



HAL
open science

Lunar perturbation of the metric associated to the averaged orbital transfer

Jérémy Rouot, Bernard Bonnard

► **To cite this version:**

Jérémy Rouot, Bernard Bonnard. Lunar perturbation of the metric associated to the averaged orbital transfer. Variational methods in Imaging and geometric control, Nov 2014, Linz, Austria. hal-02483370

HAL Id: hal-02483370

<https://inria.hal.science/hal-02483370>

Submitted on 18 Feb 2020

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Lunar perturbation of the metric associated to the averaged orbital transfer

Bernard Bonnard & Jérémy Rouot

University of Burgundy and INRIA Sophia Antipolis

Email: jeremy.rouot@inria.fr

Abstract

In a series of previous articles we introduced a Riemannian metric associated to the energy minimizing orbital transfer with low propulsion. The aim of this article is to study the deformation of this metric due to the perturbation caused by the lunar attraction. Using Hamiltonian formalism, we describe the effects of the perturbations on the orbital transfers and the deformation of the conjugate and cut loci of the original metric.

Introduction

Recent space missions like lunar Smart-1 mission, Boeing orbital transfer, using electric propulsion are innovative design features to reduce launch costs and lead to the analysis of the low thrust controlled Kepler equation using averaging techniques in optimal control. Pioneering work in this direction associated to the energy minimization problem are due to Edelbaum [4], Epenoy-Geffroy [5], and more recently to Bonnard-Caillau [1]. Under some simplifying assumption they lead to the definition of a Riemannian distance between Keplerian orbits, and this is a preliminary step in computing the time minimal or find mass maximizing solutions using numerical continuation techniques [3].

Main Objectives

1. Compute a double averaged on the lunar perturbation potential.
2. From the averaged Hamiltonian associated to the energy minimization, define a perturbed Hamiltonian related to a Zermelo navigation problem.
3. Using the Pontryagin Maximum Principle (PMP), solve numerically a boundary-value problem via a simple shooting method.
4. Study the second order optimality conditions by computing the conjugate points via an algorithm based on the Jacobi fields.
5. Compare the trajectories and cut loci of the perturbed and unperturbed problem.

The Riemannian metric

From the controlled planar Kepler equation, the ellipse of motion of a satellite can be described by the system

$$\begin{aligned} \frac{dx}{dt} &= \sum_{i=1}^2 u_i F_i(x, l) \\ \frac{dl}{dt} &= w_0(x, l) + g(x, l, u). \end{aligned} \quad (1)$$

where $x = (n, \rho, \theta)$ are the slow orbital elements of the satellite (n is the mean motion, ρ the eccentricity and θ the argument of periapsis), and l is the fast angle pointing out the position of the satellite on its orbit.

The optimal control problem associated to (1) is to find, given two boundary values (x_0, x_f) and an interval $[0, t_f]$, a control $u = (u_1, u_2)$ such that $|u| \leq 1$ and minimizing the cost is $\int_{[0, t_f]} |u(s)|^2 ds$.

Applying the Pontryagin Maximum Principle, the averaged Hamiltonian with respect to the true longitude l is given by

$$\bar{H} = \frac{1}{4n^{\frac{5}{3}}} \left[18n^2 p_n^2 + 5(1 - \rho^2) p_\rho^2 + (5 - 4\rho^2) \frac{p_\theta^2}{\rho^2} \right].$$

The Riemannian metric associated to \bar{H} is

$$g = \frac{1}{9n^{\frac{5}{3}}} dn^2 + \frac{2n^{\frac{5}{3}}}{5(1 - \rho^2)} d\rho^2 + \frac{2n^{\frac{5}{3}}}{5 - 4\rho^2} d\theta^2.$$

Model for the lunar perturbation

The perturbed dynamic can be written as

$$\begin{aligned} \frac{dx}{dt} &= F_0(x, l, l') + \sum_{i=1}^2 u_i F_i(x, l) \\ \frac{dl}{dt} &= \tilde{w}_0(x, l) + g(x, l, u) \\ \frac{dl'}{dt} &= w_1(l') \end{aligned}$$

where l' is the fast angle pointing out the position of the Moon on his orbit.

The drift F_0 corresponds to the perturbed vector field induced by the lunar attraction. This perturbation is modelled as the effect of the Moon on the two body system {Earth + satellite} where these three mass points belong to the orbital plane of the Moon. This perturbation acceleration derives from a potential $R(x, l, l')$ whose double averaged is given by

$$\bar{R}(x) = \frac{1}{4\pi^2} \int_0^{2\pi} \int_0^{2\pi} R(x, l, l') dM dM' = \frac{n'^2}{4n^{\frac{3}{4}}} \left(1 + \frac{3}{2} \rho^2 \right)$$

where n' is the mean motion of the Moon supposed constant and M, M' are the mean anomaly respectively of the satellite and the Moon. The double averaged drift vector field is deduced from the Lagrange equations

$$\bar{F}_0(n, p, \theta) = \frac{3n'^2}{4n} \sqrt{1 - \rho^2} \frac{\partial}{\partial \theta} (n, p, \theta).$$

Hamiltonian from a Zermelo navigation problem

A Zermelo navigation problem on a n -dimensional Riemannian manifold (\mathcal{X}, g) is a time minimal problem associated to the system

$$\frac{dx}{dl} = X_0(x) + \sum_{i=1}^n u_i X_i(x)$$

where F_i form an orthonormal frame for the metric g and $|u| \leq 1$. F_0 represents the current of magnitude $|F_0|_g$. Applying the PMP, the associated Hamiltonian has the form

$$G(x, p) = G_0(x, p) + \epsilon \sqrt{G(x, p)}$$

where $G_i(x, p) = \langle p, X_i(x) \rangle$.

We consider the following Hamiltonian

$$H_{pert}(x, p) = \bar{H}_0(x, p) + \epsilon \sqrt{\bar{H}(x, p)} \quad (2)$$

where $\bar{H}_0 = \langle p, \bar{F}_0 \rangle$. To such an Hamiltonian, it corresponds a Zermelo navigation problem.

The geometric concept of conjugate point

Let \vec{H}_{pert} the Hamiltonian vector field associated to H_{pert} .

- Let $z = (x, p)$ be a reference extremal solution of \vec{H}_{pert} on $[0, t_f]$. The variational equation

$$\delta \dot{z}(t) = d\vec{H}_{pert}(z(t)) \delta z(t) \quad (3)$$

is called the Jacobi equation. A Jacobi field is a non trivial solution $\delta z = (\delta x, \delta p)$ of (3) and it is said to be vertical at time t if $\delta x(t) = 0$.

- The first conjugate time t_c is a time from which the exponential map $p_0 \rightarrow \exp_{p_0}(t, p_0) = x(t, x_0, p_0)$ associated to the flow of \vec{H}_{pert} is not an immersion at $t = t_c$. $x(t_c)$ is said to be conjugate to x_0 .

This singularity can be characterised by the independence of the Jacobi fields $(\delta x_1(t), \delta x_2(t))$ vertical at $t = 0$ and such that δp_i is orthogonal to p_0 ($i = 1, 2$). In particular, at the first conjugate time t_c , the family $(\delta x_1(t), \delta x_2(t))|_{t=t_c}$ is not of rank 2.

The following test condition allow us to compute t_c for a given trajectory $x(t, x_0, p_0)$ [2]

$$\det[\delta x_1(t_c), \delta x_2(t_c), \dot{x}(t_c)] = 0. \quad (4)$$

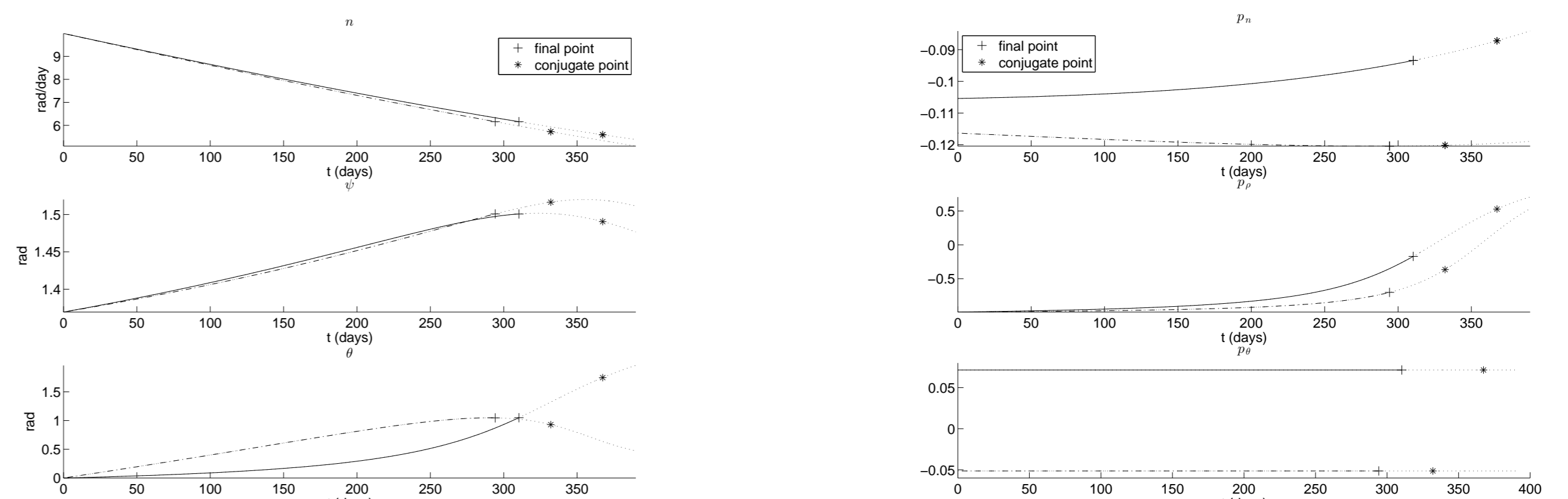
Computational results

Simulations provided below represent BC-extremals from the Hamiltonian system associated to (2). The boundary value problem is given by $x_0 = (10, 2e - 1, 0)$, $x_f = (6.16, 7e - 2, \frac{\pi}{3})$ and t_f is free. These extremals correspond to zeros of the shooting mapping

$$S : (t_f, p_0) \rightarrow (x_{t_f}(x_0, p_0) - x_f, |p(0)|^2 - 1).$$

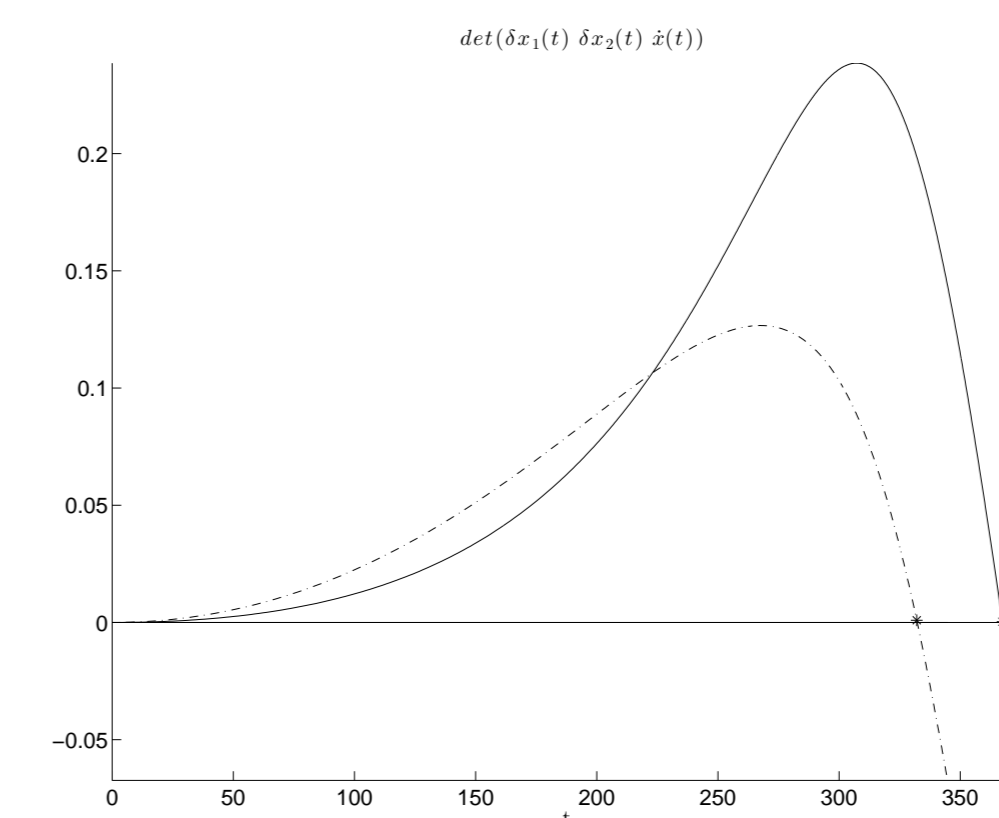
In order to check the optimality of our trajectories, conjugate points are computed via the verticality test (4). Simulations were performed via the package HAMPATH [3] which implements an indirect method (Newton) to find a zero of S .

Time evolutions of the extremal components in the perturbed case (dash-dot line) and the unperturbed case (solid line) are illustrated.



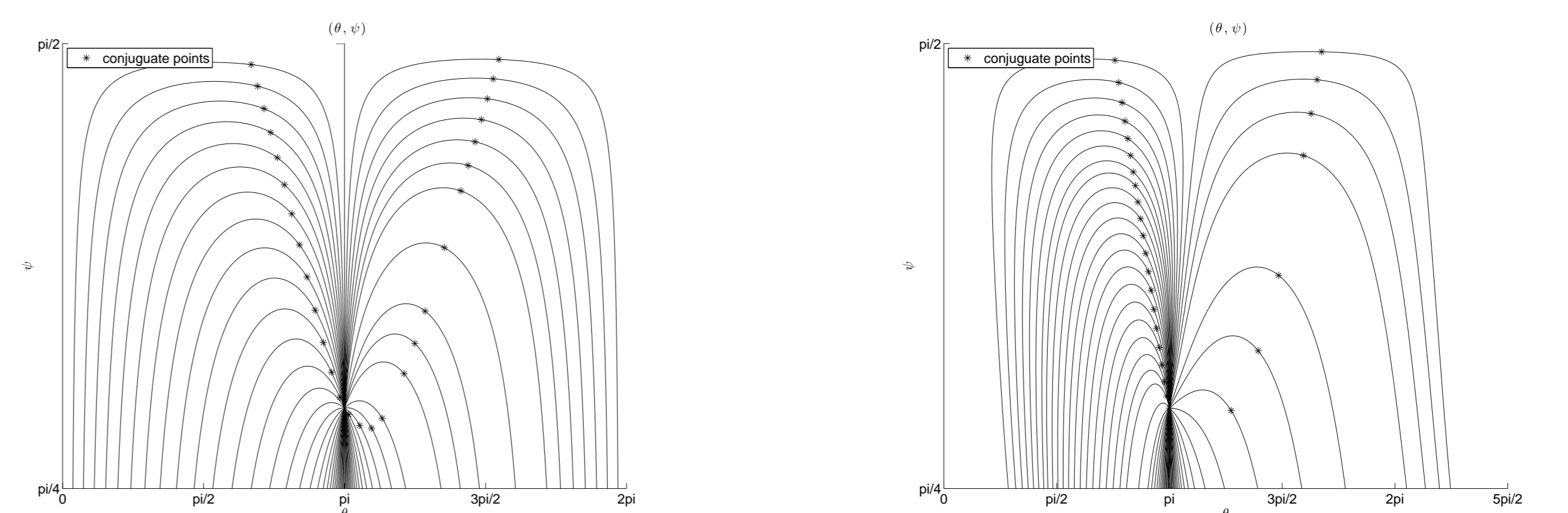
Evolution of the state vector
 $(n(t), \psi(t) = \frac{\pi}{2} - a \sin(\rho(t)), \theta(t))$.

Evolution of the adjoint vector
 $(p_n(t), p_\rho(t), p_\theta(t))$.



Rank test condition (4) for the perturbed and unperturbed problem.

The following figures represent several trajectories in the coordinates (θ, ψ) starting from $(\theta_0, \rho_0) = (\pi, 0.6)$.



Trajectories from the unperturbed case.

Trajectories from the perturbed case.

References

- [1] B. Bonnard and J.-B. Caillau. Riemannian metric of the averaged energy minimization problem in orbital transfer with low thrust. *Ann. Inst. H. Poincaré Anal. Non Linéaire*, 24(3):395–411, 2007.
- [2] J.-B. Bonnard, B.; Caillau and E. Trélat. Second order optimality conditions in the smooth case and applications in optimal control. *ESAIM Control Optim. Calc. Var.*, 13(3):207–236, 2007.
- [3] O. Cots. *Contrôle optimal géométrique: méthodes homotopiques et applications*. PhD thesis, Université de Bourgogne, 2012.
- [4] Theodore N. Edelbaum. Optimum low-thrust rendezvous and station keeping. *AIAA J.*, 2:1196–1201, 1964.
- [5] S. Geffroy and R. Epenoy. Optimal low-thrust transfers with constraints—generalization of averaging techniques. *Acta Astronautica*, 41(3):133–149, 1997.

Acknowledgements

Work supported in part by the French Space Agency the R&T action R-S13/BS-005-012 and by the region Provence-Alpes-Côte d'Azur.