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Jérémy Rouot, Bernard Bonnard, Monique Chyba, Daisuke Takagi

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Numerical Approach to the Optimal Control and Efficiency of the Copepod Swimmer.

55th IEEE Conference on Decision and Control

Bernard Bonnard, Monique Chyba, Jérémy Rouot and Daisuke Takagi

- Geometric optimal control
  - . Maximum principle
  - . Second order optimality conditions (local, conjugate points)

- Swimming at low Reynolds number
  - . sub-Riemannian (SR) geometry framework
  - . Varational approach to find trajectoires of a Hamiltonian system with periodic projections
  - . Find optimal stroke (efficiency concept)
  - . micro-device implementation (D. Takagi, M. Chyba)

$$\begin{array}{lll} \min & c(\widetilde{q}(0),\widetilde{q}(T)) & \\ \text{subject to} & \dot{q} = F(q,u), & \dot{q}^0 = L(q,u) \\ & (\widetilde{q}(0),\widetilde{q}(T)) \in B & \end{array}$$

- final time T is fixed
- $\widetilde{q} = (q, q^0)$, q state, $q^0 \in \mathbb{R}$
- $u(.) \in L^\infty([0,T],U)$, $U \subset \mathbb{R}^m$
- B : smooth closed set of $\mathbb{R}^n \times \mathbb{R}^n$, $(\widetilde{q}(0),\widetilde{q}(T)) \in B$ stands for **mixed** boundary conditions.

Ex. : $B = \{q(0) = q(T), q^0(0) = 0\}$,
$B = \{q(0) = q_0, q(T) = q_f, q^0(0) = 0\}$...

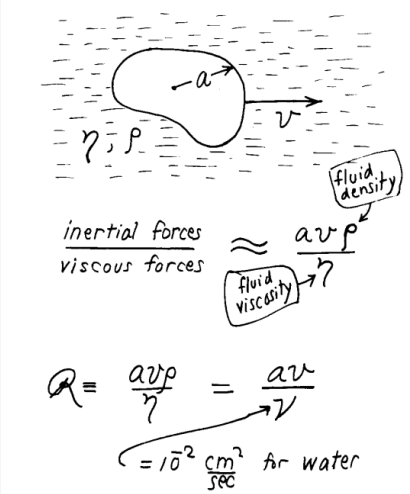
Necessary condition. $(\widetilde{q},u)$ optimal $\Longrightarrow \exists\ \widetilde{p}=(p,p^0):[0,T]\to(\mathbb{R}^n\times\mathbb{R})$ abs. continuous and $\lambda\geq 0$ cst with $(\widetilde{p},\lambda)\neq(0,0)$ and s.t.

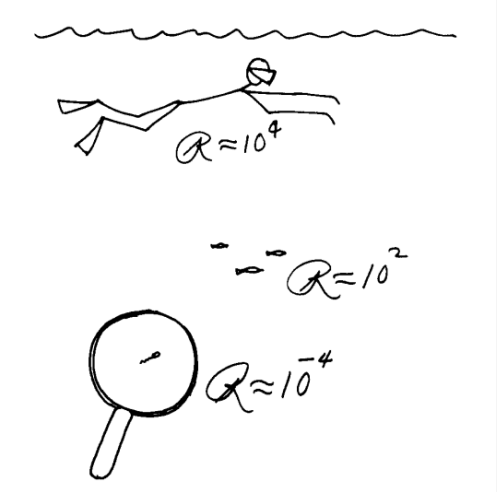
$$\dot{\widetilde{q}}=\frac{\partial H}{\partial\widetilde{p}},\quad \dot{\widetilde{p}}=-\frac{\partial H}{\partial\widetilde{q}},\quad H(\widetilde{q},\widetilde{p},u)=\max_{v\in U}\mathscr{H}(\widetilde{q},\widetilde{p},v)$$

where $\mathscr{H}(\widetilde{q},\widetilde{p},u)=p\cdot F(q,u)+p^0L(q,u)$.
Transversality conditions:

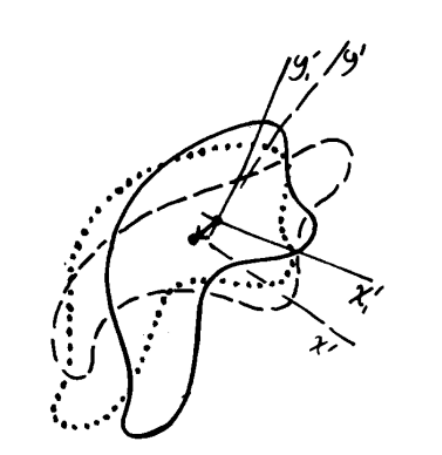
$$(\widetilde{p}(0),-\widetilde{p}(T))-\lambda\left(\frac{\partial c}{\partial\widetilde{q}(0)}(\widetilde{q}(0),\widetilde{q}(T)),\frac{\partial c}{\partial\widetilde{q}(T)}(\widetilde{q}(0),\widetilde{q}(T))\right)\in(T_{(\widetilde{q}(0),\widetilde{q}(T))}B)^{\perp}$$

Definition 1. *$(\widetilde{q},\widetilde{p},\lambda,u)$ is a normal extremal if $\lambda>0$, abnormal if $\lambda=0$. An abnormal trajectory q which is not a projection of normal extremal is said strict.*

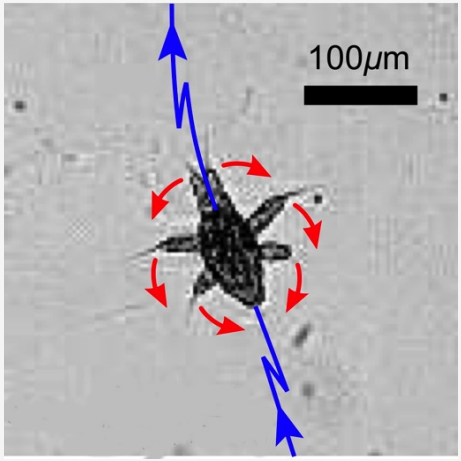




Reynolds number

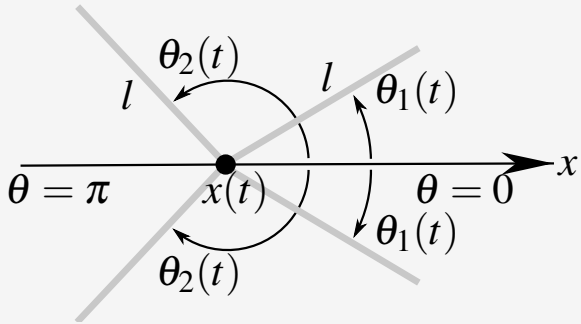


Swimmer deformations



Zooplankton

Symmetric 2-link swimmer

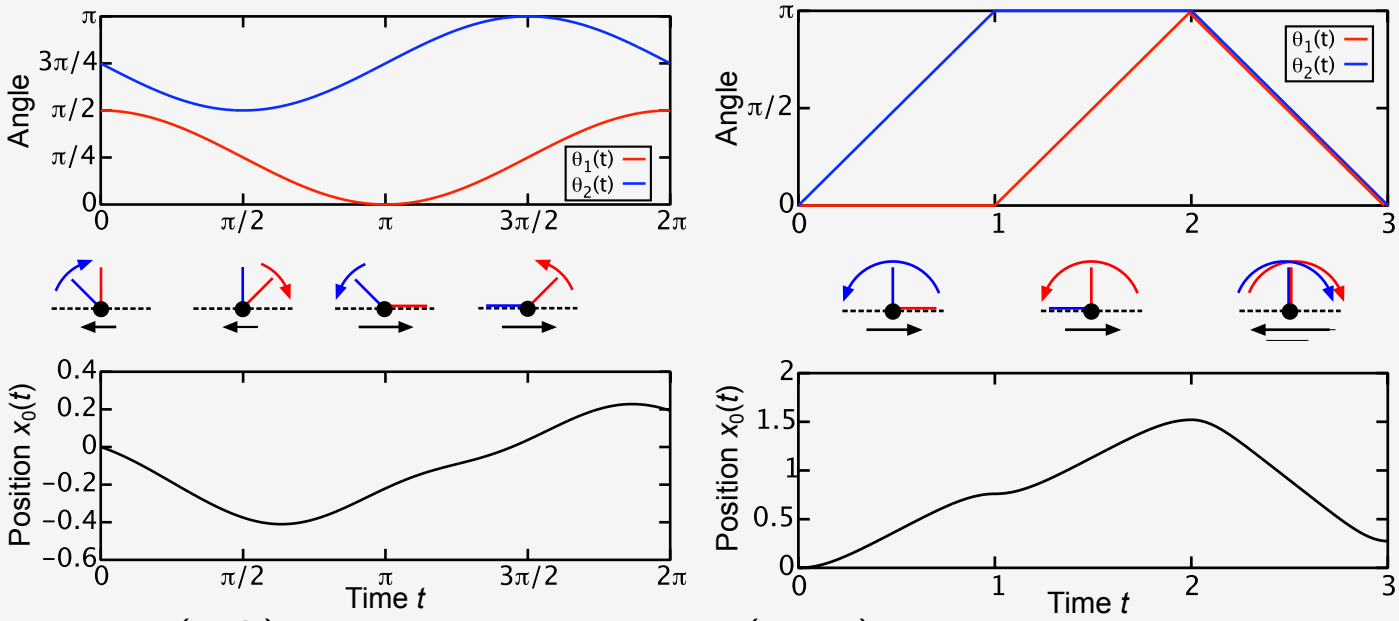


Controlled dynamic (Takagi, 2014)

$$\dot{x} = \frac{\sum_{i=1}^{2} l^2\, \dot{\theta}_i \sin(\theta_i)}{2 + \sum_{i=1}^{2} \sin^2(\theta_i)}, \quad \dot{\theta}_i = u_i,\ i = 1, 2, \quad \text{state constraints: } 0 \leq \theta_1 \leq \theta_2 \leq \pi.$$

Criterion : min of the dissipative energy: $\dot{q} S(q) \dot{q}^\intercal$ where $q = (x, \theta_1, \theta_2)$ and S is a positive-definite matrix $\Longrightarrow$ quadratic form in (u_1, u_2).

Definition 2. *An admissible stroke is a periodic motion of the shape variables* (θ_1, θ_2) *associated with a periodic control and producing a displacement of the position variable after one period* T *(we can fix* $T = 2\pi$*).*



(Left) Trigonometric controls. (Right) Piecewise controls.

$$\mathscr{E} = \textbf{Geometric Efficiency} = x(T)/l_{SR}(q)$$

SR length:

$$l_{SR}(q) = \int_0^T \sqrt{L(q,u)}\,\mathrm{d}t, \qquad L(q,u) = a(q)\,u_1^2 + 2b(q)\,u_1 u_2 + c(q)\,u_2^2$$

$$\min\, l_{SR}(q) \Longleftrightarrow \min \int_0^T |\dot{q}|_L^2\,\mathrm{d}t = \min \int_0^T L(q,u)\mathrm{d}t$$

With $c(\widetilde{q}(0),\widetilde{q}(T)) = q^0(T)$:

$$\dot{q} = \sum_{i=1}^{2} u_i F_i \leftarrow \min \int_0^T L(q,u)\mathrm{d}t,$$

$$\max\, \mathscr{E} \Longleftrightarrow \left(\min \int_0^T L(q,u)\mathrm{d}t \quad \text{with} \quad \textbf{x(T) fixed}, \text{ then } \underbrace{\text{select} \max_{x(T)} \mathscr{E}}_{\text{transversality cond.}} \right)$$

Compute points on the SR-sphere $\rightarrow$
provide candidates to maximize the efficiency.

$$\dot{q} = \sum_{i=1}^{2} u_i F_i(q), \qquad q = (x, \theta_1, \theta_2)$$

$$F_i = \frac{\sin(\theta_i)}{\Delta}\frac{\partial}{\partial x} + \frac{\partial}{\partial \theta_i}, \qquad \Delta = 2 + \sin^2(\theta_1) + \sin^2(\theta_2)$$

PMP:

$$\mathscr{H}(z,u) = u_1 H_1(z) + u_2 H_2(z) + p^0 L(q,u), \qquad z = (q,p)$$

with $H_i = p \cdot F_i$, $i = 1,2$.

- normal case: $p^0 = -1/2$ and u_i, $i = 1,2$ given by $\partial \mathscr{H}/\partial u = 0$

$$H_n = 1/2\,(a(q)H_1^2 + 2b(q)H_1H_2 + c(q)H_2^2)$$

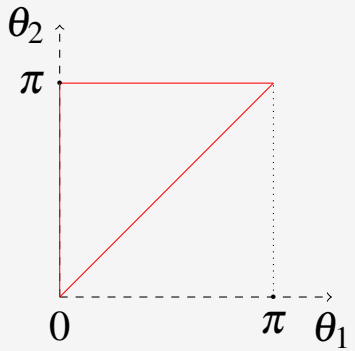
- abnormal case: $p^0 = 0$ hence $H_1 = H_2 = \{H_1, H_2\} = 0$, then

$$u_1\{\{H_1,H_2\},H_1\} + u_2\{\{H_1,H_2\},H_2\} = 0.$$

Proposition 3. *The surface* $\Sigma : \{q;\ \det(F_1(q), F_2(q), [F_1, F_2](q)) = 0\}$ *contains abnormal trajectories and is characterized by*

- $\theta_i = 0$ *or* π, *for* $i = 1, 2$,
- $\theta_1 = \theta_2$.

$\rightarrow$ **physical boundary of the state domain.**: $\theta_i \in [0, \pi]$, $i = 1, 2, \theta_1 \leq \theta_2$.



: abnormal trajectories

in the plane (θ_1, θ_2)

Boundary conditions defined by

$$x(0) = 0, \quad x(T) = x_f, \quad \theta_i(0) = \theta_i(T),\ i = 1,2$$

Transversality conditions

$$p_{\theta_i}(0) = p_{\theta_i}(T),\ i = 1,2$$

Shooting problem

$$\begin{cases} \dot{z} = \overrightarrow{H}_n(z) \\ x(0) = 0, \quad x(T) = x_f \ \textbf{(fixed)} \\ \theta_i(0) = \theta_i(T),\ i = 1,2 \\ p_{\theta_i}(0) = p_{\theta_i}(T),\ i = 1,2 \end{cases},$$

- `HamPath` *indirect method*: based on the PMP, *Newton* type algorithm and compute **necessary second order optimality conditions**, not robust w.r.t. initialization.

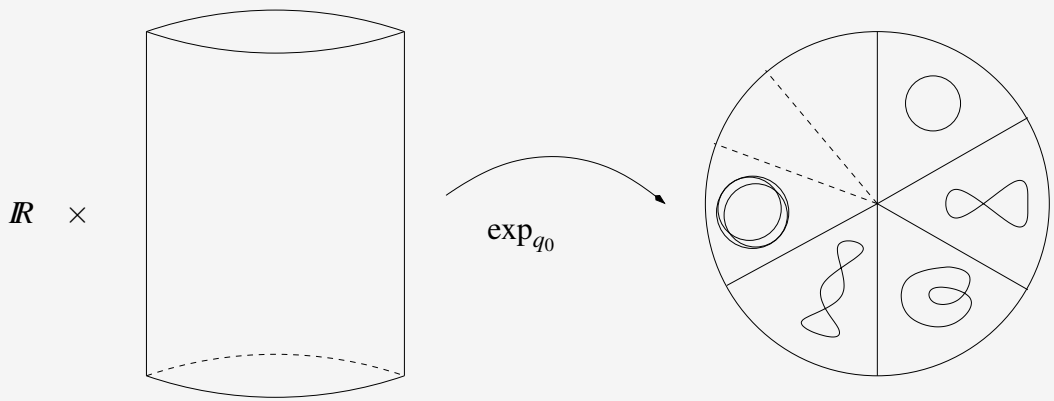
- `Bocop` *direct method*: give an initialization of the shooting algorithm of the `HamPath` code.

First conjugate time t_c: the exponential mapping

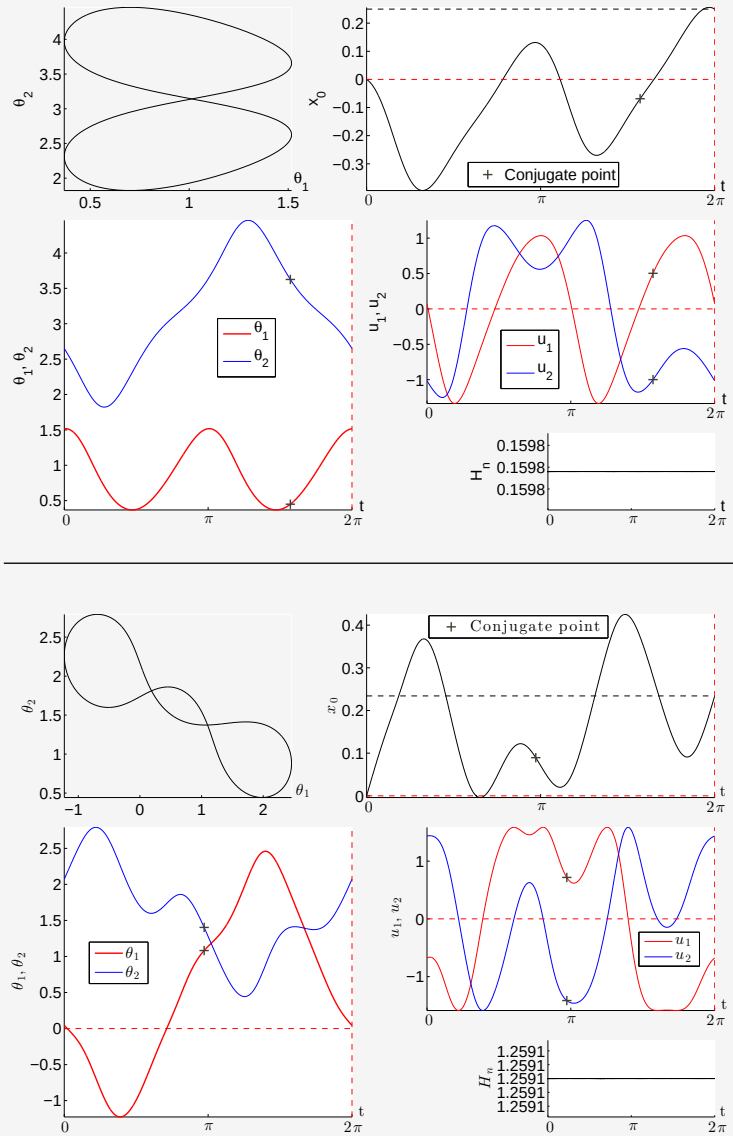
$$\exp_{q_0} : \mathbb{R} \times \mathscr{C} \to M, \quad (t, p_0) \mapsto q(t, q_0, p_0)$$

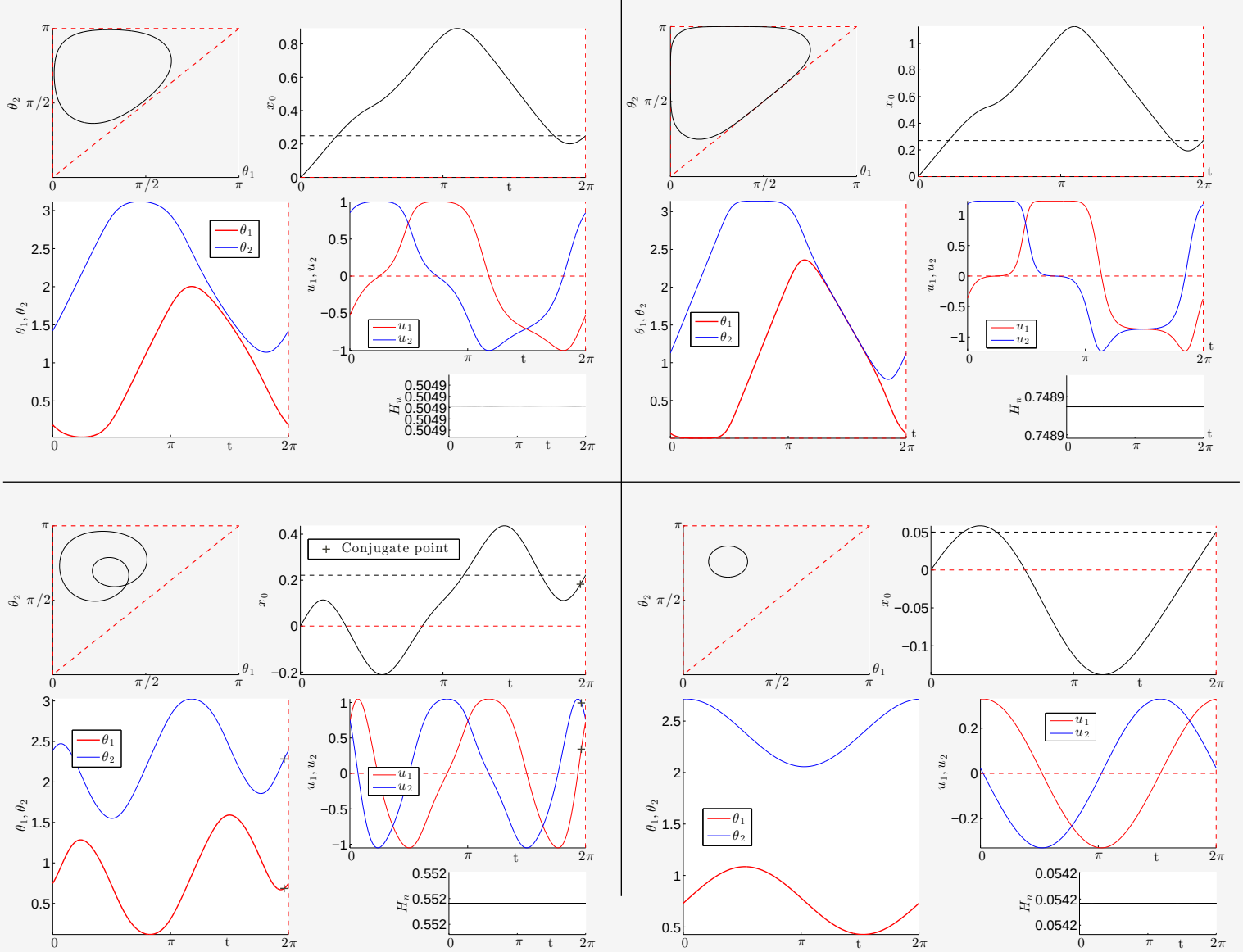
is not an immersion at (t_c, p_0).

Theorem 4. *Let $q : [0,T] \longrightarrow \mathbb{R}^n$ a strict normal stroke. If $q(.)$ has at least one conjugate point on $]0,T[$, then q* **is not a local minimizer** *for the L^∞ topology on controls, considering the fixed extremities problem.*



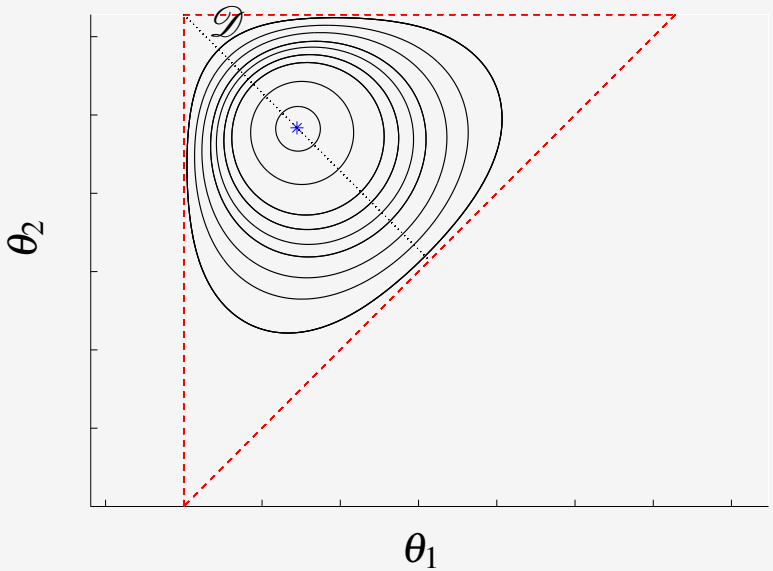
$$\mathscr{C} = \{ p_0 \mid H_1^2(q_0, p_0) + H_2^2(q_0, p_0) = 1 \}$$


Conjugate point
Conjugate point
Conjugate point
Conjugate point


θ_2
θ_1
x_0
t
θ_1, θ_2
u_1, u_2
H_α
θ_1
θ_2
u_1
u_2
+ Conjugate point

Numerical fact:
Only simple loops doesn't have a conjugate point on $]0, T[$: they are the only candidates for optimality.

Theorem 5 (Numerical result)**.** *For each fixed displacement $x(T)$, one can find a simple loop associated with an energy level $\Longrightarrow$ one parameter family of strokes.*

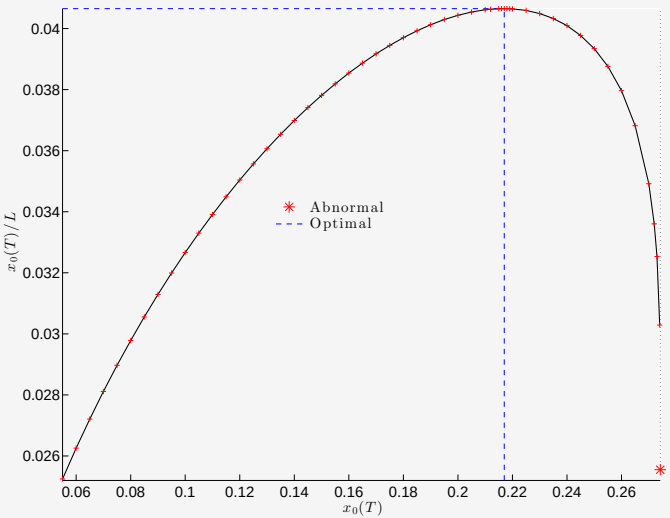


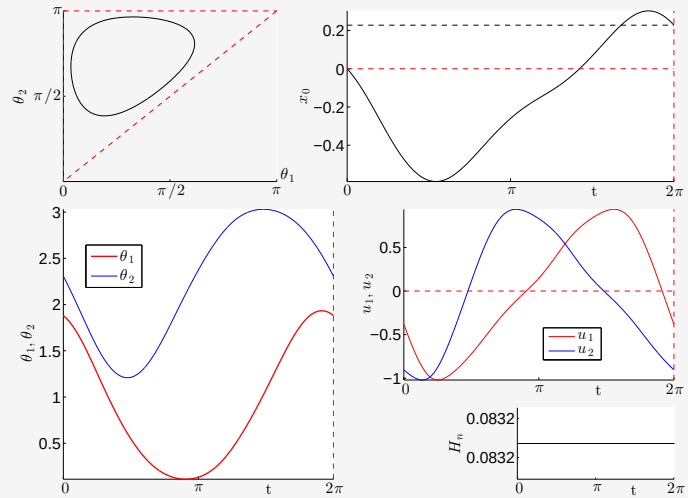
Select a stroke with transversality conditions.

Geometric efficiency, $c(\widetilde{q}(0),\widetilde{q}(T)) = -x(T)\,/\,l_{SR}(\widetilde{q})$

Transversality conditions.

$$p_x(T) = q^0(T)/x(T), \qquad p^0(T) = -1/2$$





Efficiency curve with continuation method on $x(T)$. Corresponding optimal stroke

Theorem 6. *The abnormal stroke is not minimizing for the two costs (length with fixed displacement and geometric efficiency).*

- More general models,
- Local geometric study

Nilpotent model

$$\dot{x} = u_1 \hat{F}_1 + u_2 \hat{F}_2, \qquad \min \int_0^T (u_1^2 + u_2^2)\, \mathrm{d}t$$

. **Interior point of the triangle:** weigths $x_1, x_2 \leftarrow 1$, $x_3 \leftarrow 2$

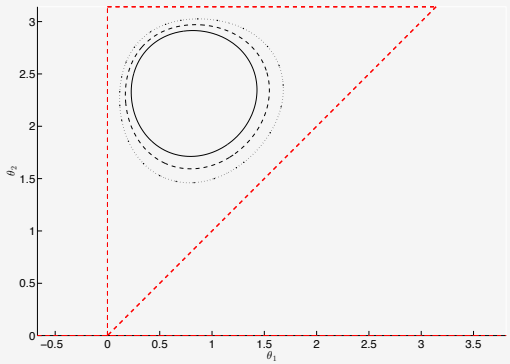
$$\hat{F}_1 = \frac{\partial}{\partial x_1} + x_2 \left(1 + Q(x_1, x_2)\right) \frac{\partial}{\partial x_3}, \qquad \hat{F}_2 = \frac{\partial}{\partial x_2} - x_1 \left(1 + Q(x_1, x_2)\right) \frac{\partial}{\partial x_3}$$

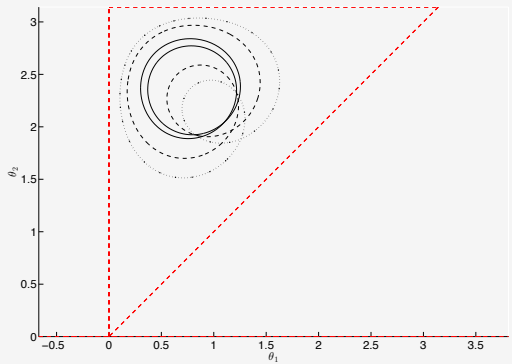
$\Longrightarrow$ simple loops and limaçons families

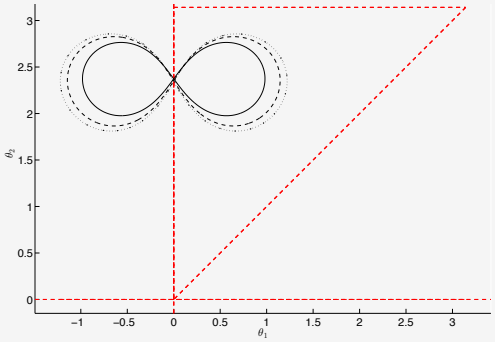
. **Point on the edges and $\neq$ vertices of the triangle:** weghts $x_1, x_2 \leftarrow 1$, $x_3 \leftarrow 3$

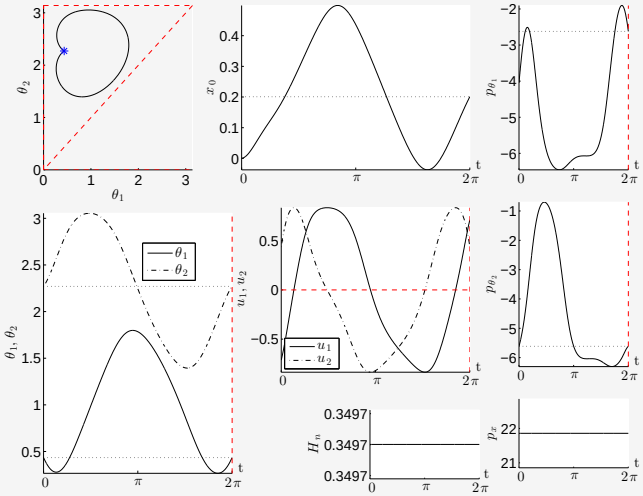
$$\hat{F}_1 = \frac{\partial}{\partial x_1} + \frac{x_2^2}{2} \frac{\partial}{\partial x_3}, \quad \hat{F}_2 = \frac{\partial}{\partial x_2}.$$

metric : $g = (1 + \alpha x_2)^2 dx_1 + (1 + \beta x_1 + \gamma x_2)^2 dx_2$.
$\Longrightarrow$ eight strokes


θ_2
θ_1


θ_2
θ_1


θ_2
θ_1


θ_2
θ_1
x_0
p_{θ_1}
θ_1, θ_2
θ_1
θ_2
u_1, u_2
u_1
u_2
p_{θ_2}
H_0
p_z
t