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A Trilevel Model for Best Response in Energy Demand-Side Management

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Abstract

Demand-side management (DSM) is a powerful tool to efficiently manage the consumption of energy. DSM relies on various techniques and means. In this work, we propose a trilevel energy market model for load shifting induced by time-of-use pricing. Four kinds of actors are involved: electricity suppliers (sell energy), local agents (buy, sell and consume), aggregators (buy and sell) and end users (consume). The interactions among these actors lead to a trilevel multi-leader-multi-follower game. Solving such games is known to be hard, thus we assume that the decision variables of all electricity suppliers but one are known and optimize the decisions of the remaining supplier. This leads to a single-leader-multi-follower game, which aims to compute the leader's best response to the decisions of his competitors.

The trilevel model is first formulated as a bilevel problem using an explicit formula for the lowest optimization level. Solution algorithms are developed in the optimistic case and in a variant named "semi-optimistic" approach leading to more robust solutions. Finally, numerical results highlight the efficiency of the methods and the sensitivity of the solutions with respect to the model parameters.

Keywords:
OR in energy, Demand-side management, Bilevel programming, Trilevel programming
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1. Introduction

Electricity production and consumption are one of the major challenges of today's society. Economical, ecological and political concerns are all at stake. In order to optimize the efficiency

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of the electricity distribution system, many approaches arose. One of them is demand-side management (DSM). The idea behind DSM is that instead of fitting the production to the demand, the demand can be adapted to the production. DSM has been an important topic for the last thirty years ,see Kreith & Goswami (2016), and became more relevant with the introduction of the smart grid paradigm, see Farhangi (2010); Fang et al. (Fourth 2012). The ever growing communication among the actors of the grid indeed allows for a better management of the energy consumption.

Works on DSM include for example Zhu et al. (2012): households possess appliances that need to receive energy with time and quantity constraints. The usage of the appliance is scheduled so as to minimize the load peak. A distributed DSM model is considered in Ramchurn et al. (2011): a case study of the UK shows that efficient DSM could lead to smaller load peaks (-17%) and less carbon gas emissions (-6%). In Arteconi et al. (2012), the use of thermal energy storage for DSM is reviewed, whereas the integration of wind power generation is the object of Moura & de Almeida (2010). DSM can be achieved through many means, but the most used one consists in pricing policies. From a general point of view, Murphy et al. (2018) aims to quantify the effects of price regulation policies. In Gottwalt et al. (2011), the reaction of a household to time-based pricing is considered. Recent works include Devine & Bertsch (2018), where a mixed complementarity problem is used to model electricity consumers offering load shedding on the market. In Laur et al. (2018), a three-stage stochastic flexibility problem is studied: in a market framework, a company wishes to purchase reserve capacities. From the point of view of the market, Le Cadre et al. (2018) is concerned with the DSO-TSO interaction (respectively distribution and transmission system operator). Finally, Tsitsiklis & Xu (2015) proposes a new pricing system, which is not based on the marginal cost, but aims to cover ancillary costs due to fluctuations in the demand.

In this work, we focus on load shifting, a technique of DSM that features a fixed overall demand. To achieve it, price incentives are determined. We thus model and solve the problem of an electricity supplier competing against other suppliers on the market to sell energy to various actors, in a context of demand-side management. More precisely, the energy supplier defines time-dependent prices for his energy, knowing that his clients will adapt by shifting their loads, either directly (local agents) or indirectly (aggregators). This sequential and hierarchical decision-making process can be adequately modeled as a trilevel problem, that is an extension of a bilevel problem.

Bilevel problems formalize the concept of Stackelberg games, and have received a great interest in the past thirty years, see e.g. Dempe (2002); Bard (2010). Let us recall that a bilevel problem is an optimization problem (called the *leader's problem*) in which at least one of the variables is constrained to be solution of another optimization problem (called the *follower's problem*). Applications of bilevel programming are numerous: toll pricing, network design prob-

lems, flight tickets pricing, see e.g. Brotcorne et al. (2008). Bilevel programming is in general NP-hard, even in its simplest form where both objectives and all constraints are linear, see Ben-Ayed & Blair (1990); Labbé et al. (1998). *Multilevel* optimization problems correspond to the more general case where more than two levels of optimization problems interlock. When there is more than one optimization problem at the leader level, the terminology used in the literature is Multi-Leader-Follower-Game (MLFG) if two additional conditions are met. First, the various leaders play in a noncooperation competition context, and second, each of the leaders' problems is at least a multilevel problem (see e.g. Leyffer & Munson (2010)). The model considered in this work is a MLFG in which the leaders' problems are trilevel. Note that this structure of model has been rarely considered in the literature, due to its complexity - see for example Migdalas et al. (1998); Gkatzikis et al. (2013).

Multilevel and MLFG are increasingly used in the energy domain, more precisely on pricing matters. For example, Afşar et al. (2016) presents a bilevel energy pricing model in order to decrease the amplitude of the peak loads, whereas Cervilla et al. (2015) considers a longer time horizon and aims to make the energy system sustainable in a situation where the consumers optimize their investments in distributed generation. In van Ackooij et al. (2018), generation companies aiming to find the cheapest possible production schedule make contracts with micro-grids that can generate or store energy, and can thus help the generation companies to cope with production surplus or lack of generation. In Ambrosius et al. (2018), multilevel problems are used to define price zones within the grid.

In Gkatzikis et al. (2013), the role of aggregators in the energy market is studied, and a trilevel model is proposed. In this model, a system operator aims to optimize his operating costs thanks to load shifts executed by end customers. Aggregators play an intermediary role between the system operator and the end customers. More precisely, the system operator offers a percentage of his benefit to each of the aggregators, and the aggregators offer pecuniary rewards to the end customers they are in contract with to induce a load shift. A heuristic to solve the trilevel model is defined. One limitation of the proposed model is that the cost function of the leader appears in the objective function of the aggregator, a situation that is not realistic for many trading situations since no aggregator is supposed to know the cost function of the supplier.

In this paper, we consider a similar interaction model, but first the cost function of the supplier only appears in his own problem, and second the actors of the intermediary level (ILAs) have the possibility to trade energy among themselves. Moreover we do not use heuristic techniques and our reformulation and resolution of the problem are theoretically thoroughly justified (see propositions 1, 2 and 3).

An important contribution of the paper is the definition of two new solution concepts that are adapted to this context of price management. Indeed, breaking with the classical optimistic approach usually made in case of nonuniqueness of the followers equilibrium solution, we define

the so-called revisited optimistic approach and semi-optimistic approach that are based on some smart selection among the followers' equilibriums, sharply taking into account the structure of the problem.

The paper is structured as follows: first, we present a *Trilevel Demand-Side Management model* (denoted by (TDSM)) and define the actors involved in the model in Section 2. Second, in Section 3, we formulate (TDSM) as a bilevel problem, thanks to an explicit resolution of the lowest level problems. In Section 4, we propose three different approaches to solve the bilevel version of (TDSM). First a classical method, consisting in replacing the lower level problems by their KKT conditions in the leader's problem leads in Subsection 4.1 to the so-called *Classical optimistic* approach. Second, in Subsection 4.2, we simplified the bilevel reformulation of the game by selecting a specific class of Generalized Nash Equilibriums (GNE) among the followers, providing thus the *revisited optimistic* approach. Third, in Subsection 4.3, we define a new class of GNE, called *semi-optimistic*, that simplifies a lot the formulation of the game and therefore its numerical treatment. Finally, in Section 5, we compare and comment the solutions computed through the three approaches.

2. Problem formulation

Four kinds of actors are involved in the (TDSM) problem: electricity suppliers $s \in \mathcal{S}$, local agents $\ell \in \mathcal{L}$, aggregators $a \in \mathcal{A}$, and end users $i \in \mathcal{I}$. Interactions among actors of (TDSM) are described in Figure 1 and are of the following nature:

- being into a "best response approach", we assume that $\mathcal{S}$ is composed of two suppliers, that is $\mathcal{S} = \{\hat{s}, \bar{s}\}$ where $\bar{s}$ (the *competitor*) is a supplier with a fixed price strategy known by $\hat{s}$ (the *leader*) that aims to determine his best strategy;
- for each period of time the suppliers propose electricity prices to the intermediate actors (aggregators and local agents);
- the intermediate actors can buy electricity from the suppliers or from other intermediate actors;
- each end user is in contract with only one aggregator and he can shift a part of his load in order to get some rewards from the aggregator.

The variables and notations of the model are as follows:

- The set of the considered time slots is denoted by H.
- Bold variables indicate vectors.
- $\mathbf{d_x} = \left(d_x^1, \ldots, d_x^{|H|}\right)$ denotes the demand variables of $x \in \mathcal{I} \cup \mathcal{L}$.

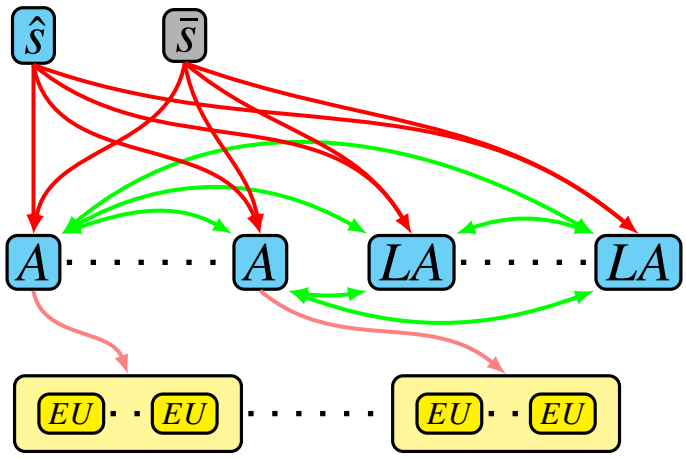

Figure 1: Structure of the model
Blue forms represent the various (dynamic) actors (the leader, the aggregators and the local agents), the grey supplier represents the static competitor, red links show unidirectional energy trades, green links designate bidirectional energy exchanges, and pink links indicate rewards for load shifting. Each aggregator is related to his own set of end users.

- Variables p^h_{xy} denote the prices of an energy unit sold by x to y at time $h \in H$.
- Variables e^h_{xy} represent the energy flow from y to x (bought by x from y) at time h.
- Variables r^h_{ai} denote the rewards given to the end user i by the aggregator a for an energy unit consumed at time $h \in H$. The set of end users that are in contract with the aggregator a is denoted by $\mathcal{I}_a$.
- W_x indicates the overall demand that needs to be satisfied for $x \in \mathcal{I} \cup \mathcal{L}$.

In this paper we denote by *Intermediate Level Actors (ILAs)* the players of the second level (Aggregators and Local agents) in the trilevel structure.

2.1. Local agents

Local agents can buy energy from the suppliers or trade energy with the other ILAs. Each local agent $\ell \in \mathcal{L}$ has a fixed overall demand W_ℓ for the horizon and an *a priori* demand vector $\left(d^{h,0}_\ell\right)_{h\in H}$. This demand can be shifted, resulting in the *real* demand splitting/vector $\left(d^h_\ell\right)_{h\in H}$. The energy bought can be used to satisfy the electricity demand or sold to the other ILAs. In case of shifting of their consumption, the local agents undergo an inconvenience cost that increases with the extent of the load shift. Local agents aim to minimize the sum of their electricity bill and the inconvenience due to the load shift. Formally, the local agent $\ell \in \mathcal{L}$ aims to solve the following

problem:

$$
(P_\ell) \qquad \min_{\mathbf{e}_{\ell\cdot},\mathbf{p}_{\ell\cdot},\boldsymbol{\alpha}_{\ell\cdot}} \sum_{h\in H}\left(\sum_{s\in\mathcal{S}} p^h_{s\ell}e^h_{\ell s} + \sum_{a\in\mathcal{A}}\left(p^h_{a\ell}e^h_{\ell a} - p^h_{\ell a}e^h_{a\ell}\right) + \sum_{\ell'\neq\ell}\left(p^h_{\ell'\ell}e^h_{\ell\ell'} - p^h_{\ell\ell'}e^h_{\ell'\ell}\right) + V^h_\ell\left(d^h_\ell\right)\right)
$$

$$
s.t.\begin{cases} \sum_{h\in H} d^h_\ell = W_\ell & \\ e^h_{\ell x} \geq 0 & \forall h\in H, x\in\mathcal{L}\cup\mathcal{A}\cup\mathcal{S}, \\ p^h_{\ell x} \leq \alpha^h_{\ell xs}p^h_{sx} + \left(1-\alpha^h_{\ell xs}\right)p^h_{s\ell} & \forall h\in H, x\in\mathcal{L}\cup\mathcal{A}, s\in\mathcal{S} \\ \alpha^h_{\ell xs}\in[0,1] & \forall h\in H, x\in\mathcal{L}\cup\mathcal{A}, s\in\mathcal{S}, \end{cases}
$$

where $V^h_\ell\left(d^h_\ell\right) = v^h_\ell\left(d^h_\ell - d^{h,0}_\ell\right)^2$ is the inconvenience caused by the load shifting and $v^h_\ell > 0$ is a fixed inconvenience coefficient. Note that, for any h, the notation d^h_ℓ stands for the demand value and thus $d^h_\ell \overset{not.}{=} \sum_{s\in\mathcal{S}} e^h_{\ell s} + \sum_{a\in\mathcal{A}}\left(e^h_{\ell a} - e^h_{a\ell}\right) + \sum_{\ell'\neq\ell}\left(e^h_{\ell\ell'} - e^h_{\ell'\ell}\right)$ since no storage is considered in this model. The inconvenience function V^h_ℓ is the same as defined in Gkatzikis et al. (2013). This function has nice mathematical properties, like convexity and differentiability, and it adequately models the real inconvenience that the consumers are undergoing. A small shift of the consumption will not represent a significant inconvenience, whereas an important shift will have strong repercussions on the consumer's comfort thanks to the square power.

The first constraint in problem (P_ℓ) ensures that ℓ's cumulated demand remains constant, whereas the second constraint avoids negative energy exchanges. The third and fourth constraints result from a deeper observation. The second (intermediate) level of (TDSM) actually constitutes a noncooperative Nash game, where the players are the ILAs. Therefore, an optimal ILA's response to the leader's decisions is a Nash equilibrium. Since the decision variables of the ILA influence the feasible domains of the other ILAs, it is precisely a Generalized Nash Equilibrium (GNE). Let us recall that a GNE is reached whenever no player can unilaterally improve his situation by modifying his decision variables (see e.g. Facchinei & Kanzow (2007)). Assume that an ILA x buys energy from another ILA y: the quantity is determined by x while the price is determined by y. Thus y can improve his revenue by unilaterally increasing the price, hence showing that without upper bounds on the prices, there cannot be energy exchanges among the ILAs in a GNE. But of course, if there is an upper bound to the prices, this upper bound is reached in any GNE, provided that there is a nonzero exchange between the affected ILAs.

For any $s\in\mathcal{S}$, a first obvious bound for p^h_{yx} (the price offered by y to x at time $h\in H$) is p^h_{sx}, the price offered to x by s. Indeed, x would never buy energy from y if he can buy energy from s at a lower price. However, this bound can lead to undesired GNE. Indeed, let us assume that x buys energy from y: y has to furnish this energy, and thus can buy it from s and if p^h_{sx} is smaller than p^h_{sy}, then y ends up paying for the energy that x needs, even though this situation is a GNE.

As a result from the previous paragraph, if y sells energy to another ILA, he must sell it at least for p^h_{sy} to avoid losses. Therefore, we must have $p^h_{yx} \geq p^h_{sy}$. Since in a GNE, the upper

bound to p^h_{yx} is reached, setting $p^h_{yx} \leq p^h_{sy}$ could make sense. However in this case, all the ILAs have access to energy at the lowest possible price, $\min_{x \in \mathcal{L} \cup \mathcal{A}} p^h_{sx}$. It is thus useless for the leader to offer different prices to different actors.

Under some competition assumptions on the market (further called *competitive case* and *nondiscriminant exogenous pricing*), the forthcoming Proposition 2 shows that thanks to the third and fourth constraints of problem (P_ℓ), if there is a nonzero exchange between two ILA x and y in a GNE of the intermediate level ($e^h_{yx} > 0$), then

$$p^h_{xy} = \max\left\{\min\left\{p^h_{\hat{s}x}, p^h_{\bar{s}x}\right\}, \min\left\{p^h_{\hat{s}y}, p^h_{\bar{s}y}\right\}\right\}.$$

One can thus wonder why to use the artificial α^h_{xys} and third and fourth constraints instead of simply using the above formula for p^h_{xy}. The answer comes from the nonsmoothness of this formula and thus the computational difficulties that it could generate. This is why the approach based on the third and fourth constraints has been preferred.

2.2. Aggregators

Aggregators can buy energy from the suppliers, and trade energy with the other ILAs, in order to satisfy the demand of the end users they are in contract with. They offer rewards to their end users as incentives to shift their consumption. The aggregators aim to minimize the sum of the rewards offered to their end users and the cost of the electricity bought from the suppliers and the other ILAs. The optimization problem of the aggregator $a \in \mathcal{A}$ can be formally expressed as:

$$(P_a) \qquad \min_{\mathbf{e}_{a\cdot}, \mathbf{r}_a, \mathbf{p}_{a\cdot}, \boldsymbol{\alpha}_{a\cdot}} \min_{\mathbf{d}^a} \sum_{h \in H} \left(\sum_{s \in \mathcal{S}} p^h_{sa} e^h_{as} + \sum_{\ell \in \mathcal{L}} \left(p^h_{\ell a} e^h_{a\ell} - p^h_{a\ell} e^h_{\ell a} \right) + \sum_{a' \neq a} \left(p^h_{a'a} e^h_{aa'} - p^h_{aa'} e^h_{a'a} \right) + \sum_{i \in \mathcal{I}_a} r^h_{ai} d^h_i \right)$$

$$s.t. \begin{cases} \sum_{s \in \mathcal{S}} e^h_{as} + \sum_{\ell \in \mathcal{L}} \left(e^h_{a\ell} - e^h_{\ell a} \right) + \sum_{a' \neq a} \left(e^h_{aa'} - e^h_{a'a} \right) = \sum_{i \in \mathcal{I}_a} d^h_i & \forall h \in H \\ \mathbf{d}^a_i \in \operatorname{argmax}(P_i) & \forall i \in \mathcal{I}_a \\ p^h_{ax} \leq \alpha^h_{axs} p^h_{sx} + \left(1 - \alpha^h_{axs}\right) p^h_{sa} & \forall h \in H, x \in \mathcal{L} \cup \mathcal{A}, s \in \mathcal{S} \\ r^h_{ai}, e^h_{ax} \geq 0 & \forall h \in H, x \in \mathcal{L} \cup \mathcal{A}, i \in \mathcal{I}_a \\ \alpha^h_{axs} \in [0, 1] & \forall h \in H, x \in \mathcal{L} \cup \mathcal{A}, s \in \mathcal{S} \end{cases}$$

where $\mathbf{d}^a$ stands for the vector $\mathbf{d}^a = \left(\mathbf{d}^a_i\right)_{i \in \mathcal{I}_a}$, with $\mathbf{d}^a_i = \left(d^h_i\right)_{h \in H}$, (P_i) is the end user i's problem and $\mathcal{I}_a$ denotes the set of end users who are in contract with aggregator a. It is here assumed that each end user is in contract with only one aggregator.

The first constraint ensures that the outcome of the energy exchanges equals the overall demand of the end users. The second constraint makes sure that the demand of the end users is optimal. The utility of the third and last constraints has been discussed in Subsection 2.1.

2.3. End users

The end users are in contract with one of the aggregators that provides energy to them. They need to receive a given amount W_i of energy during the time horizon, and they receive rewards from the aggregator as an incentive to shift their energy consumption. However, shifting their consumption induces an inconvenience, which is similar to the inconvenience for the local agents. Therefore, the problem (P_i) of the end user $i \in \mathcal{I}$ can be expressed as:

$$(P_i) \qquad \max_{\mathbf{d}_i} \sum_{h \in H} r_{ai}^h d_i^h - V_i^h\left(d_i^h\right)$$
$$s.t. \begin{cases} \displaystyle\sum_{h \in H} d_i^h = W_i \\ d_i^h \geq 0 \qquad\qquad \forall h \in H, \end{cases}$$

where d_i^h denotes i's demand at time h, W_i denotes i's overall need in electricity and $V_i^h\left(d_i^h\right) = v_i^h\left(d_i^h - d_i^{h,0}\right)^2$ is the inconvenience caused by the load shifting ($v_i^h > 0$ is fixed). As for the local agents, $\left(d_i^{h,0}\right)_h$ stands for the *a priori* demand vector of end user i.

2.4. Electricity supplier

The electricity supplier $\hat{s}$ sells energy to aggregators and to local agents. Knowing the fixed price strategy $\mathbf{p}_{\bar{s}}$ of supplier $\bar{s}$, supplier $\hat{s}$ aims to maximize his profit, which is the difference between the revenues resulting from sales and his production costs, described by the function $c_{\hat{s}}^h(\cdot)$. The electricity supplier influences the demand of the customers by setting hourly prices $p_{\hat{s}x}^h$, knowing that the clients will react optimally to these prices. Therefore, the problem of the supplier $\hat{s}$ can be expressed as:

$$(P_{\hat{s}}) \qquad \max_{\mathbf{p}_{\hat{s}\cdot}} \max_{\mathbf{e},\mathbf{r},\mathbf{p}_{x\cdot},\boldsymbol{\alpha}} \sum_{h \in H} \left(\sum_{a \in \mathcal{A}} p_{\hat{s}a}^h e_{a\hat{s}}^h + \sum_{\ell \in \mathcal{L}} p_{\hat{s}\ell}^h e_{\ell\hat{s}}^h - c_{\hat{s}}^h \left(\sum_{a \in \mathcal{A}} e_{a\hat{s}}^h + \sum_{\ell \in \mathcal{L}} e_{\ell\hat{s}}^h \right) \right)$$
$$s.t. \begin{cases} p_{\hat{s}x}^h \geq 0 & \forall h \in H, x \in \mathcal{L} \cup \mathcal{A} \\ \{\mathbf{e}_{\ell\cdot}, \mathbf{p}_{\ell\cdot}, \boldsymbol{\alpha}_{\ell\cdot}\} \in \operatorname{argmax}(P_\ell) & \forall \ell \in \mathcal{L} \\ \{\mathbf{e}_{a\cdot}, \mathbf{p}_{a\cdot}, \boldsymbol{\alpha}_{a\cdot}, \mathbf{r}_{a\cdot}\} \in \operatorname{argmax}(P_a) & \forall a \in \mathcal{A}, \end{cases}$$

We assume here that the function $c_{\hat{s}}^h : t \mapsto c_{\hat{s}}^h(t)$ is increasing and convex for all $h \in H$. According to the previous notations, (P_a) and (P_ℓ) respectively denote the optimization problems of the aggregator $a \in \mathcal{A}$ and the local agent $\ell \in \mathcal{L}$.

In this paper, we will mainly consider the following two cases:

- *competitive case*: in this case, we assume that the leader is competitive compared to the competitor, which means that the leader can take over the whole demand and make profit with lower prices than the competitor. That is, for any $h \in H$, the function $\left(d_x^h\right)_{x \in \mathcal{L} \cup \mathcal{A}} \mapsto$

$\sum_{x\in\mathcal{L}\cup\mathcal{A}} p^h_{\bar{s}x} d^h_x - c^h_{\hat{s}}\left(\sum_{x\in\mathcal{L}\cup\mathcal{A}} d^h_x\right)$ is increasing with relation to each variable. Observe in particular that when the cost function of the leader is linear with factor $k^h_{\hat{s}}$, the condition is fulfilled if for all $x \in \mathcal{L}\cup\mathcal{A}$, $p^h_{\bar{s}x} \geq k^h_{\hat{s}}$.

- *nondiscriminant exogenous pricing*: in this case, the competitor does not discriminate between the various consumers, thus for all $h \in H$ and all $x, y \in \mathcal{L}\cup\mathcal{A}$, $p^h_{\bar{s}x} = p^h_{\bar{s}y}$.

Assuming that at least one of these cases is verified allows us to prove some interesting results (see propositions 2 and 3), which lead to efficient ways of solving the leader's problem $(P_{\hat{s}})$.

3. From a trilevel model to a bilevel model

Being a trilevel problem, (TDSM) is challenging to solve. Our goal is, deeply taking advantage of the special structure of the model, to rewrite this trilevel problem as a single-level optimization problem with complementarity constraints (MPCC). First, we give explicit formulas for the end users' problems and plug them into the aggregators' problems to obtain a bilevel reformulation (Subsection 3). To solve this bilevel problem, we then apply the classical method consisting in the replacement of the followers' problems by their KKT conditions in the leader's problem.

We next build an alternative bilevel formulation $\left(P'_{\hat{s}}\right)$ of the leader's problem and prove that any solution of the bilevel problem $\left(P'_{\hat{s}}\right)$ provides a solution of the initially considered trilevel problem $(P_{\hat{s}})$. For a given $a \in \mathcal{A}$, let us first rewrite the aggregator a's minimization problem (P_a) as

$$\min_{\mathbf{e}_{a\cdot},\mathbf{p}_{a\cdot},\boldsymbol{\alpha}_{a\cdot}} \min_{\mathbf{r}_a,\mathbf{d}^a} \varphi_a\left(\mathbf{e}_a, \mathbf{p}_a, \alpha_a\right) + \sum_{h\in H}\sum_{i\in\mathcal{I}_a} r^h_{ai} d^h_i$$

$$s.t.\begin{cases} \sum_{s\in\mathcal{S}} e^h_{as} + \sum_{\ell\in\mathcal{L}}\left(e^h_{a\ell} - e^h_{\ell a}\right) + \sum_{a'\neq a}\left(e^h_{aa'} - e^h_{a'a}\right) = \sum_{i\in\mathcal{I}_a} d^h_i & \forall h \in H \\ \mathbf{d}^a_i \in \operatorname{argmax}\left(P_i\right) & \forall i \in \mathcal{I}_a \\ p^h_{ax} \leq \alpha^h_{axs} p^h_{sx} + \left(1 - \alpha^h_{axs}\right) p^h_{sa} & \forall h \in H, x \in \mathcal{L}\cup\mathcal{A}, s \in \mathcal{S} \\ r^h_{ai}, e^h_{ax} \geq 0 & \forall h \in H, x \in \mathcal{L}\cup\mathcal{A}, i \in \mathcal{I}_a \\ \alpha^h_{axs} \in [0,1] & \forall h \in H, x \in \mathcal{L}\cup\mathcal{A}, s \in \mathcal{S} \end{cases}$$

setting $\varphi_a\left(\mathbf{e}_a, \mathbf{p}_a, \alpha_a\right) = \sum_{h\in H}\left(\sum_{s\in\mathcal{S}} p^h_{sa} e^h_{as} + \sum_{\ell\in\mathcal{L}}\left(p^h_{\ell a} e^h_{a\ell} - p^h_{a\ell} e^h_{\ell a}\right) + \sum_{a'\neq a}\left(p^h_{a'a} e^h_{aa'} - p^h_{aa'} e^h_{a'a}\right)\right)$ and let

us define the associated problem (P'_a)

$$(P'_a) \qquad \min_{\mathbf{e}_{a\cdot},\mathbf{p}_{a\cdot},\boldsymbol{\alpha}_{a\cdot}} \min_{\mathbf{d}^a,\lambda^a} \ \varphi_a(\mathbf{e}_a,\mathbf{p}_a,\alpha_a) + \sum_{h\in H}\sum_{i\in\mathcal{I}_a} 2v_i^h d_i^h\left(d_i^h - d_i^{h,0}\right) + \sum_{i\in\mathcal{I}_a} W_i\hat{\lambda}_i$$

$$s.t.\begin{cases} \sum_{s\in\mathcal{S}} e_{as}^h + \sum_{\ell\in\mathcal{L}}\left(e_{a\ell}^h - e_{\ell a}^h\right) + \sum_{a'\neq a}\left(e_{aa'}^h - e_{a'a}^h\right) = \sum_{i\in\mathcal{I}_a} d_i^h & \forall h\in H \\ \sum_{h\in H} d_i^h = W_i & \forall i\in\mathcal{I}_a \\ 2v_i^h\left(d_i^h - d_i^{h,0}\right) + \lambda_i \geq 0 & \forall h\in H, \forall i\in\mathcal{I}_a \\ p_{ax}^h \leq \alpha_{axs}^h p_{sx}^h + \left(1-\alpha_{axs}^h\right)p_{sa}^h & \forall h\in H, x\in\mathcal{L}\cup\mathcal{A}, s\in\mathcal{S} \\ d_i^h, e_{ax}^h \geq 0 & \forall h\in H, x\in\mathcal{L}\cup\mathcal{A}, i\in\mathcal{I}_a \\ \alpha_{axs}^h \in [0,1] & \forall h\in H, x\in\mathcal{L}\cup\mathcal{A}, s\in\mathcal{S} \end{cases}$$

Proposition 1. *Let a belong to $\mathcal{A}$, and assume that for any $i\in\mathcal{I}_a$ and any $h\in H$, the inconvenience coefficient v_i^h is positive. If $\left(\mathbf{e}_{a\cdot},\mathbf{p}_{a\cdot},\boldsymbol{\alpha}_{a\cdot},\mathbf{d}^a,\hat{\boldsymbol{\lambda}}^a\right)$ is an optimal solution for (P'_a) then $(\mathbf{e}_{a\cdot},\mathbf{p}_{a\cdot},\boldsymbol{\alpha}_{a\cdot},\mathbf{d}^a,\mathbf{r}_a)$ is an optimal solution for (P_a), with $r_{ai}^h = 2v_i^h\left(d_i^h - d_i^{h,0}\right) + \hat{\lambda}_i$, for any $i\in\mathcal{I}_a$ and any $h\in H$.*

Proof. Observe that for all $i\in\mathcal{I}_a$, the domain of (P_i) is nonempty, compact, and convex, and the objective function of (P_i) is strictly convex. Therefore, it has a unique solution $\mathbf{d}_i^a$. Furthermore, observe that Slater's condition is verified for any $\mathbf{r}_{ai}$, since $W_i > 0$. It follows from Theorem 2.3 of Dempe & Dutta (2012) that if $\left(\mathbf{e}_{a\cdot},\mathbf{p}_{a\cdot},\boldsymbol{\alpha}_{a\cdot},\mathbf{r}_a,\mathbf{d}^a,\hat{\boldsymbol{\lambda}}^a,\boldsymbol{\mu}^a\right)$ is a global solution of

$$\left(P_a^{MPCC}\right) \qquad \min_{\mathbf{e}_{a\cdot},\mathbf{p}_{a\cdot},\boldsymbol{\alpha}_{a\cdot},\mathbf{r}^a,\mathbf{d}^a,\boldsymbol{\mu}^a,\hat{\boldsymbol{\lambda}}^a} \ \varphi_a(\mathbf{e}_a,\mathbf{p}_a,\boldsymbol{\alpha}_a) + \sum_{h\in H}\sum_{i\in\mathcal{I}_a} r_{ai}^h d_i^h$$

$$s.t.\begin{cases} \sum_{s\in\mathcal{S}} e_{as}^h + \sum_{\ell\in\mathcal{L}}\left(e_{a\ell}^h - e_{\ell a}^h\right) + \sum_{a'\neq a}\left(e_{aa'}^h - e_{a'a}^h\right) = \sum_{i\in\mathcal{I}_a} d_i^h & \forall h\in H \\ \sum_{h\in H} d_i^h = W_i & \forall i\in\mathcal{I}_a \\ \mu_i^h d_i^h = 0 & \forall h\in H \\ -r_i^h + 2v_i^h\left(d_i^h - d_i^{h,0}\right) + \hat{\lambda}_i - \mu_i^h = 0 & \forall h\in H \\ p_{ax}^h \leq \alpha_{axs}^h p_{sx}^h + \left(1-\alpha_{axs}^h\right)p_{sa}^h & \forall h\in H, x\in\mathcal{L}\cup\mathcal{A}, s\in\mathcal{S} \\ d_i^h, e_{ax}^h, r_{ai}^h, \mu_i^h \geq 0 & \forall h\in H, x\in\mathcal{S}\cup\mathcal{L}\cup\mathcal{A}, i\in\mathcal{I}_a \\ \alpha_{axs}^h \in [0,1] & \forall h\in H, x\in\mathcal{L}\cup\mathcal{A}, s\in\mathcal{S}, \end{cases}$$

then $(\mathbf{e}_{a\cdot},\mathbf{p}_{a\cdot},\boldsymbol{\alpha}_{a\cdot},\mathbf{r}_a,\mathbf{d}^a)$ is a global solution of (P_a).

We immediately deduce that, for all $h\in H$ and all $i\in\mathcal{I}_a$,

$$r_{ai}^h = 2v_i^h\left(d_i^h - d_i^{h,0}\right) + \hat{\lambda}_i - \mu_i^h,$$

and thus replacing r_{ai} in $\left(P_a^{MPCC}\right)$ yields

$$\left(P_a^{MPCC-r}\right) \qquad \min_{\mathbf{e}_{a\cdot},\mathbf{p}_{a\cdot},\boldsymbol{\alpha}_{a\cdot},\mathbf{d}^a,\boldsymbol{\mu}^a,\hat{\boldsymbol{\lambda}}^a} \quad \varphi_a\left(\mathbf{e}_a,\mathbf{p}_a,\boldsymbol{\alpha}_a\right) + \sum_{h\in H}\sum_{i\in\mathcal{I}_a}\left(2v_i^h\left(d_i^h - d_i^{h,0}\right) + \hat{\lambda}_i - \mu_i^h\right)d_i^h$$

$$s.t.\begin{cases} \sum_{s\in\mathcal{S}} e_{as}^h + \sum_{\ell\in\mathcal{L}}\left(e_{a\ell}^h - e_{\ell a}^h\right) + \sum_{a'\neq a}\left(e_{aa'}^h - e_{a'a}^h\right) = \sum_{i\in\mathcal{I}_a} d_i^h & \forall h\in H \\ \sum_{h\in H} d_i^h = W_i & \forall i\in\mathcal{I}_a \\ \mu_i^h d_i^h = 0 & \forall h\in H \\ 2v_i^h\left(d_i^h - d_i^{h,0}\right) + \hat{\lambda}_i - \mu_i^h \geq 0 & \forall h\in H \\ p_{ax}^h \leq \alpha_{axs}^h p_{sx}^h + \left(1-\alpha_{axs}^h\right)p_{sa}^h & \forall h\in H, x\in\mathcal{L}\cup\mathcal{A}, s\in\mathcal{S} \\ d_i^h, e_{ax}^h, \mu_i^h \geq 0 & \forall h\in H, x\in\mathcal{S}\cup\mathcal{L}\cup\mathcal{A}, i\in\mathcal{I}_a \\ \alpha_{axs}^h \in [0,1] & \forall h\in H, x\in\mathcal{L}\cup\mathcal{A}, s\in\mathcal{S}. \end{cases}$$

Now, assume that we have an optimal solution $\left(\mathbf{e}_{a\cdot},\mathbf{p}_{a\cdot},\boldsymbol{\alpha}_{a\cdot},\mathbf{d}^a,\boldsymbol{\mu}^a,\hat{\boldsymbol{\lambda}}^a\right)$ of the problem $\left(P_a^{MPCC-r}\right)$. Consider $\left(\mathbf{e}_{a\cdot}^* = \mathbf{e}_{a\cdot}, \mathbf{p}_{a\cdot}^* = \mathbf{p}_{a\cdot}, \boldsymbol{\alpha}_{a\cdot}^* = \boldsymbol{\alpha}_{a\cdot}, \mathbf{d}^{*a} = \mathbf{d}^a, \boldsymbol{\mu}^{*a} = 0, \hat{\boldsymbol{\lambda}}^{*a} = \hat{\boldsymbol{\lambda}}^a\right)$. Obviously, $\left(\mathbf{e}_{a\cdot}^*,\mathbf{p}_{a\cdot}^*,\boldsymbol{\alpha}_{a\cdot}^*,\mathbf{d}^{*a},\boldsymbol{\mu}^{*a},\hat{\boldsymbol{\lambda}}^{*a}\right)$ satisfies all the constraints of $\left(P_a^{MPCC-r}\right)$. Furthermore, observe that

$$\begin{aligned} \sum_{h\in H}\left(2v_i^h\left(d_i^h - d_i^{h,0}\right) + \hat{\lambda}_i - \mu_i^h\right)d_i^h &= \sum_{h\in H: d_i^h\neq 0}\left(2v_i^h\left(d_i^h - d_i^{h,0}\right) + \hat{\lambda}_i - \mu_i^h\right)d_i^h \\ &= \sum_{h\in H: d_i^h\neq 0}\left(2v_i^h\left(d_i^h - d_i^{h,0}\right) + \hat{\lambda}_i^*\right)d_i^h \\ &= \sum_{h\in H}\left(2v_i^h\left(d_i^h - d_i^{h,0}\right) + \hat{\lambda}_i^*\right)d_i^h \\ &= \sum_{h\in H} 2v_i^h d_i^h\left(d_i^h - d_i^{h,0}\right) + \hat{\lambda}_i^* W_i, \end{aligned}$$

since $\mu_i^h > 0$ implies that $d_i^h = 0$, and $\sum_{h\in H} d_i^h = W_i$. Therefore, $\left(\mathbf{e}_{a\cdot}^*,\mathbf{p}_{a\cdot}^*,\boldsymbol{\alpha}_{a\cdot}^*,\mu^{*a},\hat{\lambda}^{*a}\right)$ is an optimal solution of $\left(P_a^{MPCC-r}\right)$ too, and we can restrain the search of optimal solutions to the sets of $\left(\mathbf{e}_{a\cdot},\mathbf{p}_{a\cdot},\boldsymbol{\alpha}_{a\cdot},\mu^a,\hat{\lambda}^a\right)$ with $\mu^a = 0$, which gives rise to (P'_a). This concludes our proof. □

Note also from the proof that the optimal reward (for a) to obtain a demand $\mathbf{d}_i^a$ from the end user i at time h is

$$r_{ai}^h = 2v_i^h\left(d_i^h - d_i^{h,0}\right) + \hat{\lambda}_i \qquad \forall i\in\mathcal{I}, h\in H,$$

with $\hat{\lambda}_i$ such that $\min_{h\in H} r_{ai}^h = 0$.

Taking into account the formulation (P'_a) of the aggregators, we will now consider the fol-

lowing modified version problem of the leader, which is a bilevel problem, denoted by $\left(P'_{\hat{s}}\right)$:

$$\left(P'_{\hat{s}}\right) \quad \max_{\mathbf{p}_{\hat{s}\cdot}} \max_{\mathbf{e},\mathbf{p}_{x\cdot},\boldsymbol{\alpha},\mathbf{d},\hat{\boldsymbol{\lambda}}} \sum_{h\in H}\left(\sum_{a\in\mathcal{A}} p^h_{\hat{s}a}e^h_{a\hat{s}} + \sum_{\ell\in\mathcal{L}} p^h_{\hat{s}\ell}e^h_{\ell\hat{s}} - c^h_{\hat{s}}\left(\sum_{a\in\mathcal{A}} e^h_{a\hat{s}} + \sum_{\ell\in\mathcal{L}} e^h_{\ell\hat{s}}\right)\right)$$
$$s.t.\begin{cases} p^h_{\hat{s}x} \geq 0 & \forall h\in H, x\in\mathcal{L}\cup\mathcal{A} \\ \{\mathbf{e}_{\ell\cdot},\mathbf{p}_{\ell\cdot},\boldsymbol{\alpha}_{\ell\cdot}\} \in \operatorname{argmax}(P_\ell) & \forall \ell\in\mathcal{L} \\ \{\mathbf{e}_{a\cdot},\mathbf{p}_{a\cdot},\boldsymbol{\alpha}_{a\cdot},\mathbf{d}_i,\hat{\boldsymbol{\lambda}}\} \in \operatorname{argmax}(P'_a) & \forall a\in\mathcal{A}, \end{cases}$$

and thus according to Proposition 1 from any solution $\left(\mathbf{p}_{\hat{s}\cdot},\mathbf{e},\mathbf{p}_{x\cdot},\boldsymbol{\alpha},\mathbf{d},\hat{\boldsymbol{\lambda}}\right)$ of $\left(P'_{\hat{s}}\right)$ one obtains the solution $(\mathbf{p}_{\hat{s}\cdot},\mathbf{e},\mathbf{p}_{x\cdot},\boldsymbol{\alpha},\mathbf{d},\mathbf{r})$ of $(P_{\hat{s}})$ with $r^h_{ai} = 2v^h_i\left(d^h_i - d^{h,0}_i\right) + \hat{\lambda}_i$, for any $i\in\mathcal{I}_a$ and any $h\in H$.

4. Reformulations of the bilevel program and alternative solution concepts

As mentioned in Section 3, the resolution of the trilevel best response problem $(P_{\hat{s}})$ can be reduced to the computation of solutions of the single-leader-multi-follower problem $\left(P'_{\hat{s}}\right)$. Nevertheless, even if more simple from a structural point of view, single-leader-multi-follower games are known to be very difficult to handle.

In this section, we present three different ways to transform/simplify $\left(P'_{\hat{s}}\right)$ in order to be able to solve it. First, a somehow classical transformation of $\left(P'_{\hat{s}}\right)$ into a mathematical program with complementarity constraints (MPCC) is developed in Subsection 4.1. Second, in subsections 4.2 and 4.3, two new approaches based on a technique of selection of the GNE at the ILAs' level are proposed to simplify $\left(P'_{\hat{s}}\right)$.

4.1. First order formulation of the bilevel model

The classical method to solve a bilevel problem consists in replacing the followers' problems by their KKT conditions in the leader's problem, thus yielding a MPCC. Therefore, the bilevel reformulation of (TDSM) $\left(P'_{\hat{s}}\right)$ becomes the single level MPCC $\left(P^{cl}_{\hat{s}}\right)$:

$$\left(P^{cl}_{\hat{s}}\right) \quad \max_{\mathbf{p}_{\hat{s}\cdot}} \max_{\mathbf{e},\mathbf{p}_{x\cdot},\boldsymbol{\alpha},\mathbf{d},\hat{\boldsymbol{\lambda}}} \sum_{h\in H}\left(\sum_{a\in\mathcal{A}} p^h_{\hat{s}a}e^h_{a\hat{s}} + \sum_{\ell\in\mathcal{L}} p^h_{\hat{s}\ell}e^h_{\ell\hat{s}}e^h_{\ell\hat{s}} - c^h_{\hat{s}}\left(\sum_{a\in\mathcal{A}} e^h_{a\hat{s}} + \sum_{\ell\in\mathcal{L}} e^h_{\ell\hat{s}}\right)\right)$$
$$s.t.\begin{cases} p^h_{\hat{s}x} \geq 0 & \forall h\in H, x\in\mathcal{L}\cup\mathcal{A} \\ \{\mathbf{e}_{\ell\cdot},\mathbf{p}_{\ell\cdot},\boldsymbol{\alpha}_{\ell\cdot}\} \in \mathrm{KKT}(P_\ell) & \forall \ell\in\mathcal{L} \\ \{\mathbf{e}_{a\cdot},\mathbf{p}_{a\cdot},\boldsymbol{\alpha}_{a\cdot},\mathbf{d}^a,\hat{\boldsymbol{\lambda}}^a\} \in \mathrm{KKT}(P'_a) & \forall a\in\mathcal{A}, \end{cases}$$

where $\mathrm{KKT}(P_\ell)$ and $\mathrm{KKT}(P'_a)$ denote the solution sets of the KKT conditions. More precisely, $\mathrm{KKT}(P_\ell)$ is the set of tuples $(\mathbf{e},\mathbf{p},\boldsymbol{\alpha},\boldsymbol{\mu},\boldsymbol{\lambda})$ satisfying the primal feasibility, dual feasibility, stationarity and complementary slackness constraints related to (P_ℓ). $\mathrm{KKT}(P'_a)$ is the set of tuples $(\mathbf{e},\mathbf{d},\mathbf{p},\boldsymbol{\alpha},\boldsymbol{\mu},\boldsymbol{\lambda})$ satisfying the KKT conditions of (P'_a). All these constraints can be found in Appendix A.

It is now well known that, even if the lower level problems of (P_ℓ) and (P'_a) of the leader problems $\left(P'_{\hat{s}}\right)$ are convex, a solution of the multi-leader-follower game composed of the problems $\left(P^{cl}_{\hat{s}}\right)$ need not be a solution of the game composed with problems $\left(P'_{\hat{s}}\right)$. Indeed as proved in (Dempe & Dutta, 2012; Aussel & Svensson, 2019) some qualification conditions are required but it is beyond the scope of this work to discuss such qualification conditions.

4.2. Revisited optimistic approach

We consider here the classical optimistic approach that was considered above in its bilevel expression $\left(P'_{\hat{s}}\right)$ but we show, using the specific characteristics of the (TDSM) problem, that at optimality (Nash equilibrium) we can actually select some special GNE at the intermediary level while maintaining the optimal values of the supplier. These special GNE get rid of the energy exchanges at the intermediary level.

Proposition 2. *Consider a leader price profile* $\mathbf{p}_{\hat{s}} = \{\mathbf{p}_{\hat{s}x} \mid x \in \mathcal{L} \cup \mathcal{A}\}$*, and*

$$S(\mathbf{p}) = \left\{\left(\mathbf{e}_{xy}, \mathbf{e}_{xs}, \mathbf{p}_{xy}, \boldsymbol{\alpha}_{xy}, \mathbf{d}^a, \hat{\boldsymbol{\lambda}}^a\right) \mid x, y \in \mathcal{L} \cup \mathcal{A}, s \in \mathcal{S}, a \in \mathcal{A}, i \in \mathcal{I}_a\right\}$$

a GNE associated to $\mathbf{p}$*. Assume that, for all* $h \in H$*, at least one of the two following conditions holds:*

$(H1)$ *For all* $x \in \mathcal{L} \cup \mathcal{A}$*,* $p^h_{\hat{s}x} \leq p^h_{\bar{s}x}$*;*

$(H2)$ *For all* $x, y \in \mathcal{L} \cup \mathcal{A}$*,* $p^h_{\bar{s}x} = p^h_{\bar{s}y}$*.*

Then for any $x, y \in \mathcal{L} \cup \mathcal{A}$ *such that* $e^h_{yx} > 0$ *one has*

$$p^h_{xy} = \max\left\{\min\left\{p^h_{\hat{s}x}, p^h_{\bar{s}x}\right\}, \min\left\{p^h_{\hat{s}y}, p^h_{\bar{s}y}\right\}\right\}.$$

Let us first state the following technical lemma that will be useful to clarify the proof of Proposition 2.

Lemma 1. *Let* a, b, c, d *be real numbers, and define the two conditions*

1. $a \leq b$ *and* $c \leq d$*;*
2. $b = d$*.*

If at least one of these conditions is verified, then

$$\max\{\min\{a, b\}, \min\{c, d\}\} = \min\{\max\{a, c\}, \max\{b, d\}\}.$$

Proof of Lemma 1. Let us first assume that condition 1 holds true. In that case,

$$\max\{\min\{a, b\}, \min\{c, d\}\} = \max\{a, c\} = \min\{\max\{a, c\}, \max\{b, d\}\},$$

since by the hypothesis, $\max\{a, c\} \leq \max\{b, d\}$. Now, assume that condition 2 is verified, hence $b = d$. If $a \leq b$ and $c \leq d$, then condition 1 is verified as well, and the lemma is verified. If condition 1 is not verified, then $a > b$ or $c > d$. Since both cases are symmetric, assume without loss of generality that $a > b$. In this case,

$$\max\{\min\{a, b\}, \min\{c, d\}\} = \max\{\min\{a, b\}, \min\{c, b\}\} = \max\{b, \min\{c, b\}\} = b.$$

On the other hand, $\min\{\max\{a, c\}, \max\{b, d\}\} = \min\{\max\{a, c\}, b\} = b$, since it directly follows from the hypothesis that $\max\{a, c\} > b$. □

Proof of Proposition 2. Take $h \in H$, and $x, y \in \mathcal{L} \cup \mathcal{A}$ such that $e^h_{yx} > 0$. First, observe that since $S(\mathbf{p})$ is a GNE, and since the energy bought from x by y is a decision of the ILA y, the price p^h_{xy} takes the highest possible value. This price is upper-bounded at the same time by $\alpha^h_{xy\hat{s}} p^h_{\hat{s}x} + \left(1 - \alpha^h_{xy\hat{s}}\right) p^h_{\hat{s}y}$ and $\alpha^h_{xy\bar{s}} p^h_{\bar{s}x} + \left(1 - \alpha^h_{xy\bar{s}}\right) p^h_{\bar{s}y}$. Observe that a judicious (i.e. 0 or 1) choice of $\alpha^h_{xy\hat{s}}$ and $\alpha^h_{xy\bar{s}}$ yields that those two upper bounds become $\max\left\{p^h_{\hat{s}x}, p^h_{\hat{s}y}\right\}$ and $\max\left\{p^h_{\bar{s}x}, p^h_{\bar{s}y}\right\}$, respectively. Therefore, if $e^h_{yx} > 0$, then

$$p^h_{xy} = \min\left\{\max\left\{p^h_{\hat{s}x}, p^h_{\hat{s}y}\right\}, \max\left\{p^h_{\bar{s}x}, p^h_{\bar{s}y}\right\}\right\}.$$

Now, combining Lemma 1 with either hypothesis $(H1)$ or $(H2)$ directly implies that $p^h_{xy} = \max\left\{\min\left\{p^h_{\hat{s}x}, p^h_{\bar{s}x}\right\}, \min\left\{p^h_{\hat{s}y}, p^h_{\bar{s}y}\right\}\right\}$. □

Observe that the proof of Proposition 2 actually holds under the following hypothesis:

$(H3)$ if $e^h_{xy} > 0$ for $h \in H$ and $x, y \in \mathcal{L} \cup \mathcal{A}$, then at least one of the two conditions holds:

- $p^h_{\hat{s}x} \leq p^h_{fx}$ and $p^h_{\hat{s}y} \leq p^h_{sy}$ for all $s \neq \hat{s}$;
- $p^h_{sx} = p^h_{fy}$ for all $s \neq \hat{s}$.

The hypothesis $(H3)$ is clearly weaker than the hypothesis that for all $h \in H$, $(H1)$ or $(H2)$ holds. However, its meaning is more difficult to interpret. If $(H1)$ holds for $h \in H$, then the leader $\hat{s}$ is competitive at this time slot and offer cheaper prices than his competitors. If $(H2)$ holds, then the competitors offer prices that are not buyer-dependent, and thus do not discriminate among the ILAs.

Let us recall from Subsection 2.4 that we defined two situations: the competitive case, and nondiscriminant exogenous pricing. Clearly, if any of these two situations is true, then the assumptions of Proposition 2 are fulfilled.

Proposition 2 is crucial in our characterization of GNEs at the intermediary level. The following proposition ensures that we can restrain the search of an optimal GNE in the set $GNE^o(\mathbf{p})$,

which is the set of GNEs at the intermediary level where all energy exchanges among ILAs are equal to zero.

Proposition 3. *In a best response, optimistic context, we assume that:*

- $\mathbf{p} = \{\mathbf{p}_{\hat{s}x} \mid x \in \mathcal{L} \cup \mathcal{A}\}$ *is an optimal leader price profile;*
- $S(\mathbf{p}) = \left\{\left(\mathbf{e}_{xy}, \mathbf{e}_{xs}, \mathbf{p}_{xy}, \mathbf{d}^a, \hat{\boldsymbol{\lambda}}^a\right) | x, y \in \mathcal{L} \cup \mathcal{A}, s \in \mathcal{S}, a \in \mathcal{A}\right\}$ *is a GNE among the ILAs with relation to* $\mathbf{p}$ *that is optimal for the leader;*
- *for all* $h \in H$, $(H1)$ *or* $(H2)$ *of Proposition 2 holds;*
- *for each* $h \in H$, *the function* $\left(\mathbf{e}^h_{x\hat{s}}\right)_{x \in \mathcal{L} \cup \mathcal{A}} \mapsto \sum_{x \in \mathcal{L} \cup \mathcal{A}} p^h_{\hat{s}x} e^h_{x\hat{s}} - c^h_{\hat{s}}\left(\sum_{x \in \mathcal{L} \cup \mathcal{A}} e^h_{x\hat{s}}\right)$ *is increasing with relation to each variable;*

then one can construct a leader price profile $\mathbf{p}^*$ *and a GNE* $S^*(\mathbf{p}^*)$ *such that* $\mathbf{e}^*_{xy} = 0$ *for all* $x, y \in \mathcal{L} \cup \mathcal{A}$, *and* $z(S(\mathbf{p})) = z(S^*(\mathbf{p}^*))$, *where* z *denotes the objective function of* $\hat{s}$. *That is,* $\mathbf{p}^*$ *is an optimal price profile for the leader too.*

Proof. Let h belong to H, and for all $x \in \mathcal{L} \cup \mathcal{A}$, denote by $p^h_{x,\min} = \min\left\{p^h_{\hat{s}x}, p^h_{\bar{s}x}\right\}$ the lowest price offered to x by the suppliers.

Recall that from Proposition 2, if $e^h_{xy} > 0$, then $p^h_{yx} = \max\left\{p^h_{x,\min}, p^h_{y,\min}\right\}$. Furthermore, if $e^h_{xy} = 0$, then $p^h_{yx} \geq p^h_{x,\min}$, otherwise we would have a contradiction with $S(\mathbf{p})$ being a GNE, since x would be able to purchase cheaper energy from y than from the suppliers and thus unilaterally improve his situation by changing his energy source. Therefore, every actor $x \in \mathcal{L} \cup \mathcal{A}$ buys his energy at price $p^h_{x,\min}$, either from the supplier(s) offering this price, or from the other ILAs.

Claim 1:

For $x, y \in \mathcal{L} \cup \mathcal{A}$, if $p^h_{x,\min} < p^h_{y,\min}$, then $e^h_{xy} = 0$.

Proof:

Assume that $e^h_{xy} > 0$. It follows from Proposition 2 that $p^h_{yx} = p^h_{y,\min} > p^h_{x,\min}$, hence x has access to cheaper energy from at least one supplier than from y. It is therefore no optimal solution for x to buy energy from y and there is a contradiction with $S(\mathbf{p})$ being a GNE. Hence $e^h_{xy} = 0$ for all $x, y \in \mathcal{L} \cup \mathcal{A}$ such that $p^h_{x,\min} < p^h_{y,\min}$. Thus the claim is proved.

Claim 2:

If $S'(\mathbf{p})$ is a feasible solution for the followers satisfying

1. for all $x, y \in \mathcal{L} \cup \mathcal{A}$, if $p^h_{x,\min} \leq p^h_{y,\min}$, then $p'^h_{xy} = p^h_{y,\min}$, and if $p^h_{x,\min} < p^h_{y,\min}$, then $p'^h_{yx} > p^h_{x,\min}$,
2. for all $x \in \mathcal{L} \cup \mathcal{A}$, if $e'^h_{xy} > 0$ for $y \in \mathcal{S} \cup \mathcal{L} \cup \mathcal{A}$, then $p'^h_{yx} = p^h_{x,\min}$,

3. for all $\ell \in \mathcal{L}$,

$$d'^h_\ell = \sum_{s\in\mathcal{S}} e'^h_{\ell s} + \sum_{x\in\mathcal{L}\cup\mathcal{A}} \left(e'^h_{\ell x} - e'^h_{x\ell}\right) = \sum_{s\in\mathcal{S}} e^h_{\ell s} + \sum_{x\in\mathcal{L}\cup\mathcal{A}} \left(e^h_{\ell x} - e^h_{x\ell}\right) = d^h_\ell,$$

4. for all $i \in \mathcal{I}$, $d'^h_i = d^h_i$ and $\hat{\lambda}'_i = \hat{\lambda}_i$,

i.e. every local agent and every end user has the same demand in $S'(\mathbf{p})$ and in $S(\mathbf{p})$, then it is a GNE.

Proof:

By definition, $S'(\mathbf{p})$ is a GNE if no ILA $x \in \mathcal{L} \cup \mathcal{A}$ can unilaterally improve the value of his objective function.

The first condition ensures that $x \in \mathcal{L} \cup \mathcal{A}$ cannot buy cheaper energy from an ILA than from the cheapest supplier, and that y does not sell energy at a price too low to be profitable: otherwise, one has a contradiction with the fact that $S'(\mathbf{p})$ is a GNE. The second condition makes sure that x indeed buys the energy he needs at the lowest possible price. It is therefore not possible for x to improve his objective value by modifying his energy purchases while keeping the same consumption $\mathbf{d}_x$. On the other hand, the second condition makes sure that if x sells energy, he sells it at the maximum possible price. Hence x cannot improve his objective value by increasing his prices.

We thus deduce that the only way for x to strictly improve his objective value would imply a modification of his energy consumption $\mathbf{d}_x$. Let us assume that it is possible, with demand $\mathbf{d}''_x$. Since the energy sales of x are decision variables of the other ILAs and x buys all his energy at price $p^h_{x,\min}$, the difference in the revenue function of x is as follows:

- if x is a local agent,

$$\sum_{h\in H} \left(p^h_{x,\min}\left(d''^h_x - d'^h_x\right) + V^h_x\left(d''^h_x\right) - V^h_x\left(d'^h_x\right)\right) < 0,$$

- if x is an aggregator,

$$\sum_{h\in H} \left(p^h_{x,\min}\left(d''^h_x - d'^h_x\right) + \sum_{i\in\mathcal{I}_x} \left(r''^h_{xi} d''^h_i - r'^h_{ji} d'^h_i\right)\right) < 0,$$

with $r''^h_{xi} = 2v^h_i\left(d''^h_i - d^{h,0}_i\right) + \hat{\lambda}''_i$ and $r'^h_{xi} = 2v^h_i\left(d'^h_i - d^{h,0}_i\right) + \hat{\lambda}'_i$.

However in this case, x could also unilaterally strictly improve the value of his objective function in $S(\mathbf{p})$. If x is a local agent, it would be enough to set $\mathbf{d}_x = \mathbf{d}''_x$ and increase, respectively decrease, his energy purchases at time $h \in H$ by $(d''_x - d_x)$. The difference in the revenue function of x is the same as with $S'(\mathbf{p})$, since $\mathbf{d}_x = \mathbf{d}'_x$. If x is an aggregator, observe from

the proof of Proposition 1 that for a given demand of an end user $\mathbf{d}_i^x$, there is a unique optimal corresponding $\hat{\lambda}_i$. It follows that replacing $\mathbf{d}_i^x$ and $\hat{\lambda}^x$ by $\mathbf{d}_i'^x$ and $\hat{\lambda}'^x$, and increasing/decreasing the energy purchases at time $h \in H$ correspondingly finally yields the same revenue difference as previously, since $\mathbf{d}_x = \mathbf{d}'_x$ and thus $\hat{\lambda}_i = \hat{\lambda}'_i$. It follows from this that if x could unilaterally strictly improve the value of his objective function in $S'(\mathbf{p})$, so could he in $S(\mathbf{p})$, which is a contradiction with $S(\mathbf{p})$ being a GNE. Therefore, $S'(\mathbf{p})$ is a GNE and Claim 2 is proved.

Observe now that without loss of generality, $S(\mathbf{p})$ satisfies the conditions of Claim 2. Whereas conditions 3 and 4 are obviously verified, conditions 1 and 2 need not be valid. However, assume that condition 1 is not verified. Then there exist $x, y \in \mathcal{L} \cup \mathcal{A}$ with $p^h_{x,\min} \leq p^h_{y,\min}$ such that one of the two following cases holds true:

- $p^h_{xy} \neq p^h_{y,\min}$: it follows from the proof of Lemma 3 that $p^h_{xy} \leq p^h_{y,\min}$, thus here the inequality is strict. If $d^h_y + \sum_{x' \neq y} e^h_{x'y} > 0$, then $e^h_{x'y} > 0$ for x' offering the cheapest price to y, which is smaller than $p^h_{y,\min}$ by the assumption. This constitutes a contradiction with Lemma 3.

  Now, if $d^h_y + \sum_{x' \neq y} e^h_{x'y} = 0$, then $e^h_{yx} = 0$, thus the value of p^h_{xy} can be increased up to $p^h_{y,\min}$ without modifying the value of the objective function of x. The value of the objective function of y is not modified either, and y does not get cheaper energy with the increased p^h_{xy}. Hence y cannot unilaterally strictly improve the value of his objective function with $p^h_{xy} = p^h_{y,\min}$, and $S(\mathbf{p})$ with $p^h_{xy} = p^h_{y,\min}$ is a GNE as well.

- $p^h_{yx} < p^h_{x,\min}$: this case is similar as the previous one, considering the two cases where $d^h_x + \sum_{x' \neq x} e^h_{x'x} > 0$ and $d^h_x + \sum_{x' \neq x} e^h_{x'x} > 0$. The same reasoning leads to the same conclusions.

As a result of this reasoning, we conclude that without loss of generality, $S(\mathbf{p})$ satisfies the conditions of Claim 2.

We say that there is a *cycle* in a followers' GNE $S(\mathbf{p})$ if there are ILA $x_1, \ldots, x_n$ such that $e^h_{x_i x_{i+1}} > 0$ for $i = 1, \ldots, n$, with $x_{n+1} = x_1$.

Claim 3:

There exists a GNE $S'(\mathbf{p})$ such that there is no cycle.

Proof:

Assume there is a cycle $(x_1, \ldots, x_n)$, and define $e = \min_{i=1,\ldots,n} e^h_{x_i x_{i+1}}$. Observe that the first Claim implies that for any $i, j = 1, \ldots, n$, $p^h_{x_i,\min} = p^h_{x_j,\min}$. Set $e'^h_{x_i x_{i+1}} = e^h_{x_i x_{i+1}} - e$ for $i = 1, \ldots, n$. The followers' answer that results $S'(\mathbf{p})$ clearly satisfies the conditions of Claim 2, therefore it is a GNE, concluding the proof of Claim 3.

Notice that, in the proof of Claim 3, no variable of the form e^h_{xs} with $x \in \mathcal{L} \cup \mathcal{A}$, $s \in \{\hat{s}, \bar{s}\}$ is different in $S'(\mathbf{p})$ and $S(\mathbf{p})$; therefore the leader's profit is constant and $S'(\mathbf{p})$ is a followers' answer that is optimal for the leader.

Claim 3 allows us to assume that $S(\mathbf{p}) = S(\mathbf{p}^*)$ has no cycle. Therefore, either there is at least one ILA $x \in \mathcal{L} \cup \mathcal{A}$ such that $0 < \sum_{y \in \mathcal{L} \cup \mathcal{A}} e^h_{yx} \leq e^h_{x\hat{s}} + e^h_{x\bar{s}}$, or there are no energy exchanges among the ILAs in $S(\mathbf{p}^*)$. In the first case, choose y such that $e^h_{yx} > 0$. It follows from Claim 1 that $p^h_{y,\min} \geq p^h_{x,\min}$. Therefore, setting $e'^h_{y\hat{s}} = e^h_{y\hat{s}} + e^h_{yx}$ and $e'^h_{x\hat{s}}$ and $e'^h_{x\bar{s}}$ such that $e'^h_{x\hat{s}} + e'^h_{x\bar{s}} = e^h_{x\hat{s}} + e^h_{x\bar{s}} - e^h_{yx}$ and $e'^h_{x\hat{s}} \geq e^h_{x\hat{s}} - e^h_{yx}$ will yield a new followers' answer with $e^h_{yx} = 0$. Thanks to Claim 2, it is a GNE. Furthermore, the leader's profit cannot decrease in the process, because whenever an ILA buys less energy from $\hat{s}$, another ILA buys more energy (in greater or equal quantities) to a greater or equal price. Iterating this process will finally yield a GNE $S^*(\mathbf{p}^*)$ where $e^h_{xy} = 0$ for all $x, y \in \mathcal{L} \cup \mathcal{A}$, which concludes our proof.

□

It follows from Proposition 3 that a solution of (TDSM) can be obtained by simply solving the following revisited optimistic single-leader-multi-follower problem $\left(P^o_{\hat{s}}\right)$:

$$
\left(P^o_{\hat{s}}\right) \quad \max_{\mathbf{p}_{\hat{s}x}} \max_{\mathbf{e}_x, \mathbf{d}^a, \hat{\boldsymbol{\lambda}}^a} \sum_{h \in H} \left(\sum_{x \in \mathcal{L} \cup \mathcal{A}} p^h_{\hat{s}x} e^h_{x\hat{s}} - c^h_{\hat{s}} \left(\sum_{x \in \mathcal{L} \cup \mathcal{A}} e^h_{x\hat{s}} \right) \right)
$$
$$
s.t. \begin{cases} p^h_{\hat{s}x} \geq 0 & \forall h \in H, x \in \mathcal{L} \cup \mathcal{A} \\ (\mathbf{e}_{\ell\hat{\mathbf{s}}}, \mathbf{e}_{\ell\bar{s}}) \in \operatorname{argmax}\left(P^{el}_{\ell}\right) & \forall \ell \in \mathcal{L} \\ \left(\mathbf{e}_{a\hat{\mathbf{s}}}, \mathbf{e}_{a\bar{s}}, \mathbf{d}^a, \hat{\boldsymbol{\lambda}}^a\right) \in \operatorname{argmax}\left(P^{el}_{a}\right) & \forall a \in \mathcal{A}, \end{cases}
$$

where $\left(P^{el}_x\right)$ denotes the optimization problem of $x \in \mathcal{L} \cup \mathcal{A}$ without energy exchanges (el stands for exchangeless), that is for $x \in \mathcal{L}$:

$$
\left(P^{el}_x\right) \quad \min_{\mathbf{e}_x} \sum_{h \in H} \left(p^h_{\hat{s}x} e^h_{x\hat{s}} + p^h_{\bar{s}x} e^h_{x\bar{s}} + V^h_k \left(e^h_{x\hat{s}} + e^h_{x\bar{s}} \right) \right)
$$
$$
s.t. \begin{cases} \sum_{h \in H} e^h_{x\hat{s}} + e^h_{x\bar{s}} = W_x & \\ e^h_{x\hat{s}}, e^h_{x\bar{s}} \geq 0 & \forall h \in H, \end{cases}
$$

and for $x \in \mathcal{A}$, $\left(P^{el}_x\right)$ is as follows:

$$
\left(P^{el}_x\right) \quad \min_{\mathbf{e}_x, \mathbf{d}_i, \hat{\boldsymbol{\lambda}}_i} \sum_{h \in H} \left(p^h_{\hat{s}x} e^h_{x\hat{s}} + p^h_{\bar{s}x} e^h_{x\bar{s}} + \sum_{i \in \mathcal{I}_x} 2 d^h_i v^h_i \left(d^h_i - d^{h,0}_i \right) \right) + \sum_{i \in \mathcal{I}_x} W_i \hat{\lambda}_i
$$
$$
s.t. \begin{cases} \sum_{i \in \mathcal{I}_x} d^h_i = e^h_{x\hat{s}} + e^h_{x\bar{s}} & \forall h \in H \\ \sum_{h \in H} d^h_i = W_i & \forall i \in \mathcal{I}_x \\ d^h_i, e^h_{x\hat{s}}, e^h_{x\bar{s}} \geq 0 & \forall h \in H, i \in \mathcal{I}_x \\ 2 v^h_i \left(d^h_i - d^{h,0}_i \right) + \hat{\lambda}_i \geq 0 & \forall h \in H, i \in \mathcal{I}_x. \end{cases}
$$

4.3. Semi-optimistic approach

The developments of Section 3 and Subsection 4.2 show that the optimistic case of (TDSM) can be reformulated as a light bilevel problem, which can be efficiently solved by commercial solvers after a transformation into a MPCC. However, the optimistic approach is aptly named: if the follower has multiple solutions for a given decision of the leader, his decisions are assumed to be favorable to the leader, which might be too optimistic. At the other extreme, a possible pessimistic approach might be too pessimistic: if the follower has multiple solutions for a given decision of the leader, he will choose the least favorable for the leader. This is why we now consider a third case in between, which we call *semi-optimistic*. For the rest of this section, we will assume that we are in the competitive case, that is the leader can take over the whole electricity demand for cheaper prices than the competitor.

In the semi-optimistic case, we assume that the GNE among the ILAs will be of a specific type: at time $h \in H$, the only ILA that buys energy from the leader $\hat{s}$ is the ILA x that enjoys the lowest price, $\min_{x' \in \mathcal{A} \cup \mathcal{L}} p^h_{\hat{s}x'} \overset{not.}{=} p^h_{\min}$. The ILA y then buys the energy he needs from x at price $p^h_{\hat{s}y}$ (instead of $\hat{s}$, who offers the same price, in the optimistic case). Let us call this class of GNE $GNE^{so}(\mathbf{p})$. To motivate this choice, consider a simple case with a single time slot h and two local agents, x and y. Furthermore, assume that $p^h_{\hat{s}x} < p^h_{\hat{s}y}$, $W_x = 0$, $W_y = 1$. From the constraints of x, $p^h_{xy} \in [p^h_{\hat{s}x}, p^h_{\hat{s}y}]$. Observe that if $p^h_{xy} < p^h_{\hat{s}y}$, the optimal solution for y consists in setting $e^h_{yx} = 1$ and $e^h_{y\hat{s}} = 0$. However, if $p^h_{xy} = p^h_{\hat{s}y}$, the set of possible solutions for y is the set

$$\left\{\left(e^h_{y\hat{s}} = \alpha, e^h_{yx} = 1 - \alpha\right) \mid \alpha \in [0, 1]\right\}.$$

With the classical optimistic approach, the decision taken by y would be $e^h_{y\hat{s}} = 1$ and $e^h_{yx} = 0$. The semi-optimistic approach implies that the function $e^h_{yx}\left(p^h_{xy}\right)$ that determines the optimal value of e^h_{yx} in $\left(P_y\right)$ in function of p^h_{xy} is left-continuous in $p^h_{xy} = p^h_{\hat{s}y}$.

Another important notion to justify the semi-optimistic approach is the order in which the decisions are taken. Usually, bilevel models are used to model the response of a follower to decisions of a leader, which implies that the decisions of the leader are taken before the decisions of the follower. Consider the example given in the previous paragraph: if x sets his price p^h_{xy} to any value in $]p^h_{\hat{s}x}, p^h_{\hat{s}y}[$, he will get a strictly larger objective than with $p^h_{xy} = p^h_{\hat{s}y}$ in the optimistic approach. The semi-optimistic approach proposes a way to avoid this.

To solve (TDSM) with the semi-optimistic approach, remember that at time $h \in H$, the leader $\hat{s}$ sells energy at price $p^h_{\min}$, whereas each ILA x buys his energy at price p^h_{fx}. Thus we can

determine an optimal profile of prices of the leader with the following reformulation $\left(P^{so}_{\hat{s}}\right)$:

$$\left(P^{so}_{\hat{s}}\right) \quad \max_{\mathbf{p}_{\min},\mathbf{p}_{\hat{s}x}} \max_{\mathbf{e}_x,\mathbf{d}^a,\hat{\boldsymbol{\lambda}}^a} \sum_{h\in H} p^h_{\min}\left(\sum_{x\in\mathcal{A}\cup\mathcal{L}} e^h_{x\hat{s}}\right) - c^h_{\hat{s}}\left(\sum_{x\in\mathcal{A}\cup\mathcal{L}} e^h_{x\hat{s}}\right)$$
$$s.t.\begin{cases} p^h_{\min} \le p^h_{\hat{s}x} & \forall h\ inH, x\in\mathcal{L}\cup\mathcal{A} \\ p^h_{\hat{s}x} \le p^h_{\bar{s}x} & \forall h\in H, x\in\mathcal{L}\cup\mathcal{A} \\ (\mathbf{e}_{\ell\hat{s}},\mathbf{e}_{\ell\bar{s}}) \in \operatorname{argmax}\left(P^{el}_{\ell}\right) & \forall \ell\in\mathcal{L} \\ \left(\mathbf{e}_{a\hat{s}},\mathbf{e}_{a\bar{s}},\mathbf{d}^a,\hat{\boldsymbol{\lambda}}^a\right) \in \operatorname{argmax}\left(P^{el}_{a}\right) & \forall a\in\mathcal{A}, \end{cases}$$

where P^{el}_x is the same as in the previous subsection.

5. Numerical results

Based on the three different formulations of the best response problem of (TDSM) (the MPCC, the revisited optimistic and the semi-optimistic formulations), we provide in this section some numerical experiments to compare these approaches. The *competitive case* and the *noncompetitive case* are considered separately. The optimization problems resulting of the three reformulations lead to nonconvex problems with complementarity constraints and thus only local solutions will be obtained.

5.1. Competitive case

SPOT market prices usually range in Europe from 0.01 to 0.08 € per kWh, whereas typical prices offered by a supplier to consumers start at 0.1 € per kWh. To get a more precise idea of what happens when this is not the case, see Subsection 5.2. This assumption allows us to get rid of the variables related to the competitors by setting the competitor's prices as upper bounds for the leader's prices. Therefore, we actually solve the following problems:

- With the classical approach, we solve $\left(P^{c,KKT}_{\hat{s}}\right)$ (c stands for competitive):

$$\left(P^{c,KKT}_{\hat{s}}\right) \quad \max_{\mathbf{p}_{\hat{s}\cdot}} \max_{\mathbf{e},\mathbf{p}_{x\cdot},\boldsymbol{\alpha},\mathbf{d},\hat{\boldsymbol{\lambda}}} \sum_{h\in H}\left(\sum_{x\in\mathcal{L}\cup\mathcal{A}} p^h_{\hat{s}x}e^h_{x\hat{s}} - c^h_{\hat{s}}\left(\sum_{x\in\mathcal{L}\cup\mathcal{A}} e^h_{x\hat{s}}\right)\right)$$
$$s.t.\begin{cases} 0 \le p^h_{\hat{s}x} \le p^h_{\bar{s}x} & \forall h\in H, x\in\mathcal{L}\cup\mathcal{A} \\ \{\mathbf{e}_{\ell\cdot},\mathbf{p}_{\ell\cdot},\boldsymbol{\alpha}_{\ell\cdot}\} \in \mathrm{KKT}\left(P^c_{\ell}\right) & \forall \ell\in\mathcal{L} \\ \{\mathbf{e}_{a\cdot},\mathbf{p}_{a\cdot},\boldsymbol{\alpha}_{a\cdot},\mathbf{d}^a,\hat{\boldsymbol{\lambda}}^a\} \in \mathrm{KKT}\left(P^c_{a}\right) & \forall a\in\mathcal{A}, \end{cases}$$

  where $\mathrm{KKT}\left(P^c_{\ell}\right)$ and $\mathrm{KKT}\left(P^c_{a}\right)$ denote the solution sets of the KKT conditions respectively associated to local agents and aggregators. More precisely, $\mathrm{KKT}\left(P^c_{\ell}\right)$ is the set of tuples $(\mathbf{e},\mathbf{p},\boldsymbol{\alpha},\boldsymbol{\mu},\boldsymbol{\lambda})$ satisfying the primal feasibility, dual feasibility, stationarity and complementary slackness constraints related to problem $\left(P^c_{\ell}\right)$, which is similar to (P_ℓ), but with

$\mathcal{S} = \{\hat{s}\}$. $\mathrm{KKT}\left(P_a^c\right)$ is the set of tuples $(\mathbf{e}, \mathbf{d}, \mathbf{p}, \boldsymbol{\alpha}, \boldsymbol{\mu}, \boldsymbol{\lambda})$ satisfying the KKT conditions of (P_a^c), which is (P'_a) with $\mathcal{S} = \{\hat{s}\}$. The KKT conditions of (P_ℓ) and (P'_a) in the general case ($\mathcal{S} = \{\hat{s}, \bar{s}\}$) can be found in Subsection 3.

- With the optimistic reformulation, we solve $\left(P_{\hat{s}}^{o,KKT}\right)$:

$$\left(P_{\hat{s}}^{o,KKT}\right) \quad \max_{\mathbf{p}_{\hat{s}x}} \max_{\mathbf{e}_x, \mathbf{d}^a, \hat{\boldsymbol{\lambda}}^a, \boldsymbol{\mu}, \boldsymbol{\lambda}} \sum_{h \in H} \left(\sum_{x \in \mathcal{L} \cup \mathcal{A}} p_{\hat{s}x}^h e_{x\hat{s}}^h - c_{\hat{s}}^h \left(\sum_{x \in \mathcal{L} \cup \mathcal{A}} e_{x\hat{s}}^h \right) \right)$$
$$s.t. \begin{cases} 0 \leq p_{\hat{s}x}^h \leq p_{\bar{s}x}^h & \forall h \in H, x \in \mathcal{L} \cup \mathcal{A} \\ (\mathbf{e}_{\ell\cdot}, \boldsymbol{\mu}_\ell, \boldsymbol{\lambda}_\ell) \in \mathrm{KKT}\left(P_\ell^{el}\right) & \forall \ell \in \mathcal{L} \\ \left(\mathbf{e}_{a\cdot}, \mathbf{d}^a, \hat{\boldsymbol{\lambda}}^a, \boldsymbol{\mu}_a, \boldsymbol{\lambda}_a\right) \in \mathrm{KKT}\left(P_a^{el}\right) & \forall a \in \mathcal{A}, \end{cases}$$

  where $KKT\left(P_x^{el}\right)$ denotes the KKT conditions of the optimization program of $x \in \mathcal{L} \cup \mathcal{A}$ without energy exchanges. These KKT conditions can be found in Appendix A.

- For the semi-optimistic reformulation, we search solutions of $\left(P_{\hat{s}}^{so,KKT}\right)$:

$$\left(P_{\hat{s}}^{so,KKT}\right) \quad \max_{\mathbf{p}_{\min}, \mathbf{p}_{\hat{s}x}} \max_{\mathbf{e}_x, \mathbf{d}^a, \hat{\boldsymbol{\lambda}}^a, \boldsymbol{\mu}, \boldsymbol{\lambda}} \sum_{h \in H} \left(p_{\min}^h - k_f^h\right) \left(\sum_{x \in \mathcal{A} \cup \mathcal{L}} d_x^h \right)$$
$$s.t. \begin{cases} 0 \leq p_{\hat{s}x}^h \leq p_{\bar{s}x}^h & \forall h \in H, x \in \mathcal{L} \cup \mathcal{A}, s \neq s' \in \mathcal{S} \\ p_{\min}^h \leq p_{\hat{s}x}^h & \forall x \in \mathcal{L} \cup \mathcal{A} \\ (\mathbf{e}_{\ell\cdot}, \boldsymbol{\mu}_\ell, \boldsymbol{\lambda}_\ell) \in \mathrm{KKT}\left(P_\ell^{el}\right) & \forall \ell \in \mathcal{L} \\ \left(\mathbf{e}_{a\cdot}, \mathbf{d}^a, \hat{\boldsymbol{\lambda}}^a, \boldsymbol{\mu}_a, \boldsymbol{\lambda}_a\right) \in \mathrm{KKT}\left(P_a^{el}\right) & \forall a \in \mathcal{A}, \end{cases}$$

  where $KKT\left(P_x^{el}\right)$ represents the ILA x's program without exchanges. Observe that actually, the only differences between the optimistic and the semi-optimistic reformulations lie in the presence of $p_{\min}^h$ and the adaptation of the leader's objective function.

We solved these MPCCs with the Knitro solver included in the demo version of GAMS 24.9.2.

5.1.1. Classical optimistic versus revisited optimistic

The main interest of the revisited optimistic formulation lies in its low number of variables and constraints, compared to the classical optimistic formulation. Furthermore, the classical optimistic formulation allows degenerate cases: given tuples $\left(\mathbf{e}_{x\cdot}, \mathbf{p}_{x\cdot}, \boldsymbol{\alpha}, \left(\mathbf{d}, \hat{\boldsymbol{\lambda}}, \right) \boldsymbol{\mu}, \boldsymbol{\lambda}\right) \in KKT\left(P_x^{cl}\right)$ for all ILAs, it is possible to build infinitely many tuples satisfying the same KKT conditions by setting $e'^h_{xy} = e^h_{xy} + c$, $e'^h_{yx} = e^h_{yx} + c$, $\mu'^h_{pxy} = \mu^h_{pxy} + c$ and $\mu'^h_{pyx} = \mu^h_{pyx} + c$ for every pair of ILAs (x, y) such that $p^h_{sx} = p^h_{sy}$, with $c > 0$. Observe that such new tuples $\left(\mathbf{e}'_{x\cdot}, \mathbf{p}_{x\cdot}, \boldsymbol{\alpha}, \left(\mathbf{d}, \hat{\boldsymbol{\lambda}}, \right) \boldsymbol{\mu}', \boldsymbol{\lambda}\right)$ imply the same energy consumption for the local agents and the end users, and the same energy sales for the suppliers. With the revisited optimistic formulation of the problem, there are no exchanges

among the ILAs. Therefore, such degenerate cases are avoided. Note that it would be possible to allow unidirectional exchanges only by adding the following constraints:

$$e^h_{xy} \cdot e^h_{yx} = 0 \qquad \forall x, y \in \mathcal{L} \cup \mathcal{A}.$$

However, this method not only increases the number of constraints, but also increases the number of necessary dual variables and complementarity constraints.

As proved in Subsection 4.2, the classical optimistic and the revisited optimistic formulations should yield the same optimal value for the objective function of the leader. Concerning the number of variables and constraints, a quick analysis of the resulting MPCCs gives, for the classical approach:

$$\begin{aligned}
\#\{\text{variables}\} &= 2I + L + H\left(3I + 4A + 5L + 8A^2 + 16AL + 8L^2\right) = \Theta\left(H\left(I + A^2 + L^2\right)\right) \\
\#\{\text{constraints}\} &= 2I + L + H\left(4I + 4A + 5L + 10A^2 + 20AL + 10L^2\right) = \Theta\left(H\left(I + A^2 + L^2\right)\right),
\end{aligned}$$

where $H = |H|$, $I = |\mathcal{I}|$, $A = |\mathcal{A}|$, and $L = |\mathcal{L}|$. For the revisited optimistic formulation, these numbers reduce to:

$$\begin{aligned}
\#\{\text{variables}\} &= 2I + L + H\,(3I + A + 3L) = \Theta\,(H\,(I + A + L)) \\
\#\{\text{constraints}\} &= 2I + L + H\,(4I + A + 3L) = \Theta\,(H\,(I + A + L))\,.
\end{aligned}$$

Whereas for small instances (3 time slots, 2 local agents, 1 aggregator in contract with 2 end users), the difference between both methods might not be significant (282 variables, 342 constraints for the classical method versus 45 variables, 51 constraints for the revisited optimistic formulation), the classical method becomes intractable as soon as instances grow larger (24 time slots, 10 aggregators with 1 end user each, 10 local agents give 79710 variables and 99150 constraints, whereas the revisited optimistic formulation only needs 1710 variables and 1950 constraints).

These differences in terms of number of variables and constraints are experimentally confirmed in Table 1 summarizing some instances that have been tested. This table also highlights the computational improvement (number of iterations and time) associated to the revisited formulation.The numerical experiments have been conducted on a laptop equipped with a processor i7-4600u at 2.1 GHz and 16 Go of RAM.

The first tests below aim to study the sensitivity of the profit to the inconvenience coefficient. More precisely, we study the cases of one single local agent, and of one aggregator with a single end user. Since in both cases, there is only one actor, both approaches prove to be efficient and yield the same results, up to minor differences that can be explained by the margin of error of the commercial solver. The results of the computations can be found in Figures 2 and 3.

In both cases, we consider three time slots with base demand (4, 14, 14) kWh, which corre-

Instance	# of local agents	Problem	#var	#cons	# of iterations	comp. time (s)
1	2	opt-classic	282	342		failed
1	2	opt-revis.	45	51	17048	3,789
2	1	opt-classic	146	176	31769	46,039
2	1	opt-revis.	35	41	23165	6,928
3	1	opt-classic	146	176	56357	61,371
3	1	opt-revis.	35	41	29487	5,164

Table 1: All the tests reported here involve three hours, one aggregator and two end users. All the tests but one end up with a local optimum with a null gap. Each instance corresponds to a given set of parameters (production cost, competitor's price, base demand, inconvenience coefficients). The parameters of the instances can be found in Appendix B.

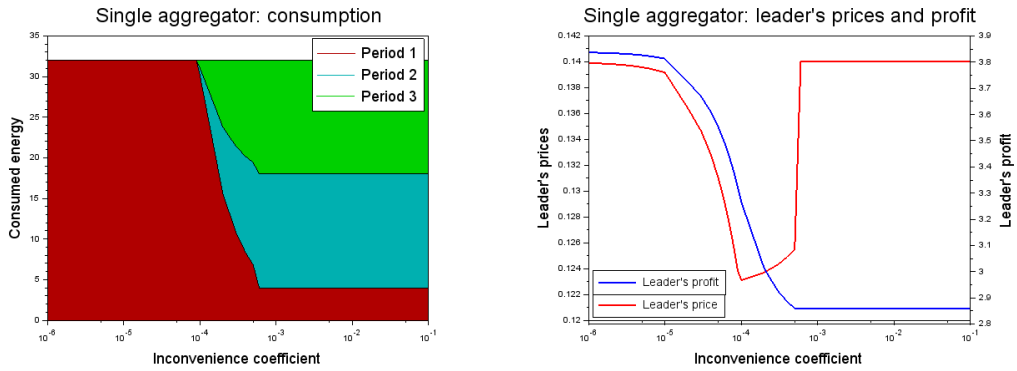

Figure 2: Situation with one aggregator
At left, the shift induced by the leader's prices. At right, the price offered by the leader during the first time period and the leader's profit.

spond to the demand of approximately ten households over one day (divided in three periods of 8 hours). The costs of energy for the leader were $(0.02, 0.06, 0.05)$ € per kWh, and the competitor's prices were assumed to be constant at 0.14 € per kWh. The inconvenience coefficients are supposed to be the same for the three time periods and vary between 10^{-6} and 0.1.

Several comments need to be highlighted. First, the energy consumption varies from the base demand to a complete shift of the whole demand on one time slot. The larger the inconvenience, the lesser the shift. When the price incentive is enough to result in a complete shift, the price curve rises towards the competitor's price: when the inconvenience coefficient decreases, a smaller incentive is needed to cause a similar shift. Then, the leader's profit increases as the inconvenience coefficients decrease. However, there is a noticeable difference between aggregators and local agents. In the aggregator case, there is a threshold: when the inconvenience coefficient reaches a certain value (around 0.0005 in this experiment), it becomes worth to incentivize load shifting with a large price difference, whereas the optimal price curve for local agents

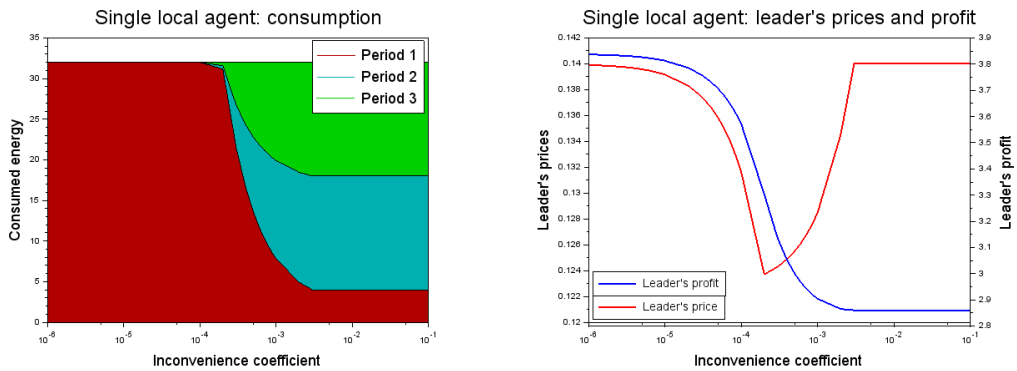

Figure 3: Situation with one local agent
At left, the shift induced by the leader's prices. At right, the price offered by the leader during the first time period and the leader's profit.

is smoother. Finally, observe that the actual interesting inconvenience coefficients are located in a narrow interval: between 10^{-4} and 10^{-2}. An inconvenience coefficient smaller than 10^{-4} will lead to higher billing costs for the consumer, whereas a coefficient larger than 10^{-2} will lead to an inability of the leader to imply load shifting with price incentives only.

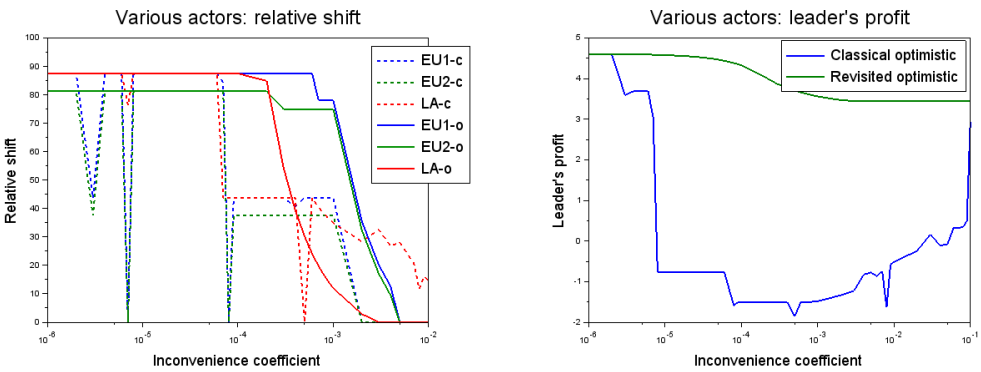

Figure 4: Situation with various actors, one local agent and one aggregator in contract with two end users
At left, the relative load shifting implied by the leader's prices for the various actors with the classical approach (c) and with the optimistic reformulation (o). At right, the profit obtained with the classical approach and with the optimistic reformulation.

After basic examples with only one ILA, we consider the case with one local agent and one aggregator, the latter in contract with two end users. The numerical results can be found in Figure 4. On the right hand side, the objective function of the leader is represented as a function of the inconvenience coefficients, assumed to be the same for all the actors. On the left hand side, the

curves indicate the relative shift in percents, according to the following formula:

$$rs_x(\mathbf{v}_x) = \frac{\sum_{h \in H} \left| d_x^h - d_x^{h,0} \right|}{2W_x},$$

where x stands for the local agent and the two end users. This value represents the percentage of energy that is consumed at another time slot than with the base demand.

These results were obtained with the multi-start option of Knitro. For each set of parameters, hundred starting points were automatically generated by the solver, and the best solution was returned. In all cases, the commercial solver returned a locally optimal solution. However, it is clear from Figure 4 that the local optima found with the classical method correspond to higher values of the objective function of the leader than the ones obtained with the local optima of the revisited optimistic formulation, and thus from the global optima of the problem. Hence the local optima found with the revisited optimistic reformulation are much better candidates for global optimality. This can be observed on the leader's profit graph: in most cases, the leader's profit obtained with the classical method is far from the leader's profit obtained with the revisited optimistic formulation. The relative shifts obtained with the classical method give interesting informations as well. Whereas in this setting, the shift should be monotonously decreasing as the inconvenience coefficient increases, it is clearly not the case. Quite the opposite, the relative shift curves obtained with the classical method are chaotic compared to the relative shift curves obtained with the revisited optimistic formulation.

Finally note that the local optima of the MPCC obtained with the revisited optimistic formulation should have equivalent local optima (in the sense of same leader prices and consumers' demands) of the MPCC obtained with the classical method. However, the solver is unable to find these equivalent local optima. All in all, the formulation of the revisited optimistic case presented in Subsection 4.2 proves to be much more efficient: the MPCC have by far less variables and constraints, and the solver is able to find much better local optima. The difficulties to obtain meaningful results with the classical method build a strong argument for a thorough analysis of the lower level.

5.1.2. Revisited optimistic versus semi-optimistic

As observed in the previous subsection, the revisited optimistic formulation is computationally much better than the classical approach. Focusing on a subset of possible GNE at the lower level allows a great decrease in the number of necessary variables and constraints, which leads to better performances of the commercial solver, in terms of solving time and of quality of the local optima. We next discuss the advantages of the semi-optimistic formulation compared to the revisited optimistic one.

First, observe that the leader's objective value is always lower with the semi-optimistic ap-

proach than with the revisited optimistic one. Indeed, for any given price profile $\mathbf{p}$ of the leader, the corresponding GNE in $GNE^o(\mathbf{p})$ yields a larger objective for the leader than the GNE in $GNE^{so}(\mathbf{p})$, since in the semi-optimistic case, at time $h \in H$, all the energy is sold at the lowest offered price $p^h_{\min}$. This is not the case in the revisited optimistic setting, where the energy bought by each client $x \in \mathcal{L} \cup \mathcal{A}$ is sold by the leader at price $p^h_{\hat{s}x} \geq p^h_{\min}$. Thus the interest of the semi-optimistic approach does not lie in its rough performance, but in its relative robustness towards the followers' reaction. The prices obtained with the semi-optimistic reformulation indeed ensure a lower bound for the leader's revenue under the mild assumption that the followers will buy energy from the leader instead of the competitors in the case of equal prices, whereas the prices obtained with the optimistic formulation only consider the leader's revenue in the most favorable case.

To have a meaningful comparison between the revisited optimistic and the semi-optimistic formulations, we need to compute the leader's revenue in the following configurations:

1. optimistic prices with the followers' response in $GNE^o(\mathbf{p})$, which yields a revenue z^o_o for the leader,
2. optimistic prices with the followers' response in $GNE^{so}(\mathbf{p})$, with revenue z^o_{so} for the leader,
3. semi-optimistic prices with the followers' response in $GNE^{so}(\mathbf{p})$, with revenue z^{so}_{so} for the leader.

The way z^o_o and z^{so}_{so} are computed is clear: it is enough to solve $\left(P^{o,KKT}_{\hat{s}}\right)$ and $\left(P^{so,KKT}_{\hat{s}}\right)$, respectively. For z^o_{so} however, there is more to do than just solve a MPCC. The prices and the energy consumption of the ILAs are the same as in the (revisited) optimistic case and are thus determined by solving $\left(P^{o,KKT}_{\hat{s}}\right)$. Nevertheless, if we assume that the followers' GNE belongs to $GNE^{so}(\mathbf{p})$, then the ILA that gets the cheapest price at time $h \in H$ from $\hat{s}$ buys all the energy that is consumed at time h. Therefore, to compute z^o_{so}, we compute the following formula:

$$\sum_{x \in \mathcal{L} \cup \mathcal{A}} p^h_{\min} d^h_x - c^h_{\hat{s}} \left(\sum_{x \in \mathcal{L} \cup \mathcal{A}} d^h_x \right),$$

where $p^h_{\min} = \min_{x \in \mathcal{L} \cup \mathcal{A}} p^h_{\hat{s}x}$ is the minimum price offered to the ILAs at time h by the leader, and for all $x \in \mathcal{L} \cup \mathcal{A}$, $\mathbf{p}_{\hat{s}x}$ and $\mathbf{d}_x$ are determined by solving $\left(P^{o,KKT}_{\hat{s}}\right)$. This represents the revenue of the leader if the followers, as an answer to the prices $\mathbf{p}_{\hat{s}x}$, choose the GNE defined as follows: if $\hat{x}$ is (one of) the ILA getting his electricity at price $p^h_{\min}$, then $e^h_{\hat{x}\hat{s}} = \sum_{x \in \mathcal{L} \cup \mathcal{A}} d^h_x$, $e^h_{x\hat{s}} = 0$ for $x \neq \hat{x}$, for $x, y \neq \hat{x}$, $e^h_{x\hat{x}} = d^h_x$ and $e^h_{xy} = 0$, and finally, $p^h_{xy} = \max\left\{p^h_{\hat{s}x}, p^h_{\hat{s}y}\right\}$. This is actually a worst-case scenario for the leader, under the assumption that no electricity is bought from the competitor. Nevertheless, it is a possible answer of the followers, since it is a GNE. Therefore, still under the assumption that no energy is bought from the competitor, GNE that are answers to the prices $\mathbf{p}_{\hat{s}}$ computed with $\left(P^{o,KKT}_{\hat{s}}\right)$ can possibly take all values between z^o_{so} and z^o_o, whereas an answer

to prices $\mathbf{p}_{\hat{s}}$ computed with $\left(P_{\hat{s}}^{so,KKT}\right)$ will yield a revenue that is at least z_{so}^{so}. These values share a common ground idea with the *rewarding* and *deceiving* solutions defined in Alves & Antunes (2016).

Clearly,

$$z_o^o \geq z_{so}^{so} \geq z_{so}^o.$$

The question here is: how bad can the optimistic prices perform when the followers choose a GNE in $GNE^{so}(\mathbf{p})$, compared to the semi-optimistic prices ? To prove the interest of the semi-optimistic approach, we study a very simple case: three time slots and two local agents x and y with respective demands $(0, 10, 0)$ and $(0, 0, 10)$. Their inconvenience coefficients are as follows: $\mathbf{v}_x = (v, v, v)$ and $\mathbf{v}_y = (100, v, v)$, with v varying between 10^{-6} and 0.1. Setting $v_y^1 = 100$ means that no energy should be consumed by y at the first time slot. The spot market prices are supposed to be $(0.01, 0.05, 0.08)$, so that the leader should incentivize its clients to consume energy as early as possible. This means that for the leader, an ideal consumption of x and y would be $(10, 0, 0)$ and $(0, 10, 0)$, respectively, considering that y should not consume during the first time slot. The competitor's prices are assumed to remain constant at $(0.14, 0.14, 0.14)$ as in the previous examples.

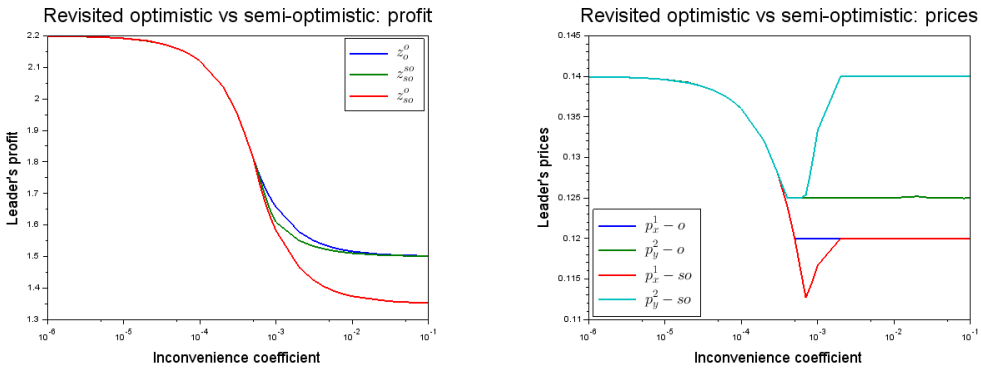

Figure 5: Comparison optimistic vs semi-optimistic reformulations
At left, the different profits: z_o^o, z_{so}^{so} and z_{so}^o. At right, the prices p_x^1 and p_y^2 with the optimistic (o) and with the semi-optimistic (so) reformulations.

It follows from the design of our example that we will have

$$p_{\hat{s}y}^1 = p_{\hat{s}x}^2 = p_{\hat{s}x}^3 = p_{\hat{s}y}^3 = 0.14.$$

Indeed, the leader does not want x or y to consume energy during these periods, therefore his price will take the highest possible value, that is $p_{\bar{s}x}^h = p_{\bar{s}y}^h = 0.14$ for all $h \in H$. The prices that will directly influence the electricity consumption of x and y are thus the prices $p_{\hat{s}x}^1$ and $p_{\hat{s}y}^2$.

In the revisited optimistic case, x and y buy their energy directly from the leader. In the semi-optimistic case, x will buy his energy from y during the second time slot, thus during this time slot, all the energy will be bought by y from the leader at price $p^2_{\hat{s}y}$. The results are presented in Figure 5. To the left, the leader's profit is represented in three cases: the blue curve depicts the result of the revisited optimistic formulation, the green curve the result of the semi-optimistic formulation, and finally the red curve illustrates the leader's profit when the leader's prices are computed with the optimistic reformulation, but the GNE of the followers is in $GNE^{so}(\mathbf{p})$. The semi-optimistic formulation yields prices that are by far more robust, in the sense that for these prices, any followers' answer will yield at least the computed profit (provided that the followers favour the leader over the competitor in case of equal prices). By contrast, the prices computed with the (revisited) optimistic approach will yield the computed profit only if the followers' answer is the computed one (except for degenerate cases). An other possible followers' answer (in particular a semi-optimistic GNE) might provide less profit. Therefore, the prices computed with this formulation ensure a certain profit in a worst-case scenario. Furthermore, the difference between the profits obtained with the revisited optimistic z^o_o or the semi-optimistic formulations z^{so}_{so} are not very large, as it can be seen in Figure 5. Therefore, computing robust prices implies lower revenues in an optimal case (i.e. the followers' GNE is in $GNE^o(\mathbf{p})$), but ensures much better results in worst cases.

Let us end this subsection with a table summarizing some instances that have been tested.

Instance	# of hours	# of end users	# of local agents	Problem	#var	#cons	Number of iterations	Comp. time (s)
4	3	2	2	opt-revis.	45	51	17048	3,789
4	3	2	2	semi-opt.	48	60	22045	5,745
5	3	3	3	opt-revis.	66	75	29895	13,341
5	3	3	3	semi-opt.	69	87	48095	13,696
6	4	3	3	opt-revis.	85	97	17176	8,146
6	4	3	3	semi-opt.	89	113	14630	6,698
7	4	6	6	opt-revis.	166	190	20140	14,613
7	4	6	6	semi-opt.	170	218	28499	23,421
8	4	8	8	opt-revis.	220	252	31846	39,883
8	4	8	8	semi-opt.	224	288	41156	42,437

Table 2: All the tests reported here involve one aggregator. All the tests end up at a local optimum with a null gap. Each instance corresponds to a given set of parameters (production cost, competitor's price, base demand, inconvenience coefficients). The parameters of the instances can be found in Appendix B.

Note that in the tests presented above, the number of iterations and the computational time are always higher for the semi-optimistic approach than for the revisited optimistic approach, except for instance 6. The behavior on this particular instance might be due to an usual structure of the instance, or more probably to the solving procedure used by Knitro.

5.2. Noncompetitive case

In the previous subsection, we assumed that the leader is competitive at any time, that is the competitor's price is larger than the production costs for any time slot. However, this highly depends on the studied data. In order to enlighten the shifting effect of the leader's pricing policy, we present a last example in which the leader is not always competitive. The data and results of the example are presented in Figure 6: we consider two time slots, and a single local agent with demand $(5, 5)$. Furthermore, the inconvenience coefficients $v^1 = v^2$ take three different values: 0.001, 0.002 and 0.003. These three values cause three very different situations: with 0.003, the inconvenience is too large to allow any shift and the optimal prices of the leader are similar to the competitor's prices. With 0.001, the inconvenience is small enough to cause, combined with the values of the optimal leader prices, a shift of the whole demand on the first time slot. The last case is intermediate: only a portion of the demand is shifted with the optimal leader prices.

First, observe that we are in an optimistic setting. The results presented here are therefore obtained with the revisited optimistic formulation. It is impossible for the leader to make any profit during the second time slot, because during this time slot, the leader's energy cost (factor of the linear cost function) is higher than the competitor's price (see top of Figure 6). Therefore, the optimal leader prices are greater than the competitor's prices during the second time slot, thus the energy that is consumed by the client at this time slot is bought from the competitor. The concern for the leader is to give an incentive to the client to shift his demand to the first time slot, where the leader can make some profit. In the base case, the profit achieved is $(0.07 - 0.01) * 5 = 0.3$: no optimization is done at all, the leader's prices are equal to the competitor's prices and thus the optimal follower's demand is the base demand. If the leader's price is too high, the follower will not shift his demand. If the leader's price is too low, the follower will shift his demand to a great extend, but the demand increase will not compensate the price decrease. The cases $v^h = 0.001$ and $v^h = 0.003$ are extreme: either the client is very shifting-averse, which leads to an optimal leader's price equal to the competitor's price and no shift, or the client undergoes a very low inconvenience with any shift, which leads to the leader offering a price small enough to induce a shift of the whole follower's demand to the first time slot. Finally, the case $v^h = 0.002$ is probably the most interesting: the client agrees to shift some of his demand, but not everything: exactly 12.5% of his demand is shifted from the second to the first time slot. In this case, the leader's profit rises to 3.125, which represents an improvement of 4.16% of the leader's profit. This case can be considered as realistic. A shift of 12.5% of the demand does not seem too difficult to achieve for the client, and the leader's profit increases enough to be worth the effort of computing the optimal prices.

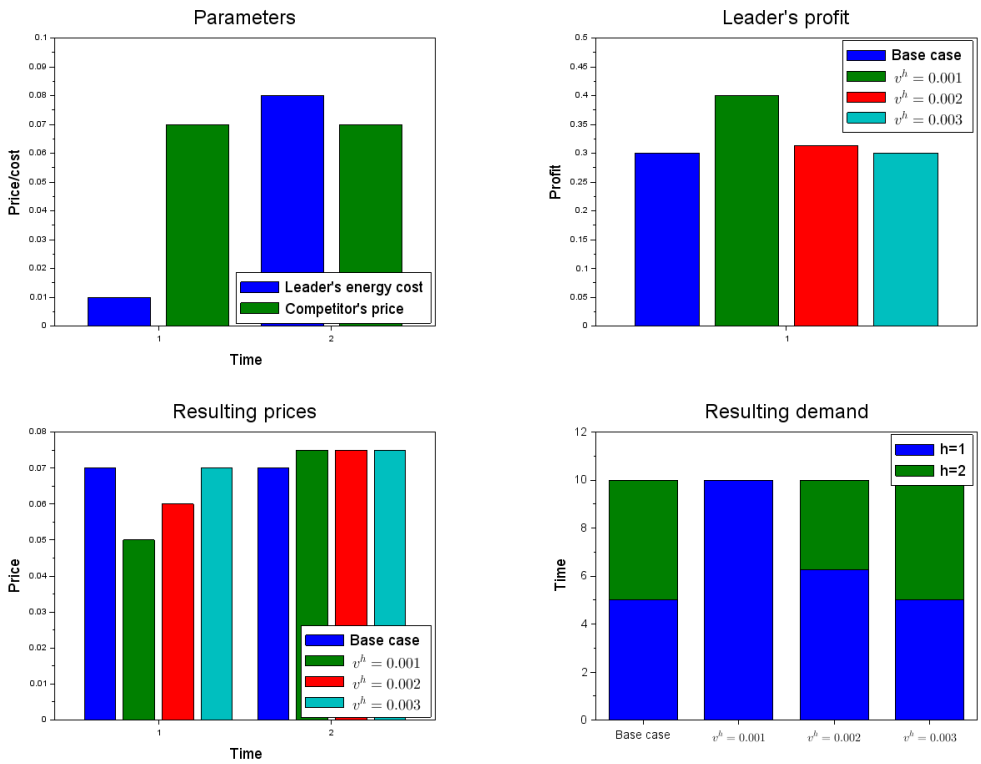

Figure 6: Noncompetitive case
On top left, the data for the example where the leader is not always competitive: the production cost and the competitor's prices. On top right, the resulting profit of the leader for the four various cases: no optimization (i.e. copying the competitor's prices), and optimization for the three possible values of v^h.
At bottom left, the optimal prices of the leader for the example where the leader is not always competitive. At bottom right, the follower's demand resulting of these prices.

6. Conclusion

In this paper, we have defined a trilevel energy pricing model for Demand-Side Management. First, by explicit resolution of the lowest level, we reduced it to a new bilevel problem. Besides the classical optimistic approach consisting in using first order reformulation of the lower level of this bilevel problem (the classical optimistic approach), we defined two new concepts of solutions of the problem. Both of those new approaches are based on a selection of special GNE of the lower level of the new model. The first one, called revisited optimistic, selects special GNE of the lower level of the new model while, theoretically, maintaining the optimal value of the leader. Actually our numerical experiments enlighten the fact that the revisited optimistic approach is clearly more stable than the classical approach, the latter providing often a lower optimal value of the supplier. Moreover the revisited formulation proved to be much more efficient than the

classical method in computational terms (number of variables, constraints and thus solving time). Further analysis of the exchanges among the ILA allowed us to define the second new concept of solution, called semi-optimistic. The special kind of GNE considered at the lowel level of the new bilevel model corresponds to an intermediate between optimistic and pessimistic concepts for the leader's problem. The numerical results obtained with the semi-optimistic approach have shown its robustness towards the followers' response to the leader's decisions while the optimal value for the supplier is only slightly impacted, thus furnishing a good compromise concept of solution. Note that the reformulations of the initial single-leader-multi-follower game lead to nonconvex optimization problems with quite complex constraints. Thus the optimization tools that are used here can only reach local solutions (at best). The development of specific and adapted optimization techniques would be needed to go further and it is out of the scope of this work.

As a conclusion, this work provides strong arguments for a careful theoretical analysis of multilevel models for DSM pricing as well as numerical experiments.

Further works might imply various notions of equilibriums at the lower level. Here, we chose to consider Nash equilibriums. However, a more realistic approach might be to consider market equilibriums in which an energy transaction is the result of a bargaining between the buyer and the seller, and not the sole fact that the buyer decides the quantity and the seller the price.

7. Acknowledgements

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Appendix A. KKT conditions

The KKT conditions of (P_ℓ) mentioned in Subsection 4.1 are as follows:

$$
\begin{cases}
\sum_{h\in H} d_\ell^h = W_\ell & (\lambda_\ell) & \\
p_{\ell x}^h \leq \alpha_{\ell xs}^h p_{sx}^h + \left(1-\alpha_{\ell xs}^h\right) p_{s\ell}^h & \left(\mu_p^{h\ell x}\right) & \forall h \in H, x \in \mathcal{L}\cup\mathcal{A}, s \in \mathcal{S} \\
e_{\ell x}^h \geq 0 & \left(\mu_e^{h\ell x}\right) & \forall h \in H, x \in \mathcal{L}\cup\mathcal{A}\cup\mathcal{S}, \\
\alpha_{\ell xs}^h \in [0,1] & \left(\mu_\alpha^{h\ell xs,\min}, \mu_\alpha^{h\ell xs,\max}\right) & \forall h \in H, x \in \mathcal{L}\cup\mathcal{A}, s \in \mathcal{S} \\
\mu_e^{h\ell x} \geq 0 & & \forall h \in H, x \in \mathcal{L}\cup\mathcal{A}\cup\mathcal{S} \\
\mu_p^{h\ell x}, \mu_\alpha^{h\ell xs,\min}, \mu_\alpha^{h\ell xs,\max} \geq 0 & & \forall h \in H, x \in \mathcal{L}\cup\mathcal{A} \\
\mu_p^{h\ell x}\left(p_{\ell x}^h + \sum_{s\in\mathcal{S}}\left(-\alpha_{\ell xs}^h p_{sx}^h + \left(\alpha_{\ell xs}^h - 1\right) p_{s\ell}^h\right)\right) = 0 & & \forall h \in H, x \in \mathcal{L}\cup\mathcal{A} \\
\mu_e^{h\ell x} e_{\ell x}^h = 0 & & \forall h \in H, x \in \mathcal{L}\cup\mathcal{A}\cup\mathcal{S} \\
\mu_\alpha^{h\ell xs,\min}\alpha_{\ell xs}^h = 0 & & \forall h \in H, x \in \mathcal{L}\cup\mathcal{A}, s \in \mathcal{S} \\
\mu_\alpha^{h\ell xs,\max}\left(1-\alpha_{\ell xs}^h\right) = 0 & & \forall h \in H, x \in \mathcal{L}\cup\mathcal{A}, s \in \mathcal{S} \\
p_{x\ell}^h + 2v_\ell^h\left(d_\ell^h - d_\ell^{h,0}\right) - \mu_e^{h\ell x} + \lambda_\ell = 0 & & \forall h \in H, x \in \mathcal{L}\cup\mathcal{A}\cup\mathcal{S} \\
-\sum_{x\in\mathcal{L}\cup\mathcal{A}} e_{x\ell}^h + \mu_p^{h\ell x} = 0 & & \forall h \in H, x \in \mathcal{L}\cup\mathcal{A} \\
\left(p_{s\ell}^h - p_{sx}^h\right)\mu_p^{h\ell x} + \sum_{s\in\mathcal{S}}\left(-\mu_\alpha^{h\ell xs,\min} + \mu_\alpha^{h\ell xs,\max}\right) = 0 & & \forall h \in H, x \in \mathcal{L}\cup\mathcal{A}.
\end{cases}
$$

The variables between parentheses denote the dual variables associated to the primal constraints.

Next, the KKT conditions of (P'_a), also mentioned in Subsection 4.1:

$$\begin{cases}
\sum_{s\in\mathcal{S}} e^h_{as} + \sum_{\ell\in\mathcal{L}} \left(e^h_{a\ell} - e^h_{\ell a}\right) + \sum_{a'\neq a} \left(e^h_{aa'} - e^h_{a'a}\right) = \sum_{i\in\mathcal{I}_a} d^h_i & (\lambda_a) & \forall h\in H \\
\sum_{h\in H} d^h_i = W_i & (\lambda_i) & \forall i\in\mathcal{I}_a \\
p^h_{ax} \leq \alpha^h_{axs} p^h_{sx} + \left(1-\alpha^h_{axs}\right) p^h_{sa} & \left(\mu^{hax}_p\right) & \forall h\in H, s\in\mathcal{S}, x\in\mathcal{L}\cup\mathcal{A} \\
2v^h_i \left(d^h_i - d^{h,0}_i\right) + \hat{\lambda}_i \geq 0 & \left(\mu^{hi}_r\right) & \forall h\in H, i\in\mathcal{I}_a \\
d^h_i, e^h_{ax} \geq 0 & \left(\mu^{hi}_i, \mu^{hax}_e\right) & \forall h\in H, x\in\mathcal{L}\cup\mathcal{A}\cup\mathcal{S}, i\in\mathcal{I}_a \\
\alpha^h_{axs} \in [0,1] & \left(\mu^{haxs,\min}_\alpha, \mu^{haxs,\max}_\alpha\right) & \forall h\in H, x\in\mathcal{L}\cup\mathcal{A}, s\in\mathcal{S} \\
\mu^{hax}_e \geq 0 & & \forall h\in H, x\in\mathcal{L}\cup\mathcal{A}\cup\mathcal{S} \\
\mu^{hax}_p,\ \mu^{haxs,\min}_\alpha,\ \mu^{haxs,\max}_\alpha,\ \mu^{hi}_r,\ \mu^{hi}_i \geq 0 & & \forall h\in H, x\in\mathcal{L}\cup\mathcal{A}, s\in\mathcal{S}, i\in\mathcal{I}_a \\
\mu^{hax}_p \left(p^h_{ax} - \alpha^h_{axs} p^h_{sx} + \left(\alpha^h_{axs}-1\right) p^h_{sa}\right) = 0 & & \forall h\in H, x\in\mathcal{L}\cup\mathcal{A} \\
\mu^{hax}_e e^h_{ax} = 0 & & \forall h\in H, x\in\mathcal{L}\cup\mathcal{A}\cup\mathcal{S} \\
\mu^{haxs,\min}_\alpha \alpha^h_{axs} = 0 & & \forall h\in H, x\in\mathcal{L}\cup\mathcal{A}, s\in\mathcal{S} \\
\mu^{haxs,\max}_\alpha \left(1-\alpha^h_{axs}\right) = 0 & & \forall h\in H, x\in\mathcal{L}\cup\mathcal{A}, s\in\mathcal{S} \\
\mu^{hi}_r \left(2v^h_i \left(d^h_i - d^{h,0}_i\right) + \hat{\lambda}_i\right) = 0 & & \forall h\in H, i\in\mathcal{I}_a \\
\mu^{hi}_i d^h_i = 0 & & \forall h\in H, i\in\mathcal{I}_a \\
p^h_{xa} - \mu^{hax}_e + \lambda_a = 0 & & \forall h\in H, x\in\mathcal{L}\cup\mathcal{A}\cup\mathcal{S} \\
-\sum_{x\in\mathcal{L}\cup\mathcal{A}} e^h_{xa} + \mu^{hax}_p = 0 & & \forall h\in H, x\in\mathcal{L}\cup\mathcal{A} \\
\left(p^h_{sa} - p^h_{sx}\right)\mu^{hax}_p + \sum_{s\in\mathcal{S}} \left(-\mu^{haxs,\min}_\alpha + \mu^{haxs,\max}_\alpha\right) = 0 & & \forall h\in H, x\in\mathcal{L}\cup\mathcal{A} \\
2v^h_i \left(2d^h_i - d^{h,0}_i - \mu^{hi}_r\right) + \hat{\lambda}_i - \lambda_a + \lambda_i - \mu^{hi}_i = 0 & & \forall h\in H, i\in\mathcal{I}_a \\
d^h_i - \mu^{hi}_r = 0 & & \forall h\in H, i\in\mathcal{I}_a.
\end{cases}$$

The KKT conditions of $\left(P^{el}_x\right)$ are as follows: for $x = \ell \in \mathcal{L}$:

$$\begin{cases}
\sum_{h\in H} e^h_{\ell\hat{s}} + e^h_{\ell\bar{s}} = W_\ell & (\lambda_\ell) & \\
e^h_{\ell s} \geq 0 & \left(\mu^h_{\ell s}\right) & \forall h\in H, s\in\mathcal{S} \\
\mu^h_{\ell s} \leq 0 & & \forall h\in H, s\in\mathcal{S} \\
\mu^h_{\ell s} e^h_{\ell s} = 0 & & \forall h\in H, s\in\mathcal{S} \\
p^h_{s\ell} + 2v^h_\ell \left(d^h_\ell - d^{h,0}_\ell\right) + \lambda_\ell + \mu^h_{\ell s} = 0 & & \forall h\in H, s\in\mathcal{S},
\end{cases}$$

and for $x = a \in \mathcal{A}$:

$$
\begin{cases}
\sum_{i \in \mathcal{I}_a} d_i^h = e_{a\hat{s}}^h + e_{a\bar{s}}^h & (\lambda_a) & \forall h \in H \\
\sum_{h \in H} d_i^h = W_i & (\lambda_i) & \forall i \in \mathcal{I}_a \\
2v_i^h \left(d_i^h - d_i^{h,0}\right) + \hat{\lambda}_i \geq 0 & \left(\mu_r^{hi}\right) & \forall h \in H, i \in \mathcal{I}_a \\
d_i^h, e_{as}^h \geq 0 & \left(\mu_d^{hi}, \mu_{as}^h\right) & \forall h \in H, i \in \mathcal{I}_a, s \in \mathcal{S} \\
\mu_d^{hi}, \mu_r^{hi}, \mu_{as}^h \leq 0 & & \forall h \in H, i \in \mathcal{I}_a, s \in \mathcal{S} \\
\mu_d^{hi} d_i^h = 0 & & \forall h \in H, i \in \mathcal{I}_a \\
\mu_r^{hi} \left(2v_i^h \left(d_i^h - d_i^{h,0}\right) + \hat{\lambda}_i\right) = 0 & & \forall h \in H, i \in \mathcal{I}_a \\
\mu_{as}^h e_{as}^h = 0 & & \forall h \in H, s \in \mathcal{S} \\
p_{sa}^h + 2v_i^h \left(2d_i^h - d_i^{h,0} + \mu_r^{hi}\right) + \mu_d^{hi} + \lambda_i + \lambda_a = 0 & & \forall h \in H, s \in \mathcal{S} \\
p_{sa}^h - \lambda_a + \mu_{as}^h = 0 & & \forall h \in H, s \in \mathcal{S} \\
W_i + \sum_{h \in H} \mu_r^{hi} = 0 & & \forall h \in H, i \in \mathcal{I}_a.
\end{cases}
$$

Appendix B. Instances parameters

This section of the appendix gathers the parameters of the instances used for the computing times in the subsections 5.1.1 and 5.1.2. First, the competitor's prices are all set to 0.14, and the inconvenience coefficients are assumed to be the same for all ILAs and all time slots: the chosen value is 0.0004.

Concerning the energy costs of the suppliers and the base demands of the local agents and end users, they are presented in the following table.

The instances are built by successive addition of local agents and end users. For example, Instance 4 involves the two first local agents ℓ_1 and ℓ_2, and the two end users i_1 and i_2, whereas the three local agents of Instance 5 are ℓ_1, ℓ_2 and ℓ_3, and its three end users are i_1, i_2, i_3. Since these two instances only involve three hours, the parameters for $h = 4$ are ignored.

Parameter	$h = 1$	$h = 2$	$h = 3$	$h = 4$
$c^h_{\hat{s}}$	0.02	0.06	0.06	0.03
$d^h_{\ell_1}$	4	14	14	9
$d^h_{\ell_2}$	4	14	14	8
$d^h_{\ell_3}$	5	12	13	5
$d^h_{\ell_4}$	9	11	15	3
$d^h_{\ell_5}$	3	8	7	9
$d^h_{\ell_6}$	2	20	19	5
$d^h_{\ell_7}$	4	14	14	8
$d^h_{\ell_8}$	5	12	13	5
$d^h_{\ell_9}$	9	11	15	3
$d^h_{i_1}$	0.4	1.4	1.4	0.9
$d^h_{i_2}$	0.4	1.4	1.4	0.55
$d^h_{i_3}$	0.4	1.4	1.2	0.8
$d^h_{i_4}$	0.3	1.5	1.6	0.5
$d^h_{i_5}$	0.7	2	1.5	1
$d^h_{i_6}$	0.2	0.9	0.8	0.3
$d^h_{i_7}$	0.4	1.4	1.2	0.8
$d^h_{i_8}$	0.3	1.5	1.6	0.5
$d^h_{i_9}$	0.7	2	1.5	1

Table B.3: The parameters of the test instances.