

Numerical Approach to the Optimal Control and Efficiency of the Copepod Swimmer

Jérémy Rouot, Bernard Bonnard, Monique Chyba, Daisuke Takagi

▶ To cite this version:

Jérémy Rouot, Bernard Bonnard, Monique Chyba, Daisuke Takagi. Numerical Approach to the Optimal Control and Efficiency of the Copepod Swimmer. 55th IEEE Conference on Decision and Control, Sep 2016, Las Vegas, United States. hal-02483321

HAL Id: hal-02483321 https://inria.hal.science/hal-02483321

Submitted on 18 Feb 2020 $\,$

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers. L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Numerical Approach to the Optimal Control and Efficiency of the Copepod Swimmer.

55th IEEE Conference on Decision and Control

Bernard Bonnard, Monique Chyba, Jérémy Rouot and Daisuke Takagi

- Geometric optimal control
 - . Maximum principle
 - . Second order optimality conditions (local, conjugate points)
- Swimming at low Reynolds number
 - . sub-Riemannian (SR) geometry framework
 - . Varational approach to find trajectoires of a Hamiltonian system with periodic projections
 - . Find optimal stroke (efficiency concept)
 - . micro-device implementation (D. Takagi, M. Chyba)

$$\begin{array}{ll} \min & c(\widetilde{q}(0),\widetilde{q}(T)) \\ \text{subject to} & \dot{q} = F(q,u), & \dot{q}^0 = L(q,u) \\ & (\widetilde{q}(0),\widetilde{q}(T)) \in B \end{array}$$

- final time T is fixed
- $\widetilde{q}=(q,q^0)$, q state, $q^0\in\mathbb{R}$
- $u(.) \in L^{\infty}([0,T],U), U \subset \mathbb{R}^m$
- B : smooth closed set of $\mathbb{R}^n \times \mathbb{R}^n$, $(\tilde{q}(0), \tilde{q}(T)) \in B$ stands for **mixed** boundary conditions.

Ex. : $B = \{q(0) = q(T), q^0(0) = 0\},\$ $B = \{q(0) = q_0, q(T) = q_f, q^0(0) = 0\}...$ **Necessary condition.** (\widetilde{q}, u) optimal $\implies \exists \ \widetilde{p} = (p, p^0) : [0, T] \rightarrow (\mathbb{R}^n \times \mathbb{R})$ abs. continuous and $\lambda \ge 0$ cst with $(\widetilde{p}, \lambda) \ne (0, 0)$ and s.t.

$$\dot{\widetilde{q}} = \frac{\partial H}{\partial \widetilde{p}}, \quad \dot{\widetilde{p}} = -\frac{\partial H}{\partial \widetilde{q}}, \quad H(\widetilde{q}, \widetilde{p}, u) = \max_{v \in U} \mathscr{H}(\widetilde{q}, \widetilde{p}, v)$$

where $\mathscr{H}(\widetilde{q}, \widetilde{p}, u) = p \cdot F(q, u) + p^0 L(q, u)$. Transversality conditions:

$$(\widetilde{p}(0), -\widetilde{p}(T)) - \lambda \left(\frac{\partial c}{\partial \widetilde{q}(0)} (\widetilde{q}(0), \widetilde{q}(T)), \frac{\partial c}{\partial \widetilde{q}(T)} (\widetilde{q}(0), \widetilde{q}(T)) \right) \in (T_{(\widetilde{q}(0), \widetilde{q}(T))} B)^{\perp}$$

Definition 1. $(\tilde{q}, \tilde{p}, \lambda, u)$ is a normal extremal if $\lambda > 0$, abnormal if $\lambda = 0$. An abnormal trajectory q which is not a projection of normal extremal is said strict.

LIFE AT LOW REYNOLDS NUMBER - PURCELL, 1977



Swimmer deformations

Zooplankton

Symmetric 2-link swimmer



Controlled dynamic (Takagi, 2014)

$$\dot{x} = \frac{\sum_{i=1}^{2} l^2 \dot{\theta}_i \sin(\theta_i)}{2 + \sum_{i=1}^{2} \sin^2(\theta_i)}, \quad \dot{\theta}_i = u_i, \ i = 1, 2, \quad \text{ state constraints: } 0 \le \theta_1 \le \theta_2 \le \pi.$$

Criterion : min of the dissipative energy: $\dot{q}S(q)\dot{q}^{\mathsf{T}}$ where $q = (x, \theta_1, \theta_2)$ and S is a positive-definite matrix \Longrightarrow quadratic form in (u_1, u_2) .

Definition 2. An admissible stroke is a periodic motion of the shape variables (θ_1, θ_2) associated with a periodic control and producing a displacement of the position variable after one period T (we can fix $T = 2\pi$).



$$\mathscr{E} =$$
Geometric Efficiency $= x(T)/l_{SR}(q)$

SR length:

With

$$l_{SR}(q) = \int_0^T \sqrt{L(q,u)} \, \mathrm{d}t, \qquad L(q,u) = a(q) \, u_1^2 + 2b(q) \, u_1 u_2 + c(q) \, u_2^2$$

$$\min l_{SR}(q) \iff \min \int_0^T |\dot{q}|_L^2 dt = \min \int_0^T L(q, u) dt$$
$$c(\tilde{q}(0), \tilde{q}(T)) = q^0(T):$$

$$\dot{q} = \sum_{i=1}^{2} u_i F_i \leftarrow \min \int_0^T L(q, u) dt,$$

$$\max \, \mathscr{E} \iff \left(\min \, \int_0^T L(q, u) dt \quad \text{with} \quad \mathbf{x}(\mathbf{T}) \text{ fixed, then } \underbrace{\text{select } \max_{x(T)} \mathscr{E}}_{\text{transversality cond.}} \right)$$

Compute points on the SR-sphere \rightarrow provide candidates to maximize the efficiency.

$$\dot{q} = \sum_{i=1}^{2} u_i F_i(q), \qquad q = (x, \theta_1, \theta_2)$$
 $F_i = \frac{\sin(\theta_i)}{\Delta} \frac{\partial}{\partial x} + \frac{\partial}{\partial \theta_i}, \qquad \Delta = 2 + \sin^2(\theta_1) + \sin^2(\theta_2)$

PMP:

with H_i

$$\mathscr{H}(z,u) = u_1 H_1(z) + u_2 H_2(z) + p^0 L(q,u), \qquad z = (q,p)$$
$$= p \cdot F_i, \ i = 1, 2.$$

- normal case: $p^0 = -1/2$ and u_i , i = 1, 2 given by $\partial \mathscr{H}/\partial u = 0$ $H_n = 1/2 (a(q)H_1^2 + 2b(q)H_1H_2 + c(q)H_2^2)$
- abnormal case: $p^0 = 0$ hence $H_1 = H_2 = \{H_1, H_2\} = 0$, then

$$u_1\{\{H_1, H_2\}, H_1\} + u_2\{\{H_1, H_2\}, H_2\} = 0.$$

Proposition 3. The surface Σ : { q; det($F_1(q), F_2(q), [F_1, F_2](q)$) = 0 } contains abnormal trajectories and is characterized by

- $\theta_i = 0$ or π , for i = 1, 2,
- $\theta_1 = \theta_2$.

 \rightarrow physical boundary of the state domain.: $\theta_i \in [0, \pi], i = 1, 2, \theta_1 \leq \theta_2$.



: abnormal trajectories

in the plane (θ_1, θ_2)

Boundary conditions defined by

$$x(0) = 0, \quad x(T) = x_f, \quad \theta_i(0) = \theta_i(T), \ i = 1, 2$$

Transversality conditions

$$p_{\theta_i}(0) = p_{\theta_i}(T), \ i = 1, 2$$

Shooting problem

$$\begin{cases} \dot{z} = \overrightarrow{H}_n(z) \\ x(0) = 0, \quad x(T) = x_f \text{ (fixed)} \\ \theta_i(0) = \theta_i(T), \ i = 1, 2 \\ p_{\theta_i}(0) = p_{\theta_i}(T), \ i = 1, 2 \end{cases}$$

- HamPath *indirect method*: based on the PMP, *Newton* type algorithm and compute **necessary second order optimality conditions**, not robust w.r.t. initialization.
- Bocop *direct method*: give an initialization of the shooting algorithm of the HamPath code.

EXPONENTIAL MAPPING

First conjugate time *t*_{*c*}: the exponential mapping

$$\exp_{q_0}: \mathbb{R} \times \mathscr{C} \to M, \quad (t, p_0) \mapsto q(t, q_0, p_0)$$

is not an immersion at (t_c, p_0) .

Theorem 4. Let $q : [0,T] \longrightarrow \mathbb{R}^n$ a strict normal stroke. If q(.) has at least one conjugate point on]0,T[, then q is not a local minimizer for the L^{∞} topology on controls, considering the fixed extremities problem.







Numerical fact:

Only simple loops doesn't have a conjugate point on]0,T[: they are the only candidates for optimality.

Theorem 5 (Numerical result). For each fixed displacement x(T), one can find a simple loop associated with an energy level \implies one parameter family of strokes.



Select a stroke with transversality conditions.

Geometric efficiency, $c(\tilde{q}(0), \tilde{q}(T)) = -x(T)/l_{SR}(\tilde{q})$ Transversality conditions.



 $p_x(T) = q^0(T)/x(T), \qquad p^0(T) = -1/2$

Efficiency curve with continuation method on x(T). Corresponding optimal stroke

Theorem 6. The abnormal stroke is not minimizing for the two costs (length with fixed displacement and geometric efficiency).

Perspectives

- More general models,
- Local geometric study

Nilpotent model

$$\dot{x} = u_1 \hat{F}_1 + u_2 \hat{F}_2, \qquad \min \int_0^1 (u_1^2 + u_2^2) dt$$

. Interior point of the triangle: weigths $x_1, x_2 \leftarrow 1$, $x_3 \leftarrow 2$

$$\hat{F}_1 = \frac{\partial}{\partial x_1} + x_2 \left(1 + Q(x_1, x_2)\right) \frac{\partial}{\partial x_3}, \qquad \hat{F}_2 = \frac{\partial}{\partial x_2} - x_1 \left(1 + Q(x_1, x_2)\right) \frac{\partial}{\partial x_3}$$

 \implies simple loops and limaçons families

. Point on the edges and \neq vertices of the triangle: weights $x_1, x_2 \leftarrow 1$, $x_3 \leftarrow 3$

$$\hat{F}_1 = rac{\partial}{\partial x_1} + rac{x_2^2}{2} rac{\partial}{\partial x_3}, \quad \hat{F}_2 = rac{\partial}{\partial x_2}.$$

metric : $g = (1 + \alpha x_2)^2 dx_1 + (1 + \beta x_1 + \gamma x_2)^2 dx_2.$ \implies eight strokes

Perspectives







