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# Learning to represent students' mathematical ideas through teacher time-outs in rehearsals

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## Abstract

*This study examines rehearsals embedded in cycles of enactment and investigation, and more specifically, the discussions the participants engage in when instruction is paused – Teacher Time-Outs (TTOs). The paper studies the ambitious teaching practices that are revealed in the discussions, as well as the aspects of these practices that the participants have the opportunity to learn. Approximately 60% of the rehearsal time was spent on teaching and 40% on TTOs. The use of representations was a key aspect in 28% of these discussions. It is particularly interesting that the discussions in the rehearsals make it possible for the participants to learn how to represent students' mathematical ideas in writing on the board while at the same time paying close attention to connections between student talk and mathematical correctness. Implications from these findings are also discussed.*

*Keywords: Mathematics teaching, rehearsals, teacher time-outs, professional development*

## Introduction

The aim of professional development (research) is to support in-service teachers (ISTs) in developing ambitious teaching practices (e.g. Lampert, Beasley, Ghouseini, Kazemi, & Franke, 2010; McDonald, Kazemi, & Kavanagh, 2013). Ambitious teaching entails mathematical meaning making, identity building and creating equitable access to learning experiences for all students. It requires that teachers know their students as learners and engage deeply with their thinking. Ambitious teaching practices enhance student learning of complex ideas and performances (Lampert et al., 2013, Ghouseini et al., 2015). Examples of ambitious practices are aiming toward goals, eliciting and responding to students' mathematical ideas and using representations (Lampert et al., 2010) (see more examples in Table 1).

Recently, teacher education and professional development (PD) have had more focus on teaching practices and pedagogies of enactment (cf. Grossman et al., 2009; Lambert et al., 2013; Gibbons, Kazemi, & Lewis, 2017). The focus of this paper is on *rehearsals*. These researchers emerging research indicates that the use of rehearsals contributes positively to the development of ISTs' ambitious teaching practices (Gibbons, Kazemi, Hintz, & Hartmann, 2017; Kazemi, Ghouseini, Cunard, & Turrou, 2016; Lampert et al., 2013; Valenta & Wæge, 2017), as well as to the development of mathematical knowledge for teaching (Ghouseini, 2017). Previous research highlights the need for more research on rehearsals (Kazemi et al., 2016; Lampert et al., 2013).

We report on findings from the PD project *Mastering ambitious mathematics teaching* (MAM). In the MAM project, ISTs are invited to collaborate in learning cycles of enactment and investigation where the overarching aim is to learn to enact the practices that constitute ambitious mathematics teaching. Rehearsals are an important element of these cycles, and we especially examine the use of teacher time-outs (TTOs) during them. The ISTs work on instructional activities (IAs) in the rehearsals. IAs are developed to highlight and support ISTs' learning of specific ambitious teaching practices and to focus on specific mathematical ideas. The IAs in the MAM project include choral counting, quick images, number strings, problem solving and games (cf. Lampert et al., 2010). The ISTs learn to teach IAs through cycles of enactment and investigation.

Lampert et al. (2013) explore what teacher educators and novice teachers do together during rehearsals so they can “learn to enact the principles, practices, and knowledge entailed in ambitious teaching” (Lampert et al., 2013, p. 226). They suggest that rehearsals provide room for novice teachers to open up their instructional decisions to one another. Kazemi et al. (2016, p. 16) suggest that rehearsals, as nested in learning cycles, provide repeated opportunities for novice teachers “to investigate, reflect on, and enact teaching through coached feedback”. Furthermore, rehearsals bring the student teachers “into interaction with one another around instructional decision making that is responsive to students' contributions”, enabling them to “come in close contact with how their colleagues make sense of and take up the practices that are being learned” (Kazemi et al., 2016, p. 28). In the context of PD, Valenta and Wæge (2017) found that the interactions between the ISTs and the teacher educator during rehearsals were mainly discussions on using mathematical representations, using a mathematical goal, paying attention to student thinking and eliciting and responding to students' mathematical ideas – all important principles and practices of ambitious mathematics teaching. They suggest that rehearsals offer ISTs “the environment and opportunity to work simultaneously on a variety of aspects of practice” (Valenta & Wæge, 2017, p. 3387).

The organisational TTO procedure has been highlighted as a promising way for ISTs to learn together in and through practice (Gibbons, Kazemi, Hintz et al., 2017). The participants can pause the instruction to think out loud together and share decision-making with one another before continuing with the enactment. For example, Fauskanger and Bjuland's (in press) analyses suggest that TTOs enable ISTs to develop a better understanding of how to elicit students' mathematical ideas by, for example, practising questions to ask. Research indicates that TTOs provide opportunities for ISTs to make changes and to shift the focus in the interactions “from one of judgment and evaluation to one of collective consideration and opportunistic experimentation in the midst of teaching mathematics” (Gibbons, Kazemi, Hintz et al., 2017, p. 29). Gibbons, Kazemi, Hintz et al. (2017, p. 48) hypothesize that TTOs support the ISTs' “ability to be adept at moment-to-moment decision-making to engage students in rich discussions” and “to cultivate learning environments where everyone [is] positioned as capable of doing substantive mathematics” at the same time as ISTs become good at “at drawing on students' multiple knowledge bases.” These researchers point out, for instance, that future research should attempt to understand more about how TTOs might support ISTs in developing their instructional practices. In this paper, we address the following research questions: (a) Which (if any) of the practices entailed in ambitious mathematics teaching are the focus of attention in the TTOs taken during rehearsals? and (b)

Bearing the TTOs in mind, which aspects of the practice of using representations do the ISTs have opportunities to learn to enact? The second research question is the main focus of this paper.

## **Methodology**

### **Design, data material and participants**

The ISTs learn to teach instructional activities (IAs) through cycles of enactment and investigation. Each cycle involves the following six steps:

1. The ISTs read an article and watch a video of a teacher's enactment of a particular IA.
2. The supervisor leads a discussion/analysis of the article and the observed video.
3. Groups of ISTs prepare to teach the IA to a group of students, guided by the supervisor.
4. One IST (the rehearsing IST) enacts the IA in a rehearsal, where the supervisor and the other ISTs act as students. During this rehearsal, all participants can ask for TTOs.
5. The IST enacts the activity with a group of students. All participants can ask for TTOs.
6. Each group of ISTs analyse the enactment, guided by the supervisor. This is followed by a guided analysis of the enactment with all the ISTs.

Thirty ISTs from 10 elementary schools participated in the project. The ISTs were divided into four groups. One group consisted of those teachers who did not want to take part in the research, whereas two out of the three remaining groups (groups 2 and 3) were randomly chosen to be part of the research reported on in this paper. The ISTs' teaching experience varies from one to 30 years. The ISTs participated in 12 sessions, which included nine cycles of enactment and investigation, over the course of two years. All the sessions were videotaped where one researcher observed each group and took notes. The data material from groups 2 and 3 consists of video recordings of the rehearsals (step 4 above) from the nine cycles of enactment, resulting in 18 recorded rehearsals.

### **Analytical approach**

A sociocultural view of ISTs' learning frames this research as the ISTs work together and learn ambitious teaching practices in a community of practice. We analysed the video recordings of the rehearsals, looking specifically for opportunities for ISTs' learning. First, all the TTOs in the rehearsals were identified according to the following definition of a TTO: The instances where the rehearsal is explicitly paused (often with a TTO hand signal) so the participants can better understand and/or act in relation to their colleagues' thinking, pedagogical choices and/or mathematical content (cf. Gibbons, Kazemi, Hintz et al., 2017). Second, we undertook a conventional content analysis (Hsieh & Shannon, 2005) of each of the TTO episodes, i.e. we used an inductive coding process with the aim of identifying aspects of ambitious teaching practices in the TTOs and which of these the ISTs have opportunities to discuss.

## **Findings**

In a previous study, we found that approximately 60% of the rehearsal time was spent on ISTs teaching the IAs and 40% on TTOs, showing the back-and-forth pattern between facilitating deliberate practice and posing questions (Wæge & Fauskanger, submitted). Moreover, we identified 175 TTOs and found five common dimensions of ambitious teaching practice that were addressed during them (Wæge & Fauskanger, submitted). An overview of these practices is given in Table 1.

Note that many of the TTOs involved simultaneous work on multiple aspects of practice, thus many of the TTOs involved more than one category.

Ambitious teaching practice	Example	Number of TTOs
Setting up a task, i.e. the TTOs consisted of questions and discussions on the practice of launching a problem.	The rehearsing IST has just posed a problem when one of the observing ISTs asks for a TTO: “There’s something I’m wondering about. Justification. Should she [the rehearsing IST] say something about that [invite the students to justify while presenting the task]?” (session eight, problem solving, group 3)	42
Using a goal to work towards, i.e. discussions on the practice of directing students’ attention on a learning goal for the lesson.	The rehearsing IST has just written the student solution “three times ten minus one” on the board: $3 \times 10 - 1$ . The supervisor intervenes by saying: “Now, the question about [the use of] parentheses becomes interesting [referring to the expression $3 \times 10 - 1$ ].” (session three, quick image, group 2)	35
Supporting mathematical discussions, e.g. using talk moves (cf. Kazemi & Hintz, 2014).	The rehearsing IST gives the ISTs (acting as students) some time to think about the problem 400 - 379, and then asks them to share their solutions when one of the observing ISTs asks: “Should we ask them [the students] to turn to each other and talk or should we let them think individually first?” (session six, strings, group 2)	23
Using representations, i.e. representing students’ mathematical ideas in writing (on the board), making connections between student talk and representations, and making connections between different kinds of representations (Lambert et al., 2010, Lambert et al., 2013).	The rehearsing IST, representing a student strategy on the board, pauses and asks: “Do I write in between (points on the number line)?” (session five, strings, group 3)	49
Organising the board, i.e. organisation on the smartboard and utilising space.	The rehearsing IST is representing a student solution on the open number line when the supervisor intervenes by saying: “I’m thinking of another thing right now and that is where to put the number line and the tasks.” (session six, strings, group 2)	27

**Table 1: An overview of the content of the 175 analysed TTOs (Wæge & Fauskanger, submitted)**

With Wæge and Fauskanger’s (in review) study as our point of departure, in this paper we focus on one dimension, namely the “use of representations”. Table 1 shows that this dimension is an important feature in many of the TTOs (shaded in Table 1).

### Use of representations

Using representations (Lampert et al., 2010) is one of the key practices in ambitious teaching. Representing students’ mathematical ideas in writing on the board, making connections between student talk and written representations, and making connections between different kinds of representations, such as the open number line, arrays and tables are important features in 28% of the TTOs. In many of these TTOs the participants discussed how to write student ideas or strategies on the board so they could represent the students’ thinking and at the same time keep mathematical

correctness in focus. For instance, they discussed the use of parentheses when representing students' ideas, the use of arrows instead of the equal sign and the degree to which they should focus on the convention related to the order of factors (in Norway, i.e.  $3 \times 4$  normally represents three groups of four). The importance of recording exactly what the students said was emphasised in several of these TTOs, however, this sometimes led to mathematical incorrectness and thus TTOs were requested.

To illustrate the TTOs in which the participants discussed how to represent the students' ideas or written representations, we provide an example from a rehearsal of a game where number sentences<sup>1</sup> presented by the students were written on the board. In this game, the students worked on finding number sentences using given numbers. Just prior to the coded exchange, the rehearsing teacher represented what one student said by writing the following two number sentences (all supposed to equal the given number 8):  $5 + 3 = 8$  and  $9 - 5 = 4 \times 3 = 12 : 6 = 2 \times 4 = 8$ . We join the rehearsal as one of the observing teachers intervenes:

- 1 OT1<sup>2</sup>: Then the question is/ (interrupted by OT2)  
 2 OT2: Then the question about [correct] notation is relevant.  
 3 S: Yes, and it can't be [written] like that (pointing to  $9 - 5 = 4 \times 3 = 12 : 6 = 2 \times 4 = 8$ )  
 4 RT: Ok, it can't be written like this? (...) <sup>3</sup> (session nine, game, group 3)

The use of the equal sign in the number sentence is not mathematically correct, and the observing ISTs suggested taking a closer look at the use of notation in the number sentences (Lines 1, 2). The supervisor supported their suggestions, pointing out that they needed to find another way to write the number sentences (Line 3). The use of arrows was introduced in the MAM project, but these ISTs do not appear to be used to writing an arrow instead of the equal sign when recording students' thinking. As the discussion continued, some of the ISTs suggested that the number sentence could be written the way it was because that was the way the student presented it. However, at the end of this TTO the participants concluded that they would replace the equal sign with arrows so the number sentences would be mathematically correct:  $9 - 5 \rightarrow 4 \times 3 \rightarrow 12 : 6 \rightarrow 2 \times 4 = 8$ .

A second representative example is taken from another group rehearsing the same IA, but now all the number sentences were to equal 6. The following was written on the board:  $5 + 1$ ,  $8 - 5 + 2 + 1$ ,  $10 - 8 + 2 + 5 + 1$  when one of the observing ISTs asked for a TTO (referring to  $10 - 8 + 2 + 5 + 1$ ):

- 1 OT1: Should we ask them [the students] what they are thinking as they do this?  
 2 RT: Yes.  
 3 OT1: Ask them to explain their thinking. (session nine, game, group 2)

In what followed, the participants discussed what to do when they wrote exactly what the students were saying and it turned out to be mathematically incorrect. After arguing back and forth whether to write exactly what the students were saying or to represent their ideas in a mathematically correct



<sup>1</sup> In their planning, the ISTs used the term equations. When analysing the rehearsals, however, we find that sometimes the "equations" are written as equality strings or false equations (e.g.  $9 - 5 = 4 \times 3 = 12 : 6 = 2 \times 4 = 8$ ) or without an equal sign (e.g.  $9 - 5$ ). For this reason we use the term "number sentence" (cf. Ghouseini, 2017).

<sup>2</sup> In the transcripts, OT represents one of the observing ISTs, RT is the rehearsing teacher and S is the supervisor.

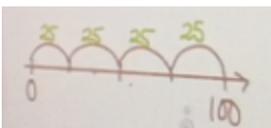
<sup>3</sup> (...) indicates that the TTO continues.

way, they agreed to use “circles” (e.g.  $10 - 8 + 2 + 5 + 1$ ). The rehearsal continued, but similar examples were presented by ISTs acting as students (e.g.  $20 - 8 = 12 + 3 = 15$ ) when the supervisor requested a TTO to ask the ISTs what they would do if these examples were presented by students in the enactment phase. In the ensuing discussion, some of the ISTs pointed out the importance of writing exactly what the students were saying, whereas others pointed to the importance of writing what was mathematically correct. This group did not reach a conclusion on this matter before continuing the rehearsal.

The next example, taken from a rehearsal of a string, shows a discussion on how to represent students’ ideas in writing. The first problem in the string was to focus on a run where four students were going to run equally long where the total distance was one hundred metres. One of the observing ISTs (acting as student) said that he knew that “they have to run 25 metres each” because “four times 25 equals one hundred”. When representing this on an open number line on the board, the rehearsing IST started by dividing a line segment into two and then by dividing each of these parts into two again thus representing  $(100 : 2) : 2$  instead of  $4 \times 25$ . The supervisor asked for a TTO:

- 1 Supervisor:           What is interesting here is that you have thought, you have explained that you were thinking four times 25 (referring to the observing IST who presented this idea). You just knew that, right. But, when you (referring to the rehearsing IST) were making the number line now, what did she do, do you recognise that [what she did represent] (looking at all the observing ISTs)?
- 2 OT1:                    She divided into two and then divided into two again.
- 3 S:                        Yes, you halved and then halved again.
- 4 RT:                     Ok.
- 5 S:                        It’s important that we are aware of this [that this represents  $(100 : 2) : 2$  and not  $4 \times 25$ ]. (...) (session six, strings, group 2)

The supervisor wanted the rehearsing IST to represent exactly what the student was saying ( $4 \times 25$ , Lines 1, 5) and she invited the observing ISTs to recognise this (Line 1). One of the observing ISTs recognised that the rehearsing IST had represented  $(100 : 2) : 2$  instead of  $4 \times 25$  (Line 2). As the discussion continued, the rehearsing IST asked if she should present the strategy  $(100 : 2) : 2$ . This was followed by a short discussion on the pros and cons relating to ISTs presenting their own ideas. The supervisor advised the rehearsing IST to represent  $4 \times 25$  on the number line as “25 and 25 and 25 and 25” (showing the four “jumps” by pointing to the number line). They agreed that the rehearsing IST in the enactment should illustrate  $4 \times 25$  as four jumps of the length 25 and in addition writing 25 above each arch as illustrated in Figure 1.



**Figure 1:  $4 \times 25$  represented on a number line**

These three examples illustrate that the TTOs in the rehearsals could support the ISTs’ learning of the practice of representing students’ ideas or strategies and their paying close attention to

connections between student talk and written representations. The ISTs could also learn more about the balance between representing students' mathematical ideas or strategies as presented by the students and representing these ideas or strategies in a way that is mathematically correct.

## **Concluding discussion**

In this study we have identified and analysed ambitious teaching practices that are revealed in TTOs in rehearsals, exploring which practices the ISTs have opportunities to learn to enact during the TTOs. The findings show that ISTs have opportunities to work on and learn to enact the practices of setting up a task, using goals to work towards, supporting mathematical discussions, organising the board and using representations. Opportunities for learning these practices have also been found in previous studies (cf. Kazemi et al., 2016; Lampert et al., 2013; Valenta & Wæge, 2017), which have also found opportunities in TTOs in the enactment phase of cycles of enactment and investigation (cf. Fauskanger, 2019; Gibbons, Kazemi, Hintz et al., 2017). In this paper, we have focused on the practice of using representations. The findings show that TTOs especially have the potential to create opportunities for ISTs to learn how to present students' mathematical ideas in writing and also show the importance of representing student ideas as accurately as possible while also focusing on the mathematical correctness of the representation. Balancing the dilemma between student talk and mathematical correctness has not previously been found in research on TTOs in rehearsals and is thus a unique finding from this study. Our findings also indicate that the ISTs might need more mathematical knowledge for teaching (MKT) and that the TTOs can be a point of departure for strengthening ISTs' MKT (cf. Ghouseini, 2017).

Previous research on TTOs in enactment (Gibbons, Kazemi, Hintz et al., 2017), as well as in rehearsals (Lampert et al., 2013), challenges future research to elaborate on how TTOs as a procedure in PD can be sustained and developed. By using the data material from the MAM project, it is possible to study the development of TTOs in each group for a two-year period. Further analyses of the data material will also make it possible to learn more about whether and how the supervisors develop their TTO expertise (cf. Gibbons, Kazemi, Hintz et al., 2017). Similar projects in new contexts are also welcome in order to study the potential of using TTOs in new contexts. In the Norwegian context the new guidelines for initial teacher education highlight ambitious teaching practices (cf. Mosvold, Fauskanger, & Wæge, 2018) and the number of days in field practice in schools has been extended. Based on the promising opportunities for ISTs to learn ambitious teaching practices through TTOs in the enactment of teaching (Fauskanger, 2019), studying TTOs in student teachers' field practice will be an important context for future TTO research.

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