Spatial Optimization of Fertilizer Application by Centrifugal Spreading

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Abstract

We study an approach for the optimization of spreading performed by centrifugal spreaders in order to minimize environmental effects due to application errors. Faced with a large scale problem, we divide the domain of study into subdomains so that each tramline is individually dealt with. In order to take into account the mechanical limits of the device, some inequality constraints are introduced. After cost function discretization, we use an augmented lagrangian algorithm associated with a L-BFGS technique to solve the problem. Results are presented for parallel and non parallel paths.

Keywords

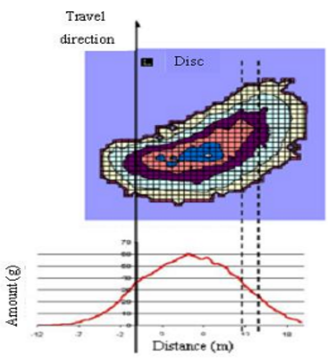
Augmented Lagrangian, Centrifugal spreaders, Discretization, Optimization.

I. INTRODUCTION

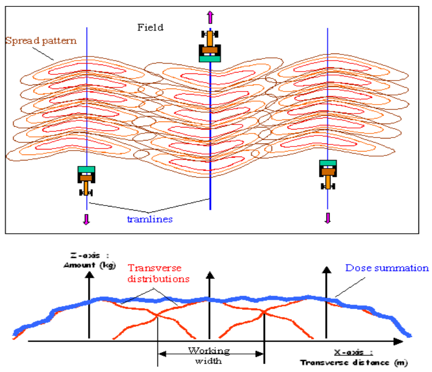
Fertilization is a regular agricultural practice which is carried on in order to apply plant nutrients to satisfy the demand of crops. In most cases, this operation is performed thanks to a centrifugal spreader because of its low cost and robustness. Unfortunately, by using unsuitable strategy, this practice can result in ill environmental effects. Indeed, an over application mostly involves an over nitrate enrichment of surface waters leading to an excessive multiplication of algae which can result in fishes dying. In the contrary, when under-dosages occur, yield losses can be very important. Therefore, mineral fertilization is today pointed out as one of the major causes of the pollution of groundwaters and watercourses. Thus, the new rules imposed by the different governments in North America and Northern Europe (Bruxelles, 2005), lead us to investigate a new strategy to improve the quality of the distribution achieved by centrifugal spreaders. This paper presents an approach for optimization of fertilizer application achieved during spreading. Instead of using the traditional method based on the best arrangement of the transverse distribution, we consider an optimization method relying on a spread pattern model developed by (Colin, 1997), (Olieslagers, 1997). Faced with a large scale problem we decompose the problem by using "sliding windows" and taking advantage of the symmetry properties of centrifugal spreading. Simulation results are presented for parallel and non parallel tramlines.

II. PROBLEM STATEMENT

Spreading regularity is achieved when the actual distributed amount of fertilizers in the field is equal to the prescribed dose determined by agronomical and pedological reasoning with respect to the crops. Here, we consider centrifugal spreaders equipped with two spinning discs which eject nutrients along each tramline in the field. The tractor-spreader combination is commonly tooled up with a GPS antenna, a radar speed sensor, an embedded computer, and an actuator. So, according to the location and speed of the tractor measured by the two first tools previously mentioned, the computer determines the actuator adjustments to control the dosage rate. The amount of applied fertilizers currently called spread pattern, has an irregular distribution shape which is often highlighted by the transverse distribution curve calculated by summing the amounts along each travel direction illustrated by the red curve in Figure (1(a)). Because of the spatial distribution heterogeneousness, the tractor follows back and forth paths to obtain an uniform deposit from transverse distributions summation for each successive travel shown in Figure (1(b)). As we can notice, transverse distributions play an important role in the fertilization strategy. Indeed, the device settings relie on their best arrangements according to the different trajectories that is to say working widths. The working width is a concept widely used in the fertilization community today and corresponds to the distance between two consecutive overlapping lines. Furthermore, when overlapping is optimal, these lines stand for symmetry axes that make two successive transverse distribution coincide. This variable is also linked to the applied mass flow rate during spreading determined by the mathematical relation $m = (Q^* \times W \times S)/600$ where m is the mass flow rate (kg/min), Q^* the prescribed dose (kg/Ha), W the working width (m) and S the speed of tractor (km/h). With this relation, we reason as if nutrients were homogeneously distributed by the machine onto a rectangular area which length is equal to the wished working width. In this case, we don't take into account the actual occurring phenomenon. Indeed, the true fertilizers global deposit in the field is due to spread pattern overlappings, and then is the result of the heterogeneous spatial distribution accumulation at each position of the spreader. Therefore, this practice can give satisfying results with regularly spaced parallel travel direction but is quickly inefficient when geometrical singularities are met in the field (end of field, irregularly spaced parallel paths, start and end of spreading) and then can produce local application errors as illustrated in Figure (2). Then, we can have over-dosage that can reach +80% and sub-dosage equivalent to -85%. Since the spreading strategy is traditionally based on the best arrangement of transverse distributions, some works were led to find appropriate trajectories with respect to these ones (Dillon *et coll.*, 2003). Unfortunately, this strategy is unsuitable when tramlines are already fixed by others agricultural practices like sowing. Therefore, it is important to know how best arrange the shape and the placement of the true distributions, that is to say spread patterns, during spreading with respect to the imposed geometrical constraints in the field. This adjustment should be continuously performed for each GPS position of the machine by changing its settings. Then, in this paper, we focus on a method which computes optimal parameters that permit to obtain best spread patterns arrangement in the presence of imposed paths. Moreover, this study deals with only optimization along the trajectories in the field and not on the boundaries of this one.

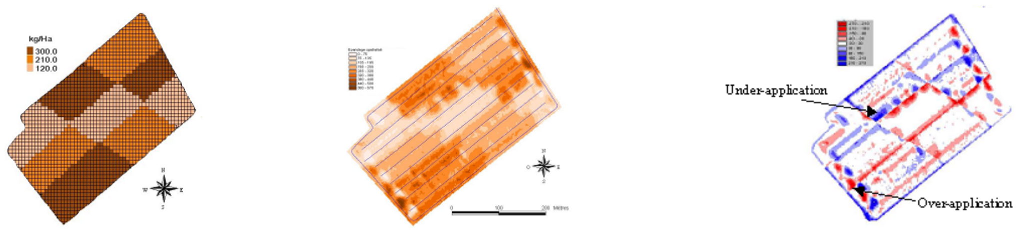


(a)



(b)

Figure 1: Centrifugal spreading characteristics: (a) Spread pattern and its tranverse distribution – (b) Fertilization strategy based on transverse distribution summation.



(a) Prescribed dose map obtained from agronomical considerations.

(b) Actual dose map obtained by applying the fertilization method based on best arrangement of transverse distribution.

(c) Application errors map calculated from the difference between the prescribed dose map and actual dose map.

Figure 2: Application errors resulting from the reasoning based on the best transverse distribution investigation and not on spread patterns overlappings.

III. COST FUNCTION AND OPTIMIZATION METHOD

In order to formalize a suitable optimization criterion, it is necessary to define at first the parameters used for the spread pattern model. So, we consider the field as a polygonal domain Ω in $\mathbb{R}^2$ and $s : (0,T) \longrightarrow \mathbb{R}^2$ a path in Ω that is to say $s(t) = (s_1(t), s_2(t)) \in \mathbb{R}^2$. $s(t)$ is assumed to be rectilinear. The mass flow rates at time t for the spinning disc on the right and the left of the spreader are respectively the functions $m : (0,T) \longrightarrow \mathbb{R}$ and $d : (0,T) \longrightarrow \mathbb{R}$. The spread pattern shown in Figure 1(a) is in most cases described by its medium radius $r(x, s(t))$ and medium angle $\theta(x, s(t))$. $r(x, s(t))$, varying with the speed of disc, is assumed to be the distance between the point x and $s(t)$, while $\theta(x, s(t))$, modifiable the fertilizers dropping point on the disc, corresponds to the angle between $\overrightarrow{s(t)x}$ and $s(t)$. To simplify notations, we will consider $r(x,t)$ and $\theta(x,t)$ instead of $r(x, s(t))$ and $\theta(x, s(t))$. Besides, we consider the following functions ρ and ξ, the medium radius concerning respectively the disc on the right and the left of the device, defined by $\rho : (0,T) \longrightarrow \mathbb{R}_+$, and $\xi : (0,T) \longrightarrow \mathbb{R}_+$. Moreover, let us assume the functions $\varphi : (0,T) \longrightarrow \mathbb{R}$, the medium angle concerning the right spinning disc, and $\psi : (0,T) \longrightarrow \mathbb{R}$, the medium angle concerning the left spinning disc.

According to (Colin, 1997), the spatial distribution pattern for the right and left spinning disc, at time t, are respectively given by the functions $q_r : \Omega \times (0,T) \longrightarrow \mathbb{R}$ and $q_l : \Omega \times (0,T) \longrightarrow \mathbb{R}$ defined by:

$$q_r(x, m(t), \rho(t), \varphi(t)) = \tau \cdot exp\left(\frac{-(r(x,t) - \rho(t))^2}{2\sigma_r^2}\right) \cdot exp\left(\frac{-(\theta(x,t) - \varphi(t))^2}{2\sigma_\theta^2}\right) \tag{1}$$

$$q_l(x, d(t), \xi(t), \psi(t)) = \kappa \cdot exp\left(\frac{-(r(x,t) - \xi(t))^2}{2\sigma_r^2}\right) \cdot exp\left(\frac{-(\theta(x,t) - \psi(t))^2}{2\sigma_\theta^2}\right) \tag{2}$$

$$\text{with} \qquad \tau = \frac{m(t)}{2\pi\sigma_r\sigma_\theta} \qquad \text{and} \qquad \kappa = \frac{d(t)}{2\pi\sigma_r\sigma_\theta}$$

In (1) and (2), σ_r and σ_θ are known parameters and stand for the standard deviations concerning respectively the medium radius and the medium angle. If we define $M(t) = (m(t), d(t)) \in \mathbb{R}^2$, $R(t) = (\rho(t), \xi(t)) \in \mathbb{R}^2$ and $\Phi(t) = (\varphi(t), \psi(t)) \in \mathbb{R}^2$, the distribution pattern is then obtained by the function $q_{tot} : \Omega \times (0,T) \longrightarrow \mathbb{R}$ defined by:

$$q_{tot}(x, M(t), R(t), \Phi(t)) = q_r(x, m(t), \rho(t), \varphi(t)) + q_l(x, d(t), \xi(t), \psi(t)) \tag{3}$$

From this model, the actual distributed dose Q during the interval of time $(0, T)$ for a single tramline is given by the function $Q : \Omega \times (0, T) \longrightarrow \mathbb{R}$ defined by:

$$Q(x, M, R, \Phi) = \int_0^T q_{tot}(x, M(t), R(t), \Phi(t))dt \tag{4}$$

For our problem, the objective is to reduce harmful fertilization effects by minimizing application error that is to say the difference between actual and desired dose for all points in the field. Then, we have to compute the optimal functions M, R and Φ, which minimize the functional:

$$F(M, R, \Phi) = \int_\Omega [Q(x, M, R, \Phi) - Q^*]^2 dx \tag{5}$$

where Q^* is the prescribed dose. It is well known that (5) cannot be analytically calculated. Thus, we use an approximative integration method. So, let us divide interval $(0, T)$ into n elements with equal length $\delta = \frac{T}{n}$ so that $t_j = j\delta$, $j = 0, 1, ..., n$, with $t_0 = 0$ and $t_n = T$. Then, we define $M_j = M(t_j)$, $R_j = R(t_j)$, $\Phi_j = \Phi(t_j)$ and

$$\boldsymbol{M} = \begin{bmatrix} M_0 \\ \vdots \\ M_n \end{bmatrix}, \quad \boldsymbol{R} = \begin{bmatrix} R_0 \\ \vdots \\ R_n \end{bmatrix}, \quad \boldsymbol{\Phi} = \begin{bmatrix} \Phi_0 \\ \vdots \\ \Phi_n \end{bmatrix}.$$

Thus, by using the trapezoidal rule, (4) can be approximated by:

$$Q(x, \boldsymbol{M}, \boldsymbol{R}, \boldsymbol{\Phi}) = \delta \left(\chi + \sum_{i=1}^{n-1} q_{tot}(x, M_i, R_i, \Phi_i \right) \tag{6}$$

$$\text{with} \qquad \chi = \frac{q_{tot}(x, M_0, R_0, \Phi_0) + q_{tot}(x, M_n, R_n, \Phi_n)}{2} \tag{7}$$

The functional to be minimized is then given as:

$$F(\boldsymbol{M}, \boldsymbol{R}, \boldsymbol{\Phi}) = \int_\Omega [Q(x, \boldsymbol{M}, \boldsymbol{R}, \boldsymbol{\Phi}) - Q^*]^2 dx \tag{8}$$

In order to take into account the mechanical limits of the device and not to untimely solicit actuators, the functions M, R, and Φ and their time derivative are subject to bound constraints. Then, the set of constraints is defined as:

$$S = \{(\boldsymbol{M}, \boldsymbol{R}, \boldsymbol{\Phi}) \in \mathbb{R}^{6(n+1)} | \, M_{min} \le \boldsymbol{M} \le M_{max}, \quad R_{min} \le \boldsymbol{R} \le R_{max}, \quad \Phi_{min} \le \boldsymbol{\Phi} \le \Phi_{max}, \\ |M_{i+1} - M_i| \le \alpha\delta, \quad |R_{i+1} - R_i| \le \alpha\delta, \quad |\Phi_{i+1} - \Phi_i| \le \beta\delta\}$$

where α, β and γ are known parameters. Therefore, we obtain the nonlinear programming problem $(\mathcal{P})$ given by:

$$(\mathcal{P}) \qquad \min_{(\boldsymbol{M}, \boldsymbol{R}, \boldsymbol{\Phi}) \in S} F(\boldsymbol{M}, \boldsymbol{R}, \boldsymbol{\Phi}) \tag{9}$$

S is a bounded closed set, then it is a compact set in $\mathbb{R}^{6(n+1)}$. Thus, according to the Weierstrass theorem, the problem $(\mathcal{P})$ has at least one local minimum. In most cases, there exist several tramlines and the actual distributed dose for all trajectories is then obtained by the summation of the applied dose for each k indexed path:

$$Q(x, M, R, \Phi) = \sum_{k=1}^{w} \int_{t_i^k}^{t_f^k} q(x, M(t^k), R(t^k), \Phi(t^k))dt \tag{10}$$

where w is the number of paths and the trajectories $s^k(t)$ are defined in the interval (t_i^k, t_f^k). If we assume that $M_j^k = M(t_j^k)$, $R_j^k = R(t_j^k)$ and $\Phi_j^k = \Phi(t_j^k)$, we can use the same discretization scheme as before. For practical reasons, like the prescribed dose map in Figure (2), the domain Ω is 1 m-gridded. In order to lose informations as little as possible, at least 2 samples per mesh must be computed. Thus, if we take the example of a farmland with only 3 tramlines 100 m long, 3600 variables should be considered. It is then clear that we cannot directly solve the problem for the entire field. Faced with this difficulty, we decompose the problem $(\mathcal{P})$ into sub-problems in order to deal with each trajectory individually. Given that the arable land contains w tramlines, Ω is at first divided into w subdomains. We use the following notations:

$$
\begin{array}{ll}
K_1 = \{k \in \mathbb{N} | \, 1 \le k \le w\}, & L_2 = \{l \in \mathbb{N} | \, \forall z \ge 2 \in \mathbb{N}, \; 2 \le l \le z\}, \\
K_2 = \{k \in \mathbb{N} | \, 1 \le k \le w-1\}, & \Omega = \bigcup_{k \in K_1} \Omega^k, \\
K_3 = \{k \in \mathbb{N} | \, 2 \le k \le w\}, & \Omega^k = \bigcup_{l \in L_1} \Omega_l^k, \\
L_1 = \{l \in \mathbb{N} | \, \forall z \ge 2 \in \mathbb{N}, \; 1 \le l \le z\}, &
\end{array}
$$

and the following definitions:

– $\Omega^k \in \mathbb{R}^2$ is the k^{th} subdomain of Ω, – $\Omega_l^k \in \mathbb{R}^2$ is the l^{th} subdomain of Ω^k.

The subdomains Ω^k are defined so that:

– $\partial\Omega^k \cap \Omega^{k+1} = s^{k+1}(t)$, $\forall\, k \in K_2$ and $\forall t \in (t_i^{k+1}, t_f^{k+1})$,

– $\partial\Omega^k \cap \Omega^{k-1} = s^{k-1}(t)$, $\forall\, k \in K_3$ and $\forall t \in (t_i^{k-1}, t_f^{k-1})$.

To make easier to understand the spatial decomposition, the figure (3) illustrates the example of three parallel tramlines in a domain Ω with a rectangular geometry.

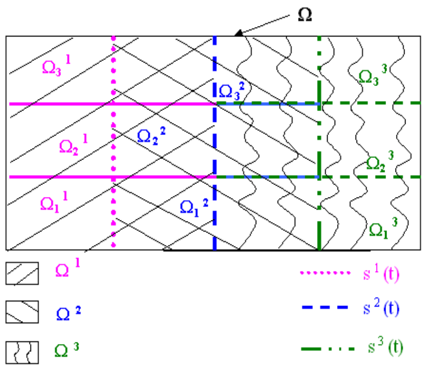


Figure 3: Rectangular domain Ω divided into 9 subdomains Ω_l^k, $1 \le l \le 3$, $1 \le k \le 3$.

From this decomposition, if the vectors $\boldsymbol{M}_l^k$, $\boldsymbol{R}_l^k$, $\boldsymbol{\Phi}_l^k$ are assumed to be respectively the restriction of $\boldsymbol{M}$, $\boldsymbol{R}$ and $\boldsymbol{\Phi}$ in Ω_l^k, we can also define S_l^k as the restriction of S in the same subdomain. Thus, by considering the symmetry properties of spreading exposed in the section **II**, we can then solve the problem $(\mathcal{P})$ by defining a new formulation $(\mathcal{P}')$ of this one given by:

$$
(\mathcal{P}') \qquad \begin{cases} \min \sum_{l=1}^{z} \sum_{k=1}^{w} J_l^k(x, \boldsymbol{M}_l^k, \boldsymbol{R}_l^k, \boldsymbol{\Phi}_l^k) \\ (\boldsymbol{M}_l^k, \boldsymbol{R}_l^k, \boldsymbol{\Phi}_l^k) \in S_l^k, \; (l,k) \in L_1 \times K_1 \end{cases} \tag{11}
$$

with

$$
J_l^k(x, \boldsymbol{M}_l^k, \boldsymbol{R}_l^k, \boldsymbol{\Phi}_l^k) = \int_{\Omega_l^k} [Q_l^k - Q^*]^2 dx \tag{12}
$$

where Q_l^k stands for the applied dose in the subdomain Ω_l^k. Then, solving the problem $(\mathcal{P}')$ is equivalent to optimize the distributed dose in each subdomain Ω_l^k, called "window", by taking into account not only the dose already applied in the domains Ω_{l-1}^k and Ω_l^{k-1}, but also the future distributed amounts in Ω_l^{k+1} predicted so that they respect the previously exposed symmetry relations. Therefore, the notion of "sliding windows" can be explained by the fact of optimizing fertilization accuracy by solving in a sequential way the problem $(\mathcal{P})$.
Given that the problem $(\mathcal{P}')$ is an optimization problem subject to inequality constraints, minimizing the functional J_l^k for $(l,k) \in L_1 \times K_1$ is equivalent to consider the following problem:

$$\mathcal{P}_{ineq} \qquad \begin{cases} \min J_l^k(\boldsymbol{M}_l^k, \boldsymbol{R}_l^k, \boldsymbol{\Phi}_l^k) \\ min_j \leq h_j(\boldsymbol{M}_l^k, \boldsymbol{R}_l^k, \boldsymbol{\Phi}_l^k) \leq max_j, \\ j = 1, 2, ...6 \times dim(\boldsymbol{M}_l^k) - 1 \end{cases} \tag{13}$$

where h_j denotes the j^{th} double inequality, min_j and max_j its lower and upper bound. To avoid to deal with the problem which consists in looking for saturated constraints, we use an augmented lagrangian algorithm which permits to penalize severely unacceptable solutions (Bertsekas, 1982). Thus, let us assume:

$$\mathcal{P}_r \qquad \begin{cases} \min_{X \in S_l^k} J_l^k(X) + \sum_j c_j(h_j(X), \lambda_j, r_j) \\ X = (\boldsymbol{M}_l^k, \boldsymbol{R}_l^k, \boldsymbol{\Phi}_l^k) \in S_l^k \\ \lambda_j \in \mathbb{R} \\ r_j \in \mathbb{R} \\ j = 1, 2, ...6 \times dim(\boldsymbol{M}_l^k) - 1 \end{cases} \tag{14}$$

where λ_j is the j^{th} lagrange multiplier, r_j the j^{th} penalty coefficient and c_j defined by:

$$c_j(h_j(X), \lambda_j, r_j) = \begin{cases} \lambda_j(h_j(X) - max_j) + \frac{r_j}{2}(h_j(X) - max_j)^2 & \text{if} \quad Z(h_j(X), \lambda_j, max_j) > 0 \\ \lambda_j(h_j(X) - min_j) + \frac{r_j}{2}(h_j(X) - min_j)^2 & \text{if} \quad Z(h_j(X), \lambda_j, min_j) < 0 \\ -\frac{\lambda_j^2}{2r_j} \quad \text{otherwise} \end{cases}$$

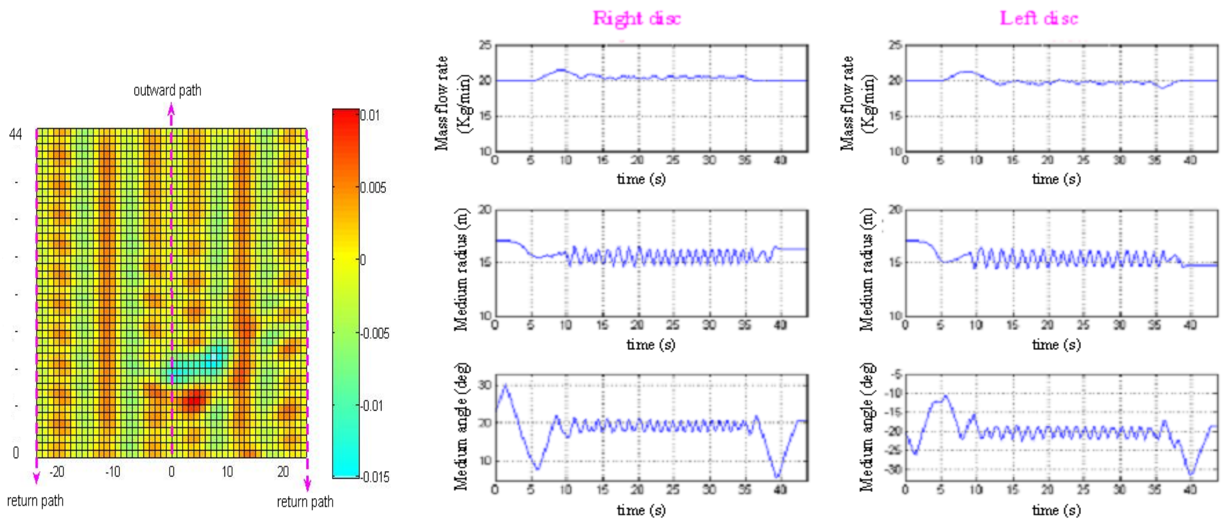
with $Z(h_j(X), \lambda_j, b(h_j)) = \lambda_j + r_j(h_j(X) - b(h_j)$ and $b(h_j)$ the bounds on h_j. Faced with a cost function and gradient which evaluations present a high computational time, we choose to apply a L-BFGS algorithm shown to be efficient in this case (Byrd *et coll.*, 1994). Then, we obtain the following algorithm:
Step 0: $k = 1, l = 1$, X in S_1^1,
Step 1: if $l \leq z$, minimize J_l^k by using the augmented lagrangian algorithm associated with the L-BFGS minimization technique, otherwise goto **Step 3**,
Step 2: $l \leftarrow l + 1$, goto **Step 1**
Step 3: $k \leftarrow k + 1$, X in S_1^k, goto **Step 1**.

IV. RESULTS

In this study, we only interest in rectangular domain. Nowadays, even in the cases where uniform rate application are considered, over and under-application are often observed within farmlands. So, here we define a constant prescribed dose fixed at 100 Kg/Ha. The speed of tractor commonly evolving around 10 Km/h, we use this average value for the machine displacement velocity. We consider two scenarios: the case of three parallel tramlines with a working width equal to 24 m and the case of two consecutive 24 m spaced parallel paths preceded by an other presenting a travel direction shift. In the first case, the domain Ω is defined by $\Omega = (-24, 24) \times (0, 44)$. We choose $z = 4$, $w = 3$ and thus divide the domain into 12 windows. According to the discretization technique, we obtain $dim(M_l^k) = 81$ for $l = 1, 2, 3, 4$ and $k = 1, 2, 3$. The number of variables is then 486 per window. After optimization with Matlab environment, we obtain in figure (4(a)) an absolute error inferior to 0.1%, wich is very satisfying. Since we study the case of 3 tramlines and because of the symmetry properties previously explained, we choose to present only the parameters for the outward path which is between the two other trajectories. By observing the optimal parameters in figure (4(b)), we can notice some oscillations for the medium radius and the medium angle. This phenomenon can be explained if we make reference to the natural circular movement which is achieved when trying to uniformly paint a surface with a circular brush. Furthermore, the mass flow rate has a mean closed to 20 Kg/min wich is the value found by applying the relation $m = (Q^* \times W \times S)/600$. So, the computed values for the other parameters are coherent with the actual ones determined by the manufacturers settings.

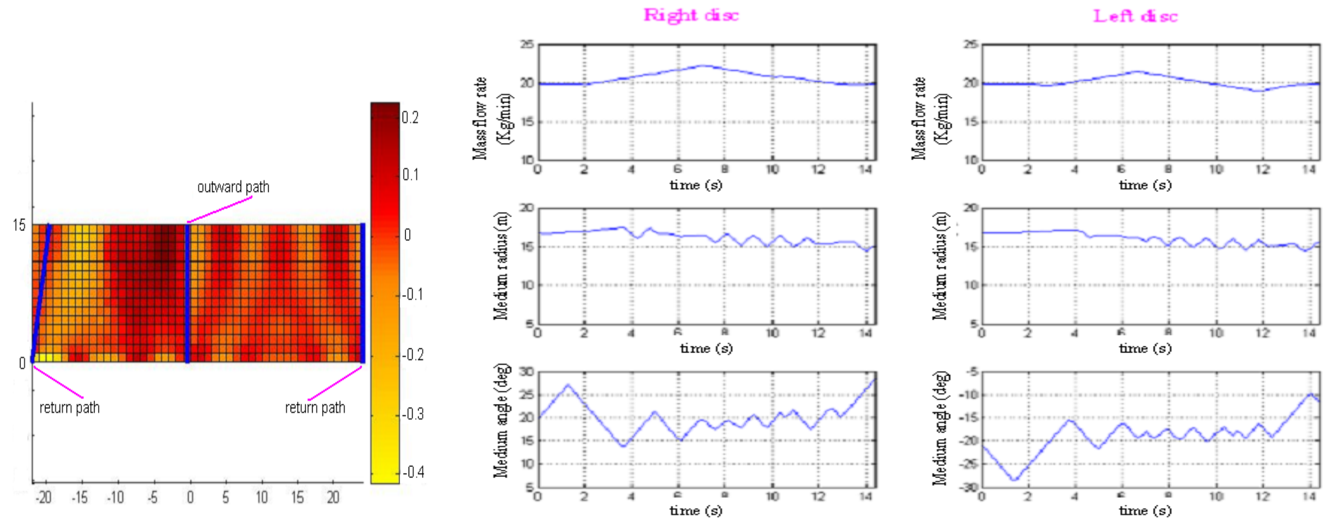
For the second case, $\Omega = (-22, 24) \times (0, 15)$ and 3 windows are considered $(w = 3, z = 1)$. By using the same discretization techniques, $v_l^k = 79$ for $l = 1$ and $k = 1, 2, 3$. The total number of variables per window raises then 474. The non parallel tramline has an angle equivalent to 81 degrees which is current in fields which boundaries are not orthogonal between them. The error after optimization and the optimal parameters for the outward path are represented by Figure (5). In this case, the absolute value of the error is slightly superior to the one mentioned above but is all the same inferior to 0.5%. Because of the narrowing occuring on the left of the outward path, we can notice a little increase in the mass flow rate while the other variables seem not to be affected by this geometrical singularity. This result suggests then that the direction shift is not large enough to cause a medium radius and angle adjustment.

The results shown in Figure 4(b) and 5(b) respecting the mechanical constraints of the machine, the set of computed parameters should be stored in the embedded computer and used as reference variables for a future control of the spreader. Therefore, the speeds of discs and granulars dropping points values could be continuously adjusted during spreading process as recommended in (Olieslagers *et coll.*, 1997) and maintain a correct distribution.



(a) Error in %   (b) Optimal parameters for the outward path

Figure 4: Error and optimal parameters for parallel tramlines, CPU=100h.



(a) Error in % (b) Optimal parameters for the outward path

Figure 5: Error and optimal parameters for non parallel tramlines, CPU=26h.

V. CONCLUSION

Because of environmental and economic pressures, an optimization method has been developed for minimizing application errors due to centrifugal spreading. Considering the great size of problem, we have divided the global spatial domain into subdomains with rectangular geometry to deal with each trajectory individually and to reduce the number of decision variables. Faced with a problem subject to inequality constraints, an augmented lagrangian algorithm combined with a l-bfgs technique has been implemented. The solutions have been computed for parallel and non parallel tramlines and permit to obtain an error inferior to 1% in either case. From these results, we could deduce the values of the disc speeds and fertilizers dropping points during time and consequently carry out the control of the applicator. This work suggests a study is needed about spreading on the boundaries in order to improve application accuracy in the whole arable land. This future study should enable to limit distributed amounts outside the field and reduce waste of fertilizers.

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