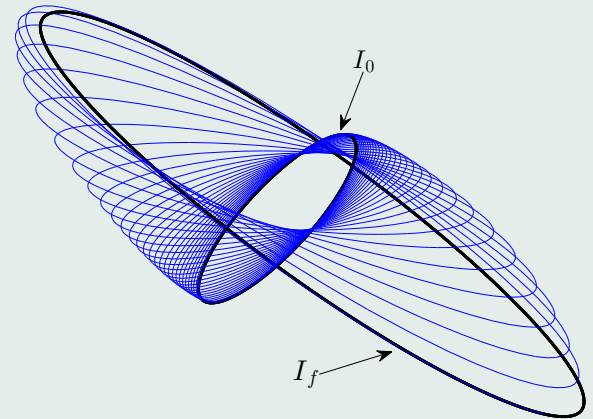
$$\min_{|u| \leq 1} t_f(u)$$

$$\text{subject to } \dot{I} = \varepsilon F_0(I, \varphi, \varepsilon) + \varepsilon \sum_{i=1}^3 u_i F_i(I, \varphi, \varepsilon), \quad \dot{\varphi} = \nu(I) + \varepsilon \sum_{i=1}^3 u_i G_i(I, \varphi, \varepsilon)$$

$$I(0) = I_0, \quad I(t_f(u)) = I_f.$$

The vector field F_0 takes into account conservative perturbations such as

- lunar and solar attraction,
- J_2 -effect.



Averaging principle for dynamical system

Let $f(X, \psi, \varepsilon), g(X, \psi, \varepsilon)$ be two smooth **2π -periodic** vector fields with respect to $\psi \in S^1$.

Perturbed system

$$\begin{aligned}\dot{X} &= \varepsilon f(X, \psi, \varepsilon) \\ \dot{\psi} &= \nu(X) + \varepsilon g(X, \psi, \varepsilon).\end{aligned}$$

Averaged system

$$\begin{aligned}\dot{Y} &= \varepsilon F(Y) \\ \text{where } F(Y) &= \frac{1}{2\pi} \int_0^{2\pi} f(Y, \psi, \varepsilon = 0) d\psi.\end{aligned}$$

Theorem 1. Assume $|\nu(X)| > R > 0$ and let $\tau > 0$ fixed.

If

- $(X_\varepsilon(\cdot), \psi_\varepsilon(\cdot))_\varepsilon$ is a family of solutions of the **perturbed system** on $[0, \frac{\tau}{\varepsilon}]$,
- $X_\varepsilon(0) \xrightarrow{\varepsilon \rightarrow 0} X_0^0$.

Then $t \rightarrow X_\varepsilon(\varepsilon t)$ uniformly converges on $[0, \tau]$ to $t \rightarrow \bar{Y}(t)$ such that

- $\bar{Y}(\cdot)$ is a solution of the **averaged system**,
- $\bar{Y}(0) = X_0^0$.

Non averaged system

Applying the PMP, a minimizing trajectory is projection of an extremal $(I(\cdot), \varphi(\cdot), p_I(\cdot), p_\varphi(\cdot))$ solution of the Hamiltonian system defined by

$$H(I, \varphi, p_I, p_\varphi, \varepsilon) = p_\varphi \nu(I) + \varepsilon K(I, \varphi, p_I, p_\varphi, \varepsilon),$$

$$K := \sqrt{\sum_{i=1}^m H_i^2}, \quad H_i(I, \varphi, p_I, p_\varphi, \varepsilon) = p_I F_i(I, \varphi, \varepsilon) + p_\varphi G_i(I, \varphi, \varepsilon), \quad i=0, \dots, m.$$

The extremal system is

$$\begin{aligned} \dot{I} &= \varepsilon \frac{\partial K}{\partial p_I}, & \dot{p}_I &= -p_\varphi \nu' - \varepsilon \frac{\partial K}{\partial I} \\ \dot{\varphi} &= \nu + \varepsilon \frac{\partial K}{\partial p_\varphi}, & \dot{p}_\varphi &= -\varepsilon \frac{\partial K}{\partial \varphi}. \end{aligned} \quad (*)$$

The adjoint vector p_I might be not a slow variable ($\dot{p}_I = O(\varepsilon)$?).

Normalization of the non averaged system

We decide to normalize on the energy level $\mathbf{H}(I, \varphi, \mathbf{p}_I, \mathbf{p}_\varphi, \varepsilon) = \varepsilon$, then one can reduce our system by setting $p_\varphi = \varepsilon \frac{K-1}{\nu}$, and locally we can write

$$\mathbf{p}_\varphi = -\varepsilon \mathbf{h}(I, \varphi, \mathbf{p}_I, \varepsilon).$$

This yields to have p_I as a **slow variable**, so is I .

→ The system can be averaged classically with respect to φ .

Boundary conditions and transversality conditions

$$I(0) = I_0, \quad p_\varphi(0) = 0, \quad I(t_f(\varepsilon)) = I_f, \quad p_\varphi(t_f(\varepsilon)) = 0, \quad H = \varepsilon.$$

Remark. *Parameterizing by φ , the non averaged extremal system (*) is hamiltonian*

$$\frac{dI}{d\varphi} = \varepsilon \frac{\partial h}{\partial p_I}, \quad \frac{dp_I}{d\varphi} = -\varepsilon \frac{\partial h}{\partial I}. \quad (\text{non averaged})$$

Averaged of the orbital transfer problem

The averaged dynamic is defined by

$$\begin{aligned}\bar{h}(I, p_I) &:= \frac{1}{2\pi} \int_0^{2\pi} h(I, \varphi, p_I, \varepsilon = 0) d\varphi \\ &= \frac{1}{2\pi} \int_0^{2\pi} \frac{K-1}{\nu} (I, \varphi, p_I, p_\varphi = 0, \varepsilon = 0) d\varphi.\end{aligned}$$

Parameterizing by $s := \varepsilon(\varphi - \varphi_0)$ leads to the **averaged boundary value problem**

$$\begin{aligned}\frac{dI}{ds} &= \frac{\partial \bar{h}}{\partial p_I}, & \frac{dp_I}{ds} &= -\frac{\partial \bar{h}}{\partial I}, & & (\text{averaged}) \\ I(0) &= I_0, & I(s_f) &= I_f, & \bar{h} &= 0.\end{aligned}$$

This system defines a distance on the set of elliptic Keplerian orbits

$$d(I_0, I_f) = s_f.$$

Singularity

The Hamiltonian H is non smooth on the set

$$\mathcal{S}_\varepsilon = \{(I, \varphi, p_I, p_\varphi) \mid H(I, \varphi, p_I, p_\varphi) - \varepsilon = 0 \text{ and } H_i(I, \varphi, p_I, p_\varphi, \varepsilon) = 0, i = 1, 2, 3\}.$$

$$\text{codim}(\mathcal{S}_\varepsilon) = 3 \text{ in } \{(I, \varphi, p_I, p_\varphi) \mid H(I, \varphi, p_I, p_\varphi) - \varepsilon = 0\}.$$

We define

$$\mathcal{S}_0 = \{(I, \varphi, p_I, p_\varphi) \mid p_\varphi = 0 \text{ and } H_i(I, \varphi, p_I, p_\varphi = 0, \varepsilon = 0) = 0, i = 1, 2, 3\}.$$

Then, the averaged Hamiltonian is non smooth on

$$\bar{\mathcal{S}} = \{(I, p_I) \mid \exists \varphi, (I, \varphi, p_I, 0) \in \mathcal{S}_0\}$$

which is of **codimension 2**.

Convergence theorem

Let V an open subset of the lifted Keplerian elliptic orbits set such that $\bar{\mathcal{S}} \cap V = \emptyset$.

Theorem 2. *If*

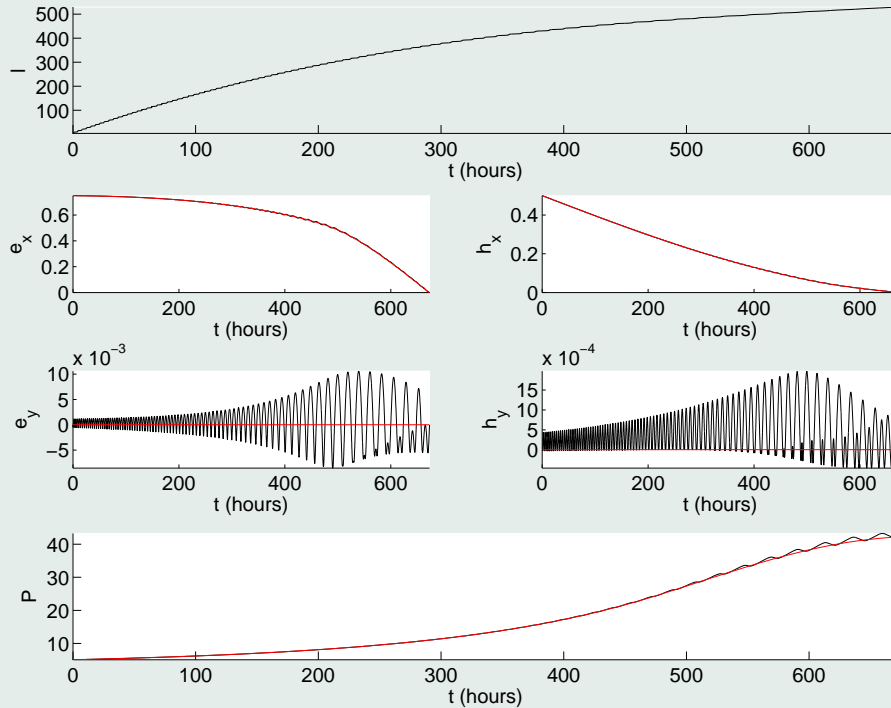
- $(I^\varepsilon(\cdot), \varphi^\varepsilon(\cdot), p_{I^\varepsilon}(\cdot), p_{\varphi^\varepsilon}(\cdot))_\varepsilon$ is a family of extremals defined on $[0, t_f(\varepsilon)]$ of the non averaged boundary value problem,
- $\forall t \in [0, t_f(\varepsilon)]$, $(I^\varepsilon(t), p_{I^\varepsilon}(t)) \in K \subset V$, K compact, so that, for ε small enough $(I^\varepsilon(t), \varphi^\varepsilon(t), p_{I^\varepsilon}(t), p_{\varphi^\varepsilon}(t)) \notin \mathcal{S}_\varepsilon$.

Then $t \rightarrow (I^\varepsilon(\varepsilon t), p_{I^\varepsilon}(\varepsilon t))$ converges uniformly to $t \rightarrow (\bar{I}(t), \bar{p}_I(t))$ on $[0, \bar{t}_f]$ which is an extremal of the averaged boundary values problem with conditions

$$\bar{I}(0) = I_0, \quad \bar{I}(\bar{t}_f) = I_f.$$

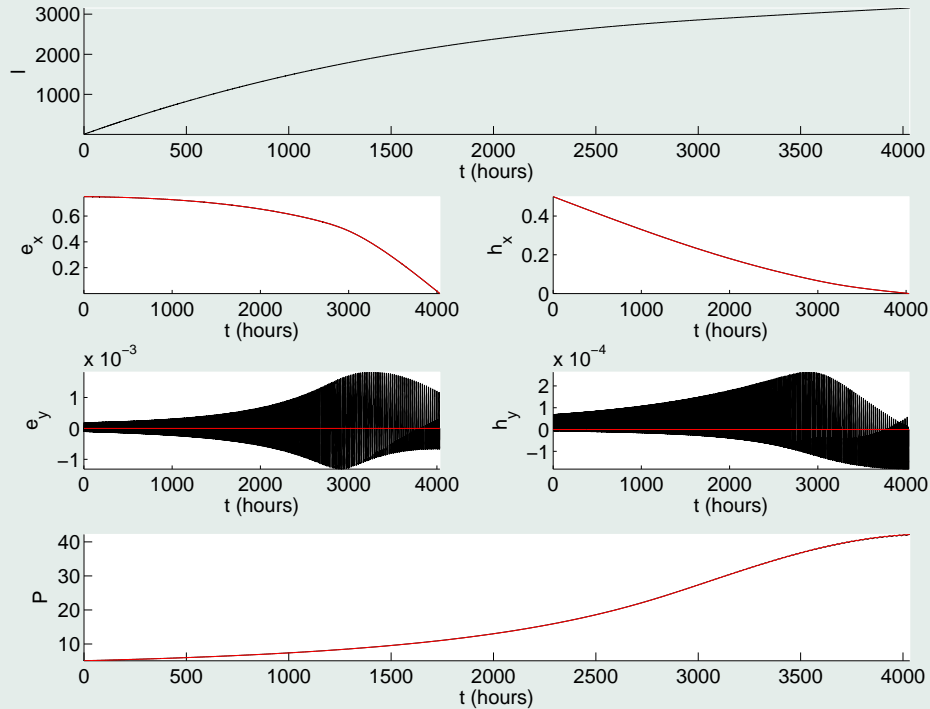
Dowód. Using the uniform convergence on $[0, T]$ of the previous averaging theorem and the equicontinuity of the family $(I^\varepsilon(\cdot))_\varepsilon$, the uniform convergence and the boundary conditions are satisfied. □

States : $I(t) = (P, e_x, e_y, h_x, h_y)(t)$ et $\varphi(t) = l(t)$



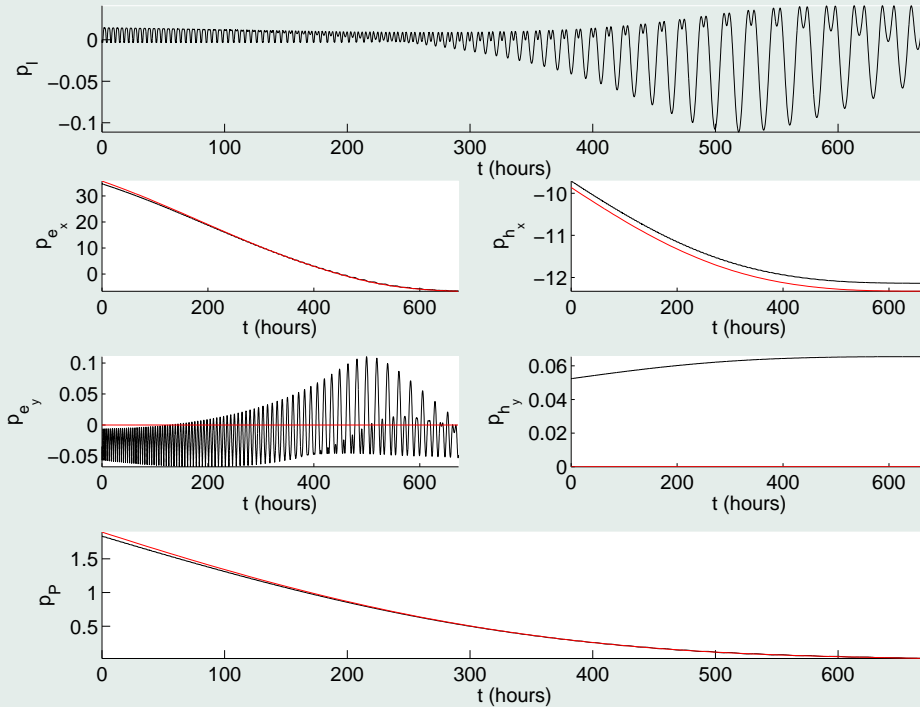
*Minimizing trajectories - Thrust $3N$, $\varepsilon \approx 2e-2$
black: non averaged, red: averaged.*

States : $I(t) = (P, e_x, e_y, h_x, h_y)(t)$ et $\varphi(t) = l(t)$



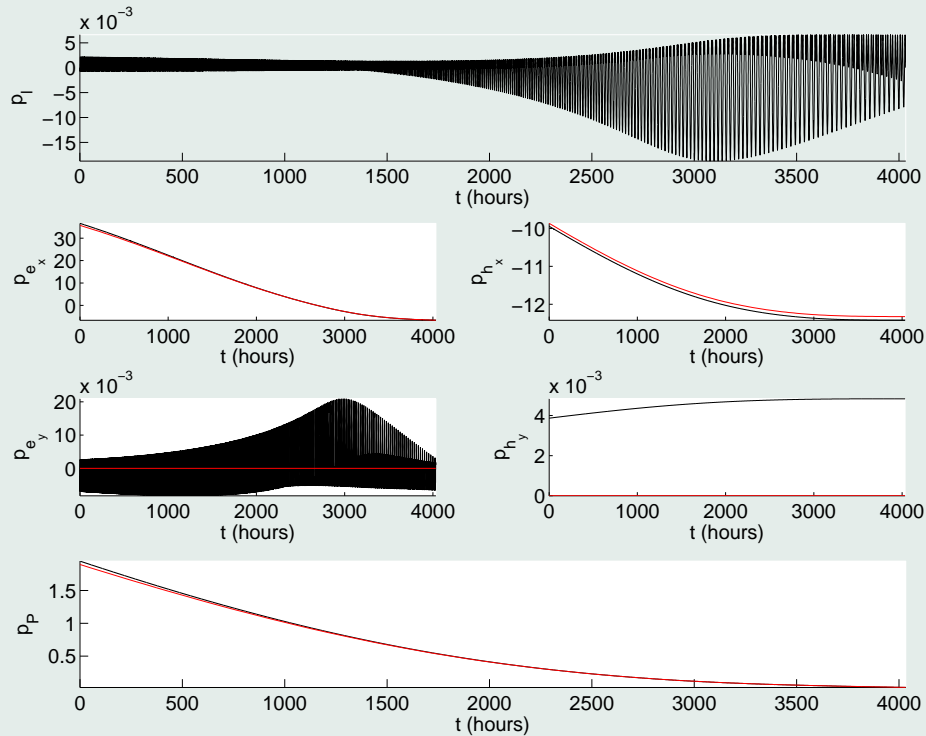
Minimizing trajectories - **Thrust 0.5N**, $\varepsilon \approx 4e-3$
black: non averaged, red: averaged.

Adjoint $p_I(t) = (p_P, p_{e_x}, p_{e_y}, p_{h_x}, p_{h_y})(t)$ et $p_\varphi(t) = p_l(t)$



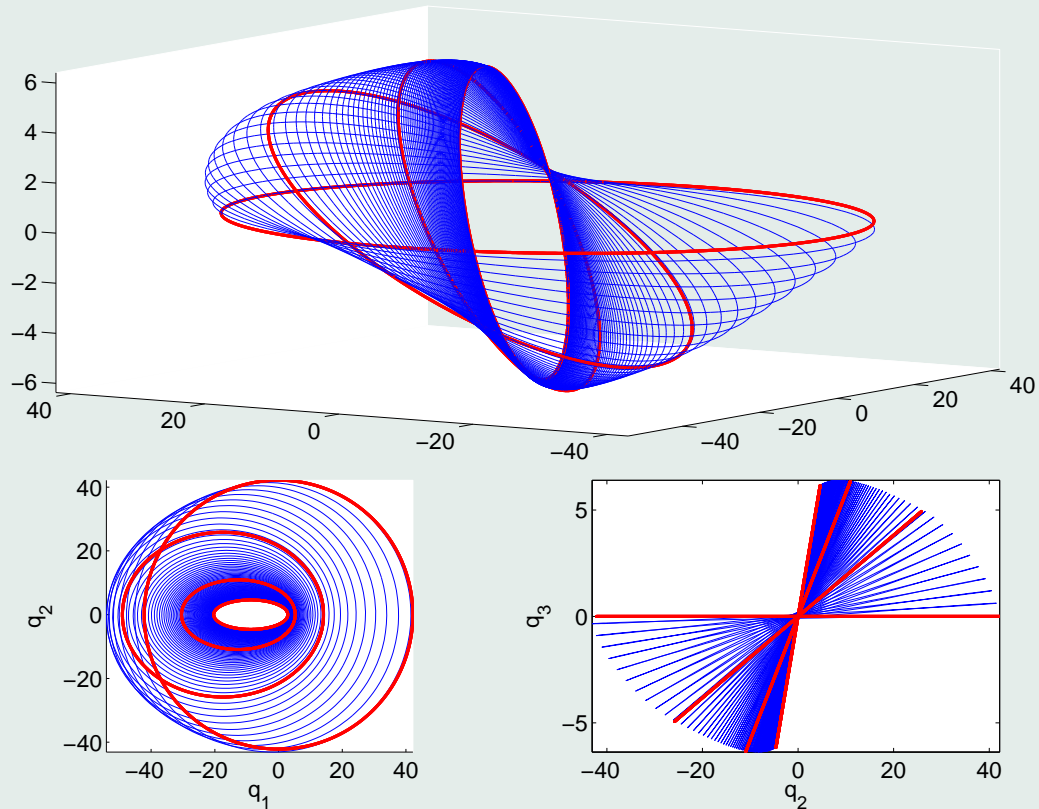
*Minimizing trajectories - Thrust 3N, $\varepsilon \approx 2e-2$
black: non averaged, red: averaged.*

Adjoint $p_I(t) = (p_P, p_{e_x}, p_{e_y}, p_{h_x}, p_{h_y})(t)$ et $p_\varphi(t) = p_l(t)$



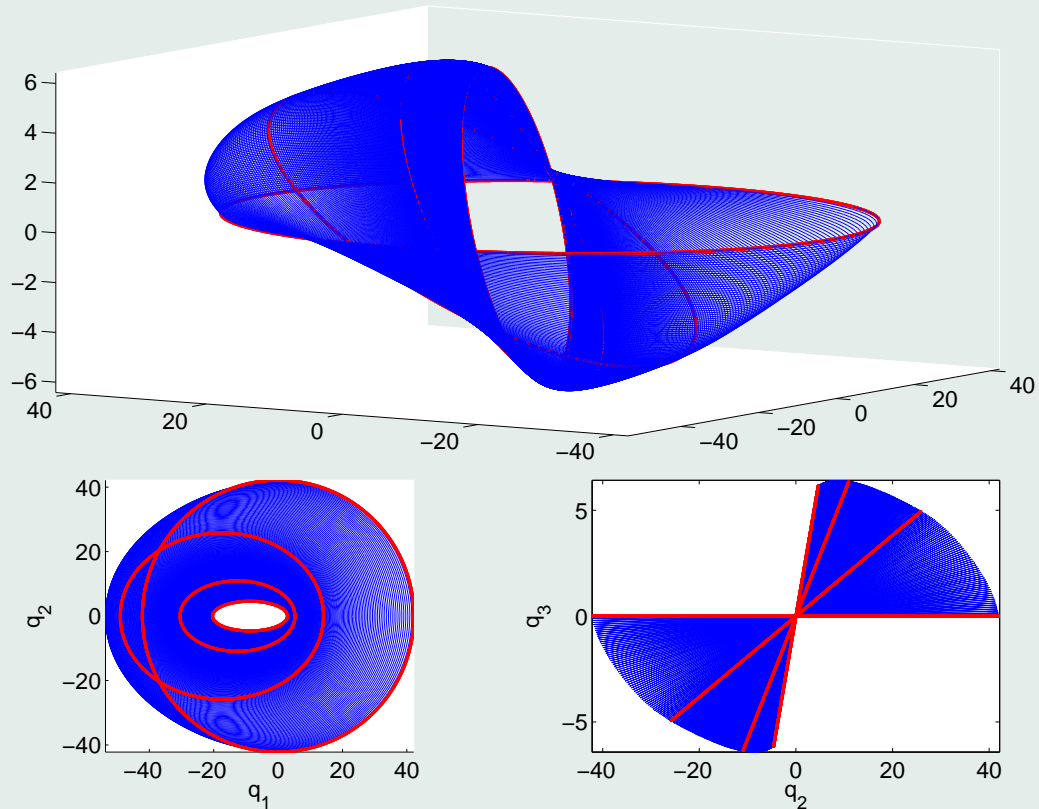
Minimizing trajectories - Thrust $0.5N$, $\varepsilon \approx 4e-3$
black: non averaged, red: averaged.

Cartesian coordinates

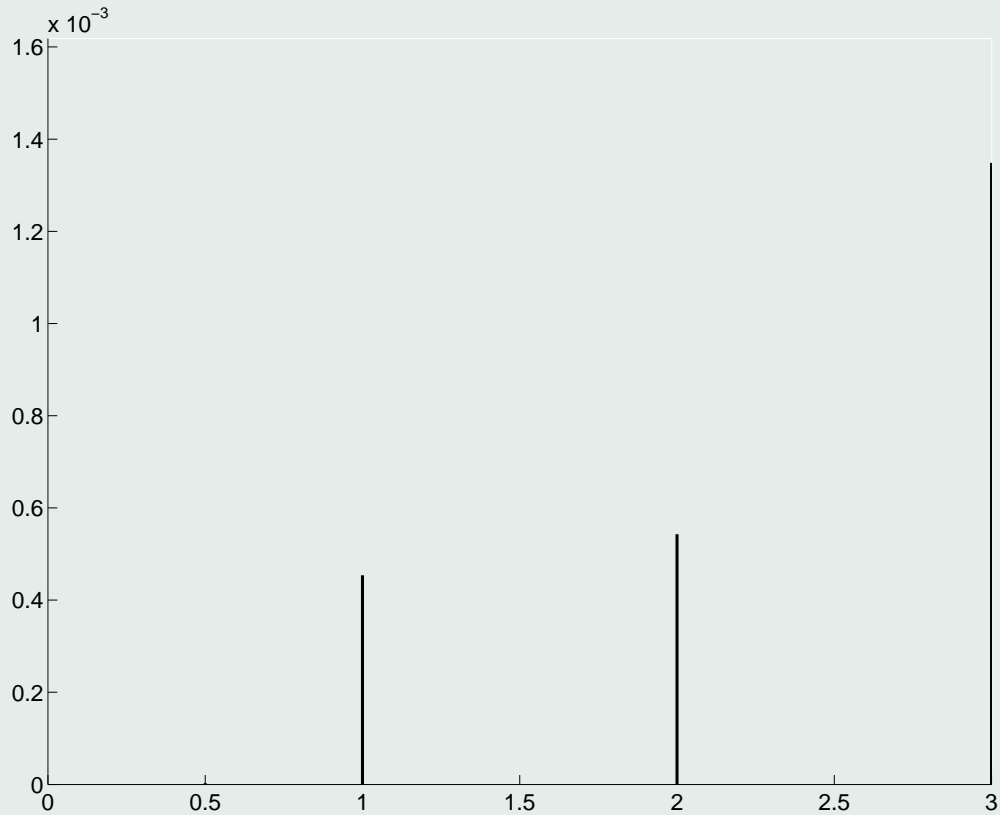


Thrust 3N - some orbits of the averaged trajectories are represented in red.

Cartesian coordinates



Thrust 0.5N - some orbits of the averaged trajectories are represented in red.



Relative error between $t_{f_{optimal}}$ (non averaged optimal time) et $t_{f_{moy}}$ (estimated from normalized optimal angle s_f of the averaged system).

References

- [1] Bombrun, Alex; Pomet, Jean-Baptiste. *The averaged control system of fast-oscillating control systems*. SIAM J. Control Optim. 51 (2013), no. 3, 2280–2305.
- [2] Thèses et habilitations / Ph.D Theses and Habilitation Theses Sophie Geffroy. *Généralisation des techniques de moyennation en contrôle optimal - Application aux problèmes de transfert et rendez-vous orbitaux à poussée faible*. Thèse de doctorat, Institut National Polytechnique de Toulouse, octobre 1997.
- [3] Bonnard, Bernard; Caillau, Jean-Baptiste. *Riemannian metric of the averaged energy minimization problem in orbital transfer with low thrust*. Ann. Inst. H. Poincaré Anal. Non Linéaire 24 (2007), no. 3, 395–411.