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Numerical Modeling of Hydraulic Control, Solitary Waves and Primary Instabilities in the Strait of Gibraltar

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Abstract

A two-dimensional, vertical section of the Strait of Gibraltar is simulated numerically with the nonhydrostatic / non-Boussinesq three-dimensional CROCO model to investigate details of small-scale dynamics. The proposed configuration is simple, computationally efficient and incorporates the configuration of sills characteristic of this region. Despite the shortcomings of a 2D representation, this configuration provides a realistic depiction of small-scale mechanisms in the strait during a typical tidal cycle: internal solitary waves generation and propagation, occurrence of hydraulic controls and hydraulic jumps at the sills and presence of active turbulent patches. In particular, the well-known eastward propagation of large amplitude internal waves is assessed using the Korteweg de Vries (KdV) propagation model for solitary waves.

As a step towards establishing a realistic three-dimensional Large Eddy Simulation (LES), the sensitivity of the configuration to various choices (e.g., resolution, amplitude of tidal forcing or numerical schemes) is investigated. Our analyses indicate that the representation of small-scale dynamics in the Strait of Gibraltar can be much improved by increasing resolution and relaxing the hydrostatic assumption. Further studies are necessary to grasp the mechanisms of mixing and/or stirring induced by this fine scale processes.

Keywords: Strait of Gibraltar, Internal Solitary Waves, nonhydrostatic

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1 **1. Introduction**

2 [Figure 1 about here.]

3 The Strait of Gibraltar connects two major basins : the Northern Atlantic
 4 and the Mediterranean Sea, over which evaporation exceeds precipitation and
 5 river run-off. To compensate the resulting loss, exchanges of mass and salt are
 6 required through the strait. Figure D.1 illustrates the rather complex exchanges
 7 occurring there. Inflowing Atlantic water is less salty (salinity $S_A \approx 36$) than
 8 the outflowing Mediterranean water ($S_M > 38$), and spreads as a surface layer
 9 in the Alboran Sea. The interface between the two water masses is distorted by
 10 undulations that are not precisely periodic with regard to the tidal cycle but
 11 exhibit regularity in some areas. One of the paper objectives is to better un-
 12 derstand the small-scale processes that lead to the Atlantic and Mediterranean
 13 water masses transformation in the vicinity of the Strait of Gibraltar.

14 To further illustrate the exchange between the Northern Atlantic and the
 15 Mediterranean, a very simple steady-state model can be expressed as a system
 16 of two basic conservation equations.

17 Volume conservation is expressed as :

$$Q_A + Q_M = E - P \tag{1}$$

18 while the conservation of salt requires:

$$Q_A S_A + Q_M S_M = 0 \tag{2}$$

19 where Q_A is the Atlantic water volume flux (positive), Q_M is the Mediterranean
 20 water volume flux (negative), both localized in the Strait of Gibraltar, and $E - P$
 21 is the space-averaged Evaporation minus Precipitation (and river runoff) water
 22 budget integrated over the whole Mediterranean Sea. $E - P$ is positive. S_A
 23 (S_M) stands for Atlantic (Mediterranean) water mean salinity and $S_M - S_A \approx 2$
 24 (Bethoux, 1979). The water budget $E - P$ is positive in the Mediterranean due
 25 to excess evaporation that correspond to a yearly averaged loss of water of about
 26 1 meter over the whole basin (Garrett et al., 1990).

27 A major dynamical feature in the Strait of Gibraltar is the so-called "flow
 28 criticality" usually characterized by the Froude number (F): it compares the
 29 internal wave phase speed with a flow characteristic velocity. Several definitions
 30 of the non-dimensional Froude number can be found in the literature: it can
 31 notably be defined for each layer, resulting in a composite number for the whole
 32 water column, as in Farmer and Armi (1988) or in Sannino et al. (2009b).

33 A "subcritical" (respectively "supercritical") regime lies in the range of small
 34 (respectively large) values of the Froude number $F < 1$ (respectively $F > 1$),
 35 with an intermediate "critical" regime for $F \approx 1$. The upstream propagation
 36 of internal waves is inhibited for supercritical flow so that a hydraulic control

37 occurs at the transition from subcritical to supercritical flow; it persists during
38 periods and within regions of large Froude numbers. As such, the hydraulic
39 regime at a given point will vary in time according to substantial currents vari-
40 ations occurring along the tidal cycle. It is, for example, well established that
41 large amplitude solitary waves in the Strait of Gibraltar can develop due to
42 the hydraulic control at Camarinal Sill (Farmer and Armi, 1988), making it a
43 crucial process to represent.

44 Several analytical models have been proposed to investigate the hydraulic
45 control in the Gibraltar region (Bryden and Stommel, 1984; Farmer and Armi,
46 1986; Garrett et al., 1990). The hydraulic control usually occurs in these models
47 at Camarinal Sill (CS), Espartel Sill (ES), and Tarifa Narrows (TN), although
48 the modelled hydraulic control location and frequency vary according to the
49 model refinement :

- 50 1. Farmer and Armi (1986)’s two-layer model accounts for the strait geometry
51 (depth and width), the exchanged volumes (Q_A and Q_M) and the salinity
52 contrast ($S_A - S_M$). This simple model is able to simulate two hydraulic
53 controls: the first one located by the sill, the other in the TN contraction,
54 defining ”maximal exchange regime” (further details are given below).
- 55 2. In a slightly more elaborated model, the inclusion of entrainment between
56 the two layers and the subsequent interfacial layer introduction modify
57 the left-hand terms of equations (1) and (2) with the introduction of hor-
58 izontal and vertical transports in the interfacial layer (Bray et al., 1995).
59 Critical conditions are changed within such two interfaces model which
60 may support two baroclinic modes and new hydraulic controls (Sannino
61 et al., 2009b).
- 62 3. Considering a three-dimensional flow, the definition of the control needs to
63 account for cross-strait variations such as the tilt of the density interface
64 in the latitudinal direction. In the maximal exchange solution, control
65 in TN may induce the detachment of the surface layer from the northern
66 coast (Sannino et al., 2009b).

67 The hydraulic control effect within the strait is illustrated in Figure D.1. The
68 flow is initially subcritical in the Strait; the propagation of internal waves is not
69 hindered at the interface between Atlantic and Mediterranean waters (denoted
70 ”a” in Figure D.1); then the tidal flood in the vicinity of the Camarinal Sill
71 becomes supercritical. In the supercritical to subcritical transition, downstream
72 of the sill, a ”hydraulic jump” (”b” in Figure D.1) may occur.

73 Hydraulic jumps are large-amplitude depressions in the regions where hy-
74 draulic controls occur. There, intense mixing between the Atlantic and Mediter-
75 ranean waters takes place as observed by Wesson and Gregg (1994). Shear flow
76 instabilities can develop in the hydraulic jump of the Camarinal Sill (denoted
77 ”c” in Figure D.1).

78 The release of hydraulic jumps generates large-amplitude, non-linear, non-
79 hydrostatic Internal Solitary Waves (ISW) trains (denoted ”d” in Figure D.1)
80 (Farmer and Armi, 1988). As the barotropic tide is constrained by the bathymetry,

81 large vertical velocities appear and induce energy transfer to several normal
82 modes of internal waves. Some observations in the Strait of Gibraltar identify
83 the largest ISW amplitude to the first baroclinic mode ; for which vertical ve-
84 locities have the same direction throughout the water column and all isopycnal
85 surface displacements are in phase. The signature of Mode 2 waves (the verti-
86 cal velocity profile exhibits one node) has also been observed in the region of
87 Gibraltar strait (Farmer and Armi, 1988; Vázquez et al., 2006). The internal
88 waves propagate at the interface of Mediterranean and Atlantic waters.

89 As the strait flow varies at various timescales during the year, some devia-
90 tion is expected in the occurrence of the hydraulic control in the strait. This
91 may have a wide impact since local flow conditions combined with the above
92 two conservation equations (1 and 2) determine the relation between the vol-
93 ume fluxes, the evaporation minus precipitation budget ($E - P$) and the salinity
94 difference ($S_A - S_M$) (Bryden and Kinder, 1991). Practically, an "overmixed"
95 solution corresponds to a minimal salinity difference and a maximal exchange of
96 water mass in the strait : it would thus constrict the formation of Mediterranean
97 waters and diapycnal mixing over the Mediterranean basin (Bryden and Stom-
98 mel, 1984; Garrett et al., 1990). Moreover, the small-scale processes occurring
99 in the strait itself can directly modify the local characteristics of Mediterranean
100 waters (García-Lafuente et al., 2011; Naranjo et al., 2015) and Atlantic waters
101 (Milot, 2014). This can affect their characteristics as they enter respectively in
102 the North Atlantic sub-basin and in the Mediterranean Sea.

103 To study the flow dynamics in the strait in further details, more realistic
104 numerical modelling is of great help. Early attempts used two-layer models
105 (Brandt et al., 1996; Izquierdo et al., 2001). The increase of computational
106 power led to 3D modelling (Sannino et al., 2004) with increasing vertical and
107 horizontal resolution, explicitly addressing the tidal cycle and flow characteris-
108 tics. More recently, even nonhydrostatic models have been used (Sánchez Gar-
109 rido et al., 2011; Sannino et al., 2014) to explicitly represent the ISW. Other
110 configurations include the Strait of Gibraltar into a Mediterranean circulation
111 model (Soto-Navarro et al., 2015). In this case, the increased resolution lo-
112 cally in the strait (Naranjo et al., 2014) — or the nesting of high-resolution
113 grids within a coarse resolved regional model (Sannino et al., 2009a) — shows
114 a clear impact on Mediterranean stratification and improves the representation
115 of convective events in the northwestern Mediterranean basin.

116 The coastal and regional ocean modelling community model (CROCO¹) is
117 based on a new nonhydrostatic and non-Boussinesq solver (Auclair et al., 2018)
118 developed within the former ROMS kernel (Shchepetkin and McWilliams, 2005),
119 for an optimal accuracy and cost efficiency. CROCO opens up new perspec-
120 tives in terms of modelling of small-scale processes (Fox-Kemper et al., 2019;
121 Lemarié et al., 2019). In this sense, the present study objectives are also nu-
122 merical: we show that a new generation of nonhydrostatic ocean models can be
123 used efficiently to simulate complex nonlinear, fine scale physics in a realistic

¹<http://www.croco-ocean.org/>

124 but computationally-affordable configuration. The complete solution of Navier-
125 Stokes equations are thus solved numerically for the very first time in a complex
126 realistic regional configuration.

127 The present configuration of the Strait of Gibraltar is based on a classi-
128 cal lock-exchange initialization (Sannino et al., 2002). A 2D vertical section of
129 the strait is adopted in order to reduce the number of parameters impacting
130 the studied dynamics. This rather simple configuration is thus of weak com-
131 putational cost and reduces the implementation burden; it allows to reach the
132 horizontal and vertical scales of the largest turbulent structures observed in this
133 area. In the strait, where most transverse dynamical feature are an order of
134 magnitude weaker, our numerical approach is some kind of ersatz of a large-
135 eddy simulation (LES²), for which at least the generation process of primary
136 instabilities is correctly represented. However, LES is a 3D concept as the route
137 to molecular dissipation differs in 2D and 3D turbulence. The present study is
138 focused on the description of the largest primary instabilities in the Strait of
139 Gibraltar ; as well as providing order of magnitudes for explicit simulations of
140 these dynamics. Along with these physical aims, the relevance of the chosen
141 numerical methods is a major concern. A quantified impact of the largest tur-
142 bulent structures on the water masses is out of the scope of what is presented
143 hereafter: it would require a fully three dimensional LES (also achievable with
144 the CROCO model), in complement with dedicated relevant experimental mea-
145 surements.

146 In Section 2, we present an overview of CROCO equations and the implemen-
147 tation for the 2D lock-exchange experiment. We describe the implementation
148 of the bathymetry profile, water masses, and the exchange and tidal flows. In
149 Section 3, we analyse the physics of the 2D configuration, comparing the model
150 solution to already published data (e.g., in Farmer and Armi (1988)). Emphasis
151 is then made on the hydraulic control (Section 3.2), the hydraulic jump (Sec-
152 tion 3.3) and the mode-1 and mode-2 ISW (non-linear internal trains of solitary
153 waves) propagation (Section 3.4). Last, the sensitivity to the tidal forcing am-
154 plitude and to the numerical choices are analysed respectively in sections 4.1
155 and 4.2, with a focus on the fine-scales dynamics listed in Figure D.1.

156 **2. Model Description and Configuration**

157 *2.1. The Numerical Modeling System*

158 The proposed numerical model of the Strait of Gibraltar simulates explicitly
159 the fine-scale processes (from tens to hundreds of meters) discussed previously.
160 This assumes that (i) a sufficient grid resolution is provided in the strait and
161 (ii) a well-suited numerical kernel is used.

²LES (Large Eddy Simulation): LES, as opposed to DNS (Direct Numerical Simulation) does not cope with the full 3D Kolmogorov energy cascade down to molecular scales. However at least the onset of this cascade (the largest turbulent structures) is explicitly represented, unlike in RANS (Reynolds Averaged Navier-Stokes).

162 The nonhydrostatic (non-Boussinesq) CROCO version is chosen for its abil-
 163 ity to allow the explicit representation of primary instabilities that cascade the
 164 kinetic energy injected at large scale down to the smaller scales. This direct
 165 transfer ends at the finest scale resolved; the subgrid dissipation of energy is
 166 performed both by the implicit mixing of the advection schemes and the ex-
 167 plicit closure schemes. The dissipation is solely performed by (quasi-)monotonic
 168 numerical advection schemes (Grinstein et al., 2007) when no parametrized tur-
 169 bulent closure scheme accounts for the sugrid-scale mixing.

170 CROCO is an extension of ROMS from which it inherited the robustness
 171 and efficiency of its time-splitting implementation, the accuracy of high-order
 172 methods, including its pressure gradient scheme for terrain-following coordi-
 173 nates, and computing performances (Shchepetkin and McWilliams, 2005; De-
 174 breu et al., 2012; Soufflet et al., 2016). In CROCO’s time-splitting algorithm,
 175 the ”slow mode” is similar to ROMS internal (baroclinic) mode (Shchepetkin
 176 and McWilliams, 2005); its ”fast mode” includes the usual external (barotropic)
 177 mode and a new pseudo-acoustic mode that allows computation of the nonhy-
 178 drostatic pressure within a non-Boussinesq formalism (Auclair et al., 2018). A
 179 two-level time-splitting kernel is thus conserved in CROCO (as opposed to the
 180 first implementation of the 3-level time-splitting by Auclair et al. (2018)) but the
 181 fast time step integrates a 3D-compressible flow. Furthermore the slow internal
 182 mode is enhanced by a prognostic equation of the vertical velocity, replacing
 183 the hydrostatic equation.

184 2.2. Continuous, Free-Surface, Compressible Equations

185 The full set of Navier-Stokes equations for a free-surface ocean is explic-
 186 itly integrated, including the continuity and momentum equations, the surface
 187 kinematic relation, the heat, salt and state equations in Cartesian coordinates :

$$\partial_t \rho = -\vec{\nabla} \cdot (\rho \vec{v}) \quad (3)$$

$$\partial_t \rho \vec{v} = -\vec{\nabla} \cdot (\rho \vec{v} \otimes \vec{v}) - 2\rho \vec{\Omega} \wedge \vec{v} - \vec{\nabla} p + \rho \vec{g} + \mu \Delta \vec{v} + \lambda \vec{\nabla} (\vec{\nabla} \cdot \vec{v}) \quad (4)$$

$$\partial_t \zeta = w_{(z=\zeta)} - \vec{v}_{(z=\zeta)} \cdot \vec{\nabla} \zeta \quad (5)$$

$$\partial_t \rho \theta = -\vec{\nabla} \cdot (\rho \theta \vec{v}) + \kappa_\theta \Delta \theta \quad (6)$$

$$\partial_t \rho S = -\vec{\nabla} \cdot (\rho S \vec{v}) + \kappa_S \Delta S \quad (7)$$

$$\rho = \varrho(\theta, S, P) \quad (8)$$

188 where $\vec{v} = (u, v, w)$ is the velocity, p the total pressure, ζ the free-surface
 189 anomaly, ρ the density, θ and S the potential temperature and salinity re-
 190 spectively. $\vec{\Omega}$ is the instantaneous earth rotation vector, \vec{g} is the acceleration of
 191 gravity and μ , λ , κ_θ and κ_S are respectively the dynamical and second (bulk)
 192 viscosity and the thermal and salinity diffusivities. $\varrho(\theta, S, P)$ is a linear approx-
 193 imation of the seawater equation of state.

194 *2.3. Density and Pressure Decomposition*

195 As part of the time-splitting, the density is splitted into one slow and one
 196 fast component based on a first-order decomposition with respect to the total
 197 pressure. In the following, s and f subscripts refer to these slow and fast com-
 198 ponents respectively.

199

$$\rho = \rho_s(\theta, S, P) + \overbrace{\left. \frac{\partial \rho}{\partial P} \right|_{\theta, S} \delta P + O(\delta P^2)}^{\delta \rho = \rho_f} \quad (9)$$

$$P = \underbrace{P_{atm} + \int_z^\zeta (\rho_s - \rho_0)g dz'}_{Slow\ mode} + \underbrace{\rho_0 g(\zeta - z) + \underbrace{\delta P}_{P_f}}_{Fast\ mode} \quad (10)$$

200 No further decomposition is required for the other variables. Note that δP
 201 is the nonhydrostatic pressure.

202 *2.4. Slow vs Fast Components*

203 Navier-Stokes equations are integrated with two different time-steps in a
 204 time-splitting algorithm. The slow mode is identical to ROMS whereas the fast
 205 mode is now 3D and includes the integration of the compressible terms of the
 206 momentum and continuity equations. The free-surface anomaly is computed
 207 through the surface kinematic condition.

$$\begin{aligned} \partial_t \rho_f &= -\partial_t \rho_s - \vec{\nabla} \cdot (\rho \vec{v}) \\ \partial_t \rho \vec{v} &= \overbrace{-\vec{\nabla} \cdot (\rho \vec{v} \otimes \vec{v}) - 2\rho \vec{\Omega} \wedge \vec{v} - \vec{\nabla} \left(\int_z^{\zeta_f} (\rho_s - \rho_0)g dz' \right) + \mu \Delta \vec{v}}^{\vec{\Lambda}_s} \\ &\quad \underbrace{-\rho_0 g \vec{\nabla} \zeta_f - \vec{\nabla} P + \rho \vec{g} + \lambda \vec{\nabla} (\vec{\nabla} \cdot \vec{v})}_{\vec{\Lambda}_f} \quad (11) \\ \partial_t \zeta_f &= w_f(z=\zeta) - \vec{v}_f(z=\zeta) \cdot \vec{\nabla} \zeta_f \\ \partial_t \rho \theta_s &= \Theta_s = -\vec{\nabla} \cdot (\rho \theta_s \vec{v}) + \kappa_\theta \Delta \theta_s \\ \partial_t \rho S_s &= \Sigma_s = -\vec{\nabla} \cdot (\rho S_s \vec{v}) + \kappa_S \Delta S_s \\ \rho_s &= \varrho(\theta_s, S_s, \zeta_f) \\ \rho_f &= c_s^{-2} P_f \end{aligned}$$

208 The momentum equations are integrated both in the slow and fast modes
 209 but the right-hand-side of the equation is split in two parts: a "slow" part ($\vec{\Lambda}_s$)

210 made of slowly varying terms (advection, Coriolis force, baroclinic pressure force
 211 and viscous dissipation) and a "fast" part ($\vec{\Lambda}_f$), made of fast-varying terms (the
 212 surface-induced and compressible pressure force, the weight and dissipation asso-
 213 ciated with bulk-viscosity). This momentum equation is numerically integrated
 214 twice, once with a large time-step keeping $\vec{\Lambda}_f$ constant, and once with a smaller
 215 time-step keeping $\vec{\Lambda}_s$ constant. This time-splitting is much more computationally
 216 efficient than integrating the whole set of equations at the same short time
 217 step.

218 The general, compressible equations set 11 can basically propagate three
 219 types of waves: internal and external gravity waves, and acoustic waves propa-
 220 gating at c_s , the speed of sound. The nonhydrostatic pressure anomaly is not
 221 a solution of a diagnostic elliptic Poisson-like equation as it is for a Boussinesq
 222 equations set. The pressure anomalies travel at the acoustic waves velocity. The
 223 acoustic solver is not global anymore (as it is in the Poisson-like set); it is now
 224 local in space, meaning that no 3D global linear system of equations needs to be
 225 inverted anymore. The price of solving fast acoustic waves is however enhanced
 226 due to a more restrictive CFL conditions. Both acoustic and surface waves are
 227 integrated in CROCO's fast mode with a smaller time-step to cope with this.
 228 As a linear set of simplified compressible equations needs to be integrated in
 229 this fast mode the whole computations remain affordable. In addition, since the
 230 sound speed is at least one order of magnitude larger than the phase-velocity
 231 of the fastest propagating waves and much larger than any ocean advection ve-
 232 locity, it can be artificially reduced. The only requirement is that the speed of
 233 sound remains larger than any other propagating wave or flow velocity in the
 234 domain. In particular, it must remain larger than the phase-velocity of long
 235 surface waves so that nonhydrostatic anomalies can be propagated vertically
 236 fast enough to set up the corresponding wave structure over the water column.
 237 Sensitivity tests show that, in this case, a slower sound speed has no impact on
 238 lower-frequency dynamics in the region of the strait: more details on that point
 239 can be found in Auclair et al. (2018).

240 2.5. Bathymetry

241 [Figure 2 about here.]

242 Figure D.2.a presents the 500-m-resolution bathymetry gathered in the frame-
 243 work of the HOMONIM project coordinated by the French Navy (SHOM) and
 244 MeteoFrance, and as provided by the French Navy (Biscara et al., 2016). The
 245 main bathymetric features as well as the localization of the studied 2D vertical
 246 section are exhibited. This section is chosen as close as possible to the transect
 247 of Farmer and Armi's Gibraltar Experiment performed in April 1986 (Farmer
 248 and Armi, 1988) and coincides in the western area with the Mediterranean wa-
 249 ters privileged path. Hereafter, u is the velocity in the longitudinal direction
 250 of the section and v the velocity in the transverse direction. Figure D.2.b presents
 251 the Gibraltar Strait width according to different reference depths. This plot
 252 shows that an averaged thirteen kilometer width can be used, featuring steep
 253 slopes at the lateral boundaries of the strait, especially in Tarifa Narrows.

254 Simulations are performed with 50m and 220m horizontal resolutions. The
255 bathymetry used in the simulations is averaged laterally to limit the unrealistic
256 effect of local seamounts in the transverse direction such as those found in TN
257 (which can end up acting as another sill in a 2D vertical section). To that
258 end, a Gaussian interpolation of the bathymetry along the section in Figure
259 D.2.a is used with a greater Gaussian radius in the transverse direction than
260 in the longitudinal direction. The Gaussian radius in the transverse direction
261 is set to 1500 m (i.e. lower than the width of the strait in figure D.2.b). As
262 a consequence, the bathymetry only reflects the deepest areas in the canal. In
263 the longitudinal direction, the Gaussian interpolation radius is set to 300 m to
264 preserve the bathymetry variability in this direction.

265 The minimum depth thus assessed at Camarinal Sill (i.e. the main sill in
266 the Strait), along the transect's path goes from the value of approximately 200
267 m to 245 m. The model bathymetry used is the one shown in Figure D.1. A
268 reference simulation (hereafter named **SimRef**) is carried out at 50-m horizontal
269 resolution with additional characteristics and parameters listed in D.1.

270

271 [Table 1 about here.]

272 [Figure 3 about here.]

273 *2.6. Initial Water Masses, Tidal Forcing and Boundary Conditions*

274 The temperature and salinity reference profiles are chosen to initialise the
275 density field of the simulations. A minimum of two profiles for each of those
276 variables is needed to initialize gradients associated with sloping isopycnal sur-
277 faces in a given direction. According to Sannino et al. (2002), a lock-exchange
278 initialization is performed with homogeneous Atlantic water initially to the west
279 of the CS and homogeneous Mediterranean water to the east. A three-day spin-
280 up described in Section 2.7) is then performed to set up the exchange flow in
281 the strait.

282 The initial temperature and salinity profiles are presented in Figure D.3. The
283 contrast in salinity between Atlantic and Mediterranean waters is noticeable,
284 with respective mean values of 35.9 and 38.2.

285 In the following, the interface between Atlantic and Mediterranean layers
286 is taken as the 37 isohaline, following Bryden et al. (1994). Density is now
287 expressed as an anomaly (written ρ') relative to a reference (mean) density ρ_0 .
288 From now on, the implicit reference density (unless contraindicated) will be
289 $\rho_0 = 1033.7 \text{ kg/m}^3$: a value reached in the pycnocline separating the two water
290 masses.

291 An idealized M2 tidal forcing (of period $T = 12.4 \text{ h}$) is prescribed at the open
292 boundaries after the spin-up period (at $t = 3 \text{ days} = 5.8 T$). It is introduced
293 thanks to a barotropic current of amplitude 0.4 m/s at the western boundary
294 (0.8 m/s at CS), corresponding to a moderate regime according to the TPXO-8
295 tidal atlas (Egbert and Erofeeva, 2002). Lateral forcing is introduced at the
296 open eastern and western boundaries through mixed active passive radiation

297 conditions (Marchesiello et al., 2001) ; cyclic conditions are imposed to the
298 northern and southern open boundaries.

299 *2.7. Initial Flow and Effect of Coriolis Force*

300 The Gibraltar strait lateral boundaries are distant of about 15 km, with
301 a clear funneling effect from the CS to the eastern end of the strait (5.4°W ;
302 see Figure D.2.b). The internal Rossby radius R (Appendix A) is usually
303 found to vary from 10 to 20 km (Bormans and Garrett, 1989; Candela et al.,
304 1990; Vlasenko et al., 2009). The width of the strait and the Rossby radius
305 R being of the same order of magnitude, rotational effects can be neglected
306 as a rather good approximation. Therefore, the momentum balance is mainly
307 between the acceleration and the pressure force in the equation (4) and the
308 geostrophic adjustment in the along-strait direction is locally neglected. In
309 their observations (Farmer and Armi, 1988) and the 3D-modeling configurations
310 (Sannino et al., 2002), the consequence of Earth’s rotation is a cross-strait shear
311 of along-strait velocity (Bormans and Garrett, 1989) and a tilt of the isopycnals:
312 along the southern boundary (i.e. along the Moroccan coast), the interfacial
313 isopycnal is deeper and the flow reaches larger velocities.

314 As the transverse flow, the coastal boundaries, and the resulting “funneling
315 effect” cannot be simulated in a 2D vertical section, it is necessary to exam-
316 ine whether completely ignoring rotational effects is viable in a 2D vertical
317 approximation. To that end, three different numerical simulations of the strat-
318 ification and the mean circulation are compared: (i) one simulation with the
319 Coriolis force activated from start to end (**SimAllCor**; $f = 8.5 \cdot 10^{-5} \text{s}^{-1}$), (ii)
320 the second one without the Coriolis force (**SimNoCor**; $f = 0$), and (iii) the
321 last one with the Coriolis force activated only after a three days spin-up period
322 (**SimRef**). Apart from the Coriolis parameter, all the three simulations were
323 performed with the characteristics given in Table D.1 for **SimRef**. During the
324 very first hours of simulation time, the ‘Lock-Exchange dam’ separating the
325 Atlantic and Mediterranean water-masses disappears; a gravity current is gen-
326 erated with dense Mediterranean waters flowing down the western slope of CS
327 and light Atlantic water spreading in the surface layer east of the sill.

328 Figure D.4 presents the field of the longitudinal velocity (u) as well as some
329 isopycnals for **SimAllCor** (a) and **SimNoCor** (b) at $t = 72$ h (i.e., the end
330 of the spinup period). The bold isopycnal surface $\rho' = -0.7 \text{ kg/m}^3$ corre-
331 sponds at that time to the 37psu-isohaline. Figures D.4.a-b also present the
332 transverse-velocity (v) isotachs ± 0.5 m/s. Figures D.4.c-d show the tidal resid-
333 ual components u and v in the water column at the two dashed vertical lines on
334 figures D.4.a-b. These locations correspond to the moorings indicated in Figure
335 1 of Candela et al. (1990). Using the 37psu-isohaline as a frontier between the
336 two water masses, the 3T time-averaged transport through the left dashed line
337 at the CS is given in the columns labelled ‘Transport’ of the Table D.2.

338 In **SimNoCor** (Figure D.4.b), a clear vertical shear of the along-section ve-
339 locity can be seen between the two water masses. The shear is still featured in
340 the 3T-averaged current profiles of Figure D.4.c and is, for station M2, in ac-
341 cordance with the observations given by the moorings at the CS. This shear has

342 decreased since initialization as progressive mixing of the two water masses re-
 343 duces the baroclinicity. The currents in **SimAllCor** and **SimRef** are weaker,
 344 with locally negative currents in the upper layer (see Figure D.4.c). This is
 345 confirmed by the layer-averaged transports presented in Table D.2, where val-
 346 ues in **SimAllCor** and **SimRef** are one order of magnitude smaller than in
 347 **SimNoCor**. Using an average strait’s width of 13 km, an approximate baro-
 348 clinic transport for the strait can be estimated from the values in Table D.2;
 349 for **SimNoCor** it would be of 0.6 Sv: it is slightly weaker than the range of
 350 0.7-1 Sv estimated from various field observations (see a review in Sammartino
 351 et al. (2015)). In **SimAllCor** and **SimRef**, the cross-section velocity is due to
 352 the inclusion of the Coriolis force. More precisely, during the spin-up phase of
 353 **SimAllCor**, the effect of rotation can no more be neglected after 6 hours. At
 354 that time, the upper Atlantic layer has spread over a distance of about 26 km
 355 and the cross-section velocity featured in Figure D.4.a is already significant.

356 [Figure 4 about here.]

357 With no coastal boundaries to hinder the geostrophic adjustment within the
 358 strait, the initially longitudinal gravity current is almost completely converted
 359 into transverse geostrophic current with spurious (non physical) consequences
 360 on the slope of the isopycnals. Geostrophy enables a thermal-wind balance for
 361 the transverse v component:

$$\frac{\partial v}{\partial z} = -\frac{g}{\rho_0 f} \frac{\partial \rho}{\partial x} \quad (12)$$

362 This is particularly apparent to the east of the CS in Figure D.4.a where the
 363 pycnocline is located at the transition between positive and negative transverse
 364 velocities. As a consequence, the pycnocline slope is $\Delta z/\Delta x = 6.10^{-3}$ (Table
 365 D.2) and the Atlantic water cannot spread further than the resulting surface
 366 front. In **SimRef**, the Coriolis force is introduced only after a 72-h-spin-up
 367 period leading to the state presented in Figure D.4.b. In this case, the resulting
 368 slope and transverse velocity are smaller than in **SimAllCor**. The pycnocline in
 369 the eastern part of the domain is deeper whereas it is shallower in the western
 370 part. However, longitudinal currents remain weak. In contrast, in **SimNo-**
 371 **Cor** the thermal-wind balance is not allowed and, away from the sills, $\Delta z/\Delta x$
 372 vanishes. In the longitudinal direction, the main balance is between the pres-
 373 sure force ($-1/\rho_0 \partial p/\partial x$) and the acceleration term. In this case, the shear of
 374 the longitudinal velocity is better represented at the two moorings. The larger
 375 transports in both layers indicate that a larger amount of Mediterranean water
 376 enters the Tangier basin than in **SimAllCor**. This is confirmed by the strati-
 377 fication (Figure D.4.a and Table D.2) since the pycnocline is shallower over the
 378 Espartel Sill (and deeper within the TN).

379 A perfect balance between the transports in the upper and lower layers is
 380 not achieved in any of these three numerical experiments. While there is no
 381 numerical challenge in achieving longer simulation, the fluxes of Mediterranean
 382 and Atlantic waters are not realistically specified to re-stratify properly the

383 water column in these academic configurations. After the spin-up period, the
 384 intense tidal mixing and other dissipative processes may end up homogenizing
 385 the initial water masses. The gap between the transports in the upper and
 386 lower layers disappears as the depth-averaged absolute transports decrease. This
 387 process is faster in **SimNoCor** than in the other configurations in which the
 388 thermal-wind balance maintains the isopycnal slopes. In this case, the depth of
 389 the 37 psu-isohaline, taken as a moving average over one tidal period at $x =$
 390 5.4°W , increases from 130 to 175 m in **SimNoCor** over three tidal periods (not
 391 shown). This impacts the large-amplitude internal waves propagation.

392 The difficulty to obtain both realistic ambient stratification and circulation is
 393 a limitation of the restriction to a 2D vertical section of the dynamical problem
 394 targeted. In the proposed implementation, an initial state is obtained by lock-
 395 exchange with a spin-up period of 72 h dedicated to the adjustment of the
 396 gravity current produced by the 'dam break'. For the remainder of this paper,
 397 the reference simulation (**SimRef**) is chosen as the simulation whose adjustment
 398 is made in a non-rotating framework. This initial state has a correct mean
 399 exchange but it is weakened as the tidal forcing is introduced and changes the
 400 stratification conditions.

401 To mitigate this problem, the rotation is restored at the end of the spin-up
 402 period: the geostrophic balance that ensues stabilizes the slopes of isopycnals
 403 by generating a transverse current. The mean exchange is nevertheless reduced
 404 but the stratification (i.e. both the slope of the isopycnals and the vertical
 405 density gradient) thus saved is crucial to the generation and propagation of the
 406 large-amplitude solitary waves. Furthermore, the small-scale processes discussed
 407 hereafter take place during the tidal cycle. At this time-scale, the barotropic
 408 exchange is dominated by the tidal currents, so that in the reference simulation,
 409 the amplitude of the baroclinic exchange is correct. Note that if rotation is
 410 activated from the beginning of the spin-up period (**SimAllCor**), it leads to
 411 unrealistically-large slopes of the isopycnals (see Table D.2).

412 **3. The Reference Simulation**

413 The reference simulation presented previously is now evaluated thanks to the
 414 observational data from the Gibraltar Experiment (Farmer and Armi, 1988). We
 415 describe the hydraulic controls, the primary instabilities and the dynamics of
 416 the ISW in this reference simulation.

417 *3.1. Comparison with in situ Observations*

418 In the present section, observations from the Gibraltar Experiment (Farmer
 419 and Armi, 1988) are investigated in order to evaluate the quality of the model
 420 solution obtained with the reference configuration **SimRef**.

421 The table D.2 presents the pycnocline depth and slope at different loca-
 422 tions along the section for the three configurations and the observational data.
 423 The depth and slope for **SimAllCor** and **SimNoCor** are calculated after 72
 424 h of simulation and correspond to the isopycnal surface $\rho' = -0.7 \text{ kg/m}^3$ in fig-
 425 ures D.4.a-b, whereas for configuration **SimRef** the same isopycnal taken in

426 a 3T-averaged stratification corresponding to Figure D.5. In **SimAllCor**, the
427 isopycnals have a greater slope than the reported observations from Gibraltar
428 Experiment (0.006 vs 0.003). In **SimNoCor**, there is no slope away from the
429 sills, while as discussed before, the slope obtained in **SimRef** is small. The
430 stratification in **SimRef** is close to that of **SimNoCor** in the eastern part
431 with an isopycnal close to the horizontal. In the western part and over the CS,
432 the pycnocline is shallower than in the other two simulations (Table D.2).

433 In the following, we further investigate small-scale dynamical processes such
434 as hydraulic jump and ISW propagation in configuration **SimRef**. This is
435 also the baseline configuration used to perform all sensitivity tests presented
436 hereafter.

437 [Table 2 about here.]

438 3.2. Tidal Currents & Hydraulic Control.

439 [Figure 5 about here.]

440 In the present study, the Froude number (F) is simply defined at each grid
441 point as the ratio between the local longitudinal velocity u and the theoretical
442 speed c_1 of the first internal wave mode, computed with the modal decomposition
443 for each point of the x-axis (see Appendix B for details).

444 For single-layer flows, hydraulic control occurs in the region of transition
445 between subcritical and supercritical flows. In this region, the condition $F > 1$
446 is met over the whole water column. For multi-layer flows, the Froude number
447 condition may be satisfied in a few layers only. In this case, the layers where
448 the flow becomes supercritical are considered as "hydraulically controlled".

449 Three areas of potential hydraulic control in Gibraltar Strait are identified
450 from previous studies: the CS, the ES and the narrowest part of the TN (near
451 5.5°W longitude in figure D.2). Farmer and Armi (1988) found persistent controls
452 for first internal wave mode at the ES, the CS and the TN. Sannino et al.
453 (2009b) found only ephemeral appearances of such controls, except at the ES
454 where it is permanent. The discrepancy is likely lying in the definition and the
455 estimation of the composite Froude number.

456 Figure D.5 shows the regions where the flow is critical. Closed contours
457 indicate the locations of critical Froude number ($F = 1$), inside which the flow
458 is supercritical. The longitudinal velocity fields (u) used to estimate F are
459 taken at maximum outflow (grey) at $t = 7.5 T$ and maximum inflow (black)
460 at $t = 8 T$. The internal waves phase speed c_1 is computed from the 3T-
461 time-averaged stratification represented in Figure D.5. No control of the first
462 mode is ever seen in TN : this is a consequence of the 2D simplification which
463 excludes the representation of the tidal flow intensification by the narrowing
464 of the strait at TN. On the other hand, hydraulic control is expected at both
465 sills. The location where it should occur alternate between the eastern (during
466 ebb) and western (during flood) sides of the sills, with a return to subcritical
467 flow when tidal current slackens. The Froude number easily goes beyond 2 in
468 the Mediterranean outflow at CS, where the flow is supercritical through most

469 of the water column, except sometime at the surface. At ES, the lower layer
 470 may become supercritical with Froude number that never exceeds 1.5. The
 471 lack of persistent control at ES may be a consequence of the crudely imposed
 472 stratification.

473 3.3. Primary Instabilities in the Hydraulic Jump Area

474 [Figure 6 about here.]

475 One manifestation of the hydraulic control is the formation of hydraulic
 476 jumps in specific regions where the flow transitions from supercritical to sub-
 477 critical. This is a complex area with steep slopes and high shears where flow-
 478 topography interaction can generate small-scale coherent structures. The largest
 479 ones are resolved in **SimRef** and are characterised here with various methods.
 480 First, the Okubo-Weiss parameter (Appendix C) is computed to investigate
 481 the presence of new 'coherent structures' in the hydraulic jump area. Their dy-
 482 namics are further investigated using Empirical Orthogonal Functions (EOFs)
 483 computed with a Singular Value Decomposition (SVD). Their typical length
 484 and velocity scales are finally compared with the expected analytical values of
 485 shear instabilities and lee waves.

486 The dynamics of the CS hydraulic jump are illustrated in Figure D.6, in
 487 which the density field (Fig. D.6.a) and the vertical velocity field (Fig. D.6.b)
 488 over the western slope of the CS are represented during flood. Several flow
 489 parameters were also computed and are depicted in Figure D.6.a and b. These
 490 parameters are :

- 491 1. The Froude number defined in Appendix B. The contours of critical
 492 Froude number $F = 1$ are shown in Figure D.6.b, inside which the flow is
 493 supercritical.
- 494 2. The Richardson gradient number defined as $Ri = N^2 / (\partial u / \partial z)^2$, with N
 495 the Brunt-Väisälä frequency defined in Eq.13. Contours of $Ri = 0.25$ are
 496 depicted in Figure D.6.a, as $Ri < 0.25$ is a required condition for the
 497 development of shear instabilities.

$$N = \sqrt{-\frac{g}{\rho_0} \frac{\partial \rho}{\partial z}} \quad (13)$$

- 498 3. The Okubo-Weiss parameter defined in Appendix C. Negative values of
 499 OW indicate vortical circulation, and so contours of $OW = -4 * 10^{-4} s^{-2}$
 500 are shown in both Figure D.6.a and b.

501 In Figure D.6.b, the supercritical flow region ($F > 1$) follows the slope of
 502 the sill where the velocity in the Mediterranean outflow is larger than 2 m/s
 503 (see the contour $F = 1$ running approximately parallel to isopycnals between
 504 5.76°W and 5.77°W). At 5.77°W , the density field in Figure D.6.a shows a sharp
 505 transition (e.g., isopycnal $\rho' = -0.5\text{kg}/\text{m}^3$ rises from 350 to 225 m depth), and
 506 forms a wedge-shaped region over the supercritical Mediterranean outflow. This
 507 is the signature of an internal hydraulic jump.

508 Downstream of the hydraulic jump, several small-scale structures are visible.
 509 Figure D.6.a shows patches of lighter water (billows) associated with areas of
 510 negative OW values at a depth of 400 m, and large-amplitude disturbances of
 511 isopycnals at 150 m. Negative OW values are also located at troughs and can
 512 reach $OW = -4 * 10^{-4} s^{-2}$. Both types of structures are propagating westward
 513 and are quite probably shed from the internal hydraulic jump at the tip of the
 514 wedge-shaped region at $5.773^{\circ}W$, with the size of billows growing rapidly as they
 515 travel down-slope. In the potential generation area, Richardson number values
 516 are less than 0.25, indicating favorable conditions for generation of primary
 517 shear instabilities.

518 Further identification of the simulated new features of Figure D.6 is achieved
 519 by proceeding with a complex singular value decomposition (SVD) (Peraud and
 520 Auclair, 2005) of the velocity field ($w+iu$) in the water column between $5.795^{\circ}W$
 521 and $5.78^{\circ}W$ longitude, during outflow conditions. This region is highlighted in
 522 Figure D.7.a, in which the time mean field of longitudinal velocity u is presented,
 523 showing the intense outflow below layers of lesser velocities. The mean density
 524 field and location of $Ri < 0.25$ are also indicated, the former showing a
 525 homogeneous area between 50-150 m above the seafloor. The SVD gives a
 526 first singular vector responsible for 28 % of the total variance corresponding
 527 to the evolution of the barotropic forcing (not shown). The remaining singular
 528 eigenvectors have lesser corresponding variance and show smaller structures with
 529 high-frequency variations in the singular right eigenvector. Two consecutive
 530 singular vectors often have very close eigenvalues and temporal variations.

531 [Figure 7 about here.]

532 Figure D.7.b shows the reconstructed field of the combination of the second
 533 and third singular vectors (respectively responsible for 13 % and 11 % of total
 534 variance) added to the mean field shown in Figure D.7.a. The Okubo-Weiss pa-
 535 rameter is again computed for the resulting velocity field, which shows two rows
 536 of y-axis vortices centered at $z = -300$ m (anti clockwise) and -400 m (clock-
 537 wise). The upper row of vortices appears to generate the observed interface
 538 oscillations.

539 Based on $OW=0$ contours, the lower clockwise vortices have an estimated
 540 horizontal scale of 200 m and a vertical scale of 150 m, corresponding to hor-
 541 izontal and vertical wavenumbers 3.10^{-2} and $4.10^{-2} m^{-1}$ respectively. Their
 542 propagation speed can be estimated as -0.7 m/s by following the center of the
 543 billows defined by areas of negative OW values. The distance between the
 544 centers of two consecutive vortices is $L = 530$ m. The centers of the upper anti-
 545 clockwise and lower clockwise vortices are shifted along flow by $L/2$, so that
 546 the extrema of their respective vertical velocities are aligned vertically. It seems
 547 that a transfer of momentum occurs between the two rows of vortices in a way
 548 reminiscent of the Vallis model of edge waves in a stratified region of a shear
 549 flow (pp 254-258 in Vallis (2006)).

550 The length scales deduced from this SVD analysis can now be compared
 551 with expected scales from simple analytical models for shear instabilities and

552 internal waves based on the general characteristics of the flow on the western
 553 slope of Camarinal.

554 Shear flow instability in a two-layer system of infinite depth results in a mixed
 555 interface of vertical extent ΔH expressed by equation 14.6 of Cushman-Roisin
 556 and Beckers (2011) :

$$\Delta H \approx \frac{1}{k_{min}} = \frac{\rho_0(u_1 - u_2)^2}{2(\rho_2 - \rho_1)g} \quad (14)$$

557 with k_{min} the wave-number of the most unstable mode in this system, taken as
 558 the scale of the primary instability that will develop. In the generation area of
 559 CS, $(u_1 - u_2)$ is in the range of 1.2 to 2 m/s and $(\rho_2 - \rho_1)$ is in the range of
 560 1.2 to 1.7 kg/m³. Additional values are $\rho_0 = 1033.7$ kg/m³ and $g = 9.81$ m²/s.
 561 This gives a range of vertical scales between 44 m and 183 m and k_{min} between
 562 $2.10^{-2} m^{-1}$ and $5.10^{-3} m^{-1}$. The scales of the simulated lower row of vortices
 563 are in the upper part of this range.

564 Lee waves are another candidate for small-scale transient flow and the inter-
 565 facial oscillations observed in our solutions. Their generation over topography is
 566 expected when tidal excursion is larger than the topographic length scale $1/k_b$,
 567 i.e. $k_b u_0 / \omega > 1$ (St. Laurent and Garrett, 2002). The slopes of Camarinal Sill
 568 are not symmetrical, as can be seen for example in Figure D.4. On the western
 569 side of the sill, the depth increases from 250 m (at 5.76°W) to 510 m (at 5.78°W)
 570 over only 1.8 km, whereas on the eastern side it increases from 250 m to 620 m
 571 (at 5.65°W) over 9.4 km. k_b is thus chosen in a range between $3.10^{-3} m^{-1}$ and
 572 $6.10^{-4} m^{-1}$. $u_0 = 0.4$ m/s is the amplitude of the barotropic tidal current away
 573 from the sill. In these conditions, the ratio $k_b u_0 / \omega$ ranges between 1.7 (over the
 574 west slope) and 8.9 (over the east slope). Based on this, we cannot rule out the
 575 possibility that lee waves are generated over the CS.

576 If the simulated small-scale structures would originate from lee waves, their
 577 phase speed would be comparable to a mode-1 internal gravity waves. This
 578 can be estimated using the same method as in Appendix B for the average
 579 stratification presented in Figure D.7.a. It yields a value of 1.4 m/s in the
 580 area down-flow of the hydraulic jump, which is twice the estimated propagation
 581 speed of both rows of simulated structures. Therefore, even though lee-wave
 582 generation is theoretically possible, the interfacial oscillations observed in the
 583 simulation appear more consistent with the stirring effect of the bottom coherent
 584 vortices, whose scales fall within the range of expected values for KH instability.

585 We conclude that coherent structures are clearly identified in our simulations.
 586 They are generated in an area of potentially unstable shear ($Ri < 0.25$) and
 587 are associated with billows of lighter mixed fluid. The deeper row of vortices
 588 can reasonably be interpreted as Kelvin-Helmholtz primary instabilities. Their
 589 downstream behaviour further corresponds to a 2D pairing of two consecutive
 590 Kelvin-Helmholtz billows. These billows are advected in a region of westward
 591 flow between the Mediterranean vein and Atlantic waters up until 5.8°W. At this
 592 location, advection is reduced as the lower layer decelerates and the billows are
 593 uplifted in a flow recirculation and mixed in the pycnocline. These small-scale
 594 features appear in the simulation for as long as the hydraulic jump is present,

595 injecting water from the pycnocline in the outflow and vigorously mixing the
596 Atlantic and Mediterranean waters until the tidal currents weaken sufficiently
597 for the flow to become subcritical.

598 However, the dynamics simulated in a 2D vertical section with no transverse
599 flow and no transverse instability may differ from the real ocean. In a fully
600 3D configuration, primary Kelvin-Helmholtz instabilities should decay faster as
601 secondary Kelvin-Helmholtz instabilities develop along the transverse rotation
602 axis of the primary billows. This is precluded in the present 2D configuration,
603 even with enhanced resolution, as only y-axis billows can occur.

604 Wesson and Gregg (1994) observed billow structures in the area of CS with
605 an extension of several tens of meters. This length scale is much smaller than the
606 one simulated in the present numerical configuration. However, these observa-
607 tions were made in shallower regions, i.e. probably closer to the generation area:
608 larger billows more in line with those simulated in the present study could thus
609 develop downstream. Further observations on site are needed, although short
610 length-scales and fast propagation speed would require adapted measurement
611 strategies.

612 3.4. Internal Tide Dynamics

613 [Figure 8 about here.]

614 In **SimRef**, two main types of large amplitude wave propagating eastward
615 can be observed. Both of them are generated at CS while tidal currents reverse
616 from westward to eastward. This is illustrated in Figure D.8.a-b. First, a mode-
617 1 wave appears as a bore over the sill’s crest approximately 2.25 hours after the
618 westward flow peaks. In Figure D.8.a, it has propagated over the eastern slope of
619 the sill’s crest and is now at 5.7°W longitude: the corresponding profile in Figure
620 D.8.c has only one maximum as expected for a mode-1 internal wave. This wave
621 rapidly steepens while a hydraulic control is maintained on the western side of
622 the sill with lower values of the Froude number. One hour and 15 minutes later,
623 the flow becomes subcritical and a large-amplitude mode-2 wave crosses the sill.
624 In Figure D.8.b, this new wave is propagating over the eastern slope of the sill;
625 its vertical velocity profile is presented in Figure D.8.c. It exhibits two maxima
626 of opposite signs and a zero-crossing at the pycnocline’s depth as expected for a
627 mode-2 internal wave. Additionally, it can be seen in this figure that the bore-
628 like wave has evolved into a train of internal solitary waves, whose propagation
629 is discussed below.

630 The propagation of internal waves can be characterized by plotting a space-
631 time evolution of a particular isopycnal. In Figure D.9.b, the depth evolution of
632 the -0.7 kg/m^3 isopycnal is represented (also in a white contour in Fig. D.8).
633 Regions of sharp horizontal density gradients can be identified periodically in the
634 region of the CS next to 5.76°W longitude. They correspond to the generation
635 of hydraulic jumps. The propagation of internal waves are identified by the tilt
636 of isolines, whose slopes provide an estimate of wave propagation speed. There
637 are differences from one tidal cycle to the next because mixing progressively
638 changes the background stratification in the domain. In the following, we focus

639 on the tidal cycle $t = 7.5 T - 8.5 T$ (second cycle after the end of the spin-up
640 phase), for which the three-tidal-cycle averaged velocity shear and stratification
641 are shown in Figures D.4 and D.5.

642 The mode-1 bore of Figure D.8.a propagates down-slope of the CS into the
643 TN, where it experiences a transition into a train of solitary waves moving at
644 a speed of 1.5 m/s. In Figure D.9.c, such a train made of four mode-1 waves
645 can be seen at 5.5°W longitude. The amplitude of the first wave reaches 100
646 m. The solitons train amplitude momentarily increases and exceeds 150 m as
647 it propagates over the slope near 5.5°W longitude in the TN (still during ebb
648 tide). From then on, the wave amplitude decreases as it propagates towards
649 the deeper region. Meanwhile the dispersion increases the number of waves as
650 well as their wavelength as noticed in the space-time diagram which shows the
651 envelop of the train of solitons expanding while it propagates eastward. The
652 dispersion as simulated in CROCO is compared with the Korteweg deVries
653 model in Appendix D.

654 Similarly, the mode-2 wave shown in Figure D.8.b propagates through the
655 shallowest part of the TN at a speed of 0.9 m/s as a new hydraulic jump is
656 being generated over the eastern slope of CS. It is located at 5.65°W longitude
657 in Figure D.9.c, with an amplitude of approximately 100 m. The propagation
658 speed of this mode-2 internal wave subsequently decreases when it reaches the
659 deepest part of the domain while simultaneously the tidal phase shifts to flood.
660 The amplitude of these waves is simultaneously strongly reduced.

661 The signatures of other large-amplitude internal waves can be seen propa-
662 gating to the west of the CS in Figure D.9.b. They are mode-1 and mode-2
663 internal waves with amplitude in the tens of meters, i.e. smaller than the east-
664 ward propagating wave. They are generated when tidal currents switch from
665 eastward to westward in the same fashion as previously described for the wave
666 train produced east of the CS (as tidal currents reverse from westward to east-
667 ward).

668 As discussed in Section 3.2, a hydraulic control also occurs at the ES. Figure
669 D.9.b shows the same variations of isopycnal depth in this area as near the
670 CS during the tidal cycle $t \in [7.5 T, 8.5 T]$, but not on the following cycle
671 anymore. In the latter case, the computed Froude number does not exceed 0.7,
672 as opposed to previous cycles ($t \in [6.5 T, 7.5 T]$ and $t \in [7.5 T, 8.5 T]$) or
673 later ones ($t \in [9.5 T, 10.5 T]$ and $t \in [10.5 T, 11.5 T]$). In these cases,
674 the variations are similar and hydraulic control occurs at ES. The sequence is
675 as in CS, with a hydraulic control briefly lost at the barotropic flow reversal.
676 Then internal mode-1 waves of 50-m amplitude at a depth of 300 m are released
677 and propagate toward the CS. In Figure D.9.c, a mode-1 wave can be found at
678 5.87°W . These waves dissipate in the area near the sill as the absolute value of
679 the barotropic current decreases.

680 In Figure D.9.c, two vertical lines are drawn in the TN. They refer to mea-
681 surements made by Farmer and Armi (1988): the lines indicate the first two
682 baroclinic modes locations three and a half hours after high tide. The right-
683 hand vertical line corresponds to a mode-1 wave and is in agreement with the
684 simulation. However, in **SimRef**, the distance between the two modes is twice

685 as large as in the observations. Based on observed wave arrivals at various sta-
686 tions, Farmer and Armi (1988) estimated the propagation speed of both mode-1
687 and mode-2 waves at about 1 to 2.5 m/s for mode 1 and 1 to 1.5 m/s for mode
688 2. The wave train they observed contained two to three large-amplitude waves,
689 the first one having an amplitude of 100 m. Additional observations by Sánchez
690 Garrido et al. (2008) in the TN region give a propagation speed for mode-1
691 waves ranging in 1.2 m/s and 2 m/s with a large variation in the velocity of two
692 consecutive wave trains due to the weight of the tidal diurnal inequality (K1
693 and O1).

694 The mode-1 dynamics simulated in **SimRef** are consistent with these ob-
695 servations reported by Farmer and Armi (1988) in terms of propagation speed
696 and longitudinal position. However, the simulated mode-2 wave seems too slow
697 and its amplitude too large. Its slower propagation might be due to an under-
698 estimation of the barotropic flow that carries the mode-2 within the TN as our
699 2D vertical approach does not catch well the tunneling effects of this narrowing.
700 This would not affect the mode-1 wave as much, because its linear propagation
701 speed is more intense and the barotropic current advection becomes compara-
702 tively small.

703 The brief hydraulic control loss observed when the tidal currents reverse
704 does not reflect the quasi-permanent control thought to be taking place at ES.
705 In this case, no internal waves packet can be emitted from the ES.

706 4. Sensitivity Testing

707 The reference configuration presented in the previous section is based on
708 several physical and numerical choices which are now investigated ; mostly the
709 impact of the forcing amplitude, momentum balance (hydrostatic approxima-
710 tion) and numerical parameters (spatial resolution, advection schemes).

711 4.1. Tidal regime

712 An additional simulation, **SimS**, is first performed similarly to **SimRef**
713 changing only the tidal forcing amplitude. The imposed tidal current amplitude
714 at the western boundary is now increased up to 0.6 m/s, so that it reaches 1.3
715 m/s over the CS. This corresponds to a spring-tide regime.

716 Figure D.10 presents a comparison between **SimS** and **SimRef**. In Figure
717 D.10.a, the contours of supercritical regions ($F > 1$) show the the CS hydraulic
718 jump extends further east in **SimS**. As a result, a mode-1 disturbance (denoted
719 "a" in Figure D.10.a) is trapped at 5.725°W longitude. It propagates eastward
720 when the flow becomes subcritical but the faster bore that is crossing the CS can
721 rapidly catch it up. The outflowing Mediterranean water vein on the westward
722 side of the sill is also thicker in **SimS** and so is the supercritical area. In addition,
723 a new supercritical region appears on the western slope of a secondary relief at
724 5.83°W longitude (denoted "b" in Figure D.10.a) with trailing lee waves.

725 the figure D.10.b shows an eastward propagating mode-1 solitons packet.
726 Since the initial stratifications are similar, linear phase velocities are the same

727 in **SimRef** and **SimS** at the beginning. The amplitude of the first trough of the
 728 train is 50-m larger in **SimS** than in **SimRef**. This should result in increased
 729 propagation speed of the solitons in **SimS**, in contradiction with a slower prop-
 730 agation seen in Figure D.10.b. It can be explained by the stronger tidal currents
 731 advection in the opposite direction in **SimS**. A mode-2 wave is also generated
 732 in both **SimS** and **SimRef**, but is only visible in **SimRef** in Figure D.10.b as
 733 it quickly dissipates in **SimS** due to stronger tidal currents. Consistent with
 734 our results, Farmer and Armi (1988) show that the ISW amplitude increases
 735 with the tidal current during the spring-tide / neap-tide cycle. In addition,
 736 two concomitant hydraulic jumps were observed in the strait during spring tide
 737 ((Sánchez Garrido et al., 2011)): they exhibit a transverse asymmetry, as the
 738 second jump only appears in the northern part of the strait. This could not be
 739 confirmed in the present 2D configuration.

740 [Figure 9 about here.]

741 4.2. Nonhydrostatic Balance and Numerical Factors

742 The consequences of several numerical choices are now investigated by run-
 743 ning five additional simulations whose differences with **SimRef** are laid out in
 744 Table D.3. In particular, the sensitivity to both vertical and horizontal resolu-
 745 tion is targeted. A hydrostatic kernel and a WENO5-Z advection scheme for
 746 momentum Borges et al. (2008) are also tested and compared with **SimRef**.
 747 We focus on the impact of these modifications on two types of small-scale pro-
 748 cesses studied in the previous sections: the primary (KH) instability generation
 749 in the hydraulic jump at the CS, and the eastward propagating solitary waves
 750 generated at the same place.

751 [Table 3 about here.]

752 4.2.1. Hydraulic Jump and Instabilities

753 [Figure 10 about here.]

754 Hydraulic controls (Section 3.2) occur in all simulations, as revealed by sys-
 755 tematic estimations of the Froude numbers. Low Richardson numbers (< 0.25)
 756 are also diagnosed for all simulations over at least part of the CS western slope
 757 during flood. However, the features that have been identified as KH instabil-
 758 ities in Section 3.3 do not appear in all simulations. They are absent in the
 759 simulations performed in the hydrostatic framework and/or with low horizontal
 760 resolution (**SimL**, **SimLH** and **SimH**). Weak horizontal vorticity tilting under
 761 the hydrostatic assumption prevents such instabilities to develop. Instead, a
 762 smooth, large recirculation appears west of the CS (Figure D.11 for **SimLH**).

763 In the remaining sensitivity experiments (**SimV** and **SimW**), KH instabil-
 764 ities are generated at the edge of the hydraulic jump and their dynamics is overall
 765 similar to the one described in Section 3.3 for **SimRef**: pairing of KH billows
 766 can occur whereas anti-clockwise vortices induce oscillations of the interface.
 767 Fine resolution (a few tens of meters in the present region) and non-hydrostatic

768 equations are both required to explicitly simulate the turbulent cascade onset
769 with KH instabilities between Mediterranean and Atlantic flows.

770 Interestingly enough, a comparison of Figures D.6 (nonhydrostatic configura-
771 tion) and D.11 (hydrostatic configuration) shows that the fine-scale solution
772 is largely filtered out by the hydrostatic assumption. Without dedicated ob-
773 servations in the area, it remains difficult to conclude that **SimRef** is more
774 realistic, although low Richardson numbers in this region lead us to expect KH
775 instability.

776 4.2.2. Large Internal Waves

777 [Figure 11 about here.]

778 As described in Section 3.4, mode-1 and mode-2 large amplitude internal
779 waves are generated in the CS vicinity and propagate eastward during each ebb
780 in all nonhydrostatic simulations. Figure D.12 presents the density fields in the
781 region of the TN at $t = 8.1 T$. A wave train with a minimum of two solitons can
782 only be identified in the nonhydrostatic experiments. In the hydrostatic cases
783 **SimH** and **SimLH**, the lack of nonhydrostatic dispersion produces internal
784 waves propagating as internal bores. In the nonhydrostatic cases, these internal
785 waves can propagate as trains of solitons with varying numbers of solitons (and
786 celerity) : 6 in **SimRef**, 8 in **SimW**, 4 in **SimV**, and 2 in **SimL**.

787 This illustrates a second aspect of the effect of hydrostatic assumptions,
788 besides the inhibition of turbulent primary instabilities (KH instabilities in the
789 region of the hydraulic jump). As already noted by previous authors (Sannino
790 et al., 2004), the dispersion needed to balance nonlinear steepening in large-
791 amplitude solitary waves is missing in hydrostatic simulations such as **SimH**
792 and **SimLH** (Figure D.12).

793 4.2.3. Evolution of Stratification

794 [Figure 12 about here.]

795 The previous results lead us to anticipate large differences in the way density
796 stratification evolves in **SimRef**, **SimW**, **SimV**, **SimH**, **SimL**, or **SimLH**. To
797 go further, Figure D.13 shows for each configuration the profiles of Brunt-Väisälä
798 frequency (N defined in eq.13) at 5.8°W (a) and at 5.55°W (b). Profiles are
799 time-averaged over one flood.

800 At 5.8°W , stratification is similar in **SimH** and **SimLH** with an interface
801 region (defined as the region where N is maximal, here $N = 8.10^{-3} \text{ s}^{-1}$) located
802 at a depth of 250 m (Figure D.13.a). The nonhydrostatic simulations **SimRef**
803 and **SimL** present an interface region at a similar depth but the vertical gra-
804 dients are larger in **SimRef** and smaller in **SimL**. **SimV** shows larger vertical
805 gradients of density ($N = 1.1 \cdot 10^{-2} \text{ s}^{-1}$) and a shallower interface at a depth
806 of 150 m. **SimW** is weakly stratified over most of the water column with no
807 clear interface between the two water masses. This result may seem surprising
808 because the WENO5 scheme is of fifth-order accuracy with more selective quasi-
809 monotonic corrections near shocks than the TVD scheme, and is thus expected

810 to produce less smoothing. This apparent contradiction can be explained by the
811 generation of more intense primary shear instabilities (i.e. with higher values of
812 associated vertical velocity and vertical velocity gradients), which has the effect
813 of diffusing density gradients.

814 In Figure D.13.b, averaged N profiles over one flood are shown at 5.55°W
815 longitude in a region subjected to intense internal wave activity. **SimH**, **SimLH**
816 and **SimL** exhibit similar profiles with N slowly decreasing below the interface
817 at 160 m. **SimRef** has a similar interface at 160 m but with higher N value,
818 although weaker stratification appear at the top of the lower layer. **SimV** and
819 **SimW** both have shallower interfaces at 130 m, with N reaching the highest
820 values in **SimV** ($N = 1.25 \cdot 10^{-2} \text{ s}^{-1}$), as the enhanced vertical resolution allows
821 the representation of stronger gradients.

822 Clear conclusions cannot yet be drawn concerning mixing. KH instabilities
823 are expected in the region of the hydraulic jump and downstream. They lead
824 to more stirring and consequently open up new opportunities to improve mod-
825 elling of the route to mixing. Further investigation and diagnostics are now
826 required to better understand the energy cascades. In particular, the present
827 comparison between WENO5 and TVD advection schemes (and their implicit
828 dissipation) indicates that numerical choices may unfortunately still have large
829 consequences. Therefore, the role of physical and numerical closure must be
830 considered comprehensively (Marchesiello et al., 2011; Soufflet et al., 2016).

831 5. Discussion and Conclusion

832 The present study focuses on small-scale dynamics in the Strait of Gibraltar
833 and on the capacity of a new split-explicit, free-surface, nonhydrostatic regional
834 oceanic model (CROCO) to represent such dynamics. Both objectives were
835 pursued in parallel and several seminal results are obtained.

836 The study confirms that the generation of large-amplitude mode-1 and mode-
837 2 internal waves in the Strait of Gibraltar as well as the onset of stratified
838 turbulence and its energy cascade can be simulated with a computationally-
839 efficient 2D vertical section. The characteristics of the simulated internal waves
840 compare qualitatively well with published observations and previous numerical
841 studies. Internal tides dynamics and shear instability in the hydraulic jump area
842 are then analysed in more details, revealing characteristics and mechanisms.

843 The results of the study rely on a new type of nonhydrostatic, non-Boussinesq,
844 free-surface kernel (Auclair et al., 2018) implemented in the CROCO model. The
845 resulting compressible oceanic model is presented in a realistic nonhydrostatic
846 configuration for the first time. Sensitivity tests confirm that a nonhydrostatic
847 (here non-Boussinesq) kernel is required (i) to simulate ISW trains and (ii) to
848 explicitly simulate the onset of stratified turbulence energy cascade, provided
849 that resolution is increased from about 200 m to 50 m. We conclude that reso-
850 lutions finer than a few hundred meters are required in addition to a refinement
851 of dynamical equations (relaxation of hydrostatic assumption) in order to solve
852 the dominant dynamical processes in a key region of the Mediterranean.

853 Detailed characteristics of the vertical 2D configuration are also given with
854 particular attention to the bathymetry and to the representation of the Coriolis
855 force (implicit representation of funnelling effect in the strait). The proposed
856 approach offers a computationally affordable way to make preliminary investi-
857 gations of internal-wave dynamics in regions where these waves are important.
858 However, the vertical 2D configuration is limited by the simplified representa-
859 tion of bathymetry and associated biases in the velocity shear between in-flowing
860 Atlantic Waters and out-flowing Mediterranean waters. The inclusion of restrat-
861 ification processes (surface and boundary forcing) would allow the model to re-
862 main accurate for a greater number of tidal cycles — the present configuration
863 is considered accurate within three days after the spin-up period, before mixing
864 starts to homogenise the water masses. Several remaining processes could not
865 be considered: the transverse propagation of ISWs in the Strait of Gibraltar
866 (Sánchez Garrido et al., 2011; Vlasenko et al., 2009) and in the Alboran Sea;
867 the hydraulic control at the TN (Farmer and Armi, 1988; Sannino et al., 2009b);
868 the boiling-water over the CS (Bruno et al., 2002) or reflections on the strait’s
869 coasts.

870 Only the “onset” of turbulence cascade could be simulated showing complex
871 dynamics occurring in the area of the hydraulic jump at CS, with some small
872 scale features identified as primary Kelvin-Helmholtz instabilities. Although
873 this 2D study highlights how interesting this area can be, there is no doubt
874 that simulation of secondary KH instabilities and subsequent energy cascade as
875 well as the long term impact of these small scale processes on Mediterranean
876 and North Atlantic circulation will require a fully 3D LES approach as well as
877 dedicated field campaigns that explore these fine-scale processes.

878 Appendix A. Evaluation of the First Internal Rossby Radius

879 The first internal Rossby radius is defined as :

$$R = \frac{\sqrt{g'h}}{f} \quad (\text{A.1})$$

880 At the Gibraltar Strait’s latitude, the Coriolis parameter is $f = 8.5 \times 10^{-5} \text{ s}^{-1}$.
881 The numerator $c^* = \sqrt{g'h}$ is the phase speed of the linear interfacial waves, into
882 which the reduced gravity is:

$$g' = g \frac{\rho_M(S_M) - \rho_A(S_A)}{\rho_0} \quad (\text{A.2})$$

883 where $\rho_M(S_M) - \rho_A(S_A) \approx 2 \text{ kg/m}^3$. If the reference density is set to $\rho_0 =$
884 1033.7 kg/m^3 and the gravity acceleration by $g = 9.81 \text{ m}^2/\text{s}$, $g' = 0.02 \text{ m/s}^{-2}$
885 which is in agreement with in situ data (cf Bryden et al. (1994)). h is a charac-
886 teristic height given by:

$$h = \frac{h_1 h_2}{h_1 + h_2} \quad (\text{A.3})$$

887 where h_1 and h_2 are respectively the upper and lower layer thicknesses. Picking
 888 up the data values from Farmer and Armi (1988) (Table D.2, first line), we
 889 obtain a range of $h = 50 - 100$ m, which combined with the previous values for
 890 g' and f leads to R ranging in 11.5 km (east of Camarinal Sill) to 16 km (west
 891 of Camarinal Sill).

892 $\sqrt{g'h}$ can be replaced by the mode-1 internal waves speed (c_1), which is es-
 893 timated in the present study (Appendix B) for the 3T-averaged stratification
 894 (**SimRef**) presented in Figure D.5. c_1 ranges between 0.8 m/s (at Camarinal
 895 Sill) and 1.8 m/s (in the eastern part of the strait), giving an estimated range
 896 of Rossby Radius c_1/f between 9.5 and 21 km for the simulated section.

897 Appendix B. Computation of a Froude Number

898 A single value of mode- n linear internal wave phase speed (c_n) can be com-
 899 puted for a flat-bottom and a linear stratification. This velocity can then be
 900 compared at each depth with the magnitude of local horizontal currents (u)
 901 to estimate when and where internal waves can propagate against currents. A
 902 diagnostic tool is the ratio of velocities, i.e. a Froude number (F_n) defined
 903 as $F_n = u/c_n$. $c_n = \omega/k_n$ where ω is the wave frequency, here the M2-tidal
 904 frequency, and k_n are eigenvalues obtained by solving numerically the Sturm-
 905 Liouville problem associated with the linear propagation equation (Gill, 1982):

$$W_n'' + k^2 \frac{N^2 - \omega^2}{\omega^2 - f^2} W_n = 0 \quad (\text{B.1})$$

906 with bottom and surface boundary conditions $W_n(0) = 0$ and $W_n(-H) = 0$. W_n
 907 gives the structures of vertical modes. For each point on the x-axis, the profile
 908 $N(z)$ is computed with Eq.13 from the 3T-averaged stratification in **SimRef**
 909 (shown in Figure D.5).

910 [Figure 13 about here.]

911 The value of the first mode phase speed $c_1(x)$ is indicated in the lower panel
 912 of Figure D.14 and compared to the longitudinal velocity $u(x, z)$ at $t = 8.5$ T
 913 plotted in the upper panel. The areas where $u(x, z) > c_1(x)$ (equivalent to
 914 Froude number F_1 larger than 1) are presented in the upper panel as well. The
 915 flow inside those contours is called supercritical.

916 Appendix C. Computation of Okubo-Weiss Parameter

917 The Okubo-Weiss parameter (OW) is defined as

$$OW = s_n^2 + s_s^2 - \Omega^2 \quad (\text{C.1})$$

918 With s_n the normal strain component, s_s the shear strain component and Ω the
 919 vorticity. Usually, it is used in a xy-plane (e.g., for tracking eddies in Chelton

920 et al. (2007)), but it is computed here in the zx-plane with the strains and
 921 vorticity expressed as :

$$s_n = \frac{\partial w}{\partial z} - \frac{\partial u}{\partial x} \quad , \quad s_s = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \quad , \quad \omega = \frac{\partial u}{\partial z} - \frac{\partial w}{\partial x} \quad (\text{C.2})$$

922 Negative values of OW indicate a greater role of rotation over deformation, and
 923 thus the presence of coherent vortices.

924 Appendix D. Comparison with Korteweg-de Vries (KdV) Model

925 [Figure 14 about here.]

926 Following many studies (Sánchez Garrido et al., 2008; Sannino et al., 2009b;
 927 Vlasenko et al., 2009), the large amplitude internal waves of Section 3.4 are
 928 termed "ISWs" for Internal Solitary Waves. To confirm that the internal waves
 929 observed in this section are ISWs, they are now compared with solutions of
 930 the Korteweg-de Vries equation which is recalled below. The solutions of this
 931 equation satisfy a balance between nonhydrostatic dispersion and nonlinear ad-
 932 vection. Nonlinear advection steepens the wave front, whereas nonhydrostatic
 933 dispersion reduces steepening by transferring energy from large to small scales,
 934 resulting in a relatively stable entity called a "solitary" wave, or soliton.

935 The Korteweg-de Vries (KdV) equation describes the evolution of an in-
 936 finitely thin interface in a two-layer system with constant bottom topography:

$$\frac{\partial \zeta}{\partial t} + c^* \frac{\partial \zeta}{\partial x} + \underbrace{\frac{3}{2} \frac{h_1 - h_2}{h_1 h_2} c^* \zeta \frac{\partial \zeta}{\partial x}}_A + \underbrace{\frac{1}{6} h_1 h_2 c^* \frac{\partial^3 \zeta}{\partial x^3}}_B - \underbrace{\frac{3}{8} \zeta^2 c^* \frac{h_1^2 + 6h_1 h_2 + h_2^2}{8(h_1 h_2)^2}}_C = 0 \quad (\text{D.1})$$

937 where c^* is the interfacial speed of small-amplitude internal waves: $c^* = \sqrt{g'h} =$
 938 $R.f$ and ζ is the vertical displacement of the interface. g' and h have already
 939 been defined in Appendix A. The first two terms on the left-hand-side of (D.1)
 940 are those involved in the classical propagation equation for a small-amplitude,
 941 linear, interfacial wave travelling in the x-direction at speed c^* . The third
 942 term (bracket A) is a first-order approximation (with respect to amplitude)
 943 of nonlinear advection. Term (B) is a dispersive term. The fifth term (C)
 944 is a higher-order non-linear term associated with a second order development
 945 for advection. The complete equation (D.1) will be referred to as "extended
 946 KdV" (or "eKdV"), whereas the same equation without term (C) will simply
 947 be referred to as "KdV" (Dossmann, 2012).

948 The simulated large amplitude waves presented in this study are compared
 949 with the solutions of the KdV or eKdV equation to gain insight into their
 950 dynamics. For optimal comparison, a new simulation, called **SimRef+** was
 951 carried out. Its characteristics are similar to **SimRef** (Table D.1), except that
 952 (i) the eastern boundary is shifted 42-km to the east into the Mediterranean sea
 953 and (ii) the tidal forcing is stopped after only 7.25 periods. The first eastward-
 954 propagating train of mode-1 waves generated by the tide at the CS is compared

955 with the propagation given by numerical integration of the KdV and eKdV
956 equations (eq. D.1).

957 The vertical displacement of isopycnal $\rho' = -0.5 \text{ kg/m}^3$ is extracted³ at
958 $t_0 = 7.5 T$ when the mode-1 ISW train propagates over a region of constant
959 depth ($H = 890 \text{ m}$) east of TN (Figure D.15), so as to get closer to the
960 KdV framework. The position of the chosen isopycnal surface at this time is
961 shown in Figure D.15.a. At that time, the distance between the first and second
962 solitons of the train is 5 km; the first soliton has an amplitude of 75 m and the
963 train includes 7 solitons. This isopycnal is chosen as initial state for the KdV
964 or eKdV model. The KdV and eKdV equations are integrated either with the
965 interfacial wave speed ($c^* \approx 1.27 \text{ m/s}$) or with mode-1 velocity ($c_1 = 1.45 \text{ m/s}$;
966 see Appendix B for details on c_1 evaluation).

967 Figures D.15.b-e compare the interface depth obtained at $t = t_0 + 0.25 T$
968 in the KdV and eKdV models with the position of the -0.5 kg/m^3 isopycnal in
969 **SimRef+**. In the latter, the distance between the first two solitons has grown
970 since $t = t_0$ and reaches 7 km with an amplitude of 70 m for the first trough.
971 The train is now made of 11 solitons: the first three have decreasing amplitudes,
972 then two smaller solitons of comparable amplitude (40 m) and one soliton of
973 greater amplitude, followed by solitons with decreasing amplitude again.

974 In the KdV solution obtained with propagation speed c^* the first solitary
975 wave is slightly slower but its amplitude is markedly larger (Figure D.15.d).
976 The remaining solitons of the train are well located, but a secondary trough is
977 generated between the first two solitons whereas it is absent in **SimRef+**.

978 Using the larger mode-1 speed c_1 instead of c^* (Figure D.15.b), the train
979 of solitons in KdV is too fast. When the eKdV equation is used with speed c^*
980 (e), the solitons are too slow, whereas this same extended equation with c_1 (c)
981 leads to a correct displacement of the solitons. Overall, the temporal evolution of
982 solitary waves in **SimRef+** is consistent with the solutions of the KdV or eKdV
983 equation (D.1). However, the KdV or eKdV framework remains an inviscid
984 simplification. Note that closer evolution of soliton amplitudes between KdV
985 or eKdV and **SimRef+** solutions can be obtained by simply adding a diffusive
986 term in KdV or eKdV equation. This also slows down propagation in KdV, and
987 to a lesser extent in eKdV (not shown).

988 The KdV or eKdV model also involves adjustable parameters, such as the
989 linear wave speed, which ranges between the interfacial speed c^* and the mode-1
990 speed c_1 , each being one particular approximation of the wave's behaviour.

991 That being said, the wave trains in **SimRef+** are appropriately modeled as
992 KdV or eKdV ISWs, which confirms (i) the propagation of interfacial troughs
993 as trains of solitons and (ii) CROCO's ability to simulate the subtle balance
994 between nonhydrostatic effects (responsible for dispersion) and nonlinear ad-
995 vection.

³Due to the extension of the domain to the east, a new value of ρ_0 is computed : $\rho_0 = 1033.9 \text{ kg/m}^3$ to optimize computations.

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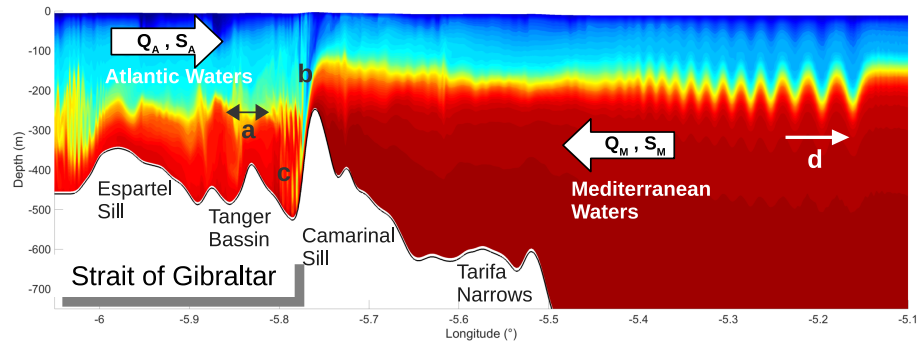


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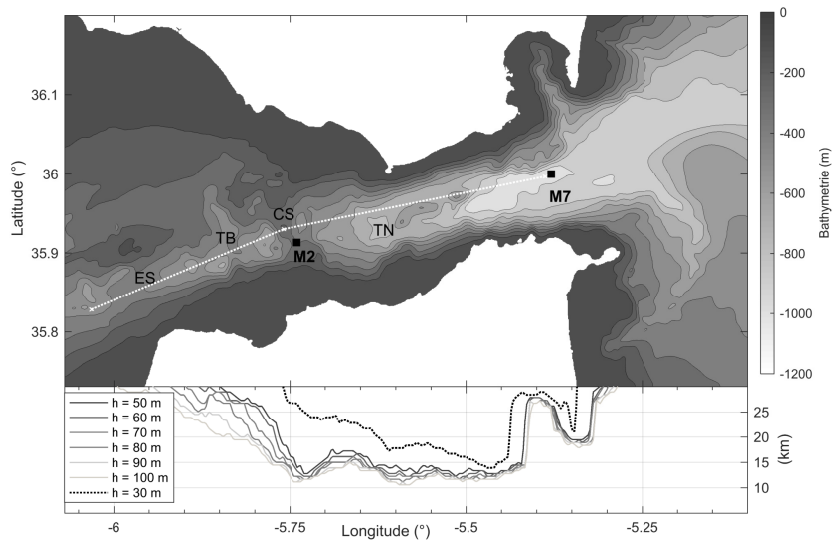


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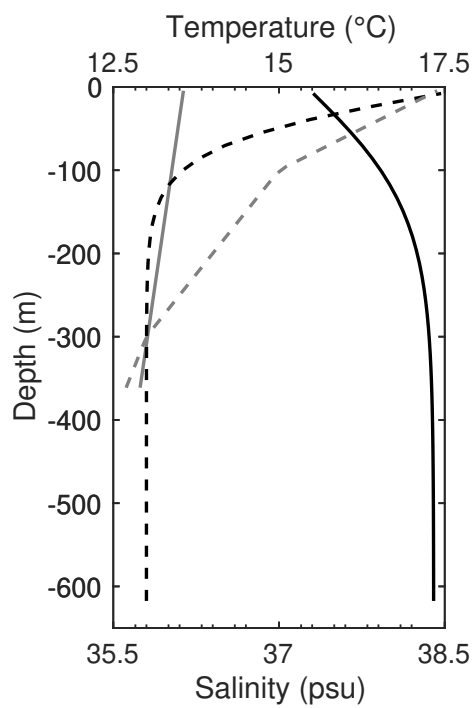


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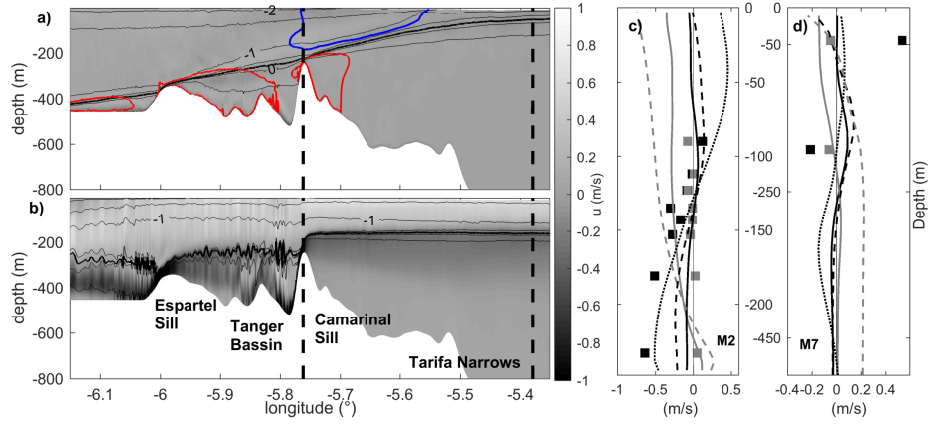


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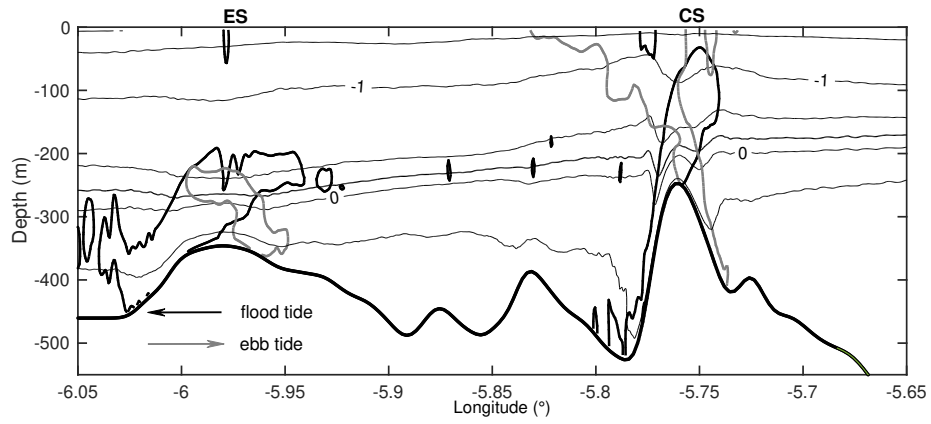


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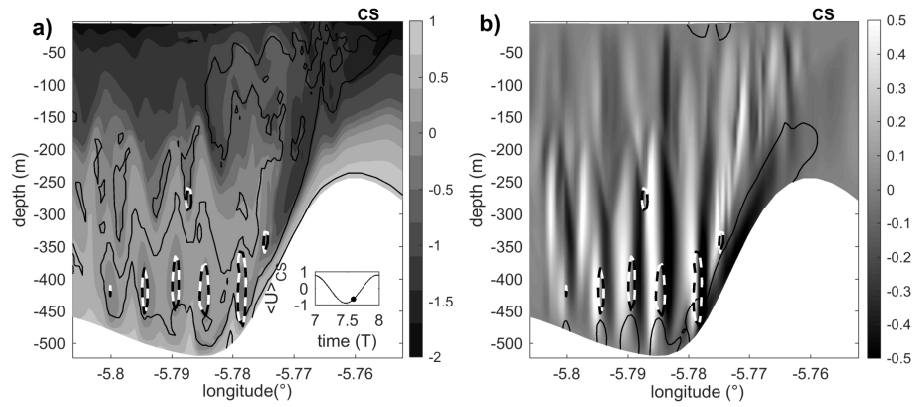


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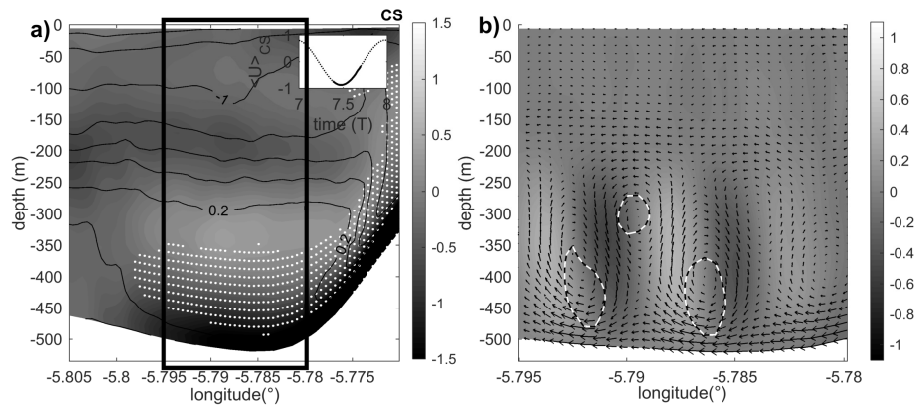


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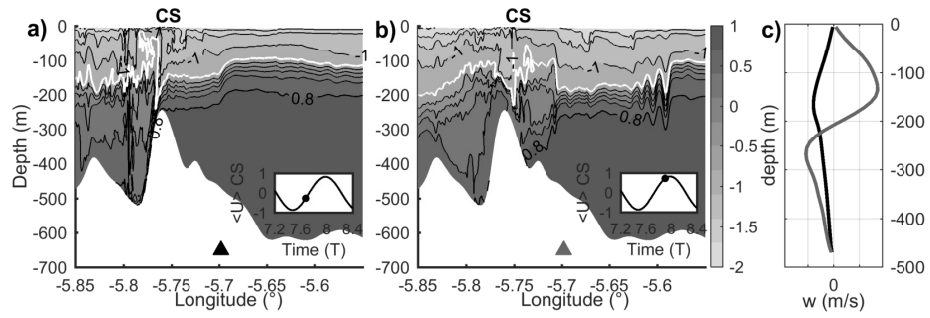


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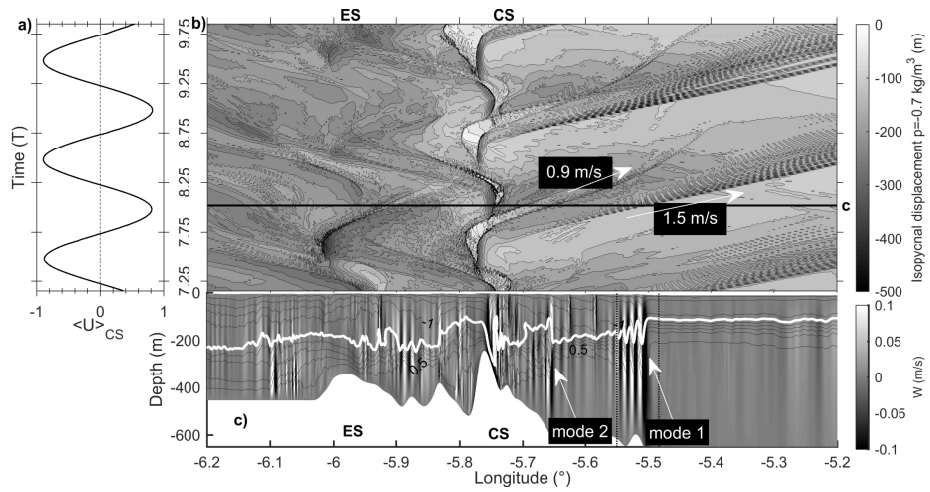


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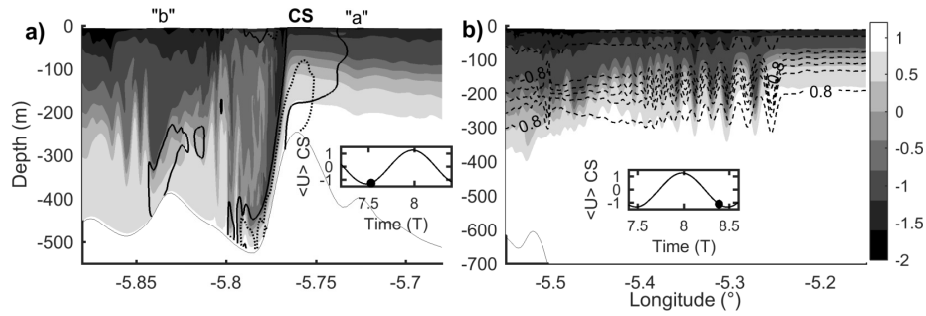


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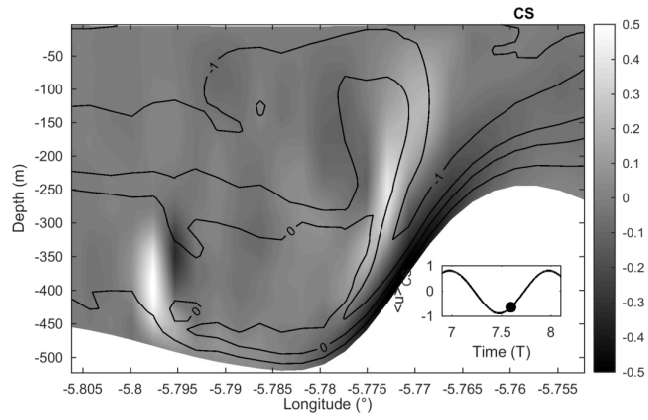


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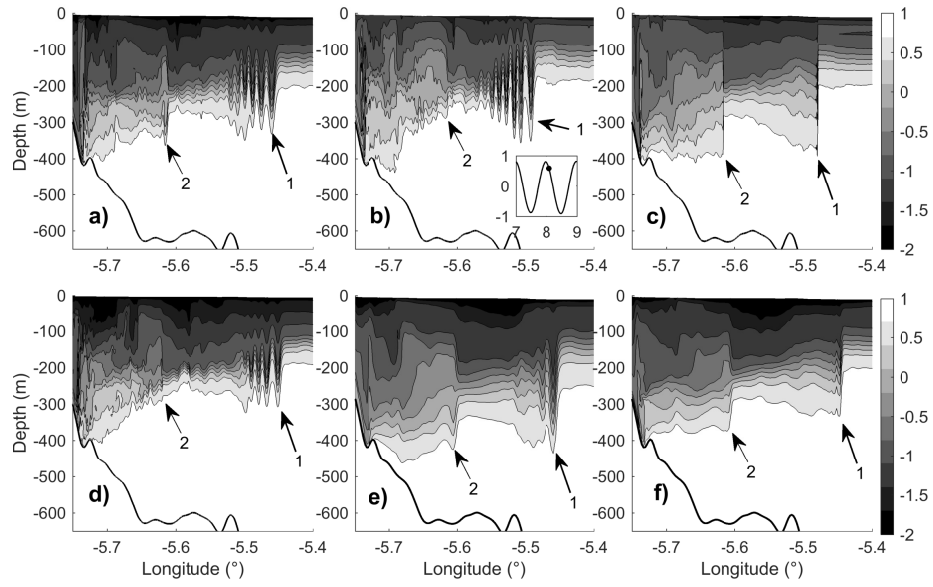


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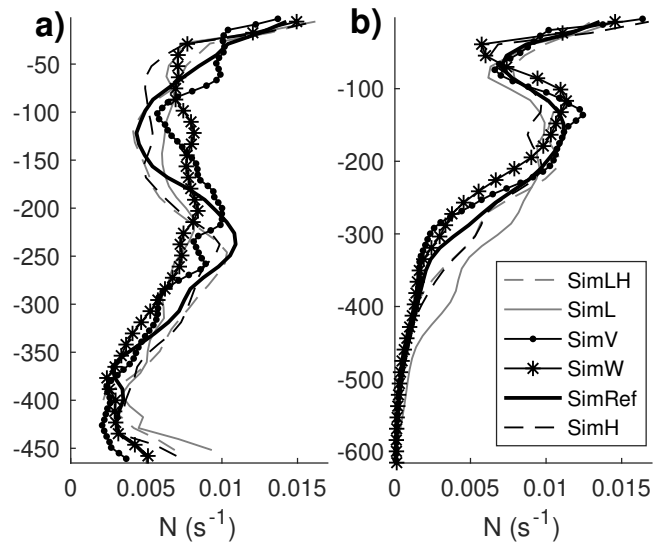


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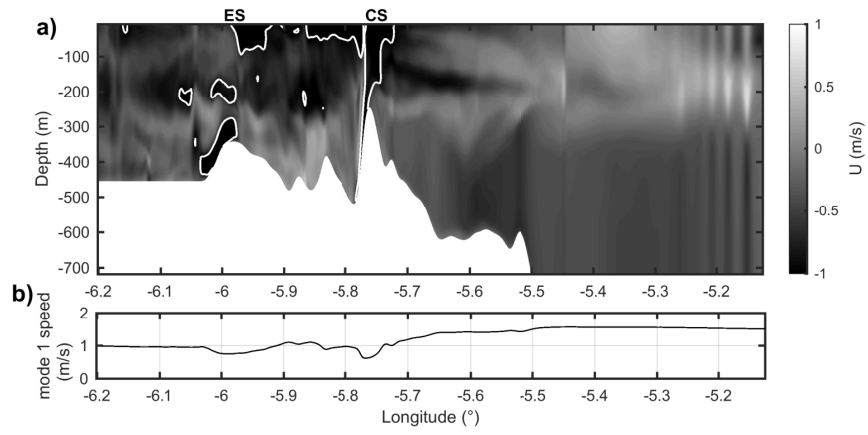


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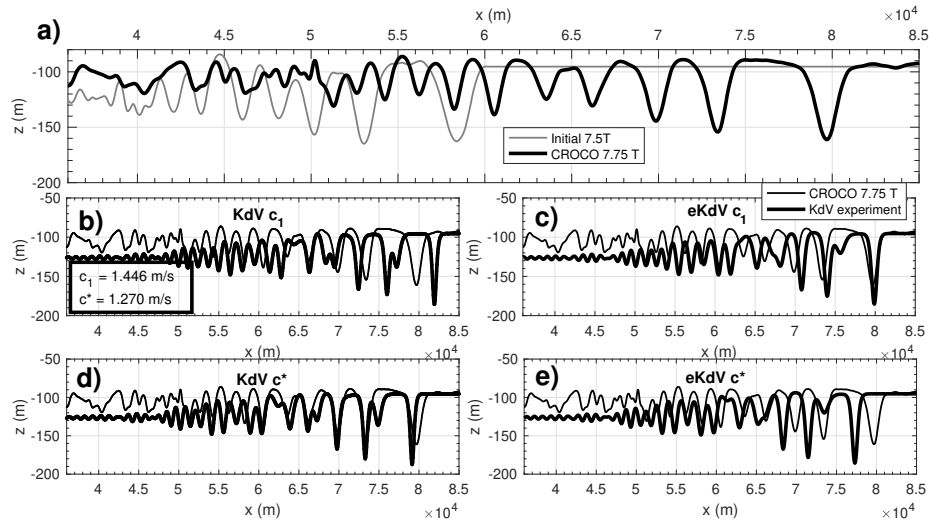


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Table D.1: Numerical parameters of simulation **SimRef**

Number of horizontal points	2661x3
Horizontal scale (Δx)	50 m
Number of vertical σ -levels	40
Depth	Min Max 247 m 900 m
Vertical scale (Δz)	6 m 23 m
Slow time step (t_s)	1 s
Fast time step (t_f)	1/8 s
Spin up period	72 h
Vertical Viscosity	10^{-6} m ² /s
Lateral Viscosity	10^{-5} m ² /s
Diffusivity	10^{-6} m ² /s
Momentum Advective Scheme	TVD - Van Leer
Turbulent Closure Scheme	none
T, S Advective Scheme	WENO5
Quadratic bottom drag coefficient	10^{-3}
Atmospheric forcing/fluxes	none

Table D.2: 3T time-averaged transports (m^2/s) at CS, depth (m) and slope of the interface.

	Transport (m^2/s)		Pycnocline Depth (m)			Pycnocline Slope	
	Upper layer	Lower layer	ES (5.91°W)	CS (5.71°W)	TN (5.52°W)	ES-CS	CS-TN
Farmer and Armi (1988)	/	/	190	125	60	0.003	0.003
SimAllCor	-5	-13	300	175	70	0.006	0.006
SimNoCor	50	-45	290	200	220	0.005	-0.001
SimRef	0.7	-6	245	175	160	0.004	0.001

Table D.3: Parameters of numerical sensitivity experiments. If not explicitly indicated, t_s and t_f are the same as in Table D.1.

SimH	Hydrostatic equations ($t_s = 0.5s$ $t_f = 0.25s$)
SimW	WENO5-Z momentum advection scheme
SimV	80 σ -levels
SimL	220-m horizontal resolution ($t_s = 4s$ $t_f = 0.5s$)
SimLH	220-m horizontal resolution with hydrostatic equations ($t_s = 2s$ $t_f = 1s$)