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► **To cite this version:**

Jean-Claude Bermond, Jean-Luc Fouquet, Michel Habib, Bernard Peroche. On linear k -arboricity. Discrete Mathematics, 1984, 52 (2-3), pp.123-132. 10.1016/0012-365X(84)90075-X . hal-02437859

HAL Id: hal-02437859

<https://inria.hal.science/hal-02437859>

Submitted on 13 Jan 2020

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ON LINEAR k -ARBORICITY

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Let us call a linear- k -forest a graph whose connected components are chains of length at most k . The linear- k -arboricity of G (denoted $la_k(G)$) is the minimum number of linear k -forests which partition $E(G)$. We study this new index in two cases: cubic graphs and complete graphs ($k = 2$ or 3).

1. Introduction

In general, we follow the graph-theoretical terminology of [3]. A *linear- k -forest* of an undirected graph G is a subgraph of G whose connected components are chains of length at most k . We define the *linear- k -arboricity* of G (denoted $la_k(G)$) as the minimum number of linear- k -forests needed to partition the edge set $E(G)$ of G .

This notion is a natural refinement of the linear-arboricity introduced by Harary [8] and studied subsequently by Akiyama, Exoo and Harary in [1].

The linear- k -arboricity has been defined by Habib and Peroche in [6], where the two authors have studied (and quite completely solved) the case of trees.

We clearly have the following inequalities (where $n = |V(G)|$):

$$q(G) = la_1(G) \geq la_2(G) \geq \dots \geq la_{n-1}(G) = la(G)$$

(where $q(G)$ is the classical chromatic index of G , and $la(G)$ is the linear arboricity).

We know by Vizing's theorem [17] that $la_1(G)$ is less than $\Delta(G) + 1$ (where $\Delta(G)$ is the maximum degree of G). This result can be improved for $k \geq 2$.

Theorem 1. Let G be a graph of maximum degree $\Delta(G)$, then for any k ($k \geq 2$), we have $la_k(G) \leq \Delta(G)$.

Proof. As noticed before, it is sufficient to show that $la_2(G)$ is less than or equal to $\Delta(G)$. We shall use an induction on $\Delta(G)$.

(1) $\Delta(G) \leq 2$. Obvious.

(2) $\Delta(G) \geq 3$. We can assume that G is regular (otherwise we can construct a graph G' regular of degree $\Delta(G)$ such that G is a subgraph of G' . In that case we clearly have $la_2(G) \leq la_2(G')$).

By using a well-known theorem of Tutte [16] (see the paper of Thomassen [15] for a very short proof of that result), G contains an almost 2-regular spanning subgraph G_0 ($\forall x \in V(G), d_{G_0}(x) = 1$ or 2). G_0 contains thus a spanning linear-2-forest F . The subgraph of G induced by $E(G) - F$ is thus a graph G' with maximum degree $\Delta(G) - 1$. By the induction hypothesis, we have $la_2(G') \leq \Delta(G) - 1$ and thus $la_2(G) \leq \Delta(G)$. \square

On the other hand, we have in an obvious way

Proposition 2.

$$la_i(G) \geq \max \left\{ \left\lceil \frac{\Delta(G)}{2} \right\rceil, \left\lceil \frac{m}{\lfloor i \cdot n / (i+1) \rfloor} \right\rceil \right\},$$

with $n = |V(G)|$ and $m = |E(G)|$.

NP-completeness. The determination of $q(G)$ was shown to be an NP-complete problem by Holyer [10]; similarly, Peroche [12] has proved the NP-completeness of the determination of $la(G)$. As a by-product, we obtain in this paper the NP-completeness of the determination of $la_3(G)$, using a result of Schaefer [14], and we conjecture that the determination of $la_i(G)$ is NP-complete for every integer.

Our aim will be now to study that index for two classes of graphs: the cubic graphs and the complete graphs.

2. Cubic graphs

Using Theorem 1 and the inequality of Proposition 2 for a cubic graph we have $la_2(G) = 3$. For the case $k = 3$, we need now some definitions.

Definition. C is a *strong matching* in a graph G , if C is a matching such that there is no edge of $E(G)$ between any two edges of C . (In other words, C is an independent set of $L^2(G)$ where $L^2(G)$ is the square of the line graph $L(G)$ of G .)

Theorem 3. Let G be a cubic graph, then the following propositions are equivalent:

- (i) $la_3(G) = 2$.
- (ii) There is a partition of $E(G)$ into two linear forests F_1 and F_2 , such that each chain of F_i ($i = 1, 2$) has length exactly 3.
- (iii) G contains a perfect matching L , and two disjoint strong-matchings C_1 and C_2 with the same order such that $C_1 \cup C_2 = L$.

Proof. (ii) \Rightarrow (i) is obvious.

(i) \Rightarrow (ii) Let F_1 and F_2 be two disjoint linear-3-forests such that $E(G) = F_1 \cup F_2$. Each vertex x of $V(G)$ being of degree 3, it is end vertex exactly once in some chain of F_1 (or F_2) and internal vertex exactly once in another chain of F_2 (respectively F_1). Therefore if $V(G) = 2s$, there are exactly s chains in $F_1 \cup F_2$, and as $|E(G)| = |F_1 \cup F_2| = 3s$, each chain is exactly of length 3.

(ii) \Rightarrow (iii) Let $C(F_i)$ ($i = 1, 2$) be the set of all middle edges of the chains of F_i . It is easy to verify that $C(F_i)$ is a strong matching.

By the proof of (i) \Rightarrow (ii) each vertex is exactly internal vertex in one chain, therefore $L = C(F_1) \cup C(F_2)$ is a perfect matching. Furthermore as the number of end vertices in a chain is the same as the number of internal vertices $|C(F_1)| = |C(F_2)|$.

(iii) \Rightarrow (ii) Let $L = C_1 \cup C_2$.

$G - L$ is a two factor of G consisting of even cycles (C_1 and C_2 are strong matchings). Let us give an arbitrary orientation to the cycles of $G - L$. To each vertex x of $V(G)$, we can associate an edge $e(x)$ of $E(G) - L$, such that x is the origin of $e(x)$ with respect to the chosen orientation of the cycle through x .

For $i = 1, 2$, let

$$F_i = \bigcup_{[x,y] \in C_i} P[x,y],$$

where $P_{[x,y]} = e(x) \cup [x,y] \cup e(y)$.

We obtain thus two linear-3-forests which partition $E(G)$, each chain being of length exactly 3. \square

Corollary 4. Let G be a cubic graph such that $la_3(G) = 2$. Then $|V(G)| \equiv 0 \pmod{4}$.

Proof. G contains a perfect matching L which is the disjoint union of two strong matchings C_1 and C_2 with the same cardinality. So $|V(G)| = 2|L| = 2|C_1 \cup C_2| = 4|C_1|$. \square

For each graph with $|V(G)| \equiv 2 \pmod{4}$ we thus have $la_3(G) = 3$. But we can have $la_3(G) = 3$ for graphs with $|V(G)| \equiv 0 \pmod{4}$ (see Fig. 1).

We now exhibit some subfamilies of cubic graphs satisfying the hypothesis of Theorem 3.

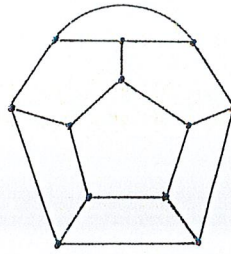


Fig. 1.

Following Jaeger [11] we shall say that G has a *square* (G^2) which is 4-chromatic if the chromatic number of G^2 is 4.

Corollary 5. *If G is a cubic graph whose square is 4-chromatic then $la_3(G) = 2$.*

Proof. Let $\{A_1, A_2, A_3, A_4\}$ be a 4-colouring of G^2 , and let us define in G

$$C_1 = \{[x, y] \mid x \in A_1, y \in A_2\}, \quad C_2 = \{[x, y] \mid x \in A_3, y \in A_4\}.$$

$C_1 \cup C_2$ is trivially a perfect matching of G . Let x be a vertex of G , a, b and c its neighbours. Let us suppose that $x \in A_1$. Then, since the square of G has a chromatic number equal to 4, each vertex a, b and c belongs to an A_i distinct from A_1 and no two of them have the same colour. So if $[x, a]$ belongs to C_1 , b and c are ends of edges of C_2 . C_1 (respectively C_2) is thus a strong matching.

By Theorem 3 the corollary is proved. \square

We shall say that a cubic planar graph is a *multi- k -gon* ($3 \leq k \leq 5$) if all of its faces have a length multiple of k .

Corollary 6. *If G is a multi- k -gon ($k = 3, 4$) then $la_3(G) = 2$.*

Proof. In [11] Jaeger has shown that those graphs have a square which are 4-chromatic. \square

The determination of $la_3(G)$ for cubic graphs with $|V(G)| \equiv 0 \pmod{4}$ is a difficult problem. Let us denote by (P) the following problem:

Instance: a cubic graph G .

Question: $la_3(G) = 2$?

Corollary 7. (P) is an NP complete problem.

Proof. In [13], Schaefer proved that the following problem (Q) is NP complete:

Instance: $G = (X, E)$.

Question: Does there exist a perfect matching L and two disjoint strong matchings C_1 and C_2 such that $L = C_1 \cup C_2$?

The NP completeness of (P) is then an easy consequence of the equivalence of (P) and (Q) for cubic graphs and the previous result of Schaefer. \square

Theorem 8. *If G is a multi-5-gon then $la_4(G) = 2$.*

Proof. In [5] it is proved that those graphs have a five strong edge-colouring (i.e., an edge-colouring of G with five colours, each colour being a strong matching). This can be done in using the following method:

(a) Let us choose a face F_0 and colour its edges with the colours 1, 2, 3, 4 and 5 in that cyclic order.

(b) The colour of any edge is thus determined uniquely if we follow the two rules R_1 and R_2 :

(R_1) Each colour must be a strong matching.

(R_2) If we run along the boundary of any face of G , we must encounter the five colours (1, 2, 3, 4, 5) in a cyclic permutation.

It is thus sufficient to extend the strong edge colouring from faces to faces. This peculiar colouring induces a lot of combinatorial properties of a multi-5-gon [5].

In our problem, we deduce from this colouring that the faces bearing the cyclic permutations (1, 2, 3, 4, 5) and (1, 5, 4, 3, 2) are separated by left-right cycles bearing the cyclic permutation (1, 4, 2, 5, 3). If in the induced two-factor we delete the edges coloured with 1, we get a linear-4-forest F_1 . It is easy to verify that the remaining edges constitute a linear-3-forest F_2 . \square

That property is illustrated on the dodecahedron (Fig. 2).

A natural question arising from linear- k -arboricity is the following:

Question. What is the least integer $i(G)$ for which $la(G) = la_i(G)$?

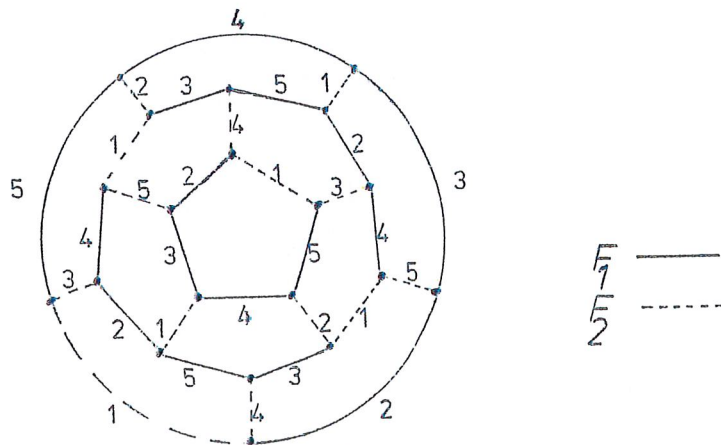


Fig. 2.

In the particular case of cubic graphs we do not know any graph G for which $i(G) \geq 6$. It can be conjectured:

Conjecture. G cubic $\Rightarrow la_5(G) = 2$.

3. Complete graphs

In this section, we shall determine the value of $la_2(K_n)$ when $n \not\equiv -1 \pmod{12}$ and $n \not\equiv -2 \pmod{12}$. More precisely, we have:

Theorem 9. For $n \not\equiv -1 \pmod{12}$ and $n \not\equiv -2 \pmod{12}$,

$$la_2(K_n) = \left\lceil \frac{n(n-1)}{2 \lfloor \frac{2}{3}n \rfloor} \right\rceil.$$

Proof. Let us call

$$l_2(n) = \left\lceil \frac{n(n-1)}{2 \lfloor \frac{2}{3}n \rfloor} \right\rceil.$$

As $l_2(n)$ is the lower bound of Proposition 2, we have only to exhibit $l_2(n)$ linear-2-forests partitioning the edges of K_n to prove the theorem.

We shall distinguish between seven cases.

Case 1. $n = 6t + 3$, $t \geq 0$. We shall use here the following result of Ray-Chaudhuri and Wilson [13]: K_n has a resolvable decomposition in K_3 if and only if $n \equiv 3 \pmod{6}$. Two subcases must be considered.

(a) $n = 12t + 9$, $t \geq 0$. In that case the problem is exactly the problem of the existence of a resolvable balanced P_3 -design (see [9, Conjecture 2]). The solution below is due to Wilson. K_n is decomposed in $(6t + 4)$ parallel classes made with $(4t + 3)$ triangles. Let us consider two distinct such classes; with these classes we can build a bipartite graph as follows: to each triangle we associate a vertex and we create an edge if and only if the two triangles have a common vertex. The bipartite graph is cubic and so have a perfect matching M . Let us consider an edge of M . The corresponding triangles of G have exactly one common vertex, and so the edges of the two triangles can be partitioned in three chains of length 2. As M is a perfect matching, the two parallel classes give rise to three linear-2-forests.

With this construction, we obtain $3 \cdot (6t + 4)/2$ linear 2-forests, and so we have the result.

(b) $n = 12t + 3$, $t \geq 0$. In this case, K_n is decomposed in $(6t + 1)$ parallel classes. By taking two by two $6t$ of these classes, as in subcase (a), we obtain $9t$ linear 2-forests. The remaining class gives two linear 2-forests and so we have our assertion.

Case 2. $n = 6t$, $t \geq 1$. We shall start here with the following result of Baker and

Wilson [2]: if F is a perfect matching of K_n , $K_n - F$ has a resolvable decomposition in K_3 if and only if $n \equiv 0 \pmod{6}$ and $n \geq 18$.

(a) If $n = 12t + 6$, $t \geq 1$, $K_n - F$ is decomposed in $(6t + 2)$ parallel classes. So, with the same method as in Case 1, these parallel classes give rise to $3(3t + 1)$ linear 2-forests. As F is also a linear 2-forest, we can conclude.

(b) $n = 12t$, $t \geq 2$. in this subcase, $K_n - F$ is decomposed in $(6t - 1)$ parallel classes. $(6t - 2)$ of them give rise to $3(3t - 1)$ linear 2-forests; the remaining class and F are partitioned by 3 linear 2-forests, so we have the result.

(c) $n = 6$. We can take the following linear 2-forests;

$$f_1 = \{[1, 3], [0, 5], [2, 4]\}, \quad f_2 = \{[4, 0, 1], [3, 5, 2]\},$$

$$f_3 = \{[0, 3, 2], [5, 1, 4]\}, \quad f_4 = \{[0, 2, 1], [5, 4, 3]\},$$

(d) $n = 12$. $la_2(K_{12}) = 9$, as proved by

$$f_1 = \{[0, 11], [1, 6], [10, 2, 7], [3, 8], [4, 9]\},$$

$$f_2 = \{[0, 1, 2], [10, 3, 9], [11, 4, 8], [5, 7, 6]\},$$

$$f_3 = \{[0, 2, 3], [1, 4, 10], [11, 5, 9], [6, 8, 7]\},$$

$$f_4 = \{[0, 3, 4], [2, 5, 1], [11, 6, 10], [7, 9, 8]\},$$

$$f_5 = \{[0, 4, 5], [3, 6, 2], [11, 7, 1], [8, 10, 9]\},$$

$$f_6 = \{[0, 5, 6], [4, 7, 3], [11, 8, 2], [9, 1, 10]\},$$

$$f_7 = \{[6, 0, 8], [9, 11, 2], [1, 3, 5], [7, 10]\},$$

$$f_8 = \{[7, 0, 9], [10, 11, 3], [2, 4, 6], [5, 8]\},$$

$$f_9 = \{[6, 9, 2], [11, 1, 8], [0, 10, 5]\}.$$

Case 3. $n = 6t + 2$, $l_2(6t + 2) = \frac{9}{2}t + 2$. But $l_2(6t + 3) = \frac{9}{2}t + 2$, so the partition in linear 2-forests obtained in Case 1 gives the result by deleting a vertex.

Case 4. $n = 12t + 7$, $l_2(12t + 7) = 9t + 6$ and, as $l_2(12t + 8) = 9t + 6$, the result of Case 3 gives a solution.

Case 5. $n = 12t + 5$, $l_2(12t + 5) = l_2(12t + 6) = 9t + 4$ so the result is a consequence of Case 2.

Case 6. $n = 12t + 4$, $l_2(12t + 4) = 9t + 3$. Let us denote $\{0, 1, \dots, 12t + 3\}$ the set vertex of K_n . K_n can be decomposed in $(6t + 2)$ hamiltonian chains C_i , $0 \leq i \leq 6t + 1$, with $C_0 = (0, 12t + 3, 1, 12t + 2, \dots, j, 12t + 3 - j, \dots, 6t + 1, 6t + 2)$, $0 \leq j \leq 6t + 1$, and where the vertices of C_i are obtained by adding i (modulo $12t + 4$) to the vertices of C_0 .

Let us consider the chains C_i and C_{3t+1+i} , for $0 \leq i \leq 3t$. With these two chains, we shall build 3 linear 2-forests in the following way (we just describe the case $i = 0$).

With C_0 , we construct a linear 2-forest F_0 , with

$$F_0 = \bigcup_{j=0}^{t-1} (P_{3j} \cup P'_{3j}) \cup \left(\bigcup_{j=0}^{t-1} P_{3j+3t+2} \cup P'_{3j+3t+2} \right) \cup (3t, 9t + 3) \cup (3t + 1, 9t + 2),$$

where

$$P_\alpha = (\alpha, 12t+3-\alpha, \alpha+1), \quad P'_\alpha = (12t+2-\alpha, \alpha+2, 12t+1-\alpha).$$

The edges of $C_0 - F_0$ are

$$\begin{aligned} a_j &= (3j+1, 12t+2-3j), \\ b_j &= (3j+3, 12t+1-3j), \quad 0 \leq j \leq t-1, \\ c_j &= (9t-3j+2, 3j+3t+2), \\ d_j &= (3t+3j+3, 9t-3j) \quad \text{and} \quad (3t+1, 9t+3). \end{aligned}$$

From C_{3t+1} , we obtain a linear 2-forest F_{3t+1} by adding $(3t+1)$ to the vertices of F_0 . The edges of $C_{3t+1} - F_{3t+1}$ are

$$\begin{aligned} a'_j &= (3t+3j+2, 3t-3j-1), \\ b'_j &= (3t+3j+4, 3t-3j-2), \quad 0 \leq j \leq t-1, \\ c'_j &= (6t+3j+3, 12t+3-3j), \\ d'_j &= (6t+3j+4, 12t+1-3j) \quad \text{and} \quad (0, 6t+2). \end{aligned}$$

The edges of $C_0 - F_0$ and $C_{3t+1} - F_{3t+1}$ form a linear 2-forest: we can concatenate

$$\begin{aligned} b_j \text{ and } d'_j &\text{ by the vertex } 12t+1-3j, \\ c_j \text{ and } a'_j &\text{ by the vertex } 3t+3j+2, \\ d_j \text{ and } c'_j &\text{ because } \{6t+3j+3/0 \leq j \leq t-1\} \equiv \{9t-3j/0 \leq j \leq t-1\}, \\ a_j \text{ and } b'_j &\text{ because } \{3j+1/0 \leq j \leq t-1\} \equiv \{3t-3j-2/0 \leq j \leq t-1\}, \end{aligned}$$

and we obtain a linear 2-forest having $(8t+2)$ edges.

Case 7. $n = 12t+1$, $t \geq 1$. $l_2(12t+1) = 9t+1$.

Let us denote $\{\infty, 0, 1, \dots, 12t-1\}$ the vertices of K_n . It is well known that K_n can be decomposed by $6t$ hamiltonian cycles H_i , $0 \leq i \leq 6t-1$, with

$$\begin{aligned} H_i &= (\infty, i, 12t-1+i, i+1, 12t-2+i, \dots, j, 12t-1+i-j, \dots, 6t+i \\ &\quad -1, 6t+i, \infty) \quad (\text{with } i \leq j \leq 6t+i-1). \end{aligned}$$

With the cycles H_i and H_{3t+i} , we can build three linear 2-forests, but two edges cannot be used. All the edges left will form together a new linear 2-forest, which will prove the result.

From H_0 , we build the linear 2-forest

$$F_0 = \bigcup_{j=0}^{2t-1} (P_{3j} \cup P'_{3j}),$$

with

$$P_\alpha = (\alpha, 12t-1-\alpha, \alpha+1), \quad P'_\alpha = (12t-2-\alpha, \alpha+2, 12t-3-\alpha).$$

$H_0 - F_0$ contains the edges

$$\begin{aligned} a_j &= (3j+1, 12t-2-3j), \quad 0 \leq j \leq 2t-1, \\ b_j &= (3j+3, 12t-3-3j), \quad 0 \leq j \leq 2t-2, \\ &(\infty, 0) \quad \text{and} \quad (\infty, 6t). \end{aligned}$$

From H_{3t} , we build the linear 2-forest

$$\begin{aligned} F_{3t} &= \left(\bigcup_{j=0}^{t-1} (Q_{6t+1+3j} \cup Q'_{3t-1+3j}) \right) \cup (9t, \infty, 3t) \\ &\quad \cup \left(\bigcup_{j=0}^{t-2} Q'_{6t+1+3j} \cup Q_{3t+2+3j} \right) \cup (0, 6t-1) \cup (6t, 12t-1), \end{aligned}$$

where $Q_\alpha = (\alpha, 6t-1-\alpha, \alpha+1)$ and $Q'_\alpha = (6t-2-\alpha, \alpha+2, 6t-3-\alpha)$.

The edges of $H_{3t} - F_{3t}$ are

$$\begin{aligned} c_j &= (6t+1+3j, 12t-1-3j), \\ d_j &= (6t+2+3j, 12t-3-3j), \\ e_j &= (3j+2, 6t-3-3j), \quad 0 \leq j \leq t-1, \\ f_j &= (3j+1, 6t-1-3j), \end{aligned}$$

and $(0, 6t)$.

With all the edges of $H_0 - F_0$ and of $H_{3t} - F_{3t}$, but $(3t, 9t)$ and $(0, 6t)$, we can build a linear 2-forest, as follows:

$$\begin{aligned} &(0, \infty, 6t), \quad (3t-1, 3t), \quad (9t-1, 9t), \\ &b_j \cup d_j, \quad 0 \leq j \leq t-2, \\ &b_{j+t} \cup e_{t-2-j}, \quad 0 \leq j \leq t-2, \\ &a_j \cup f_j, \quad 0 \leq j \leq t-1, \\ &a_{j+t} \cup c_{t-1-j}, \quad 0 \leq j \leq t-1. \end{aligned}$$

We do the same thing for the cycles H_i and H_{3t+i} , for $1 \leq i \leq 3t-1$. This method gives $3 \cdot 3t = 9t$ linear 2-forests. The remaining edges $(i, 6t+i)$ and $(3t+i, 9t+i)$, $0 \leq i \leq 3t-1$ make a perfect matching and thus a linear 2-forest so we obtained $9t+1 = l_2(12t+1)$ linear 2-forests. \square

Remarks. (a) We note that if we have proved that

$$la_2(K_n) = \left\lceil \frac{n-(n-1)}{2 \lfloor \frac{2}{3}n \rfloor} \right\rceil$$

for $n \equiv -1 \pmod{12}$, then we would have obtained immediately the same equality for $n \equiv -2 \pmod{12}$.

So only one case remains to be studied, if we want to establish Theorem 8 for

all n . But, till now, none of the techniques we have used in the proof of our theorem has succeeded.

(b) Lastly let us recall the following conjecture of two of us (Habib and Peroche [7]),

$$la_i(G) = \begin{cases} \left\lceil \Delta(G) \cdot (n+1)/2 \cdot \left\lfloor \frac{in}{i+1} \right\rfloor \right\rceil & \text{if } \Delta(G) \neq n-1, \\ \left\lceil \Delta(G)/2 \cdot \left\lfloor \frac{in}{i+1} \right\rfloor \right\rceil & \text{if } \Delta(G) = n-1. \end{cases}$$

As we have seen here this conjecture is true for cubic graphs and $i = 2, 3$. In [6] it is proved for trees, and [6, Theorem 3] proved it for complete graphs ($n \not\equiv -1 \pmod{12}$ and $\not\equiv -2 \pmod{12}$) and $i = 2$. Using techniques of resolvable designs, we can get a lot of results for complete graphs. For instance, we have

$$la_3(K_{12\rho+4}) = 8\rho + 2.$$

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