

Toward an improved simulation of ocean-atmosphere interactions



Eric Blayo

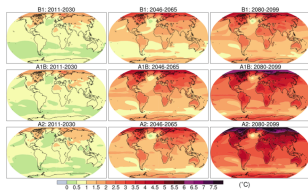
*Laboratoire Jean Kuntzmann
Univ. Grenoble Alpes and Inria, France*

Joint work with F. Lemarié, C. Pelletier and S. Thery (LJK Grenoble)
in collaboration with many colleagues from the ANR COCOA project

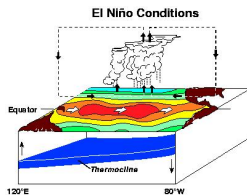


Context

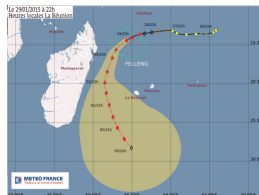
Various applications require coupling an oceanic model and an atmospheric model.



climate modeling



seasonal forecasts

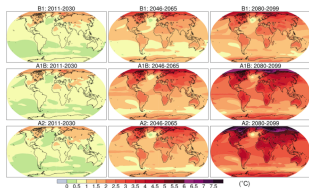


short term predictions

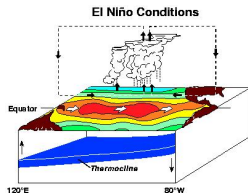
- ▶ complex interactions
- ▶ physical / mathematical / numerical / algorithmic issues

Context

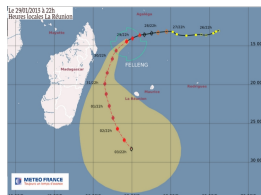
Various applications require coupling an oceanic model and an atmospheric model.



climate modeling



seasonal forecasts



short term predictions

Objectives

- ▶ Modeling aspects: improve the representation of air-sea interactions in current coupled models
- ▶ Algorithmic aspects: revisit the coupling strategies presently used in such systems

Air-sea interactions

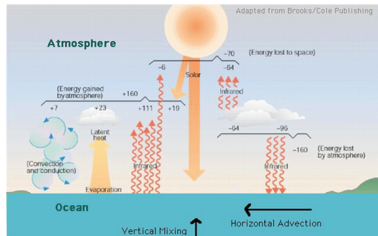
$$\begin{cases} \mathcal{L}_{\text{atm}} \mathbf{U}^a = f_{\text{atm}} & \text{in } \Omega_{\text{atm}} \times [0, T] \\ \mathcal{L}_{\text{oce}} \mathbf{U}^o = f_{\text{oce}} & \text{in } \Omega_{\text{oce}} \times [0, T] \end{cases}$$

$$\text{with } \mathbf{U}^a = \begin{pmatrix} \mathbf{u}_h^a \\ T^a \end{pmatrix} \quad \mathbf{U}^o = \begin{pmatrix} \mathbf{u}_h^o \\ T^o \end{pmatrix}$$

Interface conditions:

$$\text{momentum} \quad \rho^a K_m^a \frac{\partial \mathbf{u}_h^a}{\partial z} = \rho^o K_m^o \frac{\partial \mathbf{u}_h^o}{\partial z} = \boldsymbol{\tau} \quad \text{on } \Gamma \times [0, T]$$

$$\text{heat flux} \quad \rho^a c_p^a K_T^a \frac{\partial T^a}{\partial z} = \rho^o c_p^o K_T^o \frac{\partial T^o}{\partial z} = Q_S + \mathcal{R} \quad \text{on } \Gamma \times [0, T]$$



Air-sea interactions

$$\begin{cases} \mathcal{L}_{\text{atm}} \mathbf{U}^a = f_{\text{atm}} & \text{in } \Omega_{\text{atm}} \times [0, T] \\ \mathcal{L}_{\text{oce}} \mathbf{U}^o = f_{\text{oce}} & \text{in } \Omega_{\text{oce}} \times [0, T] \end{cases}$$

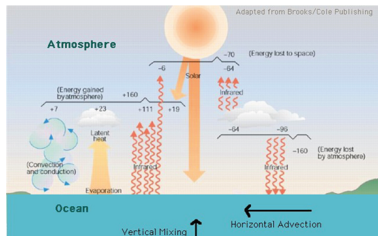
$$\text{with } \mathbf{U}^a = \begin{pmatrix} \mathbf{u}_h^a \\ T^a \end{pmatrix} \quad \mathbf{U}^o = \begin{pmatrix} \mathbf{u}_h^o \\ T^o \end{pmatrix}$$

Interface conditions:

$$\text{momentum} \quad \rho^a K_m^a \frac{\partial \mathbf{u}_h^a}{\partial z} = \rho^o K_m^o \frac{\partial \mathbf{u}_h^o}{\partial z} = \boldsymbol{\tau} \quad \text{on } \Gamma \times [0, T]$$

$$\text{heat flux} \quad \rho^a c_p^a K_T^a \frac{\partial T^a}{\partial z} = \rho^o c_p^o K_T^o \frac{\partial T^o}{\partial z} = Q_S + \mathcal{R} \quad \text{on } \Gamma \times [0, T]$$

Estimation of OA fluxes: modeling of surface boundary layer



1D primitive equations for the ocean and the atmosphere

Navier-Stokes in a rotating frame

+ standard approximations (Boussinesq, hydrostaticity, horizontal homogeneity)

+ **Reynolds average** $X = \langle X \rangle + X'$

In Ω_α , $\alpha \in \{o, a\}$:

$$\partial_t u - f v + \partial_z \langle w' u' \rangle = 0$$

$$\partial_t v + f u + \partial_z \langle w' v' \rangle = 0$$

$$\partial_z w = 0$$

$$\partial_z \tilde{p} + \tilde{\rho} g = 0$$

$$\tilde{\rho} + \rho_{0,\alpha} = \rho(\theta, S, p, q)$$

$$\partial_t \theta + \partial_z \langle w' \theta' \rangle = \mathcal{F}_\theta$$

$$\partial_t S + \partial_z \langle w' S' \rangle = 0 \quad \alpha=o$$

$$\partial_t q + \partial_z \langle w' q' \rangle = 0 \quad \alpha=a$$

unknowns

constants

turbulent scales

1D primitive equations for the ocean and the atmosphere

Navier-Stokes in a rotating frame

+ standard approximations (Boussinesq, hydrostaticity, horizontal homogeneity)

+ **turbulent closure**

In Ω_α , $\alpha \in \{o, a\}$:

$$\partial_t u - f v - \partial_z(\nu_\alpha^t \partial_z u) = 0$$

$$\partial_t v + f u - \partial_z(\nu_\alpha^t \partial_z v) = 0$$

$$\partial_z w = 0$$

$$\partial_z \tilde{p} + \tilde{\rho} g = 0$$

$$\tilde{\rho} + \rho_{0,\alpha} = \rho(\theta, S, p, q)$$

$$\partial_t \theta - \partial_z(\mathcal{K}_{\theta,\alpha}^t \partial_z \theta) = \mathcal{F}_\theta$$

$$\partial_t S - \partial_z(\mathcal{K}_{S,\alpha}^t \partial_z S) = 0 \quad \alpha=o$$

$$\partial_t q - \partial_z(\mathcal{K}_{q,\alpha}^t \partial_z q) = 0 \quad \alpha=a$$

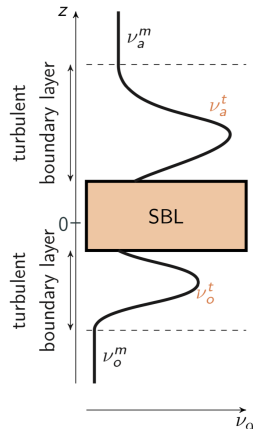
unknowns

constants

parameterizations

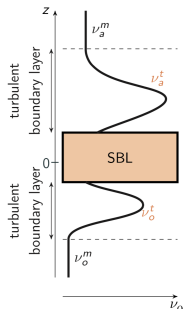
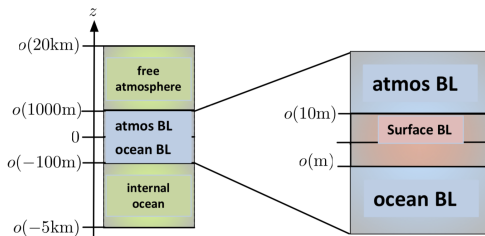
turbulent boundary layer

$$\underbrace{\nu_\alpha^m}_{\text{molecular}} \ll \underbrace{\nu_\alpha^t}_{\text{turbulent}}$$



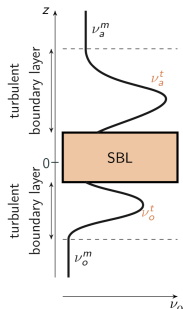
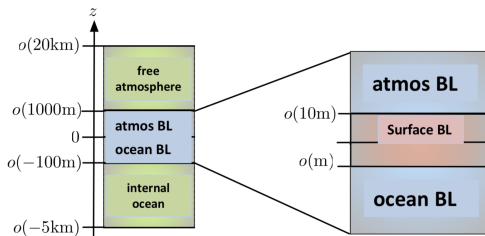
The Surface Boundary Layer

Given the values of the state variables on both sides of the surface boundary layer, what are the fluxes at the interface $z = 0$?



The Surface Boundary Layer

Given the values of the state variables on both sides of the surface boundary layer, what are the fluxes at the interface $z = 0$?



- ▶ Monin-Obukhov similarity theory (1954) (*generalized law of the wall for stratified fluids*) provides analytical vertical profiles for the state variables.

The frictional scales

- ▶ These profiles depend on so-called **frictional scales**:

$$\mathbf{x}_a^* = (u_a^*, \theta_a^*, q_a^*) \quad \text{and} \quad \mathbf{x}_o^* = (u_o^*, \theta_o^*, s_o^*)$$

- ▶ These frictional scales influence the atmospheric and oceanic boundary layers through eddy viscosities and eddy diffusivities:

$$\nu_\alpha^t = \nu_\alpha^t(\mathbf{x}_\alpha^*, z) \quad \mathcal{K}_{\alpha,x}^t = \mathcal{K}_{\alpha,x}^t(\mathbf{x}_\alpha^*, z)$$

(numerous turbulence models)

- ▶ Fluxes are also deduced from these frictional scales:

At $z = z_\alpha^1$:

$$\nu_\alpha^t \partial_z \mathbf{u}_h = \tau / \rho_{0,\alpha} = (u_\alpha^*)^2 \mathbf{e}_\tau$$

$$\mathcal{K}_{\alpha,\theta}^t \partial_z \theta = u_\alpha^* \theta_\alpha^* + \mathcal{F}_{\theta,\text{ext}}^\alpha$$

$$\mathcal{K}_{a,q}^t \partial_z q = u_a^* q_a^* + \mathcal{F}_{q,\text{ext}}^a$$

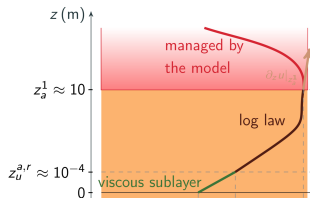
$$\mathcal{K}_{\alpha,S}^t \partial_z S = u_o^* s_o^* + \mathcal{F}_{q,\text{ext}}^o$$

$\mathbf{u}_h = (u, v)$ and $\mathcal{F}_{x,\text{ext}}^\alpha$ known ext. flux

Usual approach in OA models: the non stratified case

- ▶ Law of the wall in the atmosphere (e.g. Mohammadi et al., 1998):

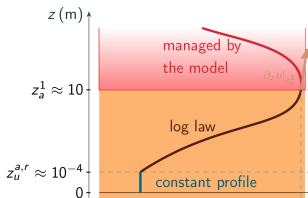
$$\left\{ \begin{array}{l} \frac{\partial u}{\partial z} = \frac{u_a^*}{\kappa z} \quad z \in (z_u^{a,r}; z_a^1) \\ \frac{\partial u}{\partial z} = \frac{u_a^*}{\kappa z_u^{a,r}} \quad z \in (0; z_u^{a,r}) \end{array} \right.$$



Usual approach in OA models: the non stratified case

- ▶ Law of the wall in the atmosphere (e.g. Mohammadi et al., 1998):

$$\begin{cases} \frac{\partial u}{\partial z} = \frac{u_a^*}{\kappa z} & z \in (z_u^{a,r}; z_a^1) \\ \frac{\partial u}{\partial z} = 0 & z \in (0; z_u^{a,r}) \end{cases}$$



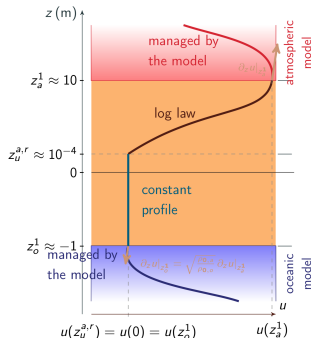
Usual approach in OA models: the non stratified case

- ▶ Law of the wall in the atmosphere (e.g. Mohammadi et al., 1998):

$$\begin{cases} \frac{\partial u}{\partial z} = \frac{u_a^*}{\kappa z} & z \in (z_u^{a,r}; z_a^1) \\ \frac{\partial u}{\partial z} = 0 & z \in (0; z_u^{a,r}) \end{cases}$$

- ▶ Constant profile in the ocean:

$$\frac{\partial u}{\partial z} = 0 \quad z \in (z_o^1; 0)$$



Usual approach in OA models: the non stratified case

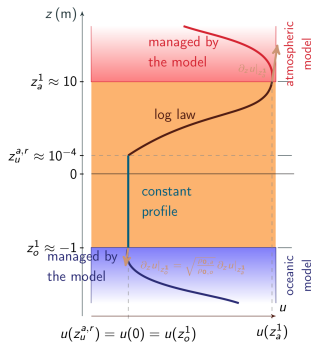
- ▶ Law of the wall in the atmosphere (e.g. Mohammadi et al., 1998):

$$\begin{cases} \frac{\partial u}{\partial z} = \frac{u_a^*}{\kappa z} & z \in (z_u^{a,r}; z_a^1) \\ \frac{\partial u}{\partial z} = 0 & z \in (0; z_u^{a,r}) \end{cases}$$

- ▶ Constant profile in the ocean:

$$\frac{\partial u}{\partial z} = 0 \quad z \in (z_o^1; 0)$$

- ▶ Viscous scale: $z_u^{a,r} = z_u^{a,r}(u_a^*)$



Usual approach in OA models: the non stratified case

- ▶ Law of the wall in the atmosphere (e.g. Mohammadi et al., 1998):

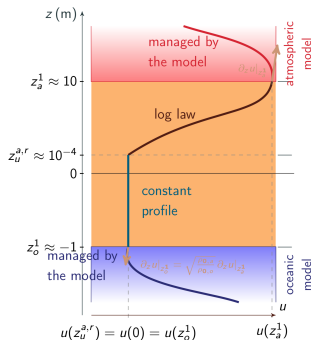
$$\begin{cases} \frac{\partial u}{\partial z} = \frac{u_a^*}{\kappa z} & z \in (z_u^{a,r}; z_a^1) \\ \frac{\partial u}{\partial z} = 0 & z \in (0; z_u^{a,r}) \end{cases}$$

- ▶ Constant profile in the ocean:

$$\frac{\partial u}{\partial z} = 0 \quad z \in (z_o^1; 0)$$

- ▶ Viscous scale: $z_u^{a,r} = z_u^{a,r}(u_a^*)$

Then: $(u(z_a^1), u(z_o^1)) \implies u_a^*$. Hence $u(z)$ in the SBL, $\tau \dots$



Usual approach in OA models: the non stratified case

- ▶ Law of the wall in the atmosphere (e.g. Mohammadi et al., 1998):

$$\begin{cases} \frac{\partial u}{\partial z} = \frac{u_a^*}{\kappa z} & z \in (z_u^{a,r}; z_a^1) \\ \frac{\partial u}{\partial z} = 0 & z \in (0; z_u^{a,r}) \end{cases}$$

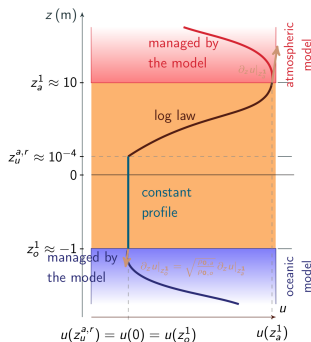
- ▶ Constant profile in the ocean:

$$\frac{\partial u}{\partial z} = 0 \quad z \in (z_o^1; 0)$$

- ▶ Viscous scale: $z_u^{a,r} = z_u^{a,r}(u_a^*)$

Then: $(u(z_a^1), u(z_o^1)) \implies u_a^*$. Hence $u(z)$ in the SBL, $\tau \dots$

An issue: lack of regularity and of symmetry



Usual approach in OA models: the non stratified case

- ▶ Law of the wall in the atmosphere (e.g. Mohammadi et al., 1998):

$$\begin{cases} \frac{\partial u}{\partial z} = \frac{u_a^*}{\kappa z} & z \in (z_u^{a,r}; z_a^1) \\ \frac{\partial u}{\partial z} = 0 & z \in (0; z_u^{a,r}) \end{cases}$$

- ▶ Constant profile in the ocean:

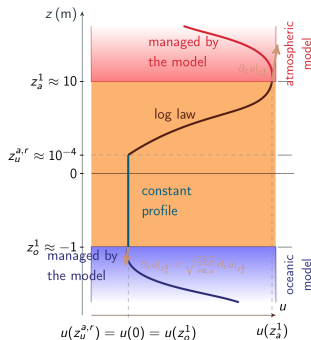
$$\frac{\partial u}{\partial z} = 0 \quad z \in (z_o^1; 0)$$

- ▶ Viscous scale: $z_u^{a,r} = z_u^{a,r}(u_a^*)$

Then: $(u(z_a^1), u(z_o^1)) \implies u_a^*$. Hence $u(z)$ in the SBL, $\tau \dots$

An issue: lack of regularity and of symmetry

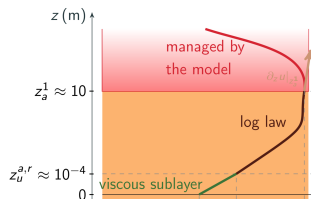
Goal: integrate these missing layers in the parameterizations (C. Pelletier, PhD thesis 2018)



An improved approach: the non stratified case

- ▶ Law of the wall in the atmosphere (e.g. Mohammadi et al., 1998):

$$\left\{ \begin{array}{l} \frac{\partial u}{\partial z} = \frac{u_a^*}{\kappa z} \quad z \in (z_u^{a,r}; z_a^1) \\ \frac{\partial u}{\partial z} = \frac{u_a^*}{\kappa z_u^{a,r}} \quad z \in (0; z_u^{a,r}) \end{array} \right.$$



An improved approach: the non stratified case

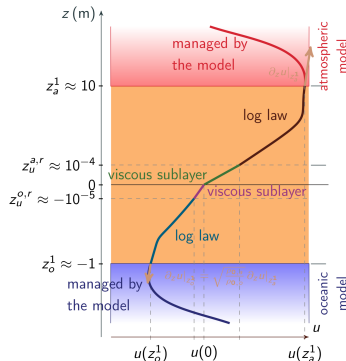
- ▶ Law of the wall in the atmosphere (e.g. Mohammadi et al., 1998):

$$\begin{cases} \frac{\partial u}{\partial z} = \frac{u_a^*}{\kappa z} & z \in (z_u^{a,r}; z_a^1) \\ \frac{\partial u}{\partial z} = \frac{u_a^*}{\kappa z_u^{a,r}} & z \in (0; z_u^{a,r}) \end{cases}$$

- ▶ the same in the ocean:

$$\begin{cases} \frac{\partial u}{\partial z} = \frac{u_o^*}{\kappa (-z_u^{o,r})} & z \in (z_u^{o,r}; 0) \\ \frac{\partial u}{\partial z} = \frac{u_o^*}{\kappa (-z)} & z \in (z_o^1; z_u^{o,r}) \end{cases}$$

with $u(0^+) = u(0^-)$ $\nu_a^m \partial_z u(0^+) = \nu_o^m \partial_z u(0^-)$



An improved approach: the non stratified case

- Law of the wall in the atmosphere (e.g. Mohammadi et al., 1998):

$$\begin{cases} \frac{\partial u}{\partial z} = \frac{u_a^*}{\kappa z} & z \in (z_u^{a,r}; z_a^1) \\ \frac{\partial u}{\partial z} = \frac{u_a^*}{\kappa z_u^{a,r}} & z \in (0; z_u^{a,r}) \end{cases}$$

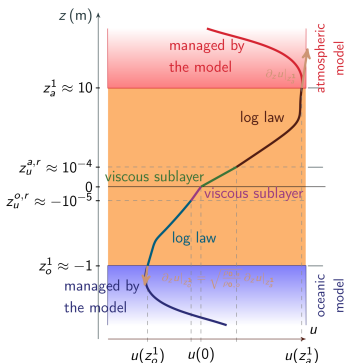
- the same in the ocean:

$$\begin{cases} \frac{\partial u}{\partial z} = \frac{u_o^*}{\kappa (-z_u^{o,r})} & z \in (z_u^{o,r}; 0) \\ \frac{\partial u}{\partial z} = \frac{u_o^*}{\kappa (-z)} & z \in (z_o^1; z_u^{o,r}) \end{cases}$$

with $u(0^+) = u(0^-)$ $\nu_a^m \partial_z u(0^+) = \nu_o^m \partial_z u(0^-)$

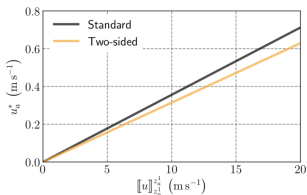
- Viscous scales: $z_u^{a,r} = z_u^{a,r}(u_a^*)$, $z_u^{o,r} = z_u^{o,r}(u_o^*)$

Then: $(u(z_a^1), u(z_o^1)) \implies u_a^*, u_o^*$. Hence $u(z)$ in the whole SBL, $\tau \dots$

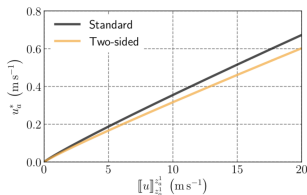


Numerical comparison

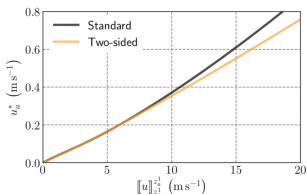
- Decrease of 10%-20% for u_a^* , i.e. 20%-40% for τ



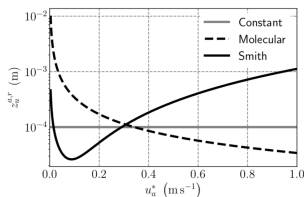
(a) $z_{it}^{a,r} = z_{it,0}^{a,r}$ (constant)



(b) $z_{it}^{a,r} = \frac{v_a^m}{\kappa u_a^*}$ ("molecular")



(c) $z_{it}^{a,r} = \frac{a}{g_0} (u_a^*)^2 + b \frac{v_a^m}{u_a^*}$ (Smith, 1988)



(d) $z_{it}^{a,r}$ for the used parameterization schemes, using realistic values for u_a^* .

Generalization: the stratified case

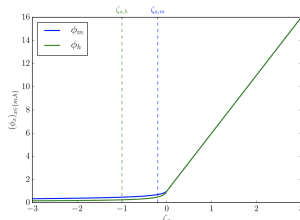
Monin-Obukhov similarity theory (1954) (dry thermodynamically-stratified surface layer)

- ▶ Generalization of the law of the wall

$$\left\{ \begin{array}{l} \frac{\partial u}{\partial z} = \frac{u_a^* e^{i\varphi}}{\kappa z} \Phi_m \left(\frac{z}{L_O^a} \right) \quad z \in (z_u^{a,r}; z_a^1) \\ \frac{\partial \theta}{\partial z} = \frac{\theta_a^*}{\kappa z} \Phi_h \left(\frac{z}{L_O^a} \right) \quad z \in (z_\theta^{a,r}; z_a^1) \end{array} \right.$$

with

- ▶ $L_O^a = \frac{(u_a^*)^2}{g\kappa} \frac{\theta(z_a^1)}{\theta_a^*}$ Obukhov length
- ▶ Φ_m, Φ_h empirical stability functions
- ▶ $\varphi \in [0, 2\pi[$ wind stress direction



Stability functions
(Large et al., 1994)

Generalization: the stratified case

Monin-Obukhov similarity theory (1954) (dry thermodynamically-stratified surface layer)

- ▶ Generalization of the log-layer

$$\begin{cases} \frac{\partial u}{\partial z} = \frac{u_a^* e^{i\varphi}}{\kappa z} \Phi_m \left(\frac{z}{L_O^a} \right) & z \in (z_u^{a,r}; z_a^1) \\ \frac{\partial \theta}{\partial z} = \frac{\theta_a^*}{\kappa z} \Phi_h \left(\frac{z}{L_O^a} \right) & z \in (z_\theta^{a,r}; z_a^1) \end{cases}$$

- ▶ Constant fluxes in the surface boundary layer

$$\begin{cases} \tau^a = \rho_a (u_a^*)^2 e^{i\varphi} & z \in (z_u^{a,r}; z_a^1) \\ Q_H^a = \rho_a C_p u_a^* \theta_a^* & z \in (z_\theta^{a,r}; z_a^1) \end{cases}$$

Generalization: the stratified case

By integration:

$$\left\{ \begin{array}{l} \left| \llbracket u \rrbracket_{z_u^{a,r}}^{z_a^1} \right| = \frac{u_a^* e^{i\varphi}}{\kappa} \left[\ln \left(\frac{z_a^1}{z_u^{a,r}} \right) - \psi_m \left(\frac{z_a^1}{L_O^a} \right) + \psi_m \left(\frac{z_u^{a,r}}{L_O^a} \right) \right] \\ \llbracket \theta \rrbracket_{z_\theta^{a,r}}^{z_a^1} = \frac{\theta_a^*}{\kappa} \left[\ln \left(\frac{z_a^1}{z_\theta^{a,r}} \right) - \psi_h \left(\frac{z_a^1}{L_O^a} \right) + \psi_h \left(\frac{z_\theta^{a,r}}{L_O^a} \right) \right] \end{array} \right.$$

Generalization: the stratified case

By integration:

$$\begin{cases} \left| \llbracket u \rrbracket_{z_u^{a,r}}^{z_a^1} \right| = \frac{u_a^* e^{i\varphi}}{\kappa} \left[\ln \left(\frac{z_a^1}{z_u^{a,r}} \right) - \psi_m \left(\frac{z_a^1}{L_O^a} \right) + \psi_m \left(\frac{z_u^{a,r}}{L_O^a} \right) \right] \\ \llbracket \theta \rrbracket_{z_\theta^{a,r}}^{z_a^1} = \frac{\theta_a^*}{\kappa} \left[\ln \left(\frac{z_a^1}{z_\theta^{a,r}} \right) - \psi_h \left(\frac{z_a^1}{L_O^a} \right) + \psi_h \left(\frac{z_\theta^{a,r}}{L_O^a} \right) \right] \end{cases}$$

which is simplified by neglecting the stratification in the viscous sublayers ($0, z_x^{a,r}$)

$$\begin{cases} \left| \llbracket u \rrbracket_{z_u^{a,r}}^{z_a^1} \right| = \frac{u_a^* e^{i\varphi}}{\kappa} \left[\ln \left(\frac{z_a^1}{z_u^{a,r}} \right) - \psi_m \left(\frac{z_a^1}{L_O^a} \right) \right] \\ \llbracket \theta \rrbracket_{z_\theta^{a,r}}^{z_a^1} = \frac{\theta_a^*}{\kappa} \left[\ln \left(\frac{z_a^1}{z_\theta^{a,r}} \right) - \psi_h \left(\frac{z_a^1}{L_O^a} \right) \right] \end{cases}$$

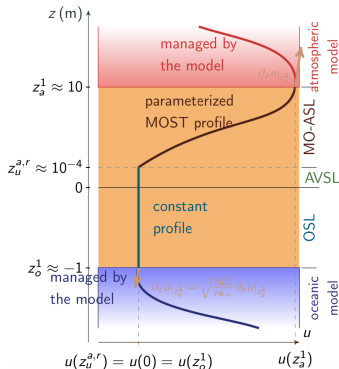
Usual practice in coupled OA models

- ▶ Constant profiles in AVSL and OSL:

$$\frac{\partial u}{\partial z} = \frac{\partial \theta}{\partial z} = 0 \quad z \in (z_o^1, z_x^{a,r})$$

- ▶ Closure parameterizations:

$$z_x^{a,r} = z_x^{a,r} (u_a^*, \theta_a^*)$$



→ nonlinear system of equations for $\mathbf{x}_a^* = (u_a^*, \theta_a^*)$

$$\begin{cases} \left| \left[\frac{u}{\theta} \right]_{z_o^1}^{z_a^1} \right| = \frac{u_a^* e^{i\varphi}}{\kappa} \left[\ln \left(\frac{z_a^1}{z_u^{a,r}(\mathbf{x}_a^*)} \right) - \psi_m \left(\frac{z_a^1}{L_O^a(\mathbf{x}_a^*)} \right) \right] \\ \left[\theta \right]_{z_o^1}^{z_a^1} = \frac{\theta_a^*}{\kappa} \left[\ln \left(\frac{z_a^1}{z_\theta^{a,r}(\mathbf{x}_a^*)} \right) - \psi_h \left(\frac{z_a^1}{L_O^a(\mathbf{x}_a^*)} \right) \right] \end{cases}$$

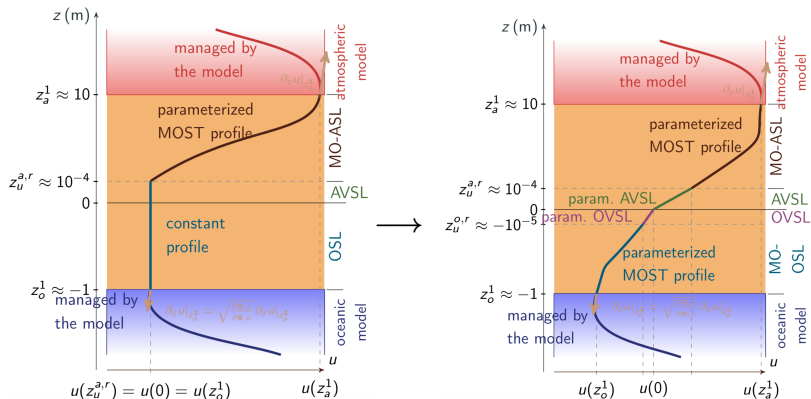
Usual practice in coupled OA models

$$\begin{cases} \left| \left[[u]_{z_o^1}^{z_a^1} \right] \right| &= \frac{u_a^* e^{i\varphi}}{\kappa} \left[\ln \left(\frac{z_a^1}{z_u^{a,r}(\mathbf{x}_a^*)} \right) - \psi_m \left(\frac{z_a^1}{L_O^a(\mathbf{x}_a^*)} \right) \right] \\ \left[[\theta]_{z_o^1}^{z_a^1} \right] &= \frac{\theta_a^*}{\kappa} \left[\ln \left(\frac{z_a^1}{z_\theta^{a,r}(\mathbf{x}_a^*)} \right) - \psi_h \left(\frac{z_a^1}{L_O^a(\mathbf{x}_a^*)} \right) \right] \end{cases}$$

- ▶ These so-called “**bulk formulas**” are solved by fixed-point iterations.
- ▶ Several such formulas have been developed (e.g. Fairall et al., 2002; Large, 2006; ECMWF, 2016), corresponding to different parameterizations for $z^{a,r}$, (ψ_m, ψ_h) , and the addition of other physical effects (gustiness, warm layers...). Calibration is done w.r.t. field experiment data.
- ▶ Typically: 20-30% uncertainty on the estimated fluxes

Improved approach

Goal: taking into account the atmospheric viscous layer and the oceanic surface layer in the bulk formulas (symmetry of the problem, regularity of the solution profile)



Toward a full parameterization of OA surface layers

AVSL Atmospheric Viscous Sub-Layer

- ▶ Introduction of a new viscous coordinate: $z \mapsto \ell_x^a(z)$, allowing a unified formalism with Monin-Obukhov theory
- ▶ Existing parameterizations of AVSL correspond to particular choices of $\ell_x^a(z)$ (e.g. Liu et al, 1979)
- ▶ Mathematical regularity is ensured

OSL Oceanic Surface Layer

- ▶ Symmetry with ASL (Monin-Obukhov + oceanic viscous coordinate $\ell_x^o(z)$)
- ▶ $\mathbf{x}_a^* \rightarrow \mathbf{x}_o^*$ via equality of turbulent fluxes: $\rho_a \nu_a^t \partial_z \mathbf{u}_h|_{z_a^1} = \rho_o \nu_o^t \partial_z \mathbf{u}_h|_{z_o^1}$
- ▶ Param. OVSL \iff AVSL

Toward a full parameterization of OA surface layers

$$\begin{aligned}
 \frac{\kappa \left\| \left[\begin{array}{c} \mathbf{u}_h \\ z_a^1 \\ z_o^1 \end{array} \right] \right\|}{u_a^*} &= \ln \left(\frac{z_a^1}{z_u^{a,r}(\mathbf{x}_a^*)} \right) - \psi_m \left(\frac{z_a^1}{L_a^a(\mathbf{x}_a^*)} \right) + e - 1 && \text{AVSL} \\
 &+ \lambda_u (e - 1) && \text{OVSL} \\
 &+ \lambda_u \ln \left(\frac{z_o^1}{z_u^{o,r}(\mathbf{x}_o^*)} \right) - \lambda_u \psi_m \left(\frac{-z_o^1}{L_o^o(\mathbf{x}_o^*)} \right) && \text{OSL}
 \end{aligned}$$

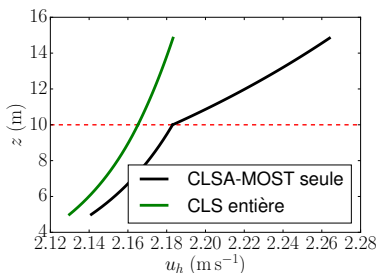
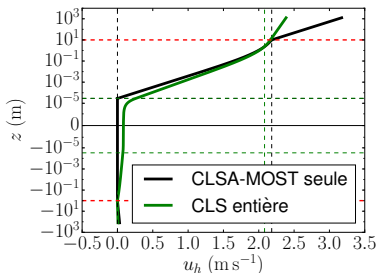
$\lambda_u = \sqrt{\rho_{0,a} / \rho_{0,o}}$

Toward a full parameterization of OA surface layers

$$\frac{\kappa \left\| \left[\begin{array}{c} \mathbf{u}_h \\ z_a^1 \\ z_o^1 \end{array} \right] \right\|}{u_a^*} = \ln \left(\frac{z_a^1}{z_{u^*,r}^a(x_a^*)} \right) - \psi_m \left(\frac{z_a^1}{L_a^a(x_a^*)} \right) + e - 1 \quad \text{AVSL}$$

$$+ \lambda_u (e - 1) \quad \text{OVSL} \quad \lambda_u = \sqrt{\rho_{0,a} / \rho_{0,o}}$$

$$+ \lambda_u \ln \left(\frac{z_o^1}{z_{u^*,r}^o(x_o^*)} \right) - \lambda_u \psi_m \left(\frac{-z_o^1}{L_o^o(x_o^*)} \right) \quad \text{OSL}$$



An example of profile of u

Toward a full parameterization of OA surface layers

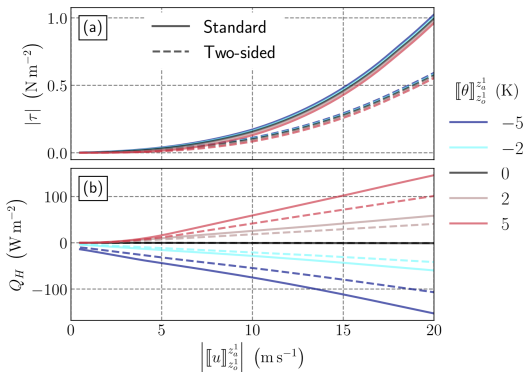
$$\frac{\kappa \left\| \left[\mathbf{u}_h \right]_{z_a^1}^1 \right\|}{u_a^*} = \ln \left(\frac{z_a^1}{z_{u,r}^a(x_a^*)} \right) - \psi_m \left(\frac{z_a^1}{L_O^a(x_a^*)} \right) + e - 1 \quad \text{AVSL}$$

$$+ \lambda_u (e - 1) \quad \text{OVSL}$$

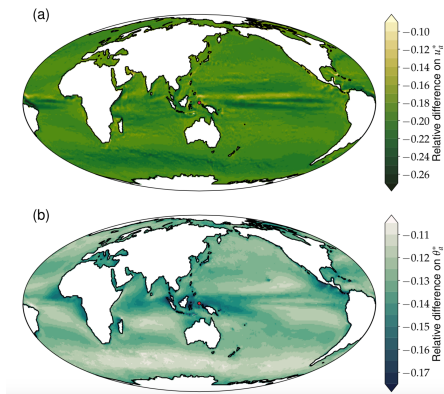
$$+ \lambda_u \ln \left(\frac{z_o^1}{z_{u,r}^o(x_o^*)} \right) - \lambda_u \psi_m \left(\frac{-z_o^1}{L_O^o(x_o^*)} \right) \quad \text{OSL}$$

$$\lambda_u = \sqrt{\rho_{0,a} / \rho_{0,o}}$$

Impact on
the fluxes



Impact: off-line tests with atmospheric and oceanic reanalyses

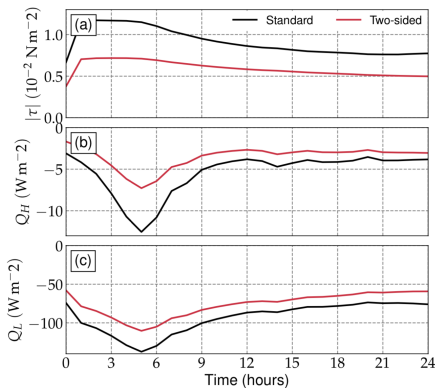


- ▶ systematic decrease (10-20%)
- ▶ more marked for winds than for temperature
- ▶ largest relative impact along the equator (weak winds)

*Yearly 2006 mean of the relative differences
on u_a^* and θ_a^**

Impact: a 1-D realistic simulation (work in progress)

- ▶ Coupled 24-hour 1-D simulation (Meso-NH / surface ocean model)
 - ▶ Allows the atmospheric BL to adjust to mitigated turbulent fluxes
 - ▶ Allows the thermodynamics to drift in time

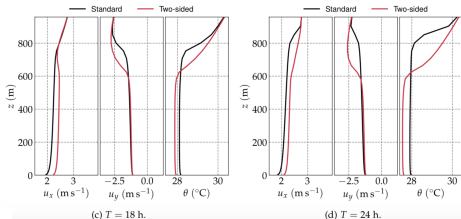
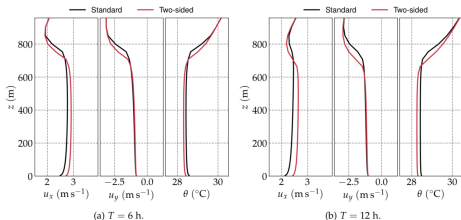


Time series of turbulent fluxes τ , Q_H and Q_L

- ▶ $\simeq 20\text{-}30\%$ weaker fluxes

Impact: a 1-D realistic simulation (work in progress)

- ▶ Coupled 24-hour 1-D simulation (Meso-NH / surface ocean model)
 - ▶ Allows the atmospheric BL to adjust to mitigated turbulent fluxes
 - ▶ Allows the thermodynamics to drift in time



- ▶ Impact in the whole turbulent layer
- ▶ Larger drift for the temperature (probably because dynamics is more constrained by lateral forcings than thermodynamics in such a 1D case)

Vertical profiles of \mathbf{u} and θ in the first 900 meters

Impact: a 1-D realistic simulation (work in progress)

- ▶ To be investigated further
- ▶ To be complemented by 3D simulations

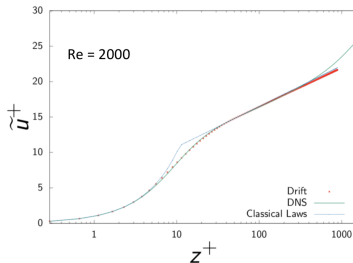
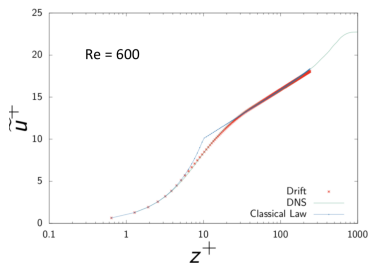
Pelletier C., F. Lemarié, E. Blayo, J.-L. Redelsperger and P.-E. Brilouet, 2019: A two-sided turbulent surface layer parameterization for the computation of air-sea fluxes. *In preparation*.

Remark: approach under location uncertainty

PhD thesis of B. Pinier, supervised by R. Lewandovski and E. Mémin (2019)

$$\frac{d\mathbf{X}_t}{dt} = \tilde{\mathbf{u}}(\mathbf{X}_t, t) + \sigma(\mathbf{X}_t, t) \frac{d\mathbf{B}_t}{dt} \quad \text{with} \quad \begin{cases} \tilde{\mathbf{u}}(\mathbf{x}, t) \text{ deterministic "large scale" velocity} \\ \sigma(\mathbf{x}, t) \text{ convolution (diffusion) kernel} \\ \mathbf{B} \text{ Brownian motion function} \end{cases}$$

- ▶ derivation of Navier-Stokes equations *under location uncertainty*
- ▶ derivation of a modified wall law expression, with a natural regular transition between the viscous sublayer and the log-layer



B. Pinier, E. Mémin, S. Laizet and R. Lewandowski, 2019: Stochastic flow approach to model the mean velocity profile of wall-bounded flows. *Phys. Rev. E* **99**, 063101.

Remark: approach under location uncertainty

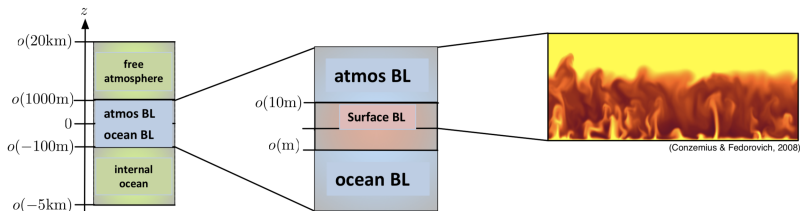
ULU approach for a new model of OA surface boundary layer?

- ▶ Extension to the stratified case
- ▶ Connexion with the turbulent BL, then extension to OA coupling
- ▶ This approach probably allows “natural” connexions all along in the vertical column
- ▶ Nothing miraculous: hypotheses/approximations on σ

ULU approach to revisit present deterministic modeling?

- ▶ What are the “deterministic” hypotheses equivalent to the hypotheses on σ ?
- ▶ Does it imply a natural buffer zone?
- ▶ Does it imply a whole regular coupled vertical profile?
- ▶ Can we draw a parallel with the previously presented approach?

Estimation of OA fluxes: in summary



$$\begin{cases} \tau = \rho_a C_D |[\mathbf{u}]_o^a| [\mathbf{u}]_o^a \\ Q_H = \rho_a C_p^a C_H |[\mathbf{u}]_o^a| [\theta]_o^a \end{cases}$$

$[\mathbf{u}]_o^a, [\theta]_o^a =$ jumps of \mathbf{u} and θ

C_D, C_H : exchange coefficients given by (complicated implicit) bulk formulas
 $= f([\mathbf{u}]_o^a, [\theta]_o^a, [q]_o^a, z_{\text{atm}}, z_{\text{oce}}, \dots)$

Keywords: Monin-Obukhov theory (wall law + stratified fluid), bulk aerodynamic formulas...

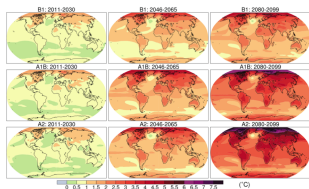
Current modeling issues

- ▶ Addition of other effects (< 1 day): gustiness, diurnal warm layers... (COCO project)
- ▶ A very challenging issue: effects of waves on turbulent fluxes
- ▶ Well-posedness issues - coherence of the different parameterizations

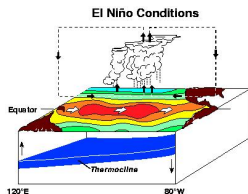
$$\left\{ \begin{array}{l} \frac{\partial u}{\partial t}(z, t) - \frac{\partial}{\partial z} \left(\nu(z, |u(z_1^a)|) \frac{\partial u}{\partial z}(z, t) \right) = f \quad z \in (z_1^a, z^\infty) \\ \nu(z_1^a, |u(z_1^a)|) \frac{\partial u}{\partial z}(z_1^a) = C_D |u(z_1^a)| u(z_1^a) \\ u(z^\infty) = u_G \end{array} \right.$$

Context

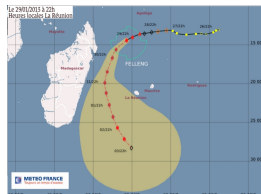
Various applications require coupling an oceanic model and an atmospheric model.



climate modeling



seasonal forecasts



short term predictions

Objectives

- ▶ Modeling aspects: improve the representation of air-sea interactions in current coupled models
- ▶ Algorithmic aspects: revisit the coupling strategies presently used in such systems

Algorithmic issues

We have to solve:

$$\begin{cases} \mathcal{L}_{\text{atm}} \mathbf{U}^{\text{a}} = f_{\text{atm}} & \text{in } \Omega_{\text{atm}} \times [0, T] \\ \mathcal{L}_{\text{oce}} \mathbf{U}^{\text{o}} = f_{\text{oce}} & \text{in } \Omega_{\text{oce}} \times [0, T] \\ \mathcal{F}_{\text{atm}} \mathbf{U}^{\text{a}} = \mathcal{F}_{\text{oce}} \mathbf{U}^{\text{o}} = F_{\text{OA}}(\mathbf{U}^{\text{a}}, \mathbf{U}^{\text{o}}, \mathcal{R}) & \text{on } \Gamma \times [0, T] \end{cases}$$

Algorithmic issues

We have to solve:

$$\begin{cases} \mathcal{L}_{\text{atm}} \mathbf{U}^{\text{a}} = f_{\text{atm}} & \text{in } \Omega_{\text{atm}} \times [0, T] \\ \mathcal{L}_{\text{oce}} \mathbf{U}^{\text{o}} = f_{\text{oce}} & \text{in } \Omega_{\text{oce}} \times [0, T] \\ \mathcal{F}_{\text{atm}} \mathbf{U}^{\text{a}} = \mathcal{F}_{\text{oce}} \mathbf{U}^{\text{o}} = F_{\text{OA}}(\mathbf{U}^{\text{a}}, \mathbf{U}^{\text{o}}, \mathcal{R}) & \text{on } \Gamma \times [0, T] \end{cases}$$

Two practical approaches:

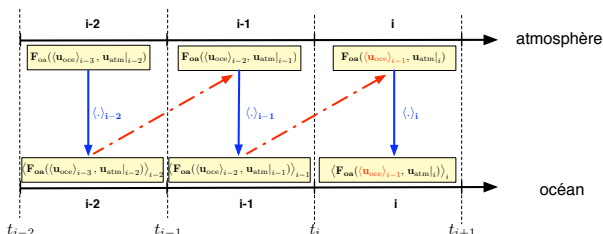
- ▶ Asynchronous coupling by time windows (averaged fluxes)
- ▶ Synchronous coupling at the time step (instantaneous fluxes)

Asynchronous coupling (time windows: $[0, T] = \cup_{i=1}^M [t_i, t_{i+1}]$)

$$\begin{cases} \mathcal{L}_{\text{atm}} \mathbf{U}^a &= f_{\text{atm}} & \text{in } \Omega_{\text{atm}} \times [t_i, t_{i+1}] \\ \mathcal{F}_{\text{atm}} \mathbf{U}^a &= F_{\text{OA}}(\langle \mathbf{U}^o \rangle_{i-1}, \mathbf{U}^a, \mathcal{R}) & \text{on } \Gamma \times [t_i, t_{i+1}] \end{cases}$$

then

$$\begin{cases} \mathcal{L}_{\text{oce}} \mathbf{U}^o &= f_{\text{oce}} & \text{in } \Omega_{\text{oce}} \times [t_i, t_{i+1}] \\ \mathcal{F}_{\text{oce}} \mathbf{U}^o &= \langle \mathcal{F}_{\text{atm}} \mathbf{U}^a \rangle_i & \text{on } \Gamma \times [t_i, t_{i+1}] \end{cases}$$



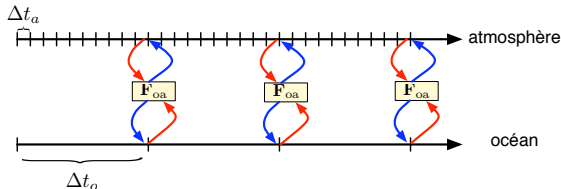
- ▶ same mean fluxes over each time window $[t_i, t_{i+1}]$
- ▶ ... but synchrony issue

Synchronous coupling at the time step

$$\begin{cases} \mathcal{L}_{\text{atm}} \mathbf{U}^a & = f_{\text{atm}} & \text{in } \Omega_{\text{atm}} \times [t_i, t_i + N \Delta t_a] \\ \mathcal{F}_{\text{atm}} \mathbf{U}^a & = F_{\text{OA}}(\mathbf{U}^o(t_i), \mathbf{U}^a(t), \mathcal{R}(t)) & \text{on } \Gamma \times [t_i, t_i + N \Delta t_a] \end{cases}$$

and

$$\begin{cases} \mathcal{L}_{\text{oce}} \mathbf{U}^o & = f_{\text{oce}} & \text{in } \Omega_{\text{oce}} \times [t_i, t_i + \Delta t_o] \\ \mathcal{F}_{\text{oce}} \mathbf{U}^o & = F_{\text{OA}}(\mathbf{U}^o(t_i), \mathbf{U}^a(t_i), \mathcal{R}(t_i)) & \text{on } \Gamma \times [t_i, t_i + \Delta t_o] \end{cases}$$



- ▶ still a (much smaller) synchrony issue
- ▶ difficult to implement efficiently
- ▶ validity issue

Stability analysis

Model problem: 1-D diffusion with variable and discontinuous coefficients

$$\begin{aligned} \frac{\partial u_{\text{atm}}}{\partial t} - \frac{\partial}{\partial z} \left(\nu_{\text{atm}}(z) \frac{\partial u_{\text{atm}}}{\partial z} \right) &= f_{\text{atm}} && \text{in } \Omega_{\text{atm}} \times [0, T] \\ \frac{\partial u_{\text{oce}}}{\partial t} - \frac{\partial}{\partial z} \left(\nu_{\text{oce}}(z) \frac{\partial u_{\text{oce}}}{\partial z} \right) &= f_{\text{oce}} && \text{in } \Omega_{\text{oce}} \times [0, T] \end{aligned}$$

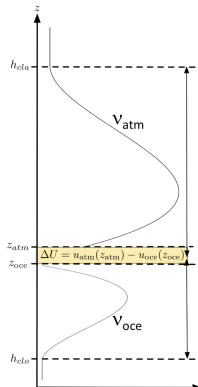
with interface conditions:

Dirichlet-Neumann:
$$\begin{cases} u_{\text{atm}}(0, t) = u_{\text{oce}}(0, t) \\ \nu_{\text{oce}}(0) \frac{\partial u_{\text{oce}}}{\partial z}(0, t) = \nu_{\text{atm}}(0) \frac{\partial u_{\text{atm}}}{\partial z}(0, t) \end{cases}$$

or

linearized bulk:

$$\nu_{\text{atm}}(0) \frac{\partial u_{\text{atm}}}{\partial z}(0, t) = \nu_{\text{oce}}(0) \frac{\partial u_{\text{oce}}}{\partial z}(0, t) = \alpha (u_{\text{atm}}(0^+, t) - u_{\text{oce}}(0^-, t))$$



Stability analysis: synchronous coupling

Consider a natural discretization scheme: Euler + implicit \rightarrow **each model is unconditionally stable.**

Stability analysis: synchronous coupling

Consider a natural discretization scheme: Euler + implicit \rightarrow **each model is unconditionally stable.**

- ▶ Stability analysis shows that **the coupled scheme is unstable for usual configurations** of ocean-atmosphere models

$$\text{stable iff } \begin{cases} \Delta z_{\text{atm}} \ll \Delta z_{\text{oce}} & \text{for Dirichlet-Neumann} \\ \alpha \leq \min \left(\frac{\nu_{\text{oce}}(0)}{\Delta z_{\text{oce}}} \frac{\rho_{\text{oce}}}{\rho_{\text{atm}}}; \frac{\nu_{\text{atm}}(0)}{\Delta z_{\text{atm}}} \right) & \text{for linearized bulk} \end{cases}$$

Lemarié F., E. Blayo and L. Debreu, 2015: Analysis of ocean-atmosphere coupling algorithms: consistency and stability issues. *Procedia Computer Science*, **51**, 2066-2075.

- ▶ This does not mean that realistic coupled OA models systematically blow up, due to other processes (additional diffusion and viscosity).

Beljaars A., E. Dutra, G. Balsamo and F. Lemarié, 2017: On the numerical stability of surface-atmosphere coupling in weather and climate models. *Geosci. Model Dev.*, **10**, 977-989.

Conclusion on current coupling methods

- ▶ Current coupling methods are simple ad-hoc algorithms, which ensure that fluxes are balanced and are computationally cheap.
- ▶ These methods are inadequate from a numerical point of view.

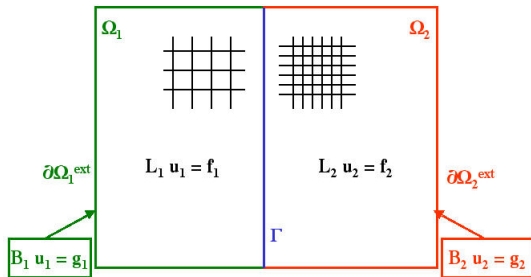
Conclusion on current coupling methods

- ▶ Current coupling methods are simple ad-hoc algorithms, which ensure that fluxes are balanced and are computationally cheap.
- ▶ These methods are inadequate from a numerical point of view.

Issues:

- ▶ Can we improve the coupling method ?
- ▶ Does it improve the physics of the coupled solution ?
- ▶ Can this be done for a reasonable CPU cost ?

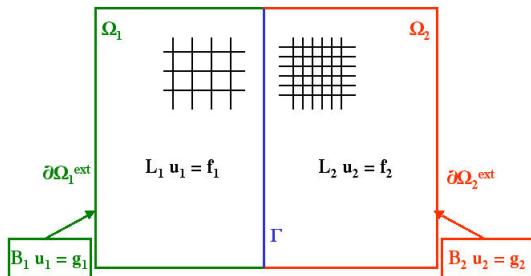
A convenient framework: Schwarz methods



$$\left\{ \begin{array}{l} L_1 u_1 = f_1 \quad \Omega_1 \times [0, T] \\ u_1 \text{ given} \quad \text{at } t = 0 \\ B_1 u_1 = g_1 \quad \partial\Omega_1^{\text{ext}} \times [0, T] \end{array} \right. \quad \left\{ \begin{array}{l} L_2 u_2 = f_2 \quad \Omega_2 \times [0, T] \\ u_2 \text{ given} \quad \text{at } t = 0 \\ B_2 u_2 = g_2 \quad \partial\Omega_2^{\text{ext}} \times [0, T] \end{array} \right.$$

+ physical constraints at the interface : $\mathcal{F}(u_1, u_2) = 0$

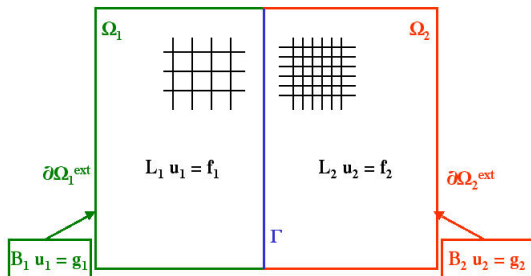
A convenient framework: Schwarz methods



$$\left\{ \begin{array}{ll} L_1 u_1 = f_1 & \Omega_1 \times [0, T] \\ u_1 \text{ given} & \text{at } t = 0 \\ B_1 u_1 = g_1 & \partial\Omega_1^{\text{ext}} \times [0, T] \\ C u_1 = C' u_2 & \Gamma \times [0, T] \end{array} \right. \quad \left\{ \begin{array}{ll} L_2 u_2 = f_2 & \Omega_2 \times [0, T] \\ u_2 \text{ given} & \text{at } t = 0 \\ B_2 u_2 = g_2 & \partial\Omega_2^{\text{ext}} \times [0, T] \\ D u_2 = D' u_1 & \Gamma \times [0, T] \end{array} \right.$$

$$\text{with } (C u_1 = C' u_2 \text{ and } D u_2 = D' u_1) \iff \mathcal{F}(u_1, u_2) = 0$$

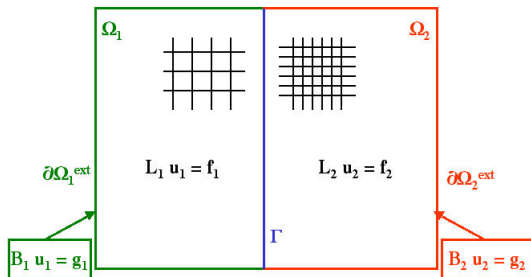
A convenient framework: Schwarz methods



$$\left\{ \begin{array}{ll} L_1 u_1^{k+1} = f_1 & \Omega_1 \times [0, T] \\ u_1^{k+1} \text{ given} & \text{at } t = 0 \\ B_1 u_1^{k+1} = g_1 & \partial\Omega_1^{\text{ext}} \times [0, T] \\ C u_1^{k+1} = C' u_2^k & \Gamma \times [0, T] \end{array} \right. \quad \left\{ \begin{array}{ll} L_2 u_2^{k+1} = f_2 & \Omega_2 \times [0, T] \\ u_2^{k+1} \text{ given} & \text{at } t = 0 \\ B_2 u_2^{k+1} = g_2 & \partial\Omega_2^{\text{ext}} \times [0, T] \\ D u_2^{k+1} = D' u_1^k & \Gamma \times [0, T] \end{array} \right.$$

$$\text{with } (C u_1 = C' u_2 \text{ and } D u_2 = D' u_1) \iff \mathcal{F}(u_1, u_2) = 0$$

A convenient framework: Schwarz methods



$$\left\{ \begin{array}{ll} L_1 u_1^{k+1} = f_1 & \Omega_1 \times [0, T] \\ u_1^{k+1} \text{ given} & \text{at } t = 0 \\ B_1 u_1^{k+1} = g_1 & \partial\Omega_1^{\text{ext}} \times [0, T] \\ C u_1^{k+1} = C' u_2^k & \Gamma \times [0, T] \end{array} \right. \quad \left\{ \begin{array}{ll} L_2 u_2^{k+1} = f_2 & \Omega_2 \times [0, T] \\ u_2^{k+1} \text{ given} & \text{at } t = 0 \\ B_2 u_2^{k+1} = g_2 & \partial\Omega_2^{\text{ext}} \times [0, T] \\ D u_2^{k+1} = D' u_1^k & \Gamma \times [0, T] \end{array} \right.$$

$$\text{with } (C u_1 = C' u_2 \text{ and } D u_2 = D' u_1) \iff \mathcal{F}(u_1, u_2) = 0$$

- Present ocean-atmosphere coupling methods correspond to **one single iteration of a Schwarz-like coupling method**

Toward iterative ocean-atmosphere coupling

$$\begin{cases} \mathcal{L}_{\text{atm}} \mathbf{U}^a = f_{\text{atm}} & \text{in } \Omega_{\text{atm}} \times [t_i, t_{i+1}] \\ \mathcal{F}_{\text{atm}} \mathbf{U}^a = F_{\text{OA}}(\mathbf{U}^o, \mathbf{U}^a, \mathcal{R}) & \text{on } \Gamma \times [t_i, t_{i+1}] \end{cases}$$

$$\begin{cases} \mathcal{L}_{\text{oce}} \mathbf{U}^o = f_{\text{oce}} & \text{in } \Omega_{\text{oce}} \times [t_i, t_{i+1}] \\ \mathcal{F}_{\text{oce}} \mathbf{U}^o = \mathcal{F}_{\text{atm}} \mathbf{U}^a & \text{on } \Gamma \times [t_i, t_{i+1}] \end{cases}$$

Toward iterative ocean-atmosphere coupling

Iterate until convergence

$$\begin{cases} \mathcal{L}_{\text{atm}} \mathbf{U}_{k+1}^{\text{a}} = f_{\text{atm}} & \text{in } \Omega_{\text{atm}} \times [t_i, t_{i+1}] \\ \mathcal{F}_{\text{atm}} \mathbf{U}_{k+1}^{\text{a}} = F_{\text{OA}}(\mathbf{U}_k^{\text{o}}, \mathbf{U}_{k+1}^{\text{a}}, \mathcal{R}_{k+1}) & \text{on } \Gamma \times [t_i, t_{i+1}] \end{cases}$$

then

$$\begin{cases} \mathcal{L}_{\text{oce}} \mathbf{U}_{k+1}^{\text{o}} = f_{\text{oce}} & \text{in } \Omega_{\text{oce}} \times [t_i, t_{i+1}] \\ \mathcal{F}_{\text{oce}} \mathbf{U}_{k+1}^{\text{o}} = \mathcal{F}_{\text{atm}} \mathbf{U}_{k+1}^{\text{a}} & \text{on } \Gamma \times [t_i, t_{i+1}] \end{cases}$$

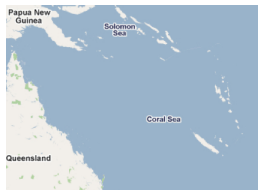
Impact on the physics

A major difficulty There is no idealized coupled ocean-atmosphere testcase with a known reference solution.

Impact on the physics

Simulation of the **tropical cyclone Erica (2003)**, by coupling

- ▶ ROMS: primitive equation ocean model (Shchepetkin-McWilliams, 2005)
- ▶ WRF: non hydrostatic atmospheric model (Skamarock-Klemp, 2007)



$$\Delta x_{\text{atm}} = 35\text{km}, \Delta t_{\text{atm}} = 180\text{s}$$

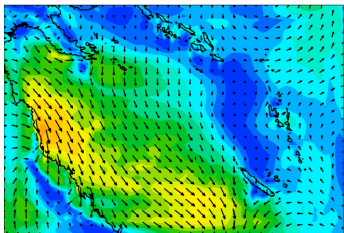
$$\Delta x_{\text{oce}} = 18\text{km}, \Delta t_{\text{oce}} = 1800\text{s}$$

15-day simulation

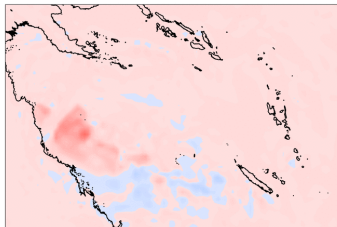
Interface conditions: vertical fluxes for momentum, heat and fresh water

Impact on the physics (cont'd)

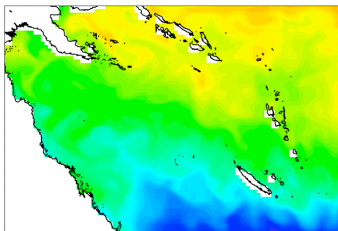
WRF - Vents (m/s) 2003-03-01_07:00:00gmt



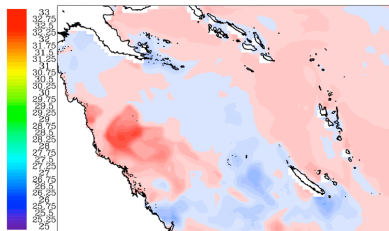
WRF Vents iter9 - iter1 (m/s) 2003-03-01_06:00:00



Sea Surface Temperature (deg C) 2003-03-01_06:00:00



SST iter9 - iter1 (deg C) 2003-03-01_06:00:00



10-meter wind (m/s) and sea surface temperature ($^{\circ}\text{C}$).

Impact on the physics (cont'd)

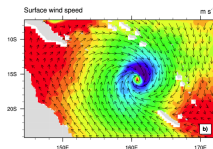
To assess the robustness of the coupled solution: **ensemble simulations**

- ▶ Initial conditions
- ▶ Length of the time windows: 6h vs 3h

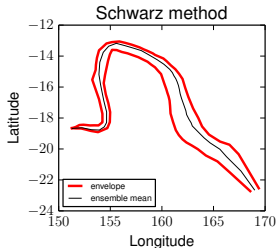
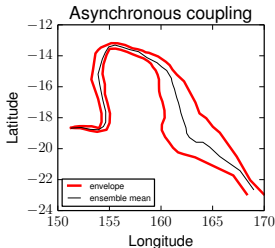
Impact on the physics (cont'd)

To assess the robustness of the coupled solution: **ensemble simulations**

- ▶ Initial conditions
- ▶ Length of the time windows: 6h vs 3h

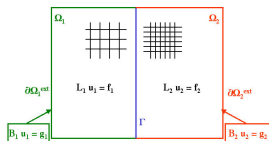


Trajectory of the cyclone



- ▶ The uncertainty on the cyclone trajectory and intensity is decreased by 30%-40%. (see Lemarié et al, 2014, for further details)

Decreasing the cost: absorbing boundary conditions

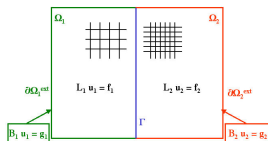


$$\left\{ \begin{array}{ll} L_1 u_1^{k+1} = f_1 & \Omega_1 \times [0, T] \\ u_1^{k+1} \text{ given} & \text{at } t = 0 \\ B_1 u_1^{k+1} = g_1 & \partial\Omega_1^{\text{ext}} \times [0, T] \\ C u_1^{k+1} = C' u_2^k & \Gamma \times [0, T] \end{array} \right. \quad \left\{ \begin{array}{ll} L_2 u_2^{k+1} = f_2 & \Omega_2 \times [0, T] \\ u_2^{k+1} \text{ given} & \text{at } t = 0 \\ B_2 u_2^{k+1} = g_2 & \partial\Omega_2^{\text{ext}} \times [0, T] \\ D u_2^{k+1} = D' u_1^k & \Gamma \times [0, T] \end{array} \right.$$

Systems satisfied by the errors $e_i^k = u_i^k - u_i$:

$$\left\{ \begin{array}{ll} L_1 e_1^{k+1} = 0 & \Omega_1 \times [0, T] \\ e_1^{k+1} = 0 & \text{at } t = 0 \\ B_1 e_1^{k+1} = 0 & \partial\Omega_1^{\text{ext}} \times [0, T] \\ C e_1^{k+1} = C' e_2^k & \Gamma \times [0, T] \end{array} \right. \quad \left\{ \begin{array}{ll} L_2 e_2^{k+1} = 0 & \Omega_2 \times [0, T] \\ e_2^{k+1} = 0 & \text{at } t = 0 \\ B_2 e_2^{k+1} = 0 & \partial\Omega_2^{\text{ext}} \times [0, T] \\ D e_2^{k+1} = D' e_1^k & \Gamma \times [0, T] \end{array} \right.$$

Decreasing the cost: absorbing boundary conditions



$$\left\{ \begin{array}{ll} L_1 u_1^{k+1} = f_1 & \Omega_1 \times [0, T] \\ u_1^{k+1} \text{ given} & \text{at } t = 0 \\ B_1 u_1^{k+1} = g_1 & \partial\Omega_1^{\text{ext}} \times [0, T] \\ C u_1^{k+1} = C' u_2^k & \Gamma \times [0, T] \end{array} \right. \quad \left\{ \begin{array}{ll} L_2 u_2^{k+1} = f_2 & \Omega_2 \times [0, T] \\ u_2^{k+1} \text{ given} & \text{at } t = 0 \\ B_2 u_2^{k+1} = g_2 & \partial\Omega_2^{\text{ext}} \times [0, T] \\ D u_2^{k+1} = D' u_1^k & \Gamma \times [0, T] \end{array} \right.$$

Systems satisfied by the errors $e_i^k = u_i^k - u_i$:

$$\left\{ \begin{array}{ll} L_1 e_1^{k+1} = 0 & \Omega_1 \times [0, T] \\ e_1^{k+1} = 0 & \text{at } t = 0 \\ B_1 e_1^{k+1} = 0 & \partial\Omega_1^{\text{ext}} \times [0, T] \\ C e_1^{k+1} = C' e_2^k & \Gamma \times [0, T] \end{array} \right. \quad \left\{ \begin{array}{ll} L_2 e_2^{k+1} = 0 & \Omega_2 \times [0, T] \\ e_2^{k+1} = 0 & \text{at } t = 0 \\ B_2 e_2^{k+1} = 0 & \partial\Omega_2^{\text{ext}} \times [0, T] \\ D e_2^{k+1} = D' e_1^k & \Gamma \times [0, T] \end{array} \right.$$

If one finds C', D' such that $C' e_2 = 0$ and/or $D' e_1 = 0$, then convergence in 2 iterations. \rightarrow **exact absorbing conditions** (Engquist & Majda, 1977)

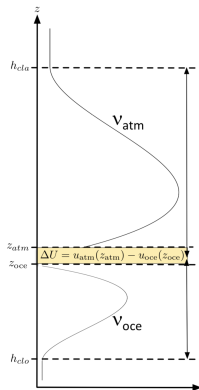
Improving the convergence speed

Model problem: coupling of two Ekman layers

$$\begin{aligned}\partial_t u_{\text{atm}} - f v_{\text{atm}} - \partial_z (\nu_{\text{atm}}(z) \partial_z u_{\text{atm}}) &= F_{\text{atm}}^x \\ \partial_t v_{\text{atm}} + f u_{\text{atm}} - \partial_z (\nu_{\text{atm}}(z) \partial_z v_{\text{atm}}) &= F_{\text{atm}}^y \quad \text{in } \Omega_{\text{atm}} \times [0, T] \\ u_{\text{atm}}(z, t = 0) &= u_0(z) \quad z \in \Omega_{\text{atm}}\end{aligned}$$

$$\begin{aligned}\partial_t u_{\text{oce}} - f v_{\text{oce}} - \partial_z (\nu_{\text{oce}}(z) \partial_z u_{\text{oce}}) &= F_{\text{oce}}^x \\ \partial_t v_{\text{oce}} + f u_{\text{oce}} - \partial_z (\nu_{\text{oce}}(z) \partial_z v_{\text{oce}}) &= F_{\text{oce}}^y \quad \text{in } \Omega_{\text{oce}} \times [0, T] \\ u_{\text{oce}}(z, t = 0) &= u_0(z) \quad z \in \Omega_{\text{oce}}\end{aligned}$$

$$\begin{pmatrix} u_{\text{atm}} \\ v_{\text{atm}} \end{pmatrix} = \begin{pmatrix} u_{\text{oce}} \\ v_{\text{oce}} \end{pmatrix} \quad \text{and} \quad \nu_{\text{atm}}(0) \partial_z \begin{pmatrix} u_{\text{atm}} \\ v_{\text{atm}} \end{pmatrix} = \nu_{\text{oce}}(0) \partial_z \begin{pmatrix} u_{\text{oce}} \\ v_{\text{oce}} \end{pmatrix} \quad \text{on } \Gamma \times [0, T]$$



Difficulties:

- ▶ coupling of u and v
- ▶ variable-in-space diffusivity + discontinuity at $z = 0$

Improving the convergence speed

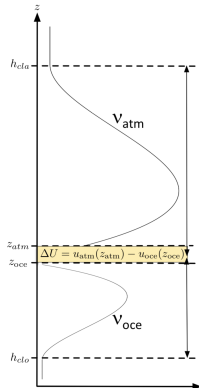
Model problem: coupling of two Ekman layers

Schwarz iteration:

$$\begin{aligned} \partial_t u_{\text{atm}}^k - f v_{\text{atm}}^k - \partial_z \left(\nu_{\text{atm}}(z) \partial_z u_{\text{atm}}^k \right) &= F_{\text{atm}}^x \\ \partial_t v_{\text{atm}}^k + f u_{\text{atm}}^k - \partial_z \left(\nu_{\text{atm}}(z) \partial_z v_{\text{atm}}^k \right) &= F_{\text{atm}}^y && \text{in } \Omega_{\text{atm}} \times \\ u_{\text{atm}}^k(z, t = 0) &= u_0(z) && z \in \Omega_{\text{atm}} \\ \mathcal{C} u_{\text{atm}}^k(0, t) &= \mathcal{C}' u_{\text{oce}}^{k-1}(0, t) && t \in [0, T] \end{aligned}$$

$$\begin{aligned} \partial_t u_{\text{oce}}^k - f v_{\text{oce}}^k - \partial_z \left(\nu_{\text{oce}}(z) \partial_z u_{\text{oce}}^k \right) &= F_{\text{oce}}^x \\ \partial_t v_{\text{oce}}^k + f u_{\text{oce}}^k - \partial_z \left(\nu_{\text{oce}}(z) \partial_z v_{\text{oce}}^k \right) &= F_{\text{oce}}^y && \text{in } \Omega_{\text{oce}} \times [0 \\ u_{\text{oce}}^k(z, t = 0) &= u_0(z) && z \in \Omega_{\text{oce}} \\ \mathcal{D} u_{\text{oce}}^k(0, t) &= \mathcal{D}' u_{\text{atm}}^k(0, t) && t \in [0, T] \end{aligned}$$

for a given $(u_{\text{oce}}^0, v_{\text{oce}}^0)$ on Γ .



Improving the convergence speed (cont'd)

		Constant diffusion	Space dependent diffusion
Steady state	No Coriolis effect	Dubois (2007) opt. Schwarz 2-D adv-diff eq. Gander-Zhang (2016) opt. Schwarz Helmholtz eq.	Lions (1990) convergence Schwarz diffusion eq.
	Coriolis effect		
Time dependent	No Coriolis effect	Gander-Halpern (2002) opt. Schwarz heat eq. Blayo-Rousseau-Tayachi (2017) lin. viscous SW eq. Bennequin-Gander-Gouarin-Halpern (2004) opt. Schwarz 2-D diff-reaction	Lemarié-Debreu-Blayo (2013) opt. Schwarz 1-D diffusion
	Coriolis effect	Martin (2003) opt. Schwarz 2-D SW Audusse-Dreyfuss-Merlet (2010) opt. Schwarz 3-D primitive eqs	

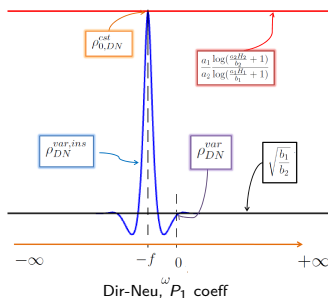
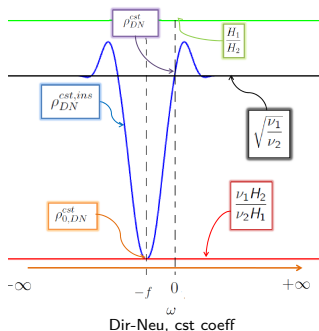
Improving the convergence speed (cont'd)

		Constant diffusion	Space dependent diffusion
Steady state	No Coriolis effect	Dubois (2007) opt. Schwarz 2-D adv-diff eq. Gander-Zhang (2016) opt. Schwarz Helmholtz eq.	Lions (1990) convergence Schwarz diffusion eq.
	Coriolis effect		
Time dependent	No Coriolis effect	Gander-Halpern (2002) opt. Schwarz heat eq. Blayo-Rousseau-Tayachi (2017) lin. viscous SW eq. Bennequin-Gander-Gouarin-Halpern (2004) opt. Schwarz 2-D diff-reaction	Lemarié-Debreu-Blayo (2013) opt. Schwarz 1-D diffusion
	Coriolis effect	Martin (2003) opt. Schwarz 2-D SW Audusse-Dreyfuss-Merlet (2010) opt. Schwarz 3-D primitive eqs	

Improving the convergence speed (cont'd)

Exact analytical expression for the convergence factor:

- ▶ P_0 , P_1 and P_2 diffusion profiles
- ▶ Dirichlet-Neumann and Robin-Robin interface conditions
- ▶ optimized coefficients for Robin-Robin conditions



Blayo E., F. Lemarié, C. Pelletier and S. They, 2019: Coupling two Ekman layers with a Schwarz algorithm. *In preparation.*

Current and future work on coupling algorithms

- ▶ Integrate all these results in a model of the three boundary layers (atmospheric, surface, oceanic)

In collaboration with climate scientists:

- ▶ Build 1D coupled reference test cases (idealized and realistic)
- ▶ Include this coupling strategy in the French climate models
- ▶ Mitigate the cost (perform the iterations only for the boundary layers, model reduction...)

