Time stepping scheme for interactive stiff simulation with changes in topology

$$\mathbb{M}(\mathbf{q})\dot{\mathbf{v}} = \mathbb{P}(t) - \mathbb{F}\left(\mathbf{q}, \mathbf{v}\right) + \mathbf{H}^T\lambda$$

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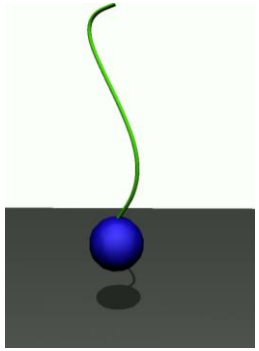

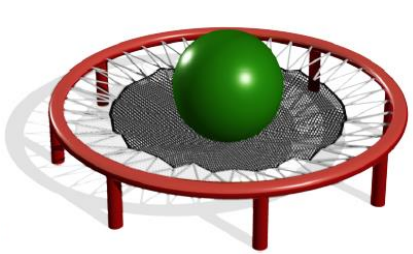

Background: stiff systems with hard/soft constraints

[Stable constraint dynamics (Tournier, Nesme, Gilles, Faure)]

PR#459 : [https://github.com/sofa-framework/sofa/pull/459]

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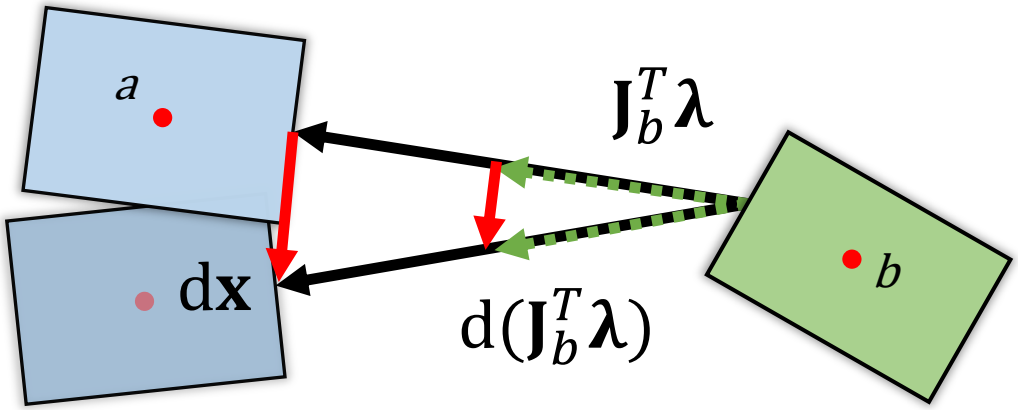
Geometric stiffness

- Stiffness which arises from the non linearity of constraint equations
- Tensor which model change in constraint direction



$$\tilde{\mathbf{K}} = \frac{\partial \mathbf{J}^T}{\partial \mathbf{x}}\,\boldsymbol{\lambda}$$

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Current implementation in FreeMotionAnimationLoop

$ApplyDJT(\lambda)$ → $\widetilde{K}$

$OdeSolve: q, v \rightarrow q_{free}, v_{free}$ → $v_{free} = A^{-1}b + v$ $q_{free} = q + hv_{free}$

$Project(v_{free})$

$ApplyJ(v_{free})$

$BuildConstraintSystem$ → J^T $JA^{-1}J^T$

$ConstraintSolve: q_{free}, v_{free} \rightarrow q^+, v^+$ → $dv = A^{-1}J^T\lambda$

$Project(q^+, v^+)$

$Apply(q^+)\ ApplyJ(v^+)$

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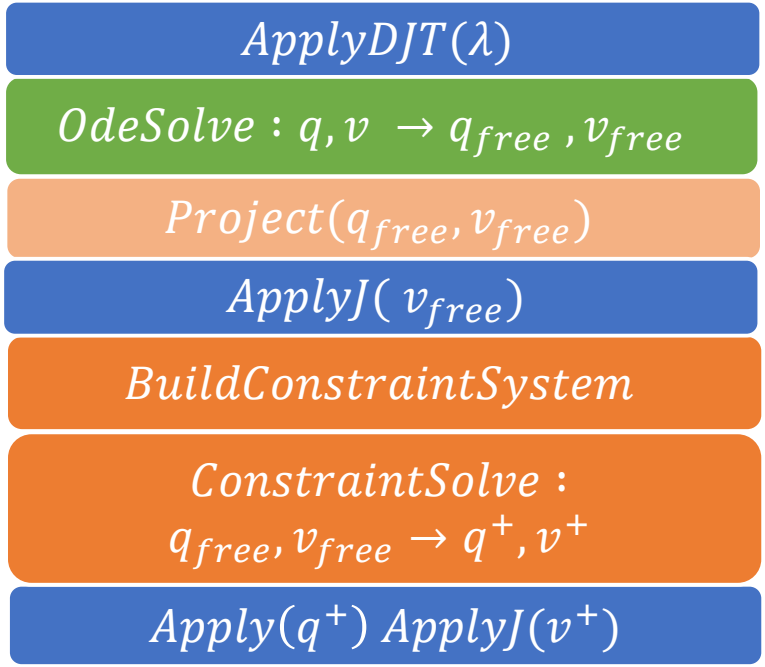
Modified FreeMotionAnimationLoop (at InSimo)

Apply projective “impulses” only on the free motion state vectors to remain compatible with the constraint formulation.
(For example to clamp the velocity of objects to a fixed upper bound)



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Introducing changes in the topology

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Possible implementation : change the topology between time steps

- Modify the topology, go through topology mappings
- Interpolate kinematic quantities (rest position, current position, velocity) : not necessarily “valid” for mapped dofs unless the mapping is linear
- Go through mechanical mappings to account for non linear relationship between independent dofs and mapped dofs generalized position and velocity
- Start a new time step

$$TopologyChange(q, v)$$

$$Apply(q_{rest})\ Apply(q)\ ApplyJ(v)$$

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Implementation at InSimo : Change the topology during the time stepping scheme

$ApplyDJT(\lambda)$

$OdeSolve : q, v \rightarrow q_{free}, v_{free}$

$Project\ (q_{free}, v_{free})$

$ApplyJ(v_{free})$

$BuildConstraintSystem$

$ConstraintSolve :$
$q_{free}, v_{free} \rightarrow q^+, v^+$

$TopologyChange\ (q^+, v^+)$

$Apply(q_{rest})$
$Apply(q^+)\ ApplyJ(v^+)$

Reduce the number of mapping traversals, which can be expensive for non linear mappings
(e.g. computing a continuous normal field and its derivative of an object boundary)

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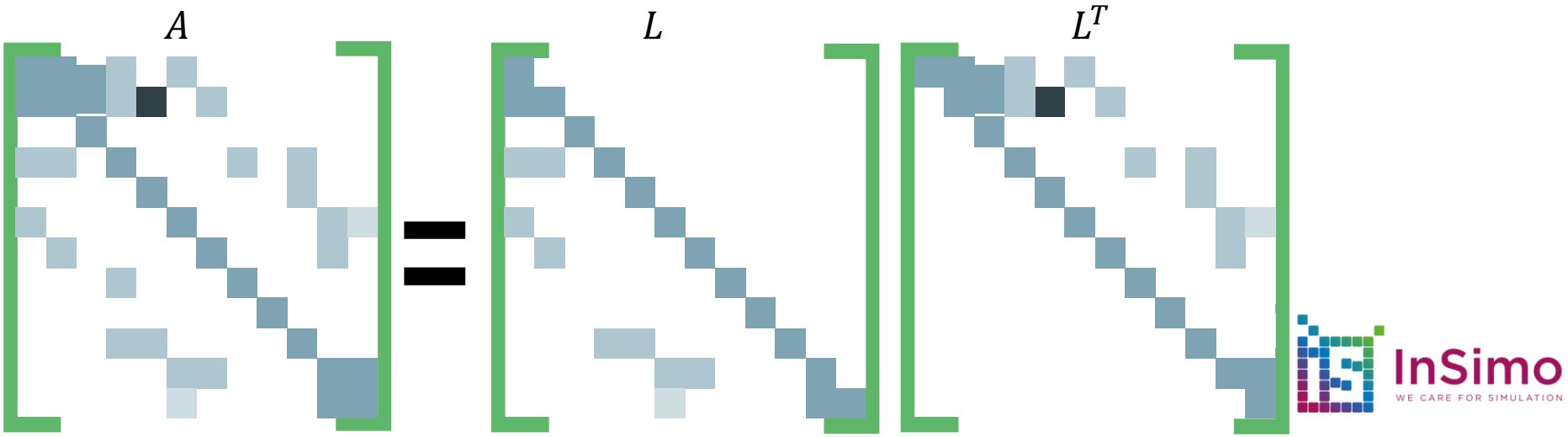
Direct solver for bilateral constraints

Idea: augment existing Cholesky factorization

- Robust treatment of bilateral constraints not really possible without using a (prohibitively) large number of iterations in Projected Gauss Seidel



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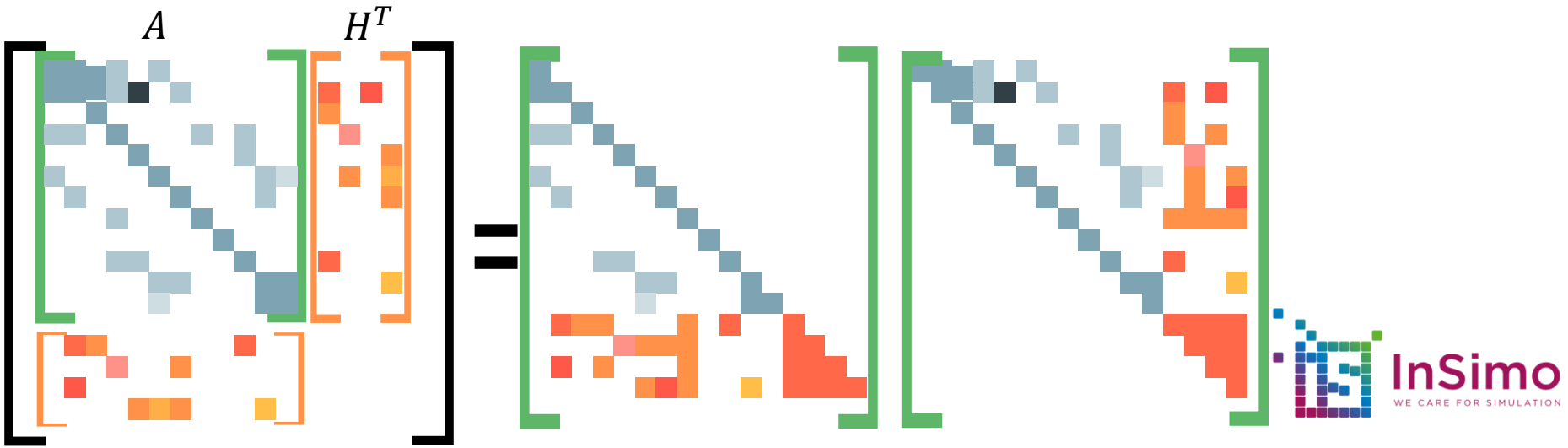
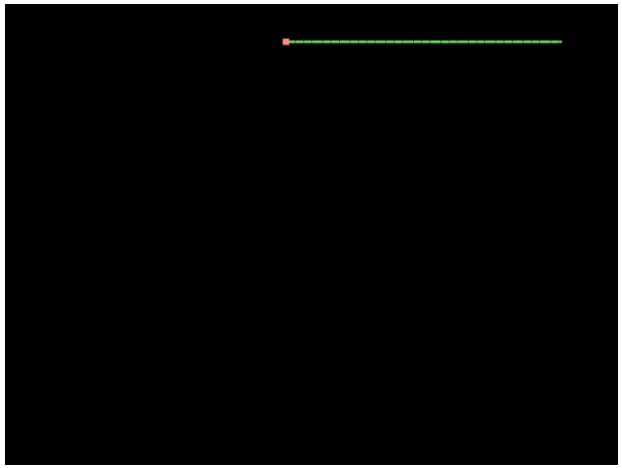
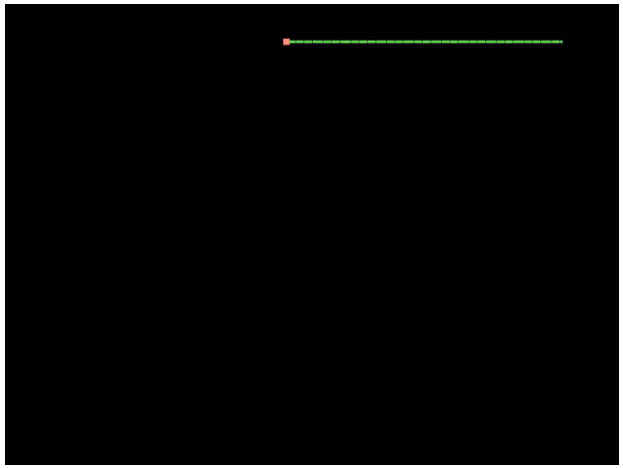


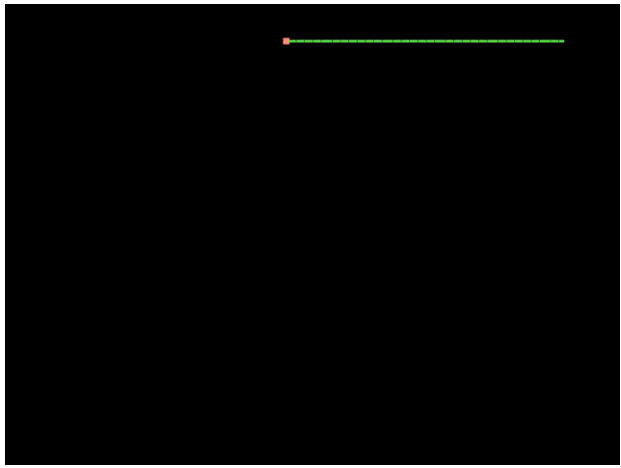
GS 100 iterations



GS 200 iterations



Bilateral factorized



20 cm rod | mass 1,5g | 100 dofs | timestep 0,01 ms

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Questions ?


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Soft constraints formulation, geometric stiffness

- Formulation where the unknown is the velocity increment

$$\begin{bmatrix} M - h^2(K + \widetilde{K}) & J^\top \\ J & \frac{1}{h^2}C \end{bmatrix} \begin{bmatrix} dv \\ -\mu^+ \end{bmatrix} = \begin{bmatrix} hf - h^2(K + \widetilde{K})v \\ \frac{-\phi}{h} - Jv \end{bmatrix}$$

- Formulation where the unknown is the new velocity

$$\begin{bmatrix} M - h^2(K + \widetilde{K}) & J^\top \\ J & \frac{1}{h^2}C \end{bmatrix} \begin{bmatrix} v^+ \\ -\mu^+ \end{bmatrix} = \begin{bmatrix} Mv + hf \\ \frac{-\phi}{h} \end{bmatrix}$$

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Soft constraints formulation, geometric stiffness

- Compute geometric stiffness using previous constraint forces
- Solve to find unconstrained velocities (free motion)

$$\begin{cases} A = M - h^2(K + \tilde{K}) \\ b = hf - h^2(K + \tilde{K})v \\ v_{free} = A^{-1}b + v \end{cases}$$

- Assemble and solve reduced system to find constraint forces

$$(JA^{-1}J^{\top} + \frac{1}{h^2}C)\mu^{+} = \frac{-1}{h}\phi - J(v + v_{free})$$

- Inject back constraint forces to find corrective motion

$$v^{+} = v_{free} + A^{-1}\mu^{+}$$

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Soft constraint : spring damper

$$\mathbf{C}\lambda^{+} - hJv^{+} - \mathbf{k}\phi = 0$$

$\boldsymbol{C}$: damping factor
- softW
- CFM (Bullet, ODE)

$\boldsymbol{k}$: position error factor
- Erp (ODE, Bullet)
- Baumgarte stabilisation

$$\ddot{x} + 2\zeta\omega_0\dot{x} + \omega_0^2 x = 0$$

Erin Catto (Box2D) Soft Constraints GDC 2011

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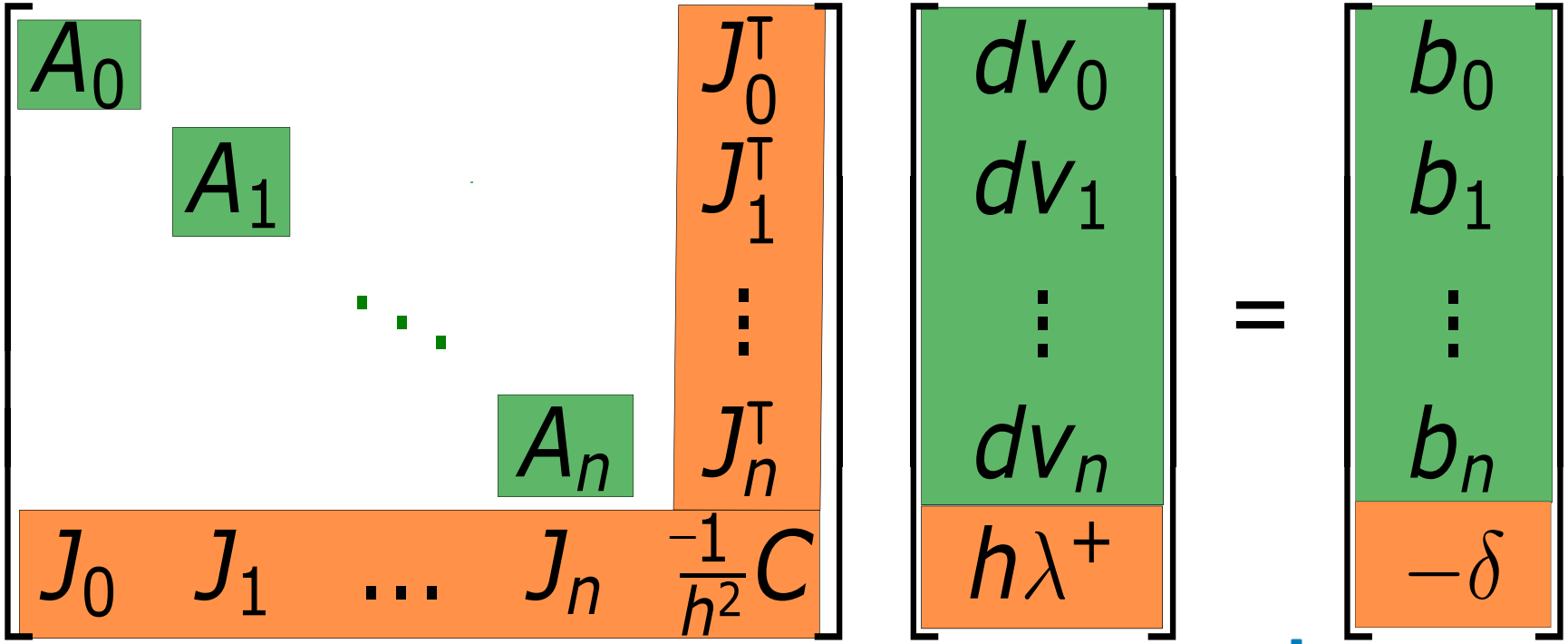

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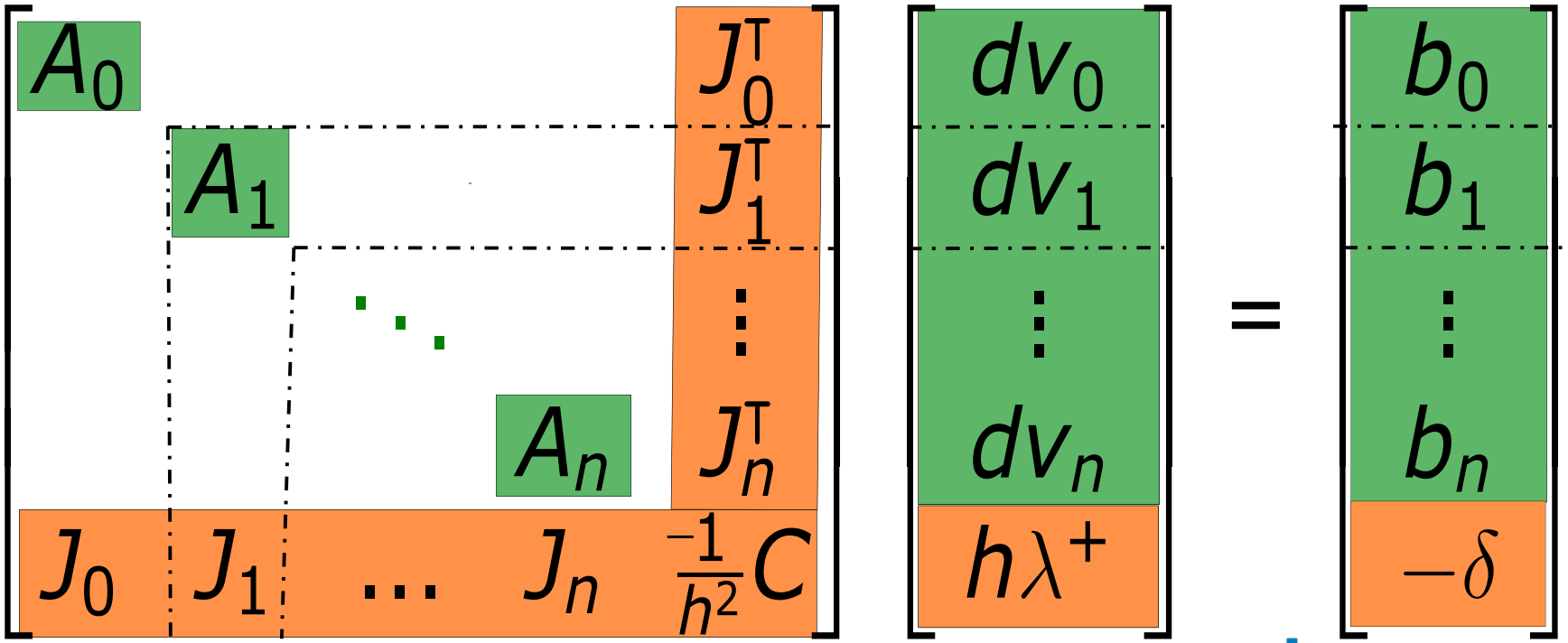

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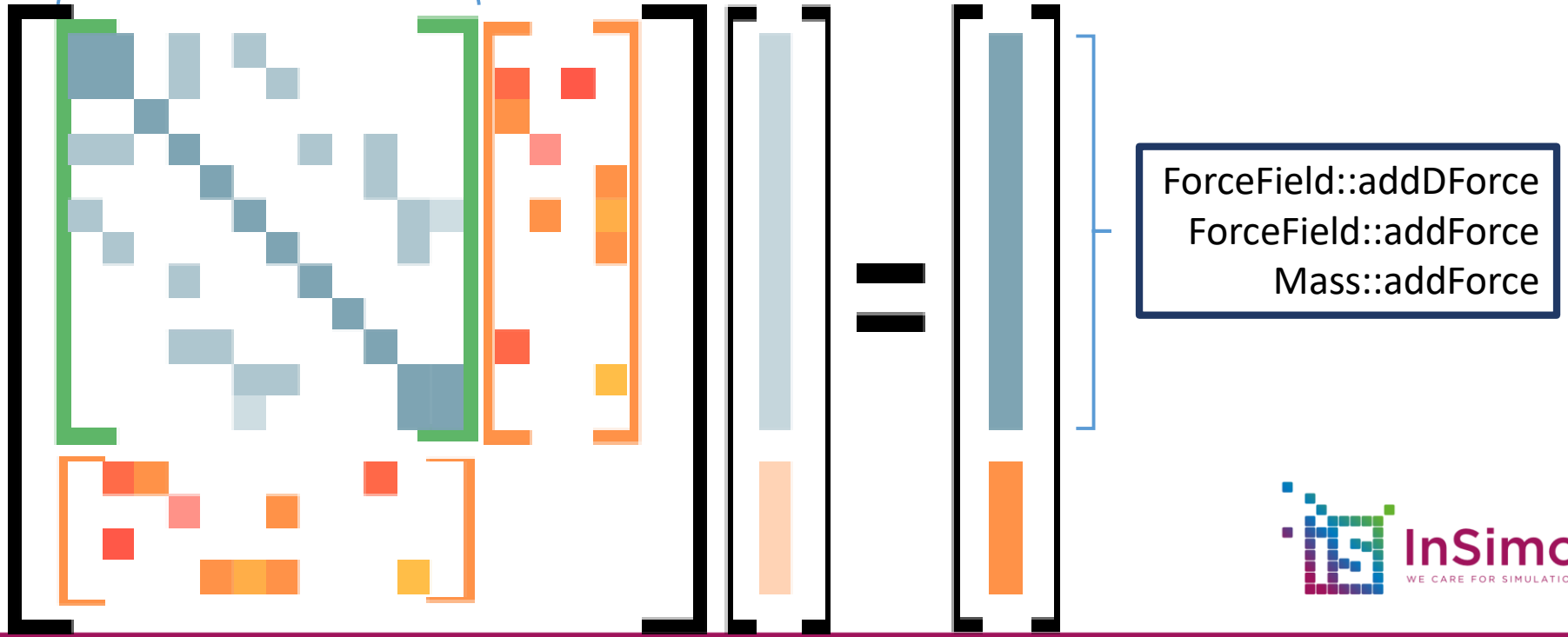
ForceField::addKtoMatrix
Mass::addMToMatrix



ForceField::addDForce
ForceField::addForce
Mass::addForce

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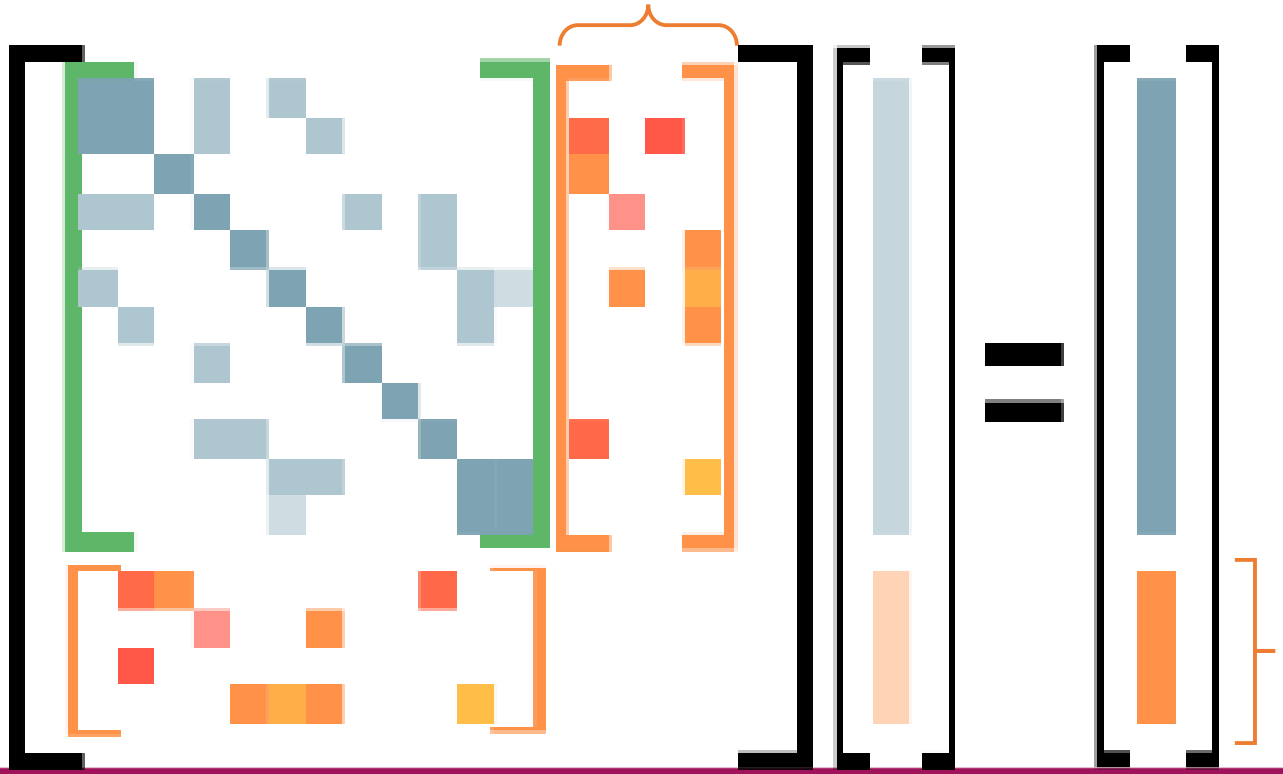
Constraint::buildConstraintMatrix
Mapping::applyJT



Constraint::
getConstraintViolation

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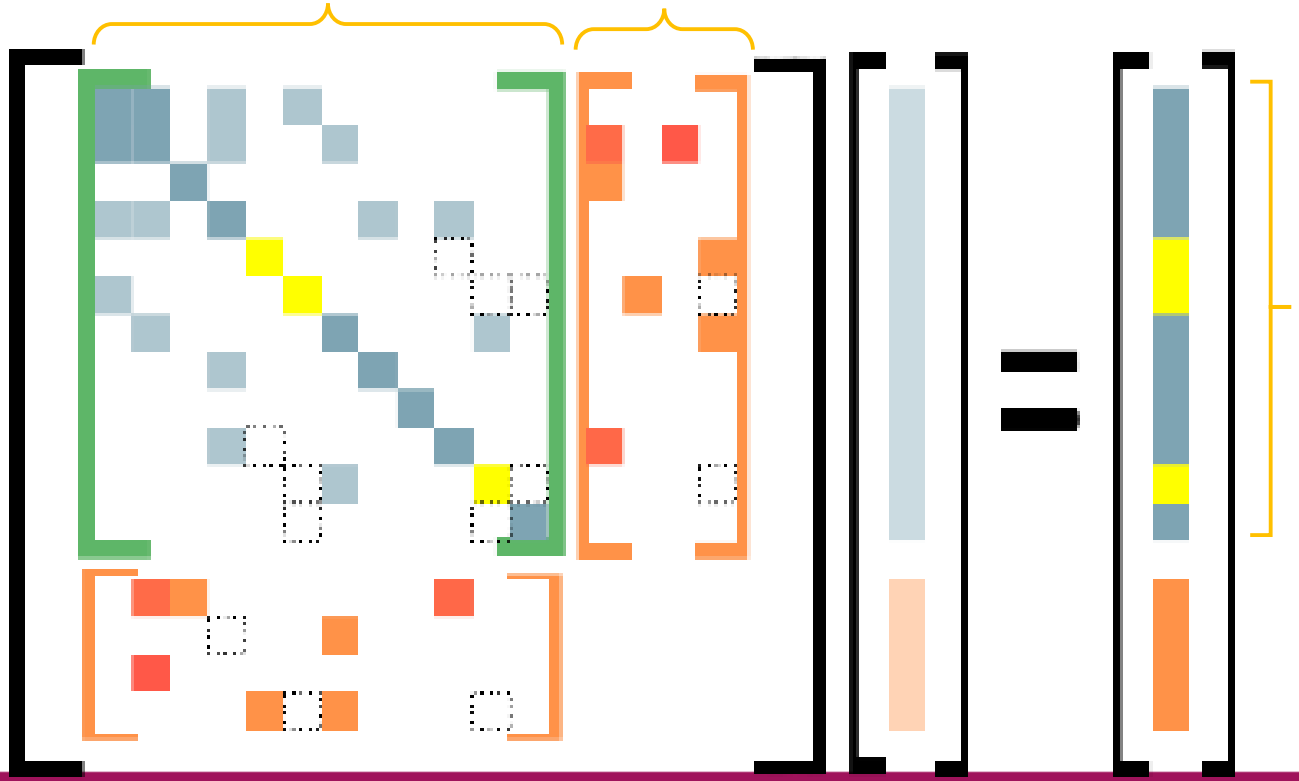
ProjectiveConstraint::
projectMatrix

ProjectiveConstraint::
projectJacobianMatrix



ProjectiveConstraint::
projectResponse

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ConstraintCorrection::addJMinvJt

$$W = \left[\;\right]\left[\;\right]^{-1}\left[\;\right]$$

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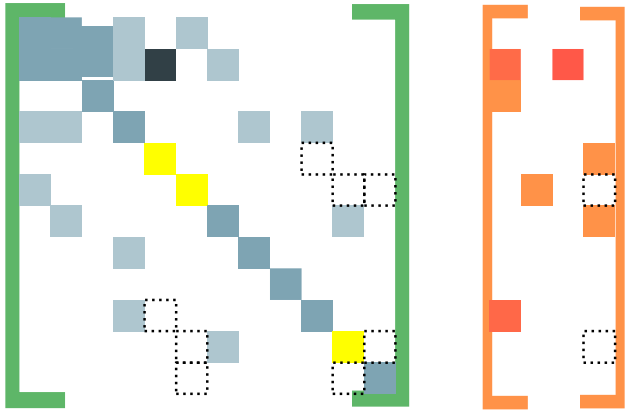
ConstraintCorrection::
computeMotionCorrectionFromLambda

$$dv = \left[\ \right]^{-1} h\lambda$$



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