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A Benders decomposition method for locating stations in a one-way electric car sharing system under demand uncertainty

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Abstract

We focus on a problem of locating recharging stations in one-way station based electric car sharing systems which operate under demand uncertainty. We model this problem as a mixed integer stochastic program and develop a Benders decomposition algorithm based on this formulation. We integrate a stabilization procedure to our algorithm and conduct a large-scale experimental study on our methods. To conduct the computational experiments, we develop a demand forecasting method allowing to generate many demand scenarios. The method is applied to real data from Manhattan taxi trips. We are able to solve problems with 100 to 500 scenarios, each scenario including 1000 to 5000 individual customer requests, under high and low cost values and 5 to 15 mins of accessibility restrictions, which is measured as the maximum walking time to the operating stations.

Keywords: Location, Urban Mobility, Electric Car Sharing, Benders Decomposition, Mixed Integer Stochastic Programming, Stochastic Demand

1. Introduction

1 The increasing number of privately owned cars causes significant amount of pollution, traffic congestion,
2 and parking problems in urban cities. The private cars are usually parked for very long hours and their
3 utilization rates are very low. The car sharing systems are based on shared usage of vehicles by multiple
4 people and the utilization rate of these vehicles, usually owned by companies or organizations, are much
5 higher compared to the private ones. They usually serve their users based on a subscription system and
6 charge each usage by traveling distance or time. Due to this kind of pricing, the users tend to drive less and
7 therefore, create less traffic. Together with the integration of environmental friendly vehicles, car sharing
8 systems have a high potential of reducing these crucial urban problems.

9 The car sharing systems can be grouped into two categories as station based and free floating depending
10 on whether the customers are required to visit certain stations or not. In station based systems, where the
11 customers need to visit stations to pick-up and drop the cars, two types of trips can be observed: i) round
12 trips which force to leave the cars to the stations that they were taken from and ii) one-way trips where the
13 cars can be dropped at stations different than the originating one. One-way systems provide more flexibility

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14 to the users but requires a more sophisticated management as otherwise there might be significant load
15 imbalances at stations. In free-floating systems, the cars can be parked anywhere within a pre-defined zone
16 and customers can pick the cars from wherever they are parked. Although they are more attractive from a
17 flexibility perspective, free floating systems do not guarantee an available parking space to the users at the
18 end of their trips and the cars need to be parked with at least a certain level of fuel or battery. Moreover,
19 these systems in general require relocation of cars in order to balance the availability at different parts of
20 the city and to perform refueling/recharging operations.

21 In this work, we focus on design of a one way station based electric car sharing (ECS) system that
22 operates under demand uncertainty. We assume that the users will be served with a fleet of identical electric
23 cars. Our main goal is to decide on the location and number of stations with limited capacities and the initial
24 level of cars at each station to operate the system in order to maximize the expected profit of the operating
25 company. The expected profit is equal to the expected revenue obtained from served customers minus the
26 fixed cost of opening stations and purchase cost of cars. The electric cars have a shorter range than the
27 conventional cars and the recharging operations take much longer. Therefore, the location of stations plays
28 a crucial role in service quality. To this end, we develop exact methods to design a system that takes into
29 account accessibility of users to located stations, availability of a car at the departure station, and availability
30 of an empty spot at the arrival station for forecasted customer requests. We measure accessibility by the
31 walking time from origin and destination points, in other words, a station is accessible by a customer if their
32 origin and destination points are within a certain walking time to some operating stations.

33 We consider general capacities (non-identical) for stations and assume that the number of charging units
34 is equal to the number of parking spots at the stations. The cars are plugged as soon as they are parked
35 to a station and they are recharged until full battery capacity before being available for the next customer.
36 This assumption enables solving real size problems with strategical level decisions [10, 14].

37 We aim to solve large scale problems of locating recharging stations without aggregation of demand. In
38 order to represent uncertainty in demand, we consider multiple scenarios with occurrence probability. Each
39 scenario consists of a certain number of customer requests and each request is associated with an origin
40 node, a destination node, and a starting time. The problem under consideration requires decisions on the
41 number and location of stations, the number of cars available at each station, the customers to be served,
42 the stations they need to visit, and the time of each visit. We assume that relocations will be done
43 outside the planning period (e.g. overnight). Therefore, we do not consider other operational decisions such
44 as relocation time/network of cars and number of operators to hire for relocation etc.

45 We formulate this problem as a two stage stochastic programming model and develop a Benders decom-
46 position algorithm based on its deterministic equivalent. In order to test our methods, we develop a demand
47 forecasting method that uses historical data to generate new requests. We evaluate the performance of
48 our algorithm under different parametric and structural settings by using problem instances obtained from
49 a real taxi trip data of Manhattan. From the experimental studies we conducted on the formulation, we
50 observe that the average linear programming relaxation (LP) gap of the formulation is less than 0.4% on
51 the instances tested. Moreover, a partial relaxation of the formulation provides the same objective value
52 with the original model for most problems with an average gap less than 0.01% in our test bed. We further
53 strengthen our Benders method with a stabilization procedure, which improves the average performance of
54 our algorithm, especially, on instances with larger number of scenarios.

55 The rest of this paper is organized as follows: In Section 2, we present a brief summary of the related works
56 in the literature. In Section 3, we provide a mathematical description of our problem with some notation
57 and provide the details of our mixed integer programming formulation. Section 4 presents the results of the
58 computational study we conducted on our formulation and its relaxations. We dedicate Section 5 to a brief
59 summary of the classical Benders decomposition method, its adaptation to pure integer programming (IP)
60 problems in the literature, and a detailed description of the Benders algorithm that we propose. Section 6
61 follows with the computational study on our Benders algorithm and a stabilized version of it. We conclude
62 in Section 7 with a summary of our results and perspectives on future research.

63 2. Related work

64 The current studies in the car sharing literature mostly focus on tactical and operational level decision
65 problems. Tactical and operational level problems concerning electric vehicles already attracted attention in
66 logistics systems and several studies in the literature focused on different variants of electric vehicle routing
67 problems [26, 28, 45, 43]. Within the context of car sharing systems, the most commonly considered problem
68 is the relocation of cars to increase availability in high demand areas and avoid surplus of cars at the low
69 demand points [4, 15, 19, 22, 35, 38, 39, 46, 33, 11]. On the other hand, as also indicated in a recent review
70 [12], the car sharing literature on strategic level decision problems involving location of stations is relatively
71 sparse.

72 Optimal location of stations in one-way car sharing systems is first addressed by Correia and Antunes [24].
73 The authors propose a mixed integer programming formulation that maximizes the profit of the operator
74 by considering several cost and revenue factors. They compare the model, which enforces service via closest
75 stations to origins and destinations, under three service strategies: (i) The operator has the possibility to
76 decide on which customer requests to serve without any restriction (ii) all requests have to be served (iii)
77 the operator can reject a request only if there is no car available at the starting station. The experiments on
78 a real data from Lisbon, Portugal, show that serving all customers might decrease the profit significantly.
79 Later, Correia et al. [25] extend the formulations of Correia and Antunes [24] to a more flexible system where
80 the assumption of service through closest stations is relaxed. The experiments in the latter study reveal that
81 this kind of flexibility increased the profit of the operator together with the introduction of vehicle stock
82 information.

83 In addition to the optimization models mentioned above, a discrete event simulation based model that
84 analyzes the impact of strategy changes in car sharing systems is introduced by Fassi et al. [27]. The strategies
85 considered include opening new stations, increasing station capacities, merging or splitting stations. The
86 model is evaluated on a car sharing data from Montreal, Canada.

87 Location of recharging stations in electric car sharing systems is first studied by Boyacı et al. [10]. The
88 authors focus on a multi-objective problem that combines strategic and tactical decisions in a one-way
89 electric car sharing system. One of the objectives is to maximize the net profit of the operating company
90 and the other one is to maximize the net benefit of the clients. The authors propose a mixed integer linear
91 programming formulation for solving this problem but due to the intractability of this formulation caused
92 by the large number of relocation variables, they employ an aggregated demand structure, where the trips
93 (demand) with the same origin and destination centers are grouped together to decrease the number of
94 decision variables. Demand centers are the clusters of origin/destination points of trips that can be accessed

95 by the same set of stations. Moreover, after each usage, the vehicles are charged for a fixed period of time
96 and depending on the demand served by vehicles, these charging periods are calculated to reach the required
97 battery level e.g. full battery. The proposed method is tested on a real data from Nice, France.

98 In a recent paper, Çalik and Fortz [16] focus on location of stations in a one-way electric car sharing where
99 the demand is known in advance. The problem includes decisions on location and number of the stations,
100 initial level of cars at operated stations, the subset of requests to serve, and their service routes. The authors
101 provide a mathematical formulation that maximizes the profit of the operator by taking into account the
102 revenue obtained from the served demand and the fixed costs of stations and car purchases. They conduct
103 experiments on large scale problem instances obtained through a real data of Manhattan (New York, USA)
104 taxi trips. In this paper, we extend the mathematical formulation of Çalik and Fortz [16] to the case where
105 there is uncertainty in demand and based on this formulation, we develop a decomposition algorithm that
106 can solve large scale problems in reasonable amount of time. Another recent paper by Biesinger et al. [7]
107 provides a bi-level heuristic algorithm that aims to find the location of recharging stations and the initial
108 number of cars in an electric car-sharing system. The algorithm is tested on real-world instances from
109 Vienna, Austria.

110 We are also aware of a few recent works [13, 14] that provide heuristics and formulations for location of
111 stations in electric car sharing systems. Among them, Brandstätter et al. [13] consider a system with perfect
112 demand information. The authors provide integer programming formulations and construction heuristics
113 which maximize the profit under budget constraints. They conduct experiments on randomly generated
114 grid-graph instances and instances from Vienna (relatively larger in size). Brandstätter et al. [14] extend
115 the methods of Brandstätter et al. [13] to the case with demand uncertainty where demand is represented
116 by multiple scenarios. They consider a maximum walking time of five minutes and seven scenarios in their
117 experiments on the Vienna instances. The model is tested on small-size grid-graph instances (maximum
118 100 requests, 50 stations, 5 scenarios) and a reduced version of the Vienna instances (1060 requests, 201
119 stations, 7 scenarios). With this model, most instances are solved to optimality within the one-week time
120 limit enforced by the authors.

121 Other than the car sharing systems, location of charging stations were also considered for private electric
122 vehicles and urban taxi providers. The literature is relatively denser for these type of problems and it is
123 possible to find both exact solution methodologies [32, 48, 21, 3, 20, 1, 2, 40] and heuristic approaches
124 [34, 36, 47] for location of public charging units.

125 3. Problem definition and formulation

126 In this section, we present a formal definition and a mathematical formulation of our problem. Given a
127 city network $G = (V, A)$ with arc set A , node set V , and set of potential stations $J \subset V$, we are required to
128 select a subset \bar{J} of J and locate an initial number of cars to the selected stations in order to satisfy a part
129 of the customer demand represented by multiple scenarios consisting of individual requests. Each request
130 must be served by a car available at a station accessible from the origin node at the time of departure
131 and the car should be left at an arrival station accessible from the destination of the request with a free
132 parking/charging spot at the time of arrival. Additionally, the battery consumption between the departure
133 and the arrival stations assigned to a request should not exceed the battery level of the car and the total
134 length (time) of the trip from the origin node to the destination should not be larger than a given threshold

135 Δ_k for request k . The traveling time and the energy consumption for each request are computed based on
 136 the shortest path between the departure and arrival stations allocated. One can also consider the case where
 137 these values are known explicitly for each request. A car that completes its service is plugged into a charger
 138 at its arrival station and it does not become available until the beginning of the first period after it has full
 139 battery level. For any scenario, the total number of cars (available or being charged) at a station should
 140 not exceed the capacity of that station at any time. The objective of the problem is to maximize the total
 141 expected profit that is equivalent to the total expected revenue obtained from the requests served minus the
 142 total cost of stations selected and cars owned.

143 Before going into the details of our mathematical formulation, we provide an initial set of parameter
 144 definitions used throughout the paper here. Upper case letters denote sets whereas lower case ones denote
 145 indices e.g. J is a set and j is the index of an element in J .

- 146 • $V = \{1, \dots, n\}$ is the set of nodes.
- 147 • $J = \{1, \dots, m\}$ is the set of potential stations where $J \subset V$.
- 148 • f_j is the fixed cost of locating a station on node $j \in J$.
- 149 • g is the cost of purchasing a car.
- 150 • C_j is the capacity of station $j \in J$.
- 151 • $T = \{0, \dots, \tau\}$ is the set of time slots (identical length)
- 152 • $S = \{1, 2, \dots, |S|\}$ is an index set corresponding to the scenarios.
- 153 • q_s is the probability that scenario $s \in S$ will occur.
- 154 • K_s is the set of requests in scenario $s \in S$, with origin $O_k \in V$, destination $D_k \in V$, starting time
 155 $T_k \in T$, and revenue p_k for $k \in K_s$. $K = \bigcup_{s \in S} K_s$.
- 156 • δ_{ij} is the battery usage (Ws) on the way from station $i \in J$ to station $j \in J$.
- 157 • β is the battery capacity (Ws) for each car .
- 158 • d_{ij} is the travel time from station $i \in J$ to station $j \in J$.
- 159 • d_{ij}^w is the walking time from node $i \in V$ to node $j \in S$.
- 160 • β^w is the maximum walking time between the origin (destination) points of requests and the departure
 161 (arrival) stations they are assigned to.
- 162 • Δ_k is the maximum time that a trip assigned to customer k can take (including the walking time).

163 We formulate our problem as a two stage stochastic program, in which location of stations and initial
 164 number of cars at each station are the first stage decisions whereas the requests served within each scenario
 165 and allocation of stations to the requests are second stage decisions. In this paper, we focus on the de-
 166 terministic equivalent of our model. We propose a path based formulation with a set of decision variables
 167 associated with the paths (ordered list of nodes to visit) allocated to the requests.

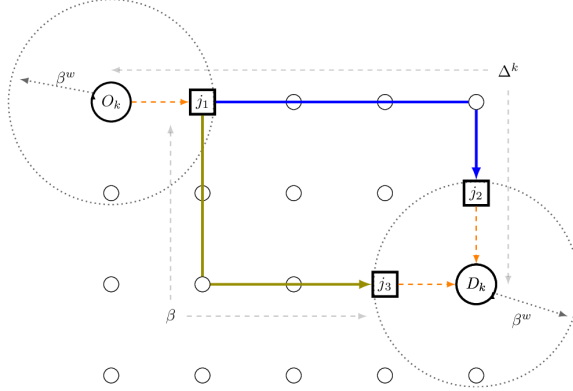


Figure 1: Feasible trips generated for a single request

168 We initially apply a preprocessing procedure to generate a set H_k of all feasible paths (trips) for each
169 request $k \in K$. A path is feasible if it satisfies the accessibility, total length, and battery restrictions. Define
170 $\bar{H}_s = \bigcup_{k \in K_s} H_k$ for scenario $s \in S$ and $H = \bigcup_{k \in K} H_k = \bigcup_{s \in S} \bar{H}_s$, then for each $h \in H$, we know the
171 stations to visit on the trip (P_h) together with the time of visit, the amount of time required to recharge the
172 car at the arrival station, thus, the time slot that the car will be ready for another customer. We illustrate
173 this procedure for a single request on a simple example in Figure 1. In this figure, we use a grid network and
174 rectilinear distances. For a request k with origin O_k and destination D_k , we illustrate two feasible trips. The
175 nodes of the grid network that fall within each dashed circle of radius β^w are the accessible points from/to
176 O_k or D_k . Squares represent potential stations and in this case, j_1 is the only potential station accessible
177 to O_k whereas j_2 and j_3 are both accessible to D_k . As we consider only a single path between two stations,
178 we can have two feasible trips for request k : Trip 1 (Path 1) is $O_k \rightarrow j_1 \rightarrow j_2 \rightarrow D_k$ if the total time spent
179 is no greater than Δ_k and $\delta_{j_1 j_2} \leq \beta$; and Trip 2 (Path 2) is $O_k \rightarrow j_1 \rightarrow j_3 \rightarrow D_k$, similarly, if the total time
180 spent is no greater than Δ_k and $\delta_{j_1 j_3} \leq \beta$. We generate all such feasible trips for each request during the
181 preprocessing.

182 Our methods can be easily adapted to more challenging variants of our problem with slight modifications
183 on this preprocessing procedure. One of these variants allows customers to visit intermediate stations when it
184 is not possible to serve a request without recharging the battery or changing the car due to the long distance
185 between the origin and destination nodes. We present a pseudo-code of our pre-processing procedure for
186 this generalized case in [16]. We do not consider visits to intermediate stations in the computational results
187 of this paper as we focus on the application inside cities where it is unlikely to have these type of requests.
188 Another possible extension would be to consider time dependent traveling times and energy consumptions
189 in the problem.

190 After the pre-processing, we retrieve the following parameters:

- 191 • $b_{hj}^t = 1$ if the car used for trip $h \in \bar{H}_s$ of scenario $s \in S$ exits station $j \in J$ at time $t \in T$, 0 otherwise.
- 192 • $\mu_{hj}^t = 1$ if the car used for trip $h \in \bar{H}_s$ of scenario $s \in S$ is being recharged at station $j \in J$ at time
193 $t \in T$, 0 otherwise.

- $\lambda_{hj}^t = 1$ if the charging of the car used for trip $h \in \bar{H}_s$ of scenario $s \in S$ is completed at station $j \in J$ at time $t \in T$, 0 otherwise.

Then, we define the following decision variables for our formulation:

- $u_h = 1$ if trip $h \in \bar{H}_s$ of scenario $s \in S$ is chosen, 0 otherwise.
- L_j^{ts} is the number of available cars at station $j \in J$ at the beginning of time $t \in T$ for scenario $s \in S$.
- L_j^0 is the number of available cars at station $j \in J$ at time 0.
- $y_j = 1$ if a station is located at node $j \in J$, 0 otherwise.

Now, we can write our path based formulation (PF) as follows:

$$(PF) \quad \max \sum_{s \in S} q_s \sum_{h \in \bar{H}_s} p_h u_h - \sum_{j \in J} f_j y_j - g \sum_{j \in J} L_j^0 \quad (1)$$

$$\text{s.t.} \quad \sum_{h \in \bar{H}_k} u_h \leq 1, \quad \forall k \in K \quad (2)$$

$$u_h \leq y_j, \quad \forall j \in J, h \in H : j \in P_h \quad (3)$$

$$\sum_{h \in \bar{H}_s} b_{hj}^t u_h \leq L_j^{ts}, \quad \forall j \in J, t \in T, s \in S \quad (4)$$

$$L_j^{ts} + \sum_{h \in \bar{H}_s} (\mu_{hj}^t - b_{hj}^t) u_h \leq C_j y_j, \quad \forall j \in J, t \in T, s \in S \quad (5)$$

$$L_j^{ts} = L_j^{(t-1)s} + \sum_{h \in \bar{H}_s} (\lambda_{hj}^t - b_{hj}^{(t-1)}) u_h, \quad \forall j \in J, t \in T : t \geq 1, s \in S \quad (6)$$

$$L_j^{0s} = L_j^0, \quad \forall j \in J, s \in S \quad (7)$$

$$0 \leq L_j^{ts} \leq C_j y_j, \quad \forall j \in J, t \in T, s \in S \quad (8)$$

$$L_j^0 \in \mathbb{Z}_0^+, \quad \forall j \in J \quad (9)$$

$$u_h \in \{0, 1\}, \quad \forall h \in H \quad (10)$$

$$y_j \in \{0, 1\}, \quad \forall j \in J \quad (11)$$

The objective function (1) maximizes the expected profit. The first term in this function gives the expected revenue obtained by serving the customers, the second term is the total fixed cost of opened stations, and the third term is total cost of purchasing cars.

By Constraints (2), a customer is served with at most one trip and by Constraints (3) every station on a selected trip is forced to be opened.

For each scenario, Constraints (4)-(8) ensure the following restrictions: Constraints (4) restrict the number of cars leaving a station with the number of available cars at that station for each time slot. Constraints (5) ensure that the capacity of each station is respected, so that parking a car is not allowed if there is no free space at the station. Constraints (6) balance the number of cars at each station at each time slot. Constraints (7) initialize the number of cars available for each station and Constraints (8) restrict this number with the capacity of that station for each time zone.

Finally, Constraints (9)-(11) give the non-negative integrality and binary restrictions.

In the following section, we present the test results on our formulation PF, its LP relaxation, and the partial relaxation RPF where we keep y variables as binary and relax the integrality of u variables as follows:

$$\begin{aligned}
 (RPF) \quad & \max (1) \\
 & \text{s.t. } (2) - (9), (11) \\
 & 0 \leq u_h \leq 1, \quad \forall h \in H \quad (12)
 \end{aligned}$$

213 4. Computational study on PF and its relaxations

214 In order to test our methods, we generate instances with multiple scenarios based on a real data obtained
 215 from Manhattan (New York, USA) taxi trips [49, 44]. The data file is based on a city network in Manhattan
 216 with 10556 nodes, 85 potential station locations, and 25592 arcs. It contains 27549 requests with origin,
 217 destination, starting time, revenue, and duration; the capacity of each station (maximum number of
 218 recharging units); the base cost of opening each station, per unit cost of installing a slow or fast charging
 219 unit, and purchase cost of three types of electric cars; charging speed for both types of charging units; and
 220 the distance and time dependent maximum traveling speed for each arc. Based on the information given in
 221 the Manhattan data file, we are able to obtain the values of all the parameters of an instance.

222 In our experiments, we consider a single car type (Smart ED), a period of 5 minutes, a capacity of a station
 223 identical to its number of charging units, and only fast charging units. Through our demand forecasting
 224 method (detailed in [17]), we generate instances that include requests with different origin-destination pairs
 225 than the ones in the original data file. We compute the revenue of these newly generated requests by using
 226 the formula of the original data (0.3 € per minute) based on the shortest path distances between the
 227 origin and the destination nodes. By using a fixed driving speed of 50 km/h and a walking speed of 1.34
 228 m/s, we obtain the shortest paths for walking and traveling times via Dijkstra’s algorithm. Each one of
 229 the scenarios includes requests coming from a one-day period. In order to see how the results change with
 230 different scenario combinations, we create four different sets of scenarios (C1-C4) generated from the same
 231 probability distribution.

232 The cost of a Smart ED in the data used in our experiments is 20000 €. Its battery capacity is 63360
 233 kW and fast charging rate is 17600 W. The base cost of stations ranges between 9000 € and 64000 € .
 234 The cost of fast charging slots ranges from 22000 € to 32000 € per unit.

235 We conduct our experiments by using IBM ILOG CPLEX 12.7 in the Java environment on an Intel(R)
 236 Xeon(R) E5-2630 v3 CPU at 2.40GHz with 16 cores and 32 GB of RAM.

237 For the Dijkstra’s algorithm, we use the implementation of the JGraphT 1.0.1 package. We impose a
 238 one hour time limit and 16 GB memory limit to each experiment. We keep CPLEX cuts and presolve on as
 239 they seemed to be efficient in our experiments.

240 In our preliminary experiments, we observe that the cost values in the original data are too high to have
 241 a profitable system. A similar issue is observed also by [14] in their case study for ECS systems in Vienna.
 242 This indicates the need for a comprehensive study on pricing and additional income resources for the initial
 243 investments of ECS systems but this remains out of the scope of our paper. In order to have solutions that
 244 open stations and serves customers, we introduce a parameter *cost factor* that divides the cost values of
 245 the original data. This way we try to preserve the distinct cost values for stations at the same proportion.
 246 This parameter can also be considered as expected repetitions of similar requests in the long run taking

247 into account the lifetime of stations and cars. In our experiments, we use two different values, namely, 10^5
248 and 10^6 as *cost factor*. We also conducted experiments with *cost factor* of 10^4 , which is the realistic value
249 obtained by considering the depreciation, but the system did not provide positive profits for such high
250 costs. Similar results were observed also in our previous study for pre-booked ECS systems [16]. We also
251 tested our instances with distinct *cost factor* values for cars and stations (10^5 for stations and less than 10^5
252 for cars), but we ended up with non-profitable systems. Therefore, we do not report them here. Moreover,
253 we use $\Delta_k = 1.1 \times d_{O_k D_k}$ for every $k \in K$. Δ_k is the time including walking times and driving times
254 between the stations assigned. The walking times are already restricted by β^w parameters and customers
255 cannot be forced/expected to walk for longer than β^w . This type of restriction allows longer driving time
256 (rather than longer walking time) if the driving time between O_k and D_k is longer.

257 In this section, we show results regarding PF, its LP relaxation, and RPF for *cost factor* values 10^5
258 (High cost) and 10^6 (Low cost) in Figures 3 - A.15. We provide tables with detailed results for one copy of
259 scenarios, namely for C1, in the Appendix A.

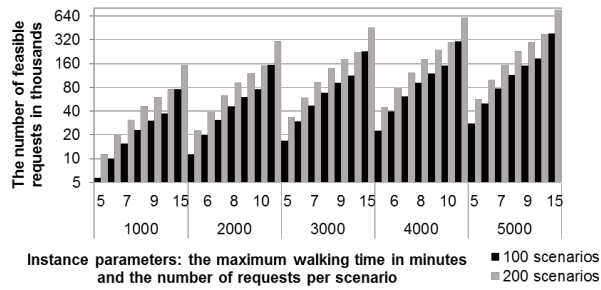
260 In these figures and tables, $|K_s|$ denotes the number of requests in each scenario generated and this
261 number is the same for every scenario of a single problem instance. The β^w column gives the maximum
262 walking time restriction in minutes, all other time values are given in seconds. The columns labeled ‘Obj’
263 give the solution value obtained from the corresponding method at termination due to optimality or resource
264 limitations (time or memory). We report two types of gap percentages: $g1(\%)$ gives the gap relative to the
265 best known PF solution value, say Obj^* , then, $g1(\%) = 100 \times [(Obj - Obj^*)/Obj^*]$; and $g2(\%)$ denotes the
266 gap provided by CPLEX at the termination. We here note that for some instances, Obj^* value might be
267 obtained via the methods that we propose in Section 5. We further indicate the instances that cannot be
268 solved to optimality within the time limit with ‘TL’ under the ‘time’ column. We do not hit the memory
269 limit in any of the instances. If we are not able to obtain a feasible solution or the gaps within the time
270 limit for an instance, we indicate this as ‘NA’. As the instances with $\beta^w = 15$, $|K_s| \geq 3000$ are intractable,
271 we omit corresponding problems for $|S| = 200$ charts and tables.

272 In Figure 2, we present the results related to pre-processing, the initial process to generate all feasible
273 trips, on problems with $|S| = 100$ and $|S| = 200$ for the first set of scenarios (similar numbers for the
274 others). On the X axis of these charts, we have instances corresponding to $\beta^w, |K_s|$ combinations. On the
275 Y-axis, we use a logarithmic scale to show $|K_a|$, the total number of requests that satisfy accessibility and
276 maximum length restrictions over all scenarios (Figure 2a); and $|H|$, the total number of trips generated i.e.
277 the number of \mathbf{u} variables in the model (Figure 2b). Here, we observe that even one minute of increase on
278 the accessibility measure (β^w) can double the number of requests that can be potentially served. The total
279 time spent during the pre-processing of these instances ranges from 30 seconds to half an hour.

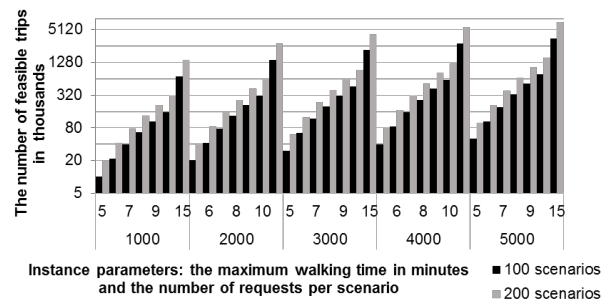
280 In Figure 3, we observe that the LP relaxations provide very tight bounds, especially, for instances of low
281 costs. The largest LP gap that we observe is 6.98% and it belongs to the C4 instance with $\beta^w = 5, |K_s| =$
282 $1000, |S| = 100$ and high costs. The average LP gap of all instances tested is 0.39%. Although there are a
283 few exceptions, usually the LP gaps tend to decrease as β^w increase.

284 When we look at Figure 4 of instances with $|S| = 100$, we can observe a pattern of increase in the
285 objective values in parallel to the increase in β^w as well as in $|K_s|$. Moreover, we see that these objective
286 values are higher when we have low cost parameters. We observe a similar pattern for C2-C4 instances.

287 We present the solving times of LP, RPF, and PF in Figures A.13-A.15 of the Appendix. As expected,



(a) The total number of requests that satisfy accessibility and maximum length restrictions.



(b) The number of feasible trips generated.

Figure 2: Preprocessing results

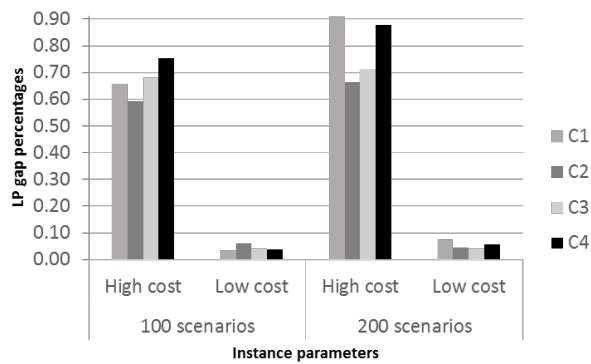


Figure 3: Average LP gaps ($g_1\%$) for C1-C4

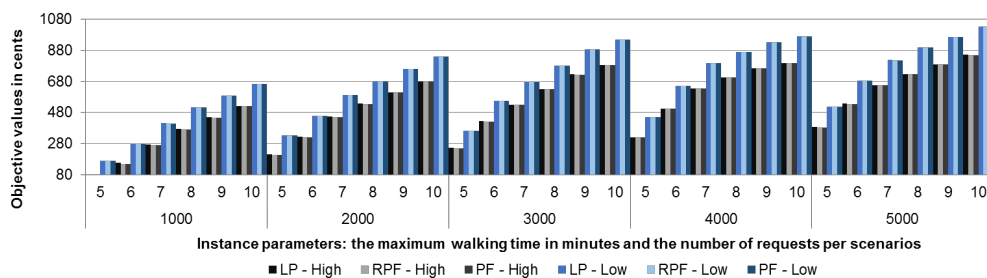


Figure 4: LP, RPF, and PF values of C1 for 100 scenarios. Blue bars for low cost, gray bars for high cost.

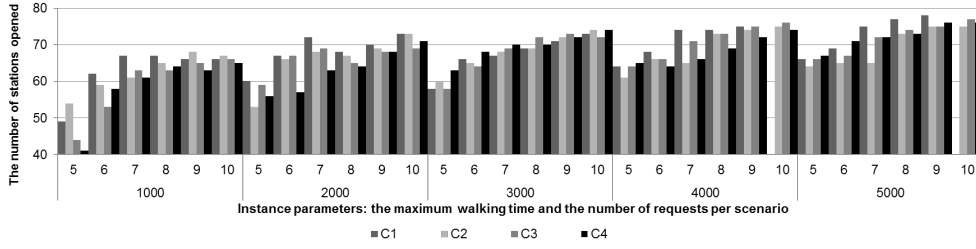


Figure 5: Number of stations opened per instance for C1-C4 for 100 scenarios and high cost.

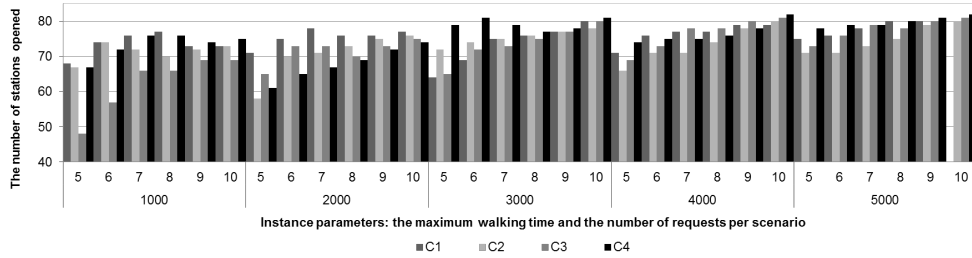


Figure 6: Number of stations opened per instance for C1-C4 for 100 scenarios and low cost.

288 PF spends a larger amount of time to solve problems with larger β^w values. PF is not able to find any
 289 solution with a positive profit value in instances with $\beta^w = 15$ mins and it spends all the time given for the
 290 presolve of CPLEX when $|K_s| \geq 2000$ for $\beta^w = 15$. For some of the instances, RPF needs more time to
 291 reach optimality than PF.

292 Moreover, we present the number of stations opened for the solution obtained from PF in Figures 5 and
 293 6 for $|S| = 100$. An important observation here is that the increase in the number of stations opened when
 294 β^w is increased from 5 minutes to 6 minutes is more significant compared to other increments, for example,
 295 from 6 minutes to 7 minutes. This is worth to express as most recent works in the literature restrict the
 296 walking time by 5 minutes but we see through our experiments that a one-minute increase of this parameter
 297 might cover a much larger portion of the demand and yield a significant increase in the expected profit.

298 Another important observation is on the number of trips that can be served by the solution of PF. We
 299 show these values for $|S| = 100$ in Figures 7 and 8. Here, we can see that the number of trips that can be
 300 served increases as β^w increases, and usually also when $|K_s|$ increases.

301 Our experiments reveal that the computational difficulty increases parallel to the number of scenarios

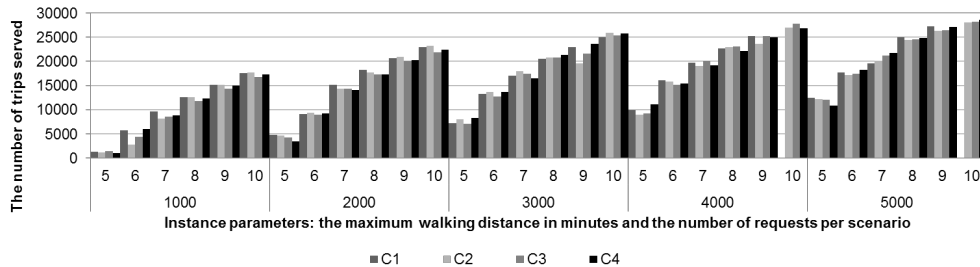


Figure 7: The number of trips that can be served per instance for C1-C4. $|S| = 100$, high cost.

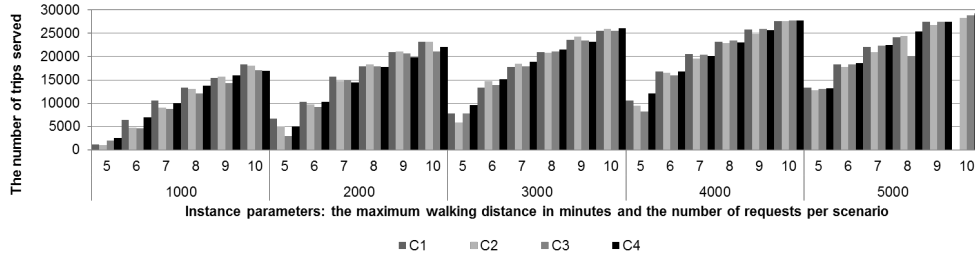


Figure 8: The number of trips that can be served per instance for C1-C4. $|S| = 100$, low cost.

302 as the number of constraints and variables increases significantly. In order to overcome this difficulty, we
 303 describe a Benders decomposition algorithm in the following section.

304 Another interesting observation on these results is that the objective value of RPF is the same as
 305 the objective value of PF for almost every instance and the difference is very small for the nonidentical
 306 ones. As it will be explained in the next section, this observation encourages us to focus on two phase
 307 solution methodologies that will first focus on solving RPF which has less number variables with integrality
 308 restrictions compared to PF and then reach to the optimality of PF by using the information obtained from
 309 RPF.

310 5. Decomposition algorithm (BAC)

311 Before presenting the details of our algorithm, we give a general description of the Benders decomposition
 312 method [6]. For a mixed integer program with a group of integer variables and a group of continuous variables,
 313 the algorithm consists of solving the formulation by temporarily removing all continuous variables. This
 314 problem is called the master problem. Then, the Benders algorithm focuses on the dual of a subproblem,
 315 which is simply a restriction of the original problem where all the integer variables are fixed to the values
 316 of the master problem. From the dual problem, a so called feasibility cut for each extreme ray and an
 317 optimality cut for each extreme point is obtained. These cuts are added to the master problem to obtain
 318 another solution and the procedure is repeated. Since enumeration of all extreme rays and extreme points
 319 is not very practical, cutting plane algorithms are commonly used in the literature.

320 The convergence rate of the classical Benders decomposition method might be very slow especially if
 321 the subproblem is difficult to solve. On the other hand, it might be very efficient if the subproblem can be
 322 decomposed further into smaller and easy-to-solve problems as in multi-commodity, multi-period, or multi-
 323 scenario problems [8]. Motivated by this fact, we aim further decomposing the second-stage problem of
 324 our two-stage recourse problem into smaller problems, each one corresponding to a single demand scenario.
 325 For this purpose, we decide to choose \mathbf{y}, \mathbf{L}^0 as first stage variables that are kept in the master problem
 326 and \mathbf{u}, \mathbf{L} as the variables of the subproblem. However, as \mathbf{u} variables are integral, we cannot apply the
 327 classical Benders decomposition to PF without any further adaptations. On the other hand, with this type
 328 of categorization of variables, RPF is a good candidate for applying the Benders procedure as \mathbf{u}, \mathbf{L} variables
 329 are continuous in this model. In this case, we need additional steps to make sure that the optimal solution
 330 of our algorithm provides an integral vector \mathbf{u} . Several successful adaptations of the classical Benders
 331 decomposition algorithm to different IP problems are available due to [9, 30, 37]. In our implementation,
 332 we use the ‘‘Combinatorial Benders cuts’’, which are first named by Codato and Fischetti [23].

333 We implemented our Benders algorithm as a two phase method. In the first phase of our algorithm, we
334 solve RPF with a classical Benders framework. Let $v(RPF)$ be the optimal value of RPF and $(\mathbf{y}^*, \mathbf{L}^*)$ be
335 the optimal values of the first stage variables $(\mathbf{y}, \mathbf{L}^0)$. Then, we solve PF by fixing values of \mathbf{y}, \mathbf{L}^0 variables
336 to $(\mathbf{y}^*, \mathbf{L}^*)$ to see if there exists an integer feasible solution for RP with objective value $v(RPF)$. If yes,
337 we stop as we have an optimal solution of PF on hand. Otherwise, we move to the second phase of the
338 algorithm. The motivation behind this first phase is the fact that RPF gives the optimal value of PF in most
339 instances; so, the values of the first stage decision variables obtained from RPF might possibly provide a
340 complete integral solution for PF and avoid excessive number of integrality checks for addition of relatively
341 weak combinatorial cuts.

342 In Phase II, we restart a branch-and-cut framework for RPF and restrict the objective value from above
343 with $v(DRPF)$, the value of the best dual bound obtained in Phase I. Then, for integer solutions of RPF
344 tree, we solve individual subproblems with integrality restrictions on \mathbf{u} variables for each scenario and add
345 combinatorial Benders cuts if these subproblems are integer infeasible. Our experimental study reveals that
346 the two-phase procedure converges faster than introducing the combinatorial cuts at the first phase in most
347 instances and it is much more efficient in overall as most problems do not require the second phase.

348 Now we present the details of Phase I and Phase II of our algorithm.

349 *Phase I:*

350 Through our experiments, we observed that the optimality cuts worsen the solving time of the algorithm
351 so, we decided to avoid them. To do so, we make a modification on our formulation so that we obtain only
352 feasibility cuts from our dual subproblem. We define a nonnegative decision variable z_s for each scenario
353 $s \in S$ and ensure that its value will be equal to the revenue obtained from that scenario as follows:

$$(PF2) \quad \max \sum_{s \in S} q_s z_s - \sum_{j \in J} f_j y_j - g \sum_{j \in J} L_j^0 \quad (13)$$

$$\text{s.t.} \quad \sum_{h \in \bar{H}_s} p_h u_h \geq z_s, \quad \forall s \in S \quad (14)$$

$$(2) - (11)$$

$$0 \leq z_s \leq \sum_{k \in K_s} p_k, \quad \forall s \in S \quad (15)$$

354 To solve (PF2) in a Benders fashion, we keep $\mathbf{z}, \mathbf{y}, \mathbf{L}^0$ variables in the master problem and deal only
355 with \mathbf{u} variables in the subproblem.

356 In order to make our master problem stronger, we introduce two sets of valid inequalities (18) and (19).
357 Thus, we solve (MP) given below as the master problem of our Benders algorithm.

$$(MP) \quad \max \sum_{s \in S} q_s z_s - \sum_{j \in J} f_j y_j - g \sum_{j \in J} L_j^0 \quad (16)$$

$$\text{s.t.} \quad \sum_{m=1}^{C_j} x_{jm} \leq y_j, \quad \forall j \in J \quad (17)$$

$$z_s \leq \sum_{k \in K_s} p_k \sum_{j \in N_{O_k}} y_j, \quad \forall s \in S \quad (18)$$

$$z_s \leq \sum_{k \in K_s} p_k \sum_{j \in N_{D_k}} y_j, \quad \forall s \in S \quad (19)$$

$$z_s \geq 0, \quad \forall s \in S \quad (20)$$

(9), (11)

In this formulation, N_{O_k} and N_{D_k} denote the set of stations accessible by, respectively, origin and destination, of request $k \in K$. Once we introduce fixed values $(\bar{\mathbf{z}}, \bar{\mathbf{y}}, \bar{\mathbf{L}}^0)$ to our subproblem, we observe that it can be decomposed into $|S|$ problems, one for each scenario, that can be solved independently.

At this point, we make some simplifications and modifications to have smaller number of constraints and variables in our primal and dual subproblems. The first observation is as follows:

Property 1. *Constraints (8) are redundant.*

Proof. (i) $L_j^{ts} \geq 0, \forall j \in J, t \in T, s \in S$:

By Constraints (4), $L_j^{ts} - \sum_{h \in \bar{H}_s} b_{hj}^t u_h \geq 0, \forall j \in J, t \in T, s \in S$. Then, by Constraints (6), $L_j^{ts} = L_j^{(t-1)s} - \sum_{h \in \bar{H}_s} b_{hj}^{(t-1)} u_h + \sum_{h \in \bar{H}_s} \lambda_{hj}^t u_h \geq 0, \forall j \in J, t \in T, s \in S$ as $\lambda_{hj}^t, u_h \in \{0, 1\}, \forall j \in J, t \in T, h \in H$.

(ii) $L_j^{ts} \leq C_j y_j, \forall j \in J, t \in T, s \in S$:

By Constraints (5), $\sum_{h \in \bar{H}_s} \mu_{hj}^t u_h \leq C_j y_j + \sum_{h \in \bar{H}_s} b_{hj}^t u_h - L_j^{ts}, \forall j \in J, t \in T, s \in S$. Moreover, $\sum_{h \in \bar{H}_s} \lambda_{hj}^{(t+1)} u_h \leq \sum_{h \in \bar{H}_s} \mu_{hj}^t u_h, \forall j \in J, t < \tau$ as the number of cars who finished recharging at $t+1$ cannot be larger than the number of cars being charged at t . Then, $\sum_{h \in \bar{H}_s} \lambda_{hj}^{(t+1)} u_h + L_j^{ts} - \sum_{h \in \bar{H}_s} b_{hj}^t u_h \leq C_j y_j$ implying that $L_j^{(t+1)s} \leq C_j y_j, \forall j \in J, t \in T : t < \tau, s \in S$ via Constraints (6).

□

Moreover, the following replacement of L_j^{ts} is possible due to Constraints (6) and (7):

$$L_j^{ts} = L_j^0 + \sum_{h \in \bar{H}_s} \left(\sum_{r=1}^t \lambda_{hj}^r - \sum_{r=0}^{t-1} b_{hj}^r \right) u_h, \quad \forall j \in J, t \in T : t \geq 1, s \in S \quad (21)$$

If we replace L_j^{ts} with the right hand side of (21) throughout PF2 and omit equality Constraints (6) and (7), we obtain a formulation with $\mathbf{z}, \mathbf{y}, \mathbf{u}, \mathbf{L}^0$ variables only. Then, for each scenario $s \in S$, we can express the corresponding subproblem (SP_s) as follows:

$$(SP_s) \quad \max 0 \quad (22)$$

$$\text{s.t.} \quad \sum_{h \in \bar{H}_s} p_h u_h \geq \bar{z}_s, \quad (23)$$

$$\sum_{h \in H_k} u_h \leq 1, \quad \forall k \in K_s \quad (24)$$

$$u_h \leq \bar{y}_j, \quad \forall h \in \bar{H}_s, j \in P_h \quad (25)$$

$$\sum_{h \in \bar{H}_s} \left(\sum_{r=0}^t b_{hj}^r - \sum_{r=1}^t \lambda_{hj}^r \right) u_h \leq \bar{L}_j^0, \quad \forall j \in J, t \in T \quad (26)$$

$$\sum_{h \in \bar{H}_s} \left(\sum_{r=1}^t \lambda_{hj}^r - \sum_{r=0}^t b_{hj}^r + \mu_{hj}^t \right) u_h \leq C_j \bar{y}_j - \bar{L}_j^0, \quad \forall j \in J, t \in T \quad (27)$$

$$u_h \in \{0, 1\}, \quad \forall h \in \bar{H}_s \quad (28)$$

377 Let LP_s denote the linear programming (LP) relaxation of SP_s , where the integrality restriction on \mathbf{u}
378 variables is removed, therefore, $0 \leq u_h \leq 1, \forall h \in \bar{H}_s$; DLP_s denote the dual of LP_s ; and $\alpha, \epsilon_k, \gamma_{hj}, \theta_{jt}$, and
379 ω_{jt} be the dual variables associated with Constraints (23)-(27), respectively. Here, we note that $u_h \leq 1$
380 restriction is already satisfied due to (24) and (25) $\forall h \in \bar{H}_s$, so, we do not include an additional set of
381 constraints for this restriction. Then, we can express DLP_s as follows:

$$(DLP_s) \quad \min -\bar{z}_s \alpha + \sum_{k \in K_s} \epsilon_k + \sum_{h \in \bar{H}_s, j \in P_h} \bar{y}_j \gamma_{hj} + \sum_{j,t} \bar{L}_j^0 \theta_{jt} \\ + \sum_{j,t} (C_j \bar{y}_j - \bar{L}_j^0) \omega_{jt} \quad (29)$$

$$\text{s.t.} \quad -p_h \alpha + \sum_{k: h \in H_k} \epsilon_k + \sum_{j \in P_h} \gamma_{hj} + \sum_{j,t} \left(\sum_{r=0}^t b_{hj}^r - \sum_{r=1}^t \lambda_{hj}^r \right) \theta_{jt} \\ + \sum_{j,t} \left(\sum_{r=1}^t \lambda_{hj}^r - \sum_{r=0}^t b_{hj}^r + \mu_{hj}^t \right) \omega_{jt} \geq 0, \quad \forall h \in \bar{H}_s \quad (30)$$

$$\alpha \geq 0, \quad (31)$$

$$\epsilon_k \geq 0, \quad \forall k \in K_s \quad (32)$$

$$\gamma_{hj} \geq 0, \quad \forall h \in \bar{H}_s, j \in P_h \quad (33)$$

$$\theta_{jt}, \omega_{jt} \geq 0, \quad \forall j \in J, t \in T \quad (34)$$

382 In our implementation, we introduce $\alpha \leq 1$ to (DLP_s) and associate unboundedness of the problem with
383 a negative objective value. Therefore, as soon as the optimal value obtained from (DLP_s) is negative, we
384 add feasibility cuts (35) to our master problem. An important property of DLP_s is that the feasible region
385 of the model does not change by the solution of the master problem. We use this property by generating
386 the individual dual problems once in our implementation and changing only the objective function at each

387 iteration. This modification in the implementation improves the efficiency of our algorithm significantly.

$$-\bar{\alpha}z_s + \sum_j \left(\sum_{h:j \in P_h} \bar{\gamma}_{hj} + C_j \sum_t \bar{\omega}_{jt} \right) y_j + \sum_j \left(\sum_t \bar{\theta}_{jt} - \sum_t \bar{\omega}_{jt} \right) L_j^0 \geq - \sum_{k \in \bar{H}_s} \bar{\epsilon}_k \quad (35)$$

388 In addition to every integral solution of the branch-and-bound tree, we add feasibility cuts (35) for vio-
 389 lated fractional solutions at the root node as well. We use the *LazyConstraintCallback* and *UserCutCallback*
 390 of CPLEX for separation of, respectively, integral and fractional solutions.

391 After this branch-and-cut first phase, the feasibility of the optimal solution is checked as follows. Let
 392 $v(RPF)$ be the optimal value of RPF and $(\mathbf{y}^*, \mathbf{L}^*)$ be the optimal values of the first stage variables $(\mathbf{y}, \mathbf{L}^0)$.
 393 Then, we solve PF by fixing values of $(\mathbf{y}, \mathbf{L}^0)$ variables to $(\mathbf{y}^*, \mathbf{L}^*)$ to see if there exists an integer feasible
 394 solution for RP with objective value $v(RPF)$. If yes, we stop as we have an optimal solution of PF on hand.
 395 Otherwise, we move to the second phase of the algorithm.

396 *Phase II:*

In order to start Phase II, we make use of $v(DRPF)$ and $(\mathbf{y}^*, \mathbf{L}^*)$ obtained from Phase I. At this stage,
 we introduce a new set of binary variables \mathbf{x} that will represent integral \mathbf{L}^0 variables with the variable
 transformation $L_j^0 = \sum_{m=1}^{C_j} mx_{jm}, j \in J$ in MP. Doing so, we can introduce combinatorial Benders cuts
 associated with these variables when needed. We further add the following constraints to MP:

$$\sum_{s \in S} q_s z_s - \sum_{j \in J} f_j y_j - g \sum_{j \in J} \sum_{m=1}^{C_j} mx_{jm} \leq v(DRPF) \quad (36)$$

$$\sum_{m=1}^{C_j} x_{jm} \leq y_j, \quad \forall j \in J \quad (37)$$

397 Similar to Phase I, we solve DLP_s for separation not only at integer solutions of the branch-and-bound
 398 tree but also at fractional solutions at the root node. But this time, the separation cut would be (38).

$$-\bar{\alpha}z_s + \sum_j \left(\sum_{h:j \in P_h} \bar{\gamma}_{hj} + C_j \sum_t \bar{\omega}_{jt} \right) y_j + \sum_j \left(\sum_t \bar{\theta}_{jt} - \sum_t \bar{\omega}_{jt} \right) \sum_{m=1}^{C_j} mx_{jm} \geq - \sum_{k \in \bar{H}_s} \bar{\epsilon}_k \quad (38)$$

399 Additionally, once DLP_s is bounded (i.e. LP_s feasible) for every $s \in S$, we check if we have an integral
 400 solution for $SPMax_s$ for the current $(\bar{\mathbf{y}}, \bar{\mathbf{x}})$ values of the master problem iteratively for $s = 1, \dots, |S|$.

$$(SP_s Max) \quad \max \sum_{h \in \bar{H}_s} p_h u_h \quad (39)$$

s.t. (24) – (28).

401 If $SPMax_s$ is infeasible for some $s \in S$, we add the combinatorial feasibility cut (40) and do not check
 402 integer feasibility for the remaining subproblems. If $SPMax_s$ is feasible but the optimal value is greater

403 than \bar{z}_s for some $s \in S$, then we add the optimality cut (41). This cut requires either the value of z_s variable
 404 to be tightened by $\sum_{h \in \bar{H}_s} p_h u_h^*$ for the same (\bar{y}, \bar{x}) solution or to change the value of y_j for some $j \in J$ or
 405 x_{jm} for some (j, m) where $j \in J, m = 1, \dots, C_j$. At the termination of this branch-and-cut procedure, we
 406 obtain an optimal solution for PF.

$$F(\bar{y}, \bar{x}) \geq 1 \tag{40}$$

$$z_s \leq \sum_{h \in \bar{H}_s} p_h u_h^* + \sum_{k \in K_s} p_k F(\bar{y}, \bar{x}) \tag{41}$$

where \mathbf{u}^* is the solution vector obtained from $SPMax_s$ of $s \in S$ and

$$F(\bar{y}, \bar{x}) = \sum_{j: \bar{y}_j=0} y_j + \sum_{j: \bar{y}_j=1} (1 - y_j) + \sum_{j, m: \bar{x}_{jm}=0} x_{jm} + \sum_{j, m: \bar{x}_{jm}=1} (1 - x_{jm}).$$

407 We here note that we also implemented a classical Benders decomposition by leaving u variables in the
 408 master problem and tested it both with the automated Benders function of CPLEX and with our manual
 409 implementation and we observed that this type of decomposition had a poor performance.

410 6. Computational Study on BAC

411 In this section, we provide a detailed analysis of the experimental study on our algorithm. Before
 412 going into this analysis, we briefly describe our demand generation method as follows: In order to generate
 413 multiple demand scenarios which follow the pattern of the available data to certain extent, we fit the data
 414 into several probability distribution (mass) functions (PDF) from the literature. Among these PDFs, we
 415 choose the one that gives the minimum of least squared errors to generate scenarios, each with a certain
 416 number of requests. Finally, we compute the probability of scenarios based on the probability of individual
 417 requests in each scenario. The details of this method can be found in [17].

418 6.1. Results

419 In addition to the metrics reported in earlier charts and tables, we also report here the number of nodes
 420 explored in the branch-and-bound tree (nodes). Moreover, regarding our algorithms, we give the number of
 421 integral and fractional solutions separated during branch-and-cut procedures via columns ‘lazy’ and ‘user’,
 422 respectively. We tried turning off the presolve option of CPLEX in our instances but decided to keep it on
 423 as the solving time performance was worse without presolve.

424 When we compare the solving times of model PF and algorithm BAC for instances with $|S| = 100, 200$
 425 (see Figures A.16-A.19 in Appendix A), we see that BAC performs better in tackling instances of C1 and
 426 C3 with large β^w values and instances with $|K_s| \geq 4000$. However, the problems with $\beta^w = 15$ become
 427 intractable also by BAC when $|K_s| \geq 3000$. When we test PF and BAC on instances with $|S| = 200$, we
 428 see that both PF and BAC needed more time on average to reach optimality. They hit the time limit for
 429 more instances but the average and maximum gaps from the best solution value were much larger for PF
 430 compared to BAC. The better performance of BAC compared to PF becomes more visible in these instances.
 431 Although the time spent by BAC for C2 and C4 seems larger compared to PF, BAC is able to find very high

432 quality solutions within one hour for most instances whereas PF struggles with finding a non-zero feasible
 433 solution in many of them (see Figure 10 for average gap percentages).

434 An interesting observation is that among the problems for which we obtain feasible solutions, less than
 435 20% need a call to Phase II. The solving times that go up to 2 hours for BAC in Figures A.16-A.19 correspond
 436 to some of these instances for which Phase II was needed.

437 On average, BAC needs to visit more branch-and-bound nodes than PF and usually, it needs to separate a
 438 larger number of fractional solutions than integer solutions even though the fractional solutions are separated
 439 only at the root node.

440 We provide a detailed comparison of PF and BAC for C1 instances in Tables A.6-A.9 of Appendix A.

441 6.2. Stabilization

442 When we compare the performance of BAC on problems with $|S| = 200$ with those of $|S| = 100$, we
 443 observe that more iterations of separation are needed in Phase I of the algorithm and Phase II iterations are
 444 needed in a larger portion of problems. To overcome these difficulties, we introduce a stabilization procedure
 445 that enables addition of potentially stronger cuts for separation of infeasible solutions at individual iterations
 446 of our Benders algorithm. This method was originally proposed by Ben-Ameur and Neto [5] for cutting-
 447 plane and column generation algorithms and it was successfully adapted to several Benders decomposition
 448 implementations [29, 18, 42].

449 In the original form of our algorithm, for each optimal solution, say $\pi^m = (z^m, y^m, L^m)$, we solve the
 450 dual of subproblem *SP1* for π^m and add a cut associated with π^m to our master problem if there is a
 451 violation and move to subproblem *SP2* to solve its dual for π^m and repeat for each subproblem. Now,
 452 instead of moving to next subproblem immediately after finding a violation with the current one, we do the
 453 following: Let π^f be a global feasible solution to the original problem PF and $\varphi \in (0, 1)$. Differently the
 454 core point of [41], π^f is not necessarily an interior point and it can be any feasible point. We find another
 455 point $\pi^1 = \varphi\pi^m + (1 - \varphi)\pi^f$ that is located on the line segment between π^m and π^f , and check if the dual
 456 of the current subproblem is violated by this point π^1 . If yes, then we obtain a potentially stronger cut
 457 that separates both π^m and π^1 , so, we add the new cut to our master problem. Then, we move to another
 458 point $\pi^2 = \varphi\pi^1 + (1 - \varphi)\pi^f$ and check if there is a violation. If there is no violation, we move to another
 459 point $\pi^3 = \varphi\pi^1 + (1 - \varphi)\pi^2$ and we keep moving on the line segment between the last feasible point and the
 460 last separation point found for certain number of iterations. In our implementation, we move for only two
 461 iterations between π^m and π^f for each subproblem and then, we move to the next subproblem to repeat the
 462 same stabilization steps. We further choose $\varphi = 0.5$ in our experiments.

463 We illustrate the stabilization procedure on a sample polyhedra in Figure 9. Here, the convex hull of
 464 the master problem is shown with the dashed blue lines and inside that, we have the intersection, initially,
 465 with the first subproblem *SP1*. The solution of the master problem π^m is infeasible for *SP1* and FC_1 is a
 466 feasibility cut that separates π^m . Now, we move to π^1 and find another feasibility cut FC_2 to separate π^1
 467 as it is also infeasible for *SP1*. Then, we move to another point π^2 , which turns out to be feasible for *SP1*.
 468 After two iterations, we add FC_1 and FC_2 to our master problem and move to the second subproblem *SP2*.
 469 As π^m is feasible for *SP2*, we do not get any feasibility cuts and we do not need any stabilization for *SP2*, so,
 470 we can move to the next subproblem if there is any; otherwise, to the next solution of the master problem.

471 A summary of the results comparing the performance of BAC with and without stabilization is presented
 472 in Figures 10 and 11. Even though there is not an absolute winner among the two methods, the stabilized

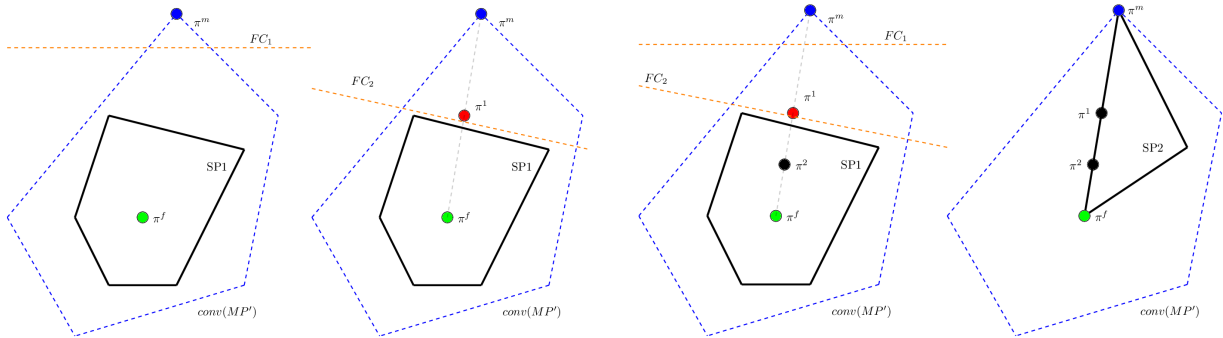


Figure 9: A sample stabilization iteration

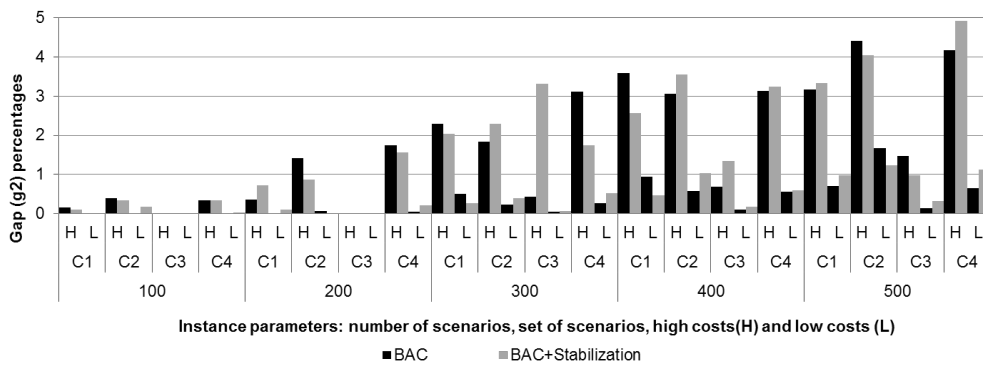


Figure 10: Average gaps ($g_2\%$) of our algorithms for C1-C4 instances

473 version performs slightly better on average solving time. It is also better on the average gap for C1 instances
 474 (1.18% vs 1.06%) whereas the non-stabilized version is better for C2-C4 instances and in overall average
 475 (1.06% vs 1.13%). Stabilization decreases the average number of *LazyConstraintCallback* iterations and it
 476 decreases the average number of *UserCutCallback* iterations by around 50% for each of C1-C4. The impact
 477 of stabilization on the number of nodes explored does not follow a clear pattern and it is difficult to draw
 478 any conclusion on this aspect (Detailed results regarding the stabilization experiments can be provided by
 479 the authors upon request).

480 6.3. Sensitivity Analysis

481 In this section, we provide a brief sensitivity analysis based on our observations regarding the effects of
 482 changes in parameters *costfactor*, β^w , $|S|$, $|K_s|$ on problem outputs.

483 When we look at the average gap % in Figure 10, we can easily see that it decreases when the cost values
 484 decrease and it usually increases when the number of scenarios increases. We do not observe a clear pattern
 485 in correlation with parameters β^w or $|K_s|$.

486 The profit of the system increases with the increase in β^w or in $|K_s|$ whereas it decreases with the increase
 487 in costs or in the number of scenarios (see Figure 4 and Tables A.2-A.9 in Appendix A).

488 We have already seen in Figures 5 and 8 in Section 4 that system tends to open more stations as we
 489 decrease cost values or increase β^w values. We do not observe any correlation with $|S|$ and although it seems
 490 that there might be a positive correlation with $|K_s|$, it is not very obvious.

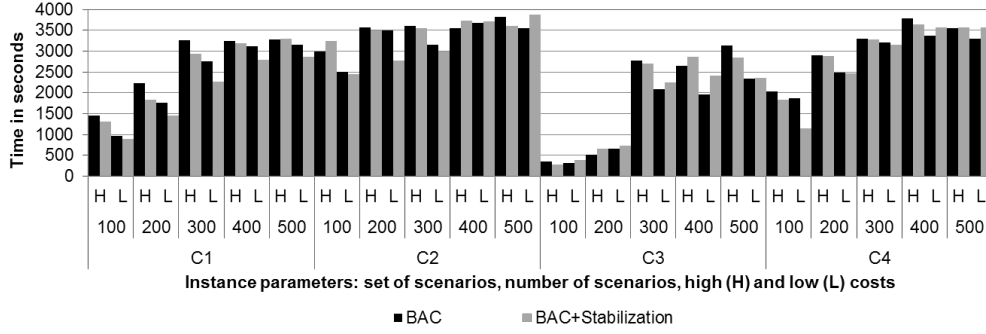
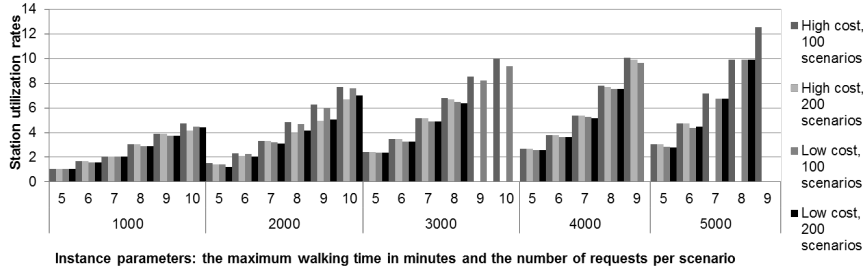
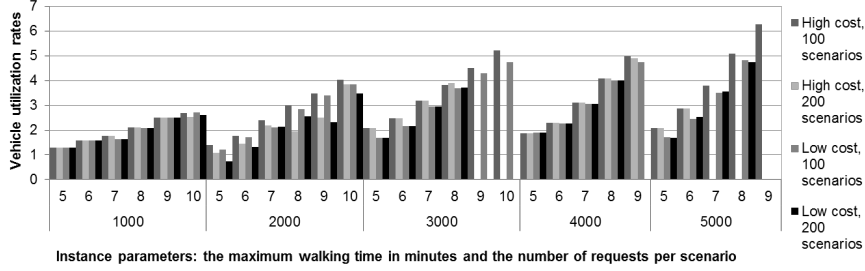


Figure 11: Average solving time (in seconds) of our algorithms for C1-C4



(a) The expected number of trips served per vehicle.



(b) The expected number of trips served per vehicle

Figure 12: Utilization rates for the stations and the vehicles.

491 As we have a multi-scenario problem, in order to provide an estimation of the utilization rates of the
492 stations and the vehicles, we calculate the expected number of trips served for each instance via the formula
493 $E[\bar{u}] = \sum_{s \in S} q_s \sum_{h \in \bar{H}_s} \bar{u}_h$. Then, we obtain the utilization rate for the stations as $E[\bar{u}]$ divided by the
494 number of stations opened, and similarly, as $E[\bar{u}]$ divided by the number cars purchased for the vehicles.
495 From Figure 12 of C3 instances, we observe that the utilization rates increase in parallel to the increase in
496 β^w and especially, in $|K_s|$, whereas they decrease when $|S|$ or the *cost factor* increases.

497 We provide a summary of our analysis in Table 1.

498 Finally, we do some additional experiments by using more variations of fixed costs for cars. These values
499 are closer to the actual costs of Smart ED in the US. We still use a *cost factor* = 10^5 to divide the fixed
500 costs of stations as otherwise the system does not provide a positive profit. We see from our experiments
501 that the maximum profit that could be obtained among all instances falls below 5000 € for $\beta^w = 10$ and
502 it is below 150 € for $\beta^w = 5$. As these values seems to be rather too small to run a profitable business, it

Table 1: Correlation between the changes on problem parameters and outputs

	$cost\ factor \uparrow$	$\beta^w \uparrow$	$ S \uparrow$	$ K_s \uparrow$
Obj	\uparrow	\uparrow	\downarrow	\uparrow
$ \bar{J} $	\uparrow	mostly \uparrow	no pattern	no pattern
PP time	not applicable	\uparrow	no pattern	mostly \uparrow
PF time	decreases in average	mostly \uparrow	\uparrow	\uparrow
BAC time	decreases in average	mostly \uparrow	\uparrow	no pattern
BAC-S time	decreases in average	mostly \uparrow	\uparrow	no pattern
BAC Phase II	no pattern	no pattern	no pattern	no pattern
BAC-S Phase II	no pattern	no pattern	no pattern	no pattern
Avg. gap %	\downarrow	no pattern	mostly \uparrow	no pattern

503 might be necessary to find additional financial means to wave the fixed costs.

504 7. Conclusion

505 In this paper, we introduce a strategic decision-making problem arising in the design of a one-way station
506 based electric car sharing system that operates under demand uncertainty. The strategic decisions include
507 the number and location of the recharging stations which need to be selected meticulously as opening of
508 these kind of facilities requires large amount of investment in money and time, and it is undesirable to
509 replace them frequently. Our model takes into account the potential customer demand based on a relevant
510 historical data and the expected revenue of the demand portion that could be served in a feasible way with
511 the opened stations. As we aim to solve this problem optimally, we develop a mathematical formulation
512 and a Benders decomposition algorithm that we further enrich with a stabilization procedure. We observe
513 that our algorithm has the potential to solve real size problems to optimality, if not we can obtain very high
514 quality solutions within very short amount of time.

515 We observe through our experimental study that the problem is harder to solve when the costs (station
516 opening and car purchasing) are higher and as expected, less number of stations are operated and less profit
517 is made. Moreover, even a small portion of increase in the maximum walking time for customers increases the
518 profit significantly. Therefore, the accessibility measures should be selected very carefully when operating
519 the system. The accessibility measures can also be used as a parameter in formation of pricing strategies.
520 Another parameter that needs to be taken into account in design or operation of electric car sharing systems
521 is the number of different demand scenarios. As we observe in this study, considering too few scenarios
522 in taking decisions might cause an overestimation of the actual profit or misplacement of stations. The
523 length of the planning horizon is also very effective in the profit of the system constructed. We observe that
524 the profit increases as the number of requests per scenario increases, which can be interpreted as a longer
525 planning period. A future study could focus on deciding the optimal length of the planning horizon for
526 strategic decisions such as location of stations or recharging units.

527 The main focus of this work is on exact methods but the methods we propose can be easily modified or
528 combined with practical procedures to obtain near optimal solutions.

529 As mentioned in Section 3, we can easily adapt our methods to more complicated variations of the
530 problem we proposed here. These variations include considering time-dependent traveling time and energy
531 consumption as well as visiting intermediate stations. An interesting and more challenging extension would

532 be to relax full battery restriction for availability and develop methods that can ensure sufficient battery at
533 the beginning of each trip. In this case, an additional index for each operating car would be needed and
534 the mathematical models might have problems in tackling large scale problems. On the other hand, some
535 meta-heuristic methods might be developed to solve these problems heuristically.

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Table A.2: Model results for $|S| = 100$ and $costfactor = 10^5$ - gaps & solution values

$ K_s $	β^w	LP		RPF		PF		time
		Obj	g1(%)	Obj	g2(%)	Obj	g2(%)	
1000	5	7342.31	1.71	7218.65	0.00	7218.65	0.00	4.12
1000	6	15605.92	3.88	15022.45	0.00	15022.45	0.00	15.76
1000	7	27573.03	1.80	27085.46	0.00	27085.46	0.00	46.71
1000	8	37518.81	1.45	36980.87	0.00	36980.87	0.00	247.10
1000	9	44984.52	0.58	44723.11	0.00	44723.11	0.00	656.09
1000	10	52304.87	0.28	52158.64	0.00	52158.64	0.00	723.22
1000	15	89960.52	0.13	0.00	NA	0.00	NA	TL
2000	5	21016.22	0.85	20839.93	0.00	20839.93	0.00	11.72
2000	6	32538.52	1.90	31931.30	0.00	31931.30	0.00	33.56
2000	7	45264.42	0.80	44907.15	0.00	44907.15	0.00	97.92
2000	8	53763.37	0.32	53593.97	0.00	53593.97	0.00	294.15
2000	9	61039.66	0.06	61004.03	0.00	61004.03	0.00	850.56
2000	10	68256.41	0.12	68173.08	0.00	68173.08	0.00	2987.40
2000	15	120772.52	0.05	0.00	NA	NA	NA	TL
3000	5	25213.00	1.74	24781.00	0.00	24781.00	0.00	18.78
3000	6	42570.33	0.87	42204.00	0.00	42204.00	0.00	86.56
3000	7	53092.93	0.24	52964.00	0.00	52964.00	0.00	794.61
3000	8	63073.87	0.22	62933.00	0.00	62933.00	0.00	784.47
3000	9	72583.38	0.15	72480.00	0.00	72476.00	0.04	TL
3000	10	78374.00	0.02	78358.00	0.00	78358.00	0.00	TL
3000	15	141114.40	NA	0.00	NA	NA	NA	TL
4000	5	32185.67	0.64	31980.00	0.00	31980.00	0.00	27.40
4000	6	50607.83	0.32	50444.00	0.00	50444.00	0.00	94.88
4000	7	63640.95	0.25	63485.00	0.00	63485.00	0.00	1720.41
4000	8	70676.08	0.14	70577.00	0.00	70577.00	0.00	2042.30
4000	9	76461.20	0.04	76346.00	0.12	76431.00	0.01	1655.19
4000	10	79728.28	0.03	79666.00	0.00	79708.00	0.00	2329.47
4000	15	149452.50	NA	0.00	NA	NA	NA	TL
5000	5	38549.29	0.99	38169.82	0.00	38169.82	0.00	68.72
5000	6	53705.84	0.41	53488.83	0.00	53488.83	0.00	312.48
5000	7	65663.05	0.17	65554.10	0.00	65554.10	0.00	1877.70
5000	8	72589.00	0.05	72554.02	0.00	72554.02	0.00	2357.71
5000	9	79130.76	0.16	79005.40	0.00	79005.40	0.04	TL
5000	10	85073.95	0.09	84986.33	0.00	84998.90	0.00	TL
5000	15	161011.19	NA	NA	NA	NA	NA	TL
Avg.:			0.64		0.00		0.00	1518.65
Max:			3.88		0.12		0.04	3777.61

Table A.3: Model results with $|S| = 100$ and $costfactor = 10^6$ - gaps & solution values

$ K_s $	β^w	LP		RPF		PF		time
		Obj	g1(%)	Obj	g2(%)	Obj	g2(%)	
1000	5	16980.75	0.03	16975.83	0.00	16975.83	0.00	5.59
1000	6	27698.48	0.08	27675.73	0.00	27675.73	0.00	14.35
1000	7	41089.05	0.14	41030.45	0.00	41030.45	0.00	42.35
1000	8	51404.74	0.05	51380.12	0.00	51380.12	0.00	116.04
1000	9	58869.93	0.05	58838.80	0.00	58838.80	0.00	543.88
1000	10	66417.96	0.02	66402.34	0.00	66402.34	0.00	521.08
1000	15	106736.62	0.00	0.00	NA	0.00	NA	TL
2000	5	33364.02	0.04	33349.13	0.00	33349.13	0.00	15.77
2000	6	45939.10	0.07	45904.90	0.00	45904.90	0.00	41.48
2000	7	59443.25	0.07	59399.62	0.00	59399.62	0.00	139.59
2000	8	67929.84	0.04	67903.56	0.00	67903.56	0.00	1096.80
2000	9	75852.91	0.01	75843.23	0.00	75843.23	0.00	TL
2000	10	83831.89	0.01	83820.27	0.00	83820.27	0.00	TL
2000	15	139678.53	0.00	0.00	NA	NA	NA	TL
3000	5	36036.00	0.09	36003.00	0.00	36003.00	0.00	23.38
3000	6	55492.75	0.05	55466.00	0.00	55466.00	0.00	96.77
3000	7	67553.75	0.04	67527.00	0.00	67527.00	0.00	248.83
3000	8	78213.28	0.01	78209.00	0.00	78209.00	0.00	701.07
3000	9	88457.67	0.01	88446.00	0.00	88446.00	0.00	2693.32
3000	10	94913.75	0.01	94909.00	0.00	94909.00	0.00	2726.03
3000	15	160490.00	NA	0.00	NA	NA	NA	TL
4000	5	45154.33	0.04	45137.00	0.00	45137.00	0.00	32.39
4000	6	65262.25	0.02	65252.00	0.00	65252.00	0.00	115.26
4000	7	79767.07	0.02	79753.00	0.00	79753.00	0.00	1746.54
4000	8	86871.00	0.00	86868.00	0.00	86868.00	0.00	1035.95
4000	9	93277.00	0.00	93277.00	0.00	93277.00	0.00	1655.19
4000	10	96947.00	0.00	96947.00	0.00	96947.00	0.00	1824.23
4000	15	168970.00	NA	0.00	NA	NA	NA	TL
5000	5	51641.00	0.05	51614.58	0.00	51614.58	0.00	53.21
5000	6	68400.66	0.06	68362.75	0.00	68362.75	0.00	423.10
5000	7	81719.65	0.03	81695.37	0.00	81695.37	0.00	1223.83
5000	8	89801.18	0.02	89786.21	0.00	89786.21	0.00	2706.08
5000	9	96651.38	0.01	96645.94	0.00	96645.70	0.00	3337.07
5000	10	103253.62	0.00	103252.48	0.00	103252.48	0.00	3024.76
5000	15	180913.30	NA	0.00	NA	NA	NA	TL
Avg.:			0.03		0.00		0.00	1491.68
Max:			0.14		0.00		0.00	3888.52

Table A.4: Model results for $|S| = 200$ and $costfactor = 10^5$ - gaps & solution values

$ K_s $	β^w	LP		RPF		PF		time
		Obj	g1(%)	Obj	g2(%)	Obj	g2(%)	
1000	5	6969.82	2.00	6832.88	0.00	6832.88	0.00	14.46
1000	6	15444.12	3.74	14887.28	0.00	14887.28	0.00	62.97
1000	7	27442.90	1.66	26994.52	0.00	26994.52	0.00	265.89
1000	8	37319.10	1.50	36740.66	0.18	36766.38	0.00	2550.04
1000	9	44926.41	0.64	0.00	NA	44496.02	0.38	TL
1000	10	52354.37	0.30	0.00	NA	0.00	NA	TL
1000	15	0.02	NA	0.00	NA	NA	NA	TL
2000	5	17369.70	0.69	17251.00	0.00	17251.00	0.00	31.43
2000	6	32614.25	1.22	32220.00	0.00	32220.00	0.00	147.28
2000	7	44027.19	1.16	43229.00	0.74	43524.00	0.09	TL
2000	8	52368.69	0.14	0.00	NA	52294.00	0.00	2088.65
2000	9	62119.02	1.55	0.00	NA	0.00	NA	TL
2000	10	68569.30	1.74	0.00	NA	0.00	NA	TL
2000	15	0.00	NA	0.00	NA	NA	NA	TL
3000	5	25213.00	1.74	24781.00	0.00	24781.00	0.00	63.67
3000	6	42570.33	0.87	42204.00	0.00	42204.00	0.00	485.12
3000	7	53092.93	0.24	52964.00	0.00	52964.00	0.00	3286.40
3000	8	63073.87	2.76	0.00	NA	0.00	NA	TL
3000	9	72583.38	1.42	0.00	NA	0.00	NA	TL
3000	10	78374.00	0.75	0.00	NA	0.00	NA	TL
4000	5	32046.28	0.64	31842.00	0.00	31842.00	0.00	160.84
4000	6	50500.08	0.32	50337.72	0.00	50337.72	0.00	575.42
4000	7	63537.28	0.25	63005.95	0.62	0.00	NA	TL
4000	8	70579.57	0.14	0.00	NA	0.00	NA	TL
4000	9	76396.01	0.03	0.00	NA	0.00	NA	TL
4000	10	79660.24	0.05	0.00	NA	NA	NA	TL
5000	5	38280.05	0.91	37934.12	0.00	37934.12	0.00	673.09
5000	6	53494.06	0.36	53216.43	0.16	53300.98	0.00	1903.53
5000	7	65459.66	0.15	0.00	NA	0.00	NA	TL
5000	8	72521.60	0.05	0.00	NA	0.00	NA	TL
5000	9	79074.33	0.16	0.00	NA	0.00	NA	TL
5000	10	85072.10	0.09	0.00	NA	NA	NA	TL
Avg.:			0.91		0.11		0.03	2438.34
Max:			3.74		0.74		0.38	3739.77

Table A.5: Model results with $|S| = 200$ and $costfactor = 10^6$ - gaps & solution values

$ K_s $	β^w	LP		RPF		PF		time
		Obj	g1(%)	Obj	g2(%)	Obj	g2(%)	
1000	5	16766.10	0.00	16766.10	0.00	16766.10	0.00	12.64
1000	6	27615.35	0.07	27597.19	0.00	27597.19	0.00	30.69
1000	7	41037.19	0.11	40992.58	0.00	40992.58	0.00	79.90
1000	8	51395.72	0.05	51371.33	0.00	51371.33	0.00	354.34
1000	9	58943.52	0.07	58855.20	0.11	58899.58	0.01	TL
1000	10	66533.64	0.01	66524.12	0.01	66528.32	0.00	1782.33
1000	15	0.21	NA	0.00	NA	NA	NA	TL
2000	5	27767.50	0.03	27758.00	0.00	27758.00	0.00	24.26
2000	6	44523.50	0.09	44485.00	0.00	44485.00	0.00	89.46
2000	7	56945.73	0.04	56921.00	0.00	56921.00	0.00	1051.88
2000	8	65833.38	0.01	65824.00	0.00	65824.00	0.00	TL
2000	9	76688.18	0.80	0.00	NA	0.00	NA	TL
2000	10	83826.47	0.21	0.00	NA	0.00	NA	TL
2000	15	0.01	NA	0.00	NA	NA	NA	TL
3000	5	36036.00	0.09	36003.00	0.00	36003.00	0.00	134.59
3000	6	55492.75	0.05	55466.00	0.00	55466.00	0.00	388.46
3000	7	67553.75	0.04	67527.00	0.00	67527.00	0.00	1150.57
3000	8	78213.28	0.01	78206.00	0.00	78209.00	0.00	1877.87
3000	9	88457.67	0.16	0.00	NA	0.00	NA	TL
3000	10	94913.75	0.22	0.00	NA	0.00	NA	TL
4000	5	45063.29	0.04	45047.24	0.00	45047.24	0.00	177.75
4000	6	65174.09	0.02	65164.01	0.00	65164.01	0.00	747.65
4000	7	79657.46	0.02	79643.34	0.00	79643.34	0.01	TL
4000	8	86775.66	0.01	86768.81	0.00	0.00	NA	TL
4000	9	93220.05	0.00	93220.05	0.00	0.00	NA	TL
4000	10	96908.38	0.01	0.00	NA	0.00	NA	TL
5000	5	51525.43	0.03	51508.09	0.00	51508.09	0.00	429.57
5000	6	68232.66	0.03	68215.50	0.00	68215.50	0.00	1099.12
5000	7	81548.30	0.03	81521.92	0.00	81521.92	0.00	2349.23
5000	8	89735.76	0.02	89720.81	0.00	89714.20	0.02	TL
5000	9	96601.81	0.00	0.00	NA	NA	NA	TL
5000	10	103249.59	0.00	0.00	NA	0.00	NA	TL
Avg.:			0.07		0.01		0.00	2081.68
Max:			0.80		0.11		0.02	3768.34

Table A.6: Comparison of PF with BAC for $|S| = 100$ and $costfactor = 10^5$

$ K_s $	β^w	PF			BAC		BAC Phase I			BAC Phase II			$ \bar{J} $
		g1(%)	time	nodes	g1(%)	time	lazy	user	nodes	lazy	user	nodes	
1000	5	0.00	4.12	0	0.00	36.44	24	44	17				49
1000	6	0.00	15.76	0	0.00	95.51	42	82	104				62
1000	7	0.00	46.71	4	0.00	80.21	15	63	3				67
1000	8	0.00	247.10	500	0.00	1518.94	148	87	27853				67
1000	9	0.00	656.09	40	0.00	2826.21	21	60	241	6	53	197	66
1000	10	0.00	723.22	83	0.00	498.66	19	53	328				66
1000	15	100.00	3629.30	0	0.00	2942.05	8	19	15	6	0	0	76
2000	5	0.00	11.72	0	0.00	6.96	4	1	0				60
2000	6	0.00	33.56	0	0.00	18.87	4	1	0				67
2000	7	0.00	97.92	0	0.00	25.42	2	0	0				72
2000	8	0.00	294.15	0	0.00	52.37	4	1	0				68
2000	9	0.00	850.56	0	0.00	120.18	4	1	0				70
2000	10	0.00	2987.40	1068	0.00	604.06	9	7	13	3	0	0	72
2000	15	NA	TL	NA	0.00	1763.26	1	6	0				81
3000	5	0.00	18.78	0	0.81	TL	130	353	58812	165	1040	8146	58
3000	6	0.00	86.56	0	0.40	TL	134	634	7781				66
3000	7	0.00	794.61	230	0.19	TL	51	493	0				67
3000	8	0.00	784.47	20	1.96	TL	16	84	0				69
3000	9	0.00	TL	511	1.90	TL	11	81	0				74
3000	10	0.00	TL	969	0.00	3481.80	21	84	985				76
3000	15	NA	TL	NA	NA	TL	1	4	0				NA
4000	5	0.00	27.40	0	0.00	11.66	4	1	0				64
4000	6	0.00	94.88	0	0.00	22.17	4	1	0				68
4000	7	0.00	1720.41	1111	0.00	184.13	21	6	64				74
4000	8	0.00	2042.30	525	0.00	262.33	10	6	47				74
4000	9	0.00	1655.19	967	0.00	385.09	4	7	5				75
4000	10	0.00	2329.47	99	0.00	636.52	6	7	3				76
4000	15	NA	TL	NA	NA	TL	0	6	0				NA
5000	5	0.00	68.72	36	0.00	62.78	23	38	75				66
5000	6	0.00	312.48	66	0.00	106.57	14	22	80				69
5000	7	0.00	1877.70	1177	0.00	359.93	27	20	354				75
5000	8	0.00	2357.71	2052	0.00	481.01	10	13	35				77
5000	9	0.00	TL	942	0.00	637.82	6	11	22				78
5000	10	0.00	TL	16	0.00	1087.72	13	8	60				79
5000	15	NA	TL	NA	NA	TL	NA	NA	NA				NA
Avg.:		3.23	1518.65		0.16	1460.36							
Max:		100.00	3777.61		1.96	7208.06							

Table A.7: Comparison of PF with BAC for $|S| = 100$ and $costfactor = 10^6$

$ K_s $	β^w	PF			BAC		BAC Phase I			BAC Phase II			$ \bar{J} $
		g1(%)	time	nodes	g1(%)	time	lazy	user	nodes	lazy	user	nodes	
1000	5	0.00	5.59	0	0.00	9.70	19	22	0				68
1000	6	0.00	14.35	0	0.00	14.74	19	27	0				74
1000	7	0.00	42.35	2	0.00	35.82	34	44	44				76
1000	8	0.00	116.04	11	0.00	49.91	9	23	10				77
1000	9	0.00	543.88	14	0.00	90.99	10	41	24				73
1000	10	0.00	521.08	0	0.00	184.31	15	24	10				73
1000	15	100.00	TL	0	0.00	1691.83	6	17	0	6	0	0	80
2000	5	0.00	15.77	0	0.00	7.06	3	0	0				71
2000	6	0.00	41.48	0	0.00	14.47	4	1	0				75
2000	7	0.00	139.59	0	0.00	24.15	6	2	0				78
2000	8	0.00	1096.80	1557	0.00	63.43	10	5	10				76
2000	9	0.00	TL	75	0.00	134.13	6	7	9				76
2000	10	0.00	TL	858	0.00	284.24	8	6	56				77
2000	15	NA	TL	NA	0.00	2633.47	3	10	0				NA
3000	5	0.00	23.38	0	0.00	962.87	60	1004	179				64
3000	6	0.00	96.77	0	0.00	1371.60	50	765	331				69
3000	7	0.00	248.83	0	0.00	1876.95	50	695	1084				75
3000	8	0.00	701.07	0	0.03	TL	42	368	0				76
3000	9	0.00	2693.32	0	0.01	TL	17	219	0				77
3000	10	0.00	2726.03	24	0.00	TL	21	145	62				80
3000	15	NA	TL	NA	NA	TL	1	4	0				NA
4000	5	0.00	32.39	0	0.00	9.56	3	0	0				71
4000	6	0.00	115.26	0	0.00	21.85	3	1	0				76
4000	7	0.00	1746.54	1017	0.00	70.61	5	7	18				77
4000	8	0.00	1035.95	0	0.00	177.57	8	8	0				77
4000	9	0.00	1655.19	0	0.00	218.38	3	2	0				79
4000	10	0.00	1824.23	0	0.00	300.02	1	2	0				79
4000	15	NA	TL	NA	NA	TL	0	5	0				NA
5000	5	0.00	53.21	0	0.00	53.51	22	82	50				75
5000	6	0.00	423.10	90	0.00	82.80	21	28	24				76
5000	7	0.00	1223.83	17	0.00	141.18	14	16	36				78
5000	8	0.00	2706.08	546	0.00	201.82	2	8	55				80
5000	9	0.00	3337.07	28	0.00	492.26	11	7	0	5	0	0	80
5000	10	0.00	3024.76	0	0.00	437.84	2	1	0				81
5000	15	NA	TL	NA	NA	TL	0	0	0				NA
Avg.:		3.23	1485.64		0.00	959.53							
Max:		100.00	3888.52		0.03	3699.71							

Table A.8: Comparison of PF with BAC for $|S| = 200$ and $costfactor = 10^5$

$ K_s $	β^w	PF			BAC		BAC Phase I			BAC Phase II			$ \bar{J} $
		g1(%)	time	nodes	g1(%)	time	lazy	user	nodes	lazy	user	nodes	
1000	5	0.00	14.46	0	0.00	52.38	31	35	195				48
1000	6	0.00	62.97	0	0.00	268.96	31	70	206	9	0	0	62
1000	7	0.00	265.89	3	0.00	155.95	25	46	11	10	0	0	68
1000	8	0.00	2550.04	1079	0.00	2606.79	98	76	21161	11	0	0	66
1000	9	0.32	TL	3	0.00	969.98	46	56	1194				66
1000	10	100.00	TL	0	0.00	866.58	18	45	581				66
1000	15	NA	TL	0	NA	TL	0	12	0				NA
2000	5	0.00	31.43	0	0.00	1280.23	51	417	489				53
2000	6	0.00	147.28	0	1.52	TL	58	569	0				62
2000	7	0.00	TL	173	0.44	TL	133	974	1103				66
2000	8	0.00	2088.65	0	0.21	TL	43	589	10				64
2000	9	100.00	TL	0	0.00	TL	21	203	0				71
2000	10	100.00	TL	0	0.00	TL	13	53	0				70
2000	15	NA	TL	0	NA	TL	1	3	0				NA
3000	5	0.00	63.67	0	5.26	TL	170	368	23932	61	1186	0	58
3000	6	0.00	485.12	0	0.81	TL	42	439	0				66
3000	7	0.00	TL	144	2.75	TL	30	190	0				67
3000	8	100.00	TL	0	0.00	TL	14	63	0				71
3000	9	100.00	TL	0	0.00	TL	12	51	0				72
3000	10	100.00	TL	0	0.00	TL	7	36	0				76
4000	5	0.00	160.84	0	0.00	27.79	5	2	0				64
4000	6	0.00	575.42	0	0.00	45.32	3	3	0				68
4000	7	100.00	TL	0	0.00	264.77	17	8	47				74
4000	8	100.00	TL	0	0.00	335.08	5	6	86				74
4000	9	100.00	TL	0	0.00	3494.47	6	5	7	4	7	51	75
4000	10	NA	TL	0	0.00	867.07	3	6	71				76
5000	5	0.00	673.09	25	0.00	69.24	14	30	92				66
5000	6	0.00	1903.53	0	0.00	264.90	14	25	50				69
5000	7	100.00	TL	0	0.00	463.73	17	21	151				75
5000	8	100.00	TL	0	0.00	TL	0	11	37	10	12	9	75
5000	9	100.00	TL	0	0.00	813.33	6	9	35				77
5000	10	NA	TL	0	0.00	TL	0	6	11	8	9	0	78
Avg.:		42.87	2438.34		0.37	2236.12							
Max:		100.00	3739.77		5.26	7207.33							

Table A.9: Comparison of PF with BAC for $|S| = 200$ and $costfactor = 10^6$

$ K_s $	β^w	PF			BAC		BAC Phase I			BAC Phase II			$ \bar{J} $
		g1(%)	time	nodes	g1(%)	time	lazy	user	nodes	lazy	user	nodes	
1000	5	0.00	12.64	0	0.00	9.75	18	14	0				68
1000	6	0.00	30.69	0	0.00	61.61	36	42	87	8	0	0	75
1000	7	0.00	79.90	0	0.00	66.85	28	28	13	9	0	0	77
1000	8	0.00	354.34	0	0.00	209.72	17	24	56	10	0	0	77
1000	9	0.01	TL	0	0.00	TL	26	51	91200	18	29	7044	76
1000	10	0.00	1782.33	0	0.00	484.39	7	25	45				74
1000	15	NA	TL	0	0.00	TL	9	13	158	15	0	0	80
2000	5	0.00	24.26	0	0.00	804.65	56	641	83				56
2000	6	0.00	89.46	0	0.00	2546.39	57	1520	416				64
2000	7	0.00	1051.88	0	0.00	2201.05	47	861	243				68
2000	8	0.00	TL	147	0.00	3081.48	45	626	333				68
2000	9	100.00	TL	0	0.00	TL	18	101	0				73
2000	10	100.00	TL	0	0.00	TL	25	109	0				76
2000	15	NA	TL	0	NA	TL	1	4	0				NA
3000	5	0.00	134.59	0	0.00	1719.39	50	890	84				64
3000	6	0.00	388.46	0	0.00	2744.97	43	856	314				69
3000	7	0.00	1150.57	7	0.00	3051.89	45	553	2983				74
3000	8	0.00	1877.87	0	0.05	TL	40	302	0				76
3000	9	100.00	TL	0	0.00	TL	22	121	0				79
3000	10	100.00	TL	0	0.00	TL	13	44	0				83
4000	5	0.00	177.75	0	0.00	74.96	32	0	184				71
4000	6	0.00	747.65	0	0.00	66.64	5	9	0				76
4000	7	0.00	TL	361	0.00	138.63	7	8	25				77
4000	8	100.00	TL	0	0.00	306.16	5	9	38				78
4000	9	100.00	TL	0	0.00	343.61	3	3	0				80
4000	10	100.00	TL	0	0.00	500.06	2	2	0				79
5000	5	0.00	429.57	153	0.00	154.87	32	64	22				77
5000	6	0.00	1099.12	0	0.00	730.80	14	29	40	13	34	123	76
5000	7	0.00	2349.23	180	0.00	488.06	19	13	117	10	0	0	79
5000	8	0.01	TL	0	0.00	756.91	8	8	36				80
5000	9	NA	TL	0	0.00	922.29	3	3	0	10	0	0	80
5000	10	NA	TL	0	0.00	1404.04	3	3	0				81
Avg.:		25.00	2081.68		0.00	1758.44							
Max:		100.00	3768.34		0.05	7226.77							

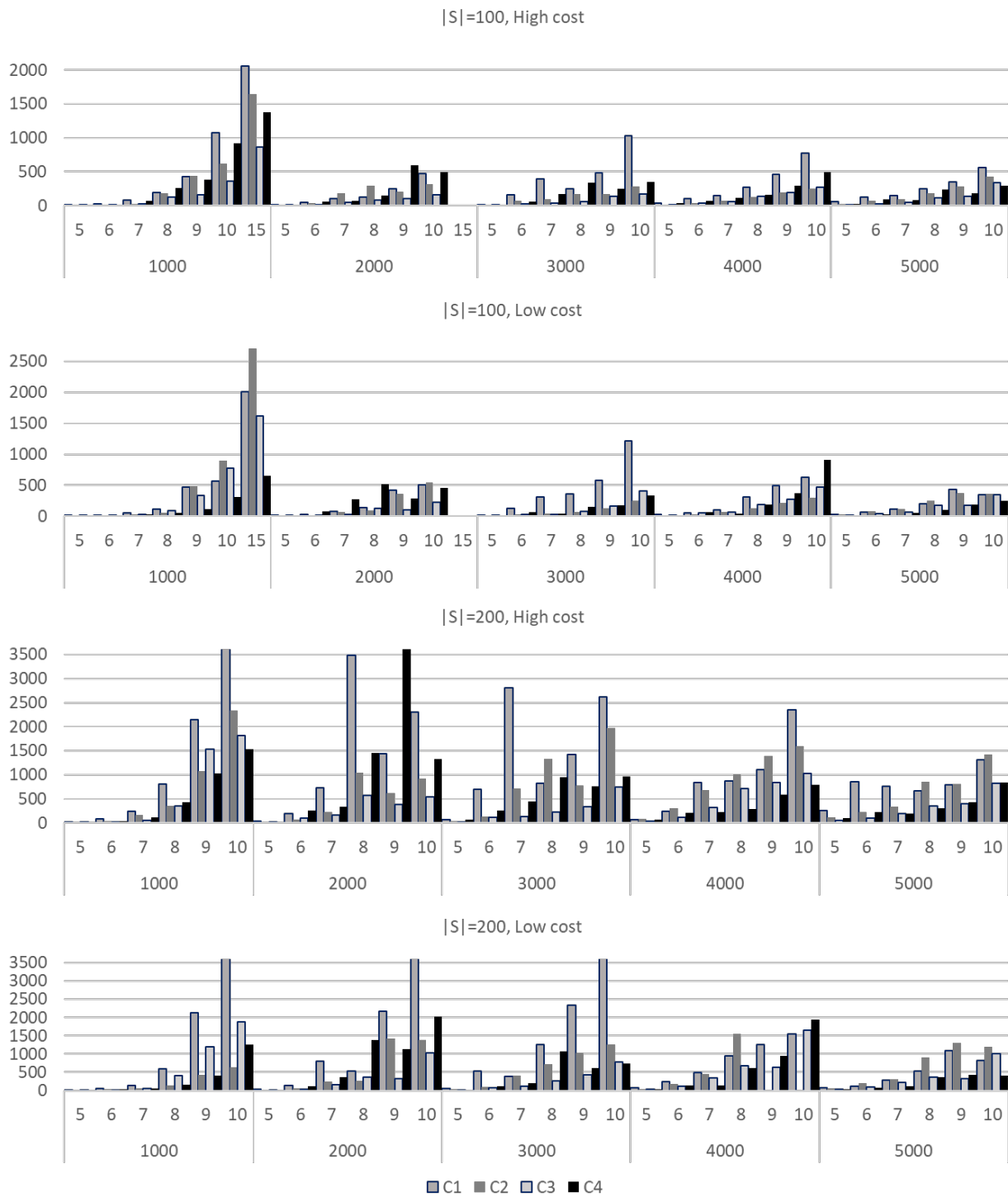


Figure A.13: LP solving times per instance for C1-C4

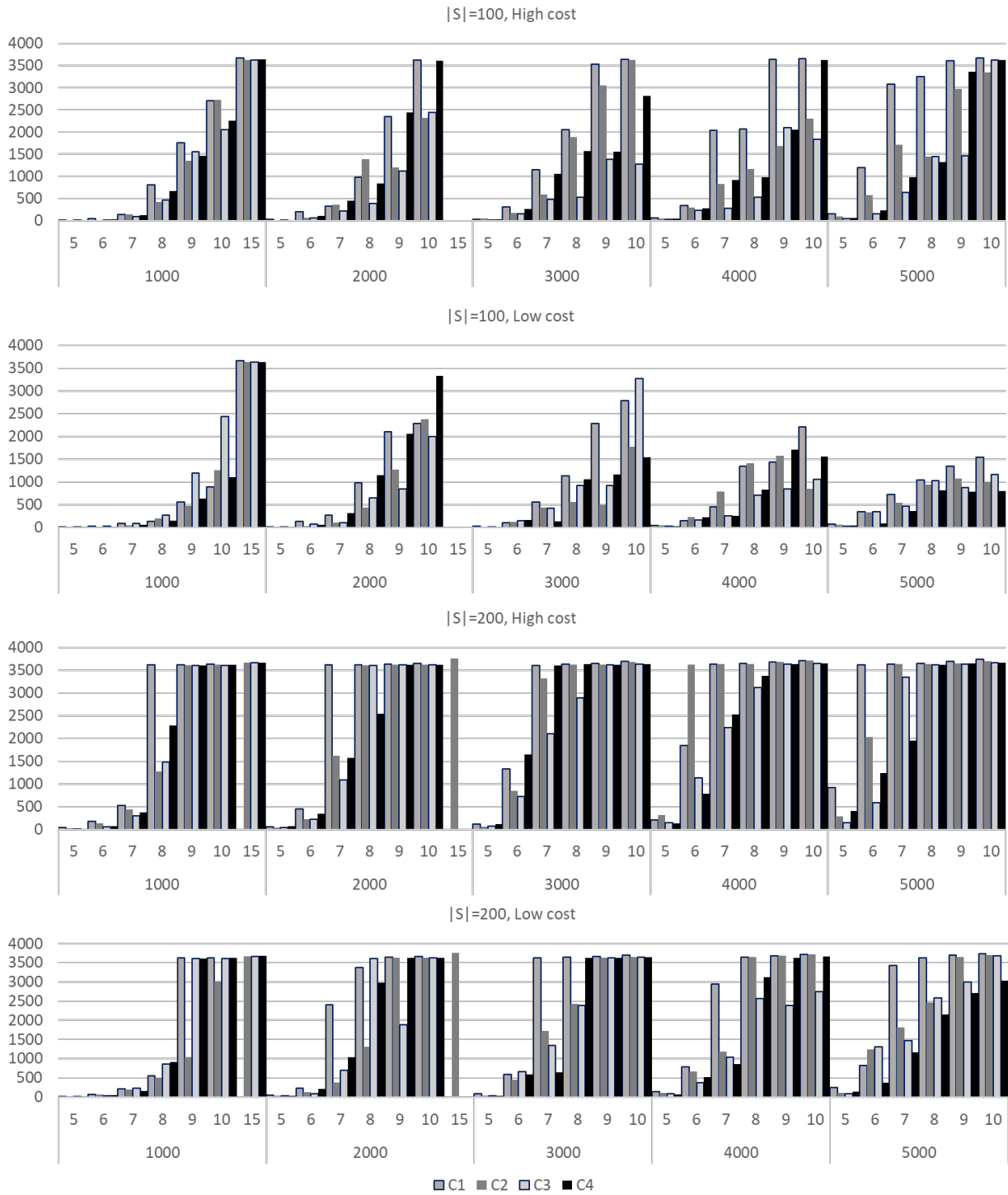


Figure A.14: RPF solving times per instance for C1-C4

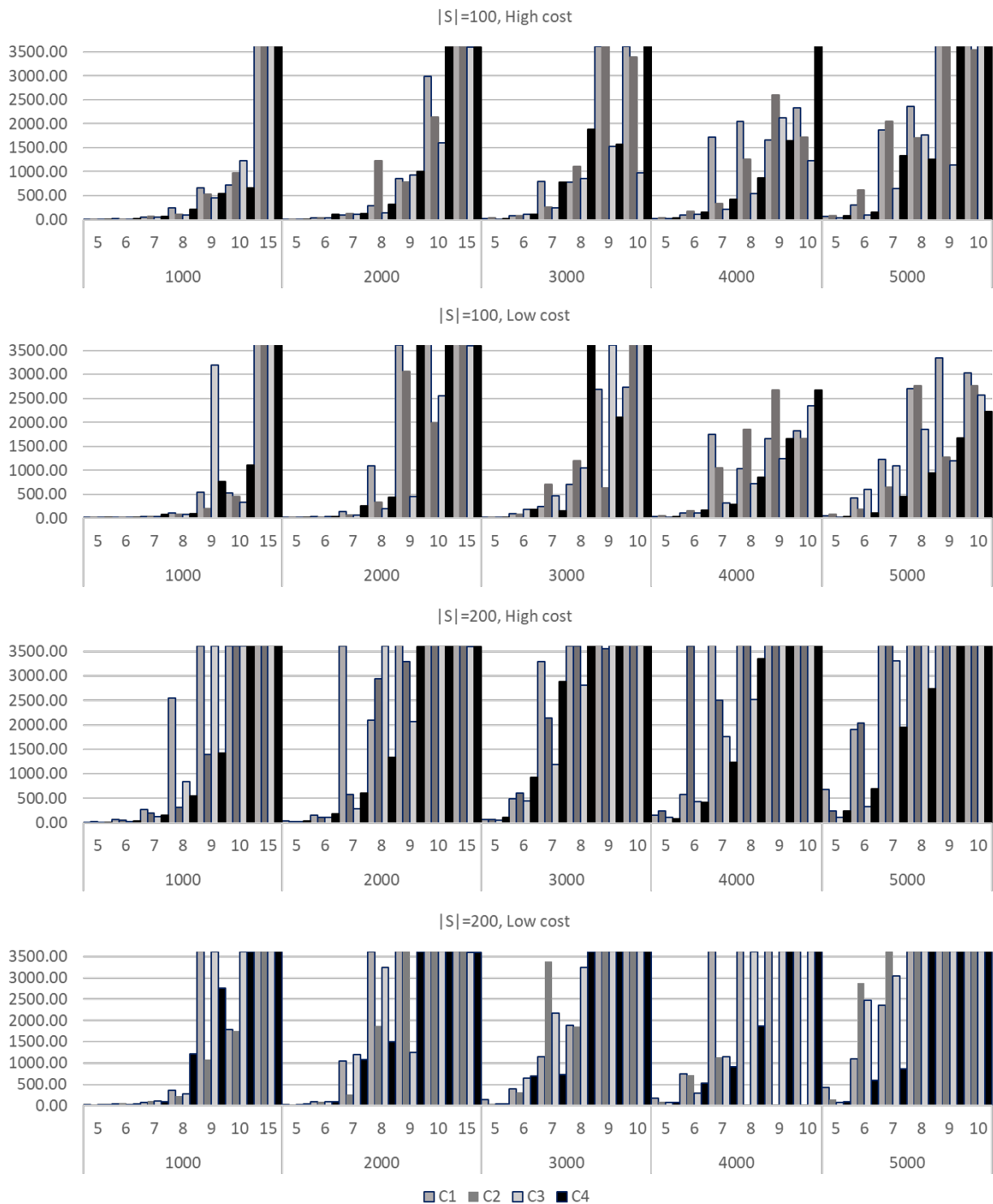


Figure A.15: PF solving times per instance for C1-C4

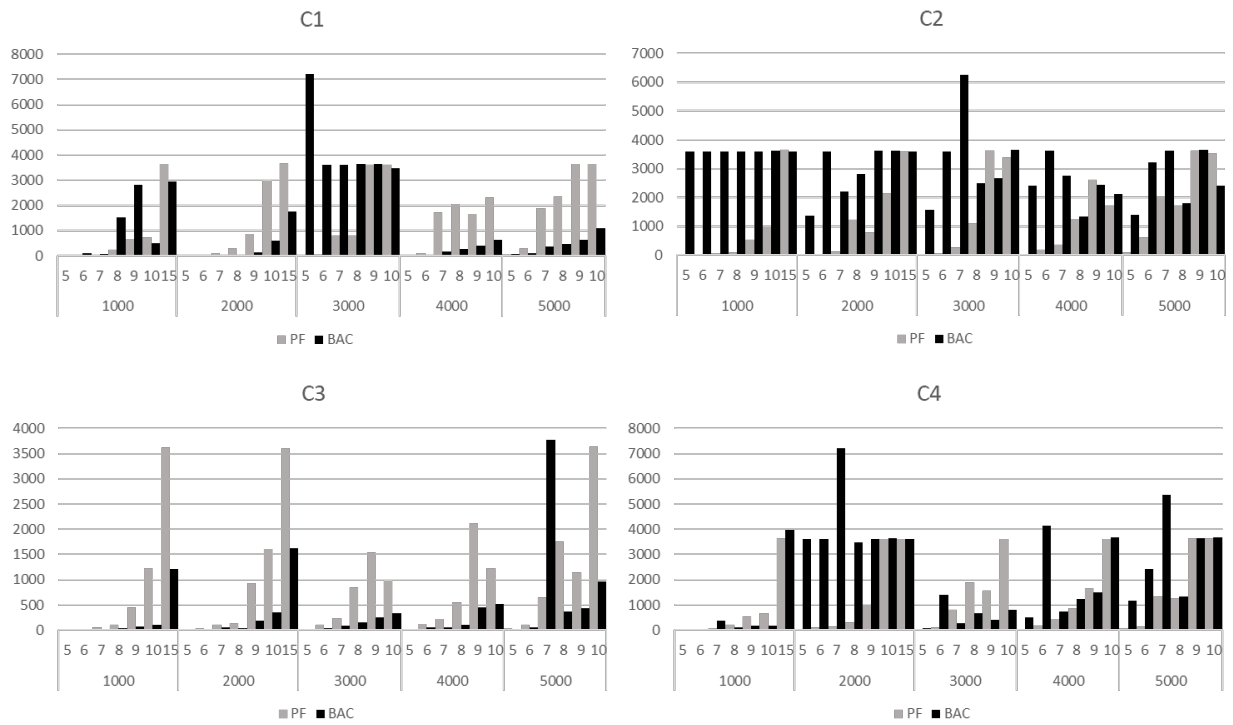


Figure A.16: Solving time of PF and BAC for $|S| = 100$, High cost

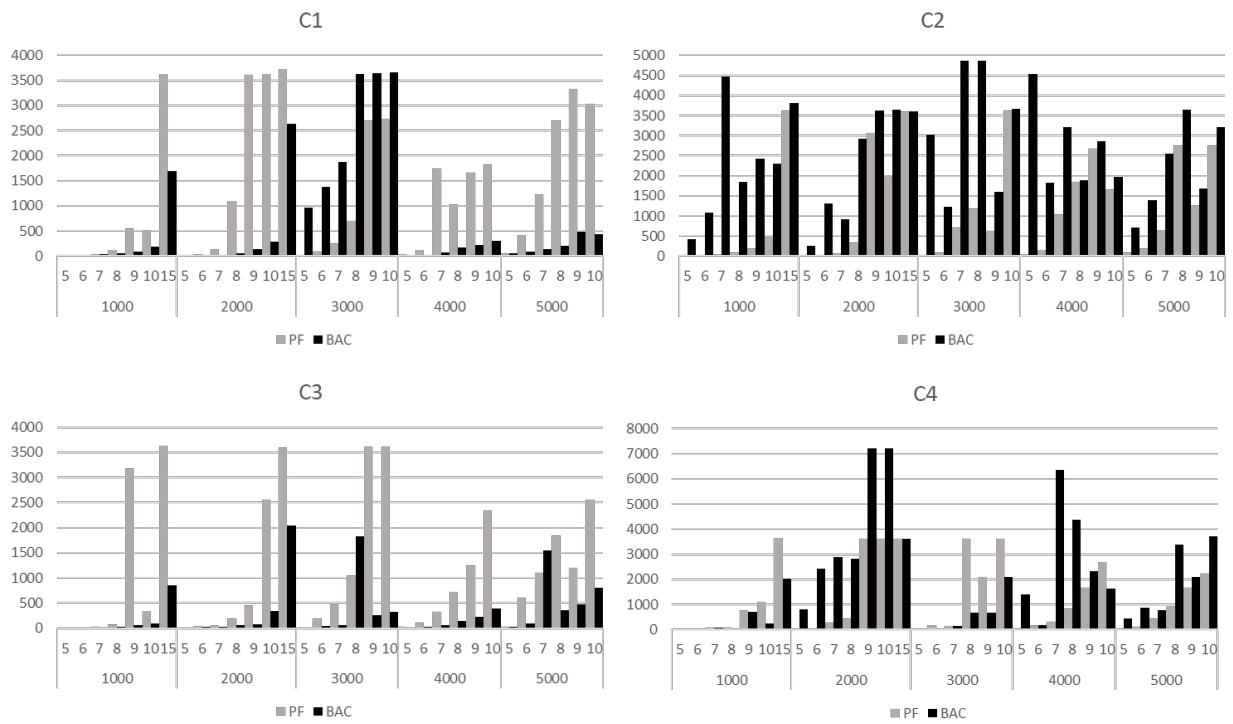


Figure A.17: Solving time of PF and BAC for $|S| = 100$, Low cost

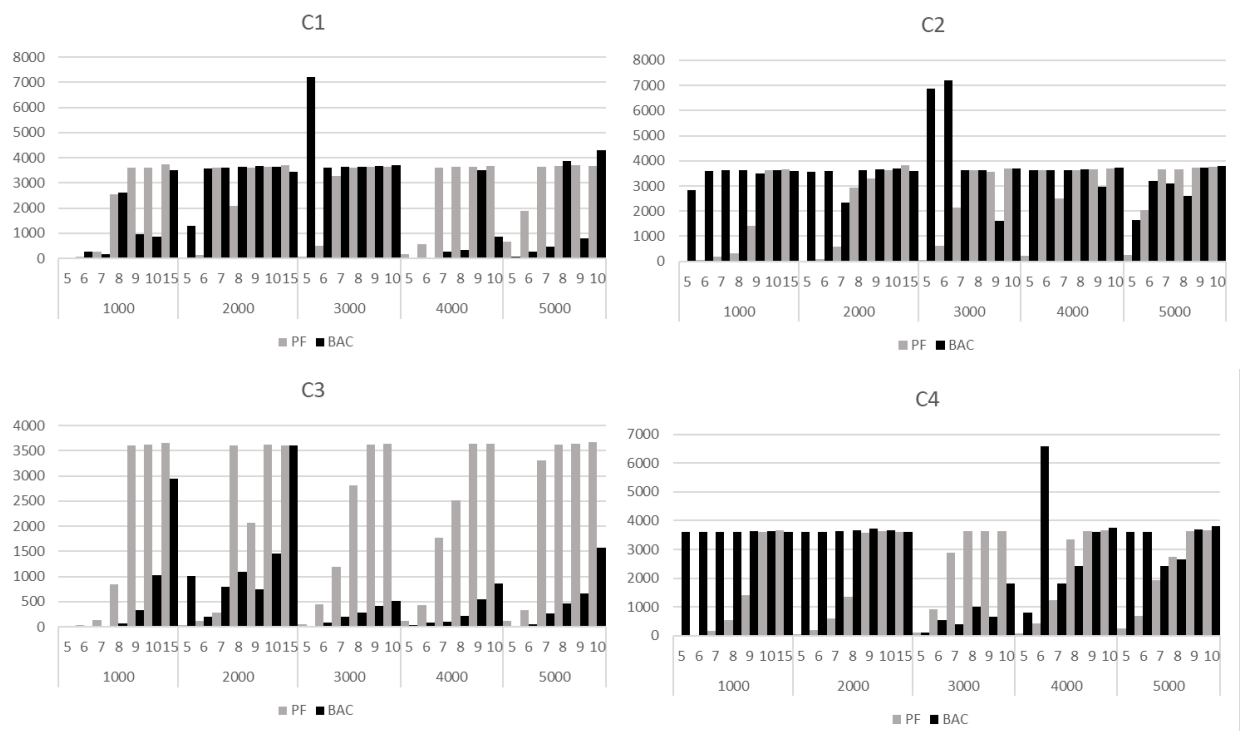


Figure A.18: Solving time of PF and BAC for $|S| = 200$, High cost

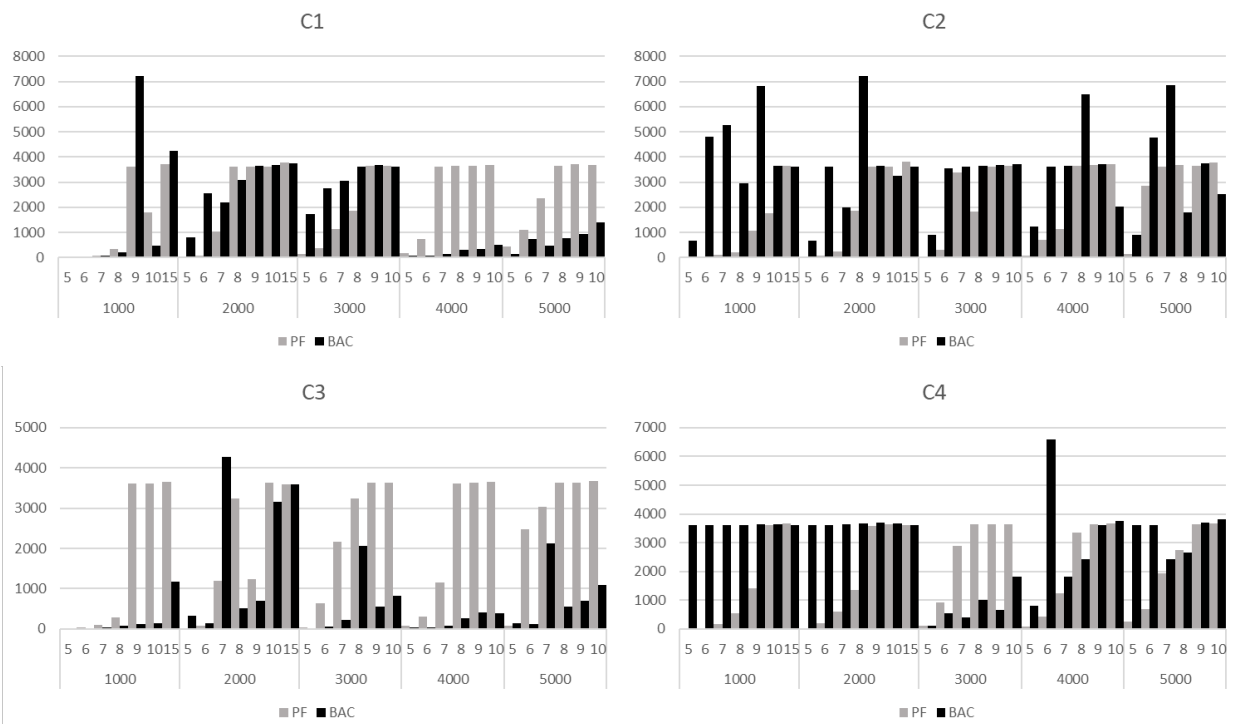


Figure A.19: Solving time of PF and BAC for $|S| = 200$, Low cost