



**HAL**  
open science

## On deciding stability of high frequency amplifiers

Laurent Baratchart, Sébastien Fueyo, Gilles Lebeau, Jean-Baptiste Pomet

► **To cite this version:**

Laurent Baratchart, Sébastien Fueyo, Gilles Lebeau, Jean-Baptiste Pomet. On deciding stability of high frequency amplifiers. IFAC 2019 - 15th IFAC Workshop on Time Delay Systems, Sep 2019, Sinaia, Romania. hal-02437112v2

**HAL Id: hal-02437112**

**<https://inria.hal.science/hal-02437112v2>**

Submitted on 8 Apr 2020

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

# On deciding stability of high frequency amplifiers

L. Baratchart, S. Fueyo, G. Lebeau, J.-B. Pomet

September 10, 2019  
*15th IFAC Workshop on Time Delay Systems*

The logo for Inria, consisting of the word "Inria" written in a red, cursive script font.

## Motivation

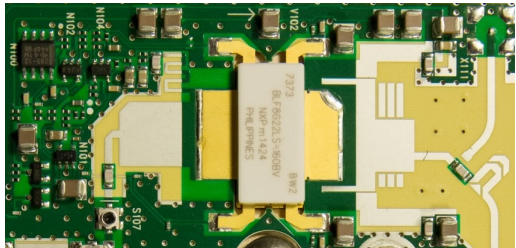
- Amplifiers at high frequency are ubiquitous (Cell phones, relays...). They need to be quick to design and produced in large quantities.
- Computer-assisted design (CAD) and simulation before production.
- Powerful “frequency simulation” tools give a reliable prediction of the response, but that response might be unstable.
- **Need** for a tool to predict stability/unstability in the frequency domain.

# Inside

An amplifier is made of interconnected

- resistors, inductors, capacitors,
- diodes/transistors,
- lossless transmission lines which cannot be neglected at high frequency inducing delays.

Forcing periodic signal ►► periodic solution in the amplifier ►► amplified signal.



# Harmonic Balance

The Harmonic Balance method, through Fourier development, Laplace transform and fixed point methods permits to :

- approximate the periodic solution of the circuit,
- linearize the circuit around the periodic solution,
- give a frequency response to a periodic signal which disturbs the linearized circuit.

Our focus : Structure of the harmonic transfer function, its singularities, links with stability.

# Summary

- 1 Equations and stability
- 2 Harmonic transfer function
- 3 conclusion

General system,  $T$ -periodic :

$$\left\{ \begin{array}{l} \frac{dx(t)}{dt} = A_1(t)x(t) + \sum_{i=0}^N B_{1,i}(t)y(t - \tau_i) \\ y(t) = \sum_{i=1}^N B_{2,i}(t)y(t - \tau_i) + A_2(t)x(t), \quad t \geq s, \end{array} \right.$$

- $L^2 := \mathbb{R}^n \times L^2([-\tau_N, 0], \mathbb{R}^k)$ .
- Solution operator  $U(t, s) : L^2 \rightarrow L^2$
- Monodromy operator  $U(T, 0)$
- $L^2$  exponential stability  $\} \Leftrightarrow \left\{ \begin{array}{l} Sp(U(T, 0)) \text{ included in} \\ \text{disc of radius } r < 1 \end{array} \right.$

General system,  $T$ -periodic :

$$\left\{ \begin{array}{l} \frac{dx(t)}{dt} = A_1(t)x(t) + \sum_{i=0}^N B_{1,i}(t)y(t - \tau_i) \\ y(t) = \sum_{i=1}^N B_{2,i}(t)y(t - \tau_i) + A_2(t)x(t), \quad t \geq s, \end{array} \right.$$

- $L^2 := \mathbb{R}^n \times L^2([- \tau_N, 0], \mathbb{R}^k)$ .
- Solution operator  $U(t, s) : L^2 \rightarrow L^2$
- Monodromy operator  $U(T, 0)$
- $\left. \begin{array}{l} L^2 \text{ exponential} \\ \text{stability} \end{array} \right\} \Leftrightarrow \left\{ \begin{array}{l} Sp(U(T, 0)) \text{ included in} \\ \text{disc of radius } r < 1 \end{array} \right.$



## Behaviour at high frequency

High frequency system :

$$\begin{cases} x(t) = 0 \\ y(t) = \sum_{i=1}^N B_{2,i}(t)y(t - \tau_i), \quad t \geq s, \end{cases}$$

- $\tilde{L}^2 := \{0_n\} \times L^2([-\tau_N, 0], \mathbb{R}^k)$ .
- Solution operator  $V(t, s) : \tilde{L}^2 \rightarrow \tilde{L}^2$ .
- Monodromy operator  $V(T, 0)$ .

- 

$$\left. \begin{array}{l} L^2 \text{ exponential} \\ \text{stability} \end{array} \right\} \Leftrightarrow \left\{ \begin{array}{l} Sp(V(T, 0)) \text{ included in} \\ \text{disc of radius } r < 1 \end{array} \right.$$

## Compact perturbation

### Lemma

*We have :*

$$U(t, s) = V(t, s)P + K(t, s), \quad t \geq s$$

*with  $K(t, s)$  compact operator  $L^2 \rightarrow L^2$  for all  $t, s$  and  $P$  the canonical projection  $L^2 \rightarrow \tilde{L}^2$ .*

### Theorem

*If the high frequency system is exponentially stable then the monodromy operator  $U(T, 0)$  possesses at most a finite number of eigenvalues  $\zeta_1, \dots, \zeta_n$  outside a disk of a radius strictly less than 1.*

## $L^2$ stability equivalent to $C^0$ stability

Proposition (Chitour and al 2016, BFLP 2019)

*A periodic delay system is  $L^2$  exponentially stable if and only if it is  $C^0$  exponentially stable.*

Theorem

*Assuming the system at high frequency is  $L^2$  exponentially stable. Then the general system is  $L^2$  exponentially stable if and only if it is  $C^0$  exponentially stable.*

**Assumption : High frequency system exponentially stable**  
 **$\Leftarrow$  always true for "realistic" circuit.**

## Input-Output system

$$\left\{ \begin{array}{l} \frac{dx(t)}{dt} = A_1(t)x(t) + \sum_{i=0}^N B_i^1(t)y(t - \tau_i) + C_1(t)u(t) \\ y(t) = \sum_{i=1}^N B_i^2(t)y(t - \tau_i) + A_2(t)x(t) + C_2(t)u(t) \\ z(t) = \sum_{i=1}^N B_i^3(t)y(t - \tau_i) + A_3(t)x(t) + C_3(t)u(t), \quad t \geq 0, \end{array} \right.$$

- $x(t), y(t), z(t) = 0$  for  $t < 0$ ,
- Input  $u \in L_{loc}^2([0, +\infty), \mathbb{R})$  current perturbation, output  $z$  the voltage,
- All coefficients are  $T$ -periodic.

- $X(t, \alpha)$  response at time  $t$  to an impulse at time  $\alpha$   
 $z(t) = \int_0^t X(t, \alpha) u(\alpha) d\alpha$
- $G(s, t) = \int_0^{+\infty} X(t, t - \alpha) e^{-s\alpha} d\alpha$  : Laplace Transform
- $G_k(s) = \frac{1}{T} \int_0^T G(s, t) e^{ik\omega_0 t} dt$  with  $\omega_0 := \frac{2\pi}{T}$

### Definition (Harmonic Transfer Function HTF)

The infinite matrix  $H(s)$  defined by  $H_{m,n}(s) := G_{n-m}(s + \frac{2i\pi m}{T})$  for  $s \in \mathbb{C}$  is called the harmonic transfer function.

$$Z(s) := \int_0^{+\infty} z(t)e^{-st} dt \text{ and } U(s) := \int_0^{+\infty} u(t)e^{-st} dt.$$

$$\begin{pmatrix} \vdots \\ Z(s+i\omega_0) \\ Z(s) \\ Z(s-i\omega_0) \\ \vdots \end{pmatrix} = \begin{pmatrix} \ddots & \vdots & \vdots & \vdots & \ddots \\ \cdots & G_2(s-i\omega_0) & G_1(s) & G_0(s+i\omega_0) & \cdots \\ \cdots & G_1(s-i\omega_0) & G_0(s) & G_{-1}(s+i\omega_0) & \cdots \\ \cdots & G_0(s-i\omega_0) & G_{-1}(s) & G_{-2}(s+i\omega_0) & \cdots \\ \ddots & \vdots & \vdots & \vdots & \ddots \end{pmatrix} \begin{pmatrix} \vdots \\ U(s+i\omega_0) \\ U(s) \\ U(s-i\omega_0) \\ \vdots \end{pmatrix}$$

- HTF is an operator valued analytic map  
 (values: continuous ops  $l^2(\mathbb{Z}) \rightarrow l^2(\mathbb{Z})$ )
- its entries  $\{G_n\}$  are complex valued analytic maps

# Structure of the Harmonic Transfer Function

Define  $z_{j,k} = \frac{\ln(\zeta_j) + 2ik\pi}{T}$  for  $j$  in  $\{1\dots n\}$ ,  $k$  in  $\mathbb{Z}$ .

## Theorem

In  $\{s \in \mathbb{C}, \Re(s) \geq \gamma\}$  for some  $\gamma < 0$ ,

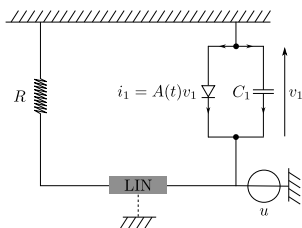
- $H$  is a meromorphic operator  $l^2(\mathbb{Z}) \rightarrow l^2(\mathbb{Z})$  with possibly poles at  $\{z_{j,k}, j \in \{1\dots n\}, k \in \mathbb{Z}\}$ .

Under observability/controllability assumptions,

- for all  $j$ , there is at least a  $k$  such that  $z_{j,k}$  is a pole of  $H$ , and also a pole of one  $G_n$ .

If no **pole** in right half plane, exponential  $C^0$  stability.

## Example : Brayton 1976



### Theorem

*If  $T/\tau_1 \notin \mathbb{Q}$ , all points of a certain vertical line in the left-half plane are essential singularities of the HTF, as an operator valued analytic map.*

But are they singularities for some  $G_n$  ?



## Contribution

- Math. foundation of HF amplifiers stability decision in CAD
- Projection on the unstable part and rational approximation to find the poles
- Advances in stability of periodic delay systems

## Open questions

- For fixed  $j$ , which  $z_{j,k}$  is a pole of which  $G_n$ ?  
(In practice, few  $G_n$  are computed.)
- Bound on the number of unstable poles?
- May the (stable) singularities of the  $G_n$ 's be other than poles ?