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# Bin Packing Problem with Generalized Time Lags: A Branch-Cut-and-Price Approach

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# Bin Packing With Time Lags Problem

## Classic Bin Packing Problem

- ▶ Set of items to pack into bins.
- ▶ Items have positive weight, and bins have capacity.
- ▶ Objective: Minimize number of bins used.

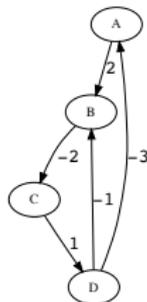
## Bin Packing Problem with Time Lags

- ▶ Bins are assigned to time periods.
- ▶ Number of bins in each period is unbounded
- ▶ Pairs of items have precedence constraints with lags.

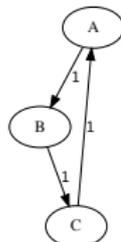
## Precedence Constraints

- ▶ Precedences are represented by a directed graph  $G = (I, A)$ .
- ▶ Each arc  $(i, j) \in A$  has a lag  $l_{ij} \in \mathbb{Z}$ .
- ▶ Bins are assigned to time periods, and items are assigned to the time period of the bin it belongs to.
- ▶ Each lag  $l_{ij}$  imposes the following constraint: The time period that item  $j$  is assigned must be at least  $l_{ij}$  time periods after the time period item  $i$  is assigned.

The graph is not necessarily acyclic.



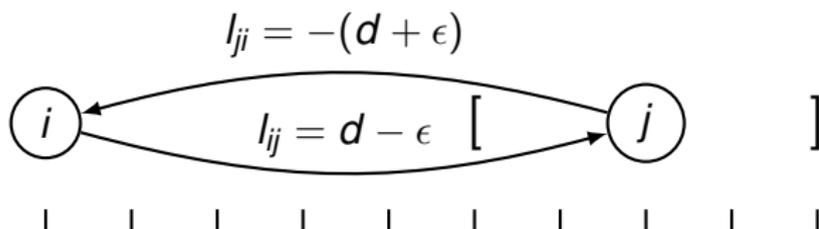
An instance is infeasible if and only if there is a cycle of positive length in the graph.



# Motivation

## Applications

- ▶ Performing a set of periodic tasks using rented capacitated resources



- ▶ Flexible periodic vehicle routing (generalisation)

## Special cases

- ▶ Simple Assembly Line Balancing Problem of type 1 ( $l_{ij} = 0$ ) [Becker and Scholl, 2006]
- ▶ Bin Packing with Precedences ( $l_{ij} = 1$ ) [Pereira, 2016]
- ▶ Bin Packing with Generalized Precedences ( $l_{ij} \geq 0$ ) [Kramer et al., 2017]

# An IP formulation: variables and objective

## Notation

- ▶ The bin capacity  $W \in \mathbb{Z}^+$ .
- ▶ A weight  $w_i \in \mathbb{Z}^+$ ,  $w_i \leq W$ , for each  $i \in V$ .
- ▶  $\mathcal{B} = \{1, 2, \dots, B\}$  the set of potential bins in a period.
- ▶  $\mathcal{T} = \{1, 2, \dots, T\}$  the set of time periods.

## Variables

- ▶  $x_{ibt} \in \{0, 1\}$  for each  $i \in V, j \in \mathcal{B}, t \in \mathcal{T}$ . Takes value 1 iff item  $i$  is assigned to bin  $b$  of time period  $t$ .
- ▶  $u_{bt} \in \{0, 1\}$  for each  $j \in \mathcal{B}, t \in \mathcal{T}$ . Takes value 1 iff bin  $b$  of time period  $t$  is in use.

## Objective

$$\min \sum_{b \in \mathcal{B}} \sum_{t \in \mathcal{T}} u_{bt}$$

# An IP formulation: constraints

## Basic Structure

$$\sum_{b \in \mathcal{B}} \sum_{t \in \mathcal{T}} x_{ibt} = 1 \quad \forall i \in I,$$

$$x_{ibt}, u_{bt} \in \{0, 1\} \quad \forall i \in I, b \in \mathcal{B}, t \in \mathcal{T}.$$

## Bin use and capacity

$$\sum_{i \in I} w_i x_{ibt} \leq W u_{bt} \quad \forall b \in \mathcal{B}, t \in \mathcal{T}.$$

## Precedence Constraints

$$l_{ij} + \sum_{t \in \mathcal{T}} t \cdot \sum_{b \in \mathcal{B}} x_{ibt} \leq \sum_{t \in \mathcal{T}} t \cdot \sum_{b \in \mathcal{B}} x_{jbt} \quad \forall (i, j) \in A.$$

## Symmetry-breaking constraints

$$u_{b-1,t} \geq u_{bt} \quad \forall t \in \mathcal{T}, \forall b \in \mathcal{B} \setminus \{1\}.$$

# Suitable partitions

## Suitable partition

Partition  $\mathcal{P}$  of  $I$  is **suitable** if graph  $G'_{\mathcal{P}} = (I, A \cup A'_{\mathcal{P}})$  has **no cycle of positive length**, where  $A'_{\mathcal{P}}$  contains arcs  $(i, j)$  with  $l_{ij} = 0$  for all  $i, j \in P, P \in \mathcal{P}$ .

## Proposition

Partition  $\mathcal{P}$  induces a feasible solution **if and only if**

- ▶  $\mathcal{P}$  contains all items in  $I$
- ▶  $\mathcal{P}$  is a suitable partition.
- ▶  $\sum_{i \in P} w_i \leq W$  for each  $P \in \mathcal{P}$ .

## Distance

$d_{ij}$  — the total lag of the longest directed path from  $i$  to  $j$  in  $G$ .  
If no path between  $i$  and  $j$  in  $G$ ,  $d_{ij} = -\infty$ .

## Sufficient condition

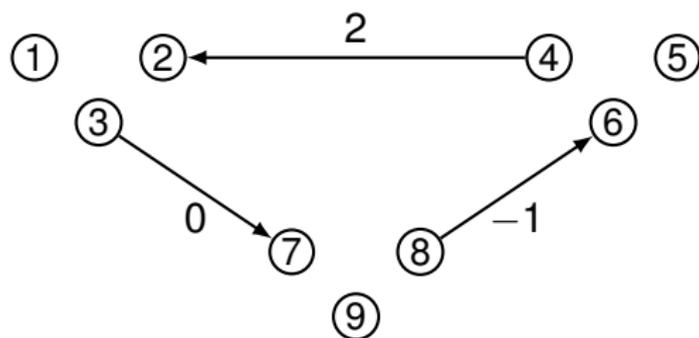
Any partition  $\mathcal{P}$  containing set  $B \supseteq \{i, j\}$ ,  $d_{ij} > 0$ , is non-suitable

## Set partitioning formulation

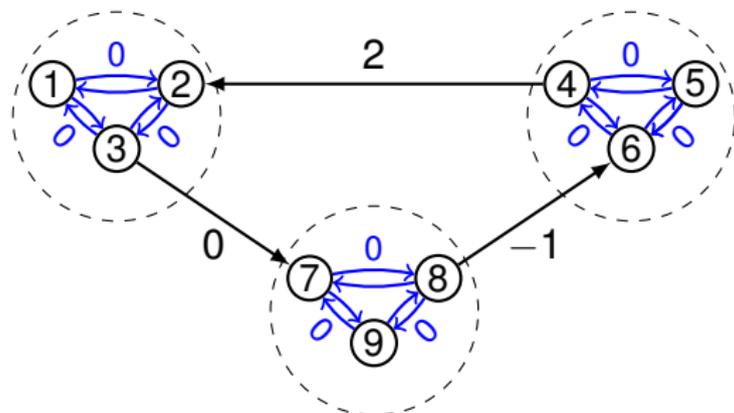
- ▶  $\mathcal{B}$  — set of all items set which can be put to the same bin
- ▶ Variable  $\lambda_B$ ,  $B \in \mathcal{B}$ , — whether set  $B$  is put to the same bin
- ▶  $\mathbb{1}_B(i) = 1 \Leftrightarrow i \in B$
- ▶  $\mathcal{N} \subset \mathcal{B}$  — set of **non-suitable** partitions

$$\begin{aligned} \min \quad & \sum_{B \in \mathcal{B}} \lambda_B \\ \text{s.t.} \quad & \sum_{B \in \mathcal{B}} \mathbb{1}_B(i) \lambda_B = 1, & \forall i \in I, \\ & \sum_{B \in \mathcal{P}} \lambda_B \leq |\mathcal{P}| - 1, & \forall \mathcal{P} \in \mathcal{N}, \\ & \lambda_B \in \{0, 1\}, & \forall B \in \mathcal{B}. \end{aligned}$$

## Characterising non-suitable partitions

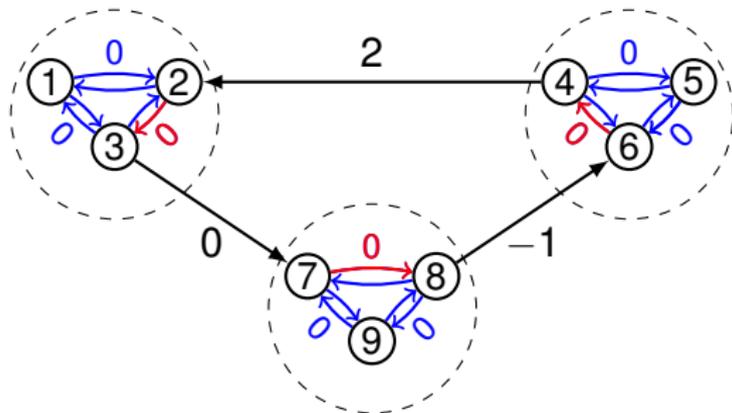


## Characterising non-suitable partitions



- ▶ Partition  $\mathcal{P}$  is non-suitable  $\Rightarrow$  there is a cycle of positive length in graph  $G'_{\mathcal{P}} = (I, A \cup A'_{\mathcal{P}})$ .

## Characterising non-suitable partitions



- ▶ Partition  $\mathcal{P}$  is non-suitable  $\Rightarrow$  there is a cycle of positive length in graph  $G'_\mathcal{P} = (I, A \cup A'_\mathcal{P})$ .
- ▶ Let  $C_\mathcal{P} \subseteq A \cup A'_\mathcal{P}$  be such a cycle, and  $F_\mathcal{P} = (C_\mathcal{P} \setminus A) \subseteq A'_\mathcal{P}$  be the set of arcs in the cycle induced by the partition
- ▶ Then constraint  $\sum_{B \in \mathcal{P}} \lambda_B \leq |\mathcal{P}| - 1$  can be replaced by

$$\sum_{(i,j) \in F_\mathcal{P}} \sum_{\substack{B \in \mathcal{B}: \\ \{i,j\} \in B}} \lambda_B \leq |F_\mathcal{P}| - 1$$

## Pricing problem

- ▶  $\pi_i, i \in I$ , — dual values from the set partitioning constraints
- ▶  $\mu_{\mathcal{P}}, \mathcal{P} \in \bar{\mathcal{N}}$ , — dual values from the active “suitability” constraints

## Binary knapsack problem with hard and soft conflicts

$$\max \sum_{i \in I} \pi_i z_i + \sum_{\mathcal{P} \in \bar{\mathcal{N}}} \sum_{(i,j) \in F_{\mathcal{P}}} \mu_{\mathcal{P}} y_{ij}$$

$$\text{s.t. } \sum_{i \in I} w_i z_i \leq W,$$

$$z_i + z_j \leq 1,$$

$$z_i + z_j \leq 1 + y_{ij},$$

$$z_i \in \{0, 1\},$$

$$y_{ij} \geq 0,$$

$$\forall i, j \in I, d_{ij} > 0,$$

$$\forall \mathcal{P} \in \bar{\mathcal{N}}, \forall (i, j) \in F_{\mathcal{P}},$$

$$\forall i, j \in I.$$

$$\forall \mathcal{P} \in \bar{\mathcal{N}}, \forall (i, j) \in F_{\mathcal{P}}.$$

Solution is using a MIP solver.

# Separation of “non-suitability” constraints

## Integer solution $\mathcal{P}$

We search for a positive cycle in  $G'_{\mathcal{P}}$  in  $O(|I|^2)$  time.

## Fractional solution $(\bar{\mathcal{P}}, \bar{\lambda})$

1. We create valued directed graph  $\bar{G}'_{\bar{\mathcal{P}}} = (I, A \cup A'_{\bar{\mathcal{P}}})$ :

$$v_{ij} = \begin{cases} 1 - \sum_{B \in \bar{\mathcal{P}}: \{i,j\} \in B} \bar{\lambda}_B, & (i,j) \in A'_{\bar{\mathcal{P}}}, \\ 0, & (i,j) \in A. \end{cases}$$

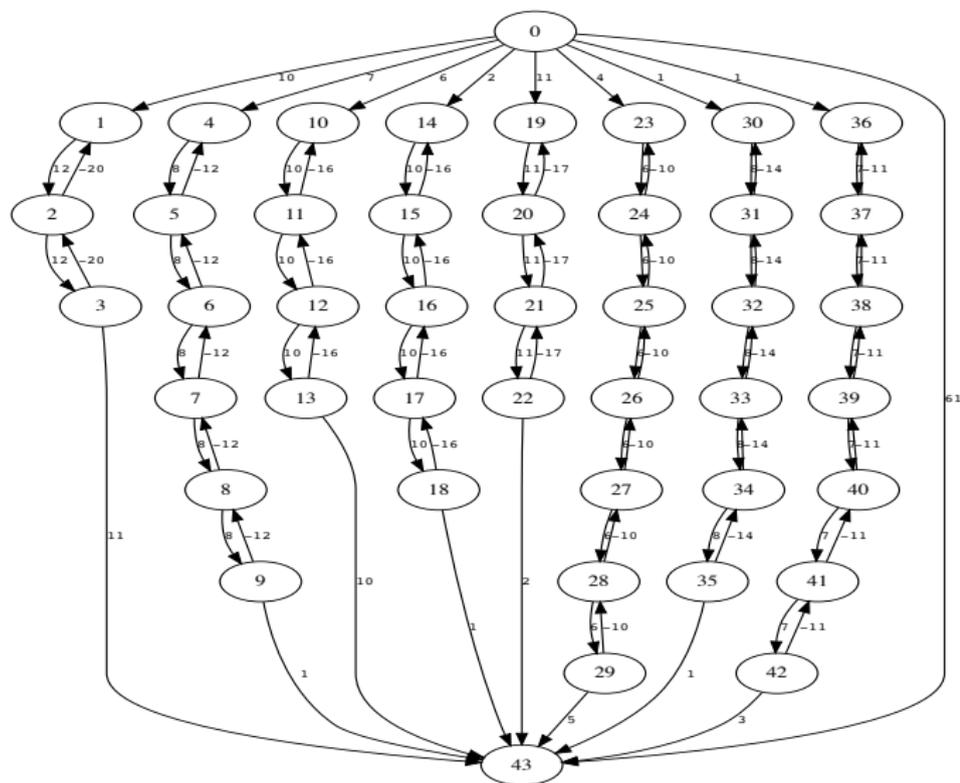
2. We search (by enumeration) in  $\bar{G}'_{\bar{\mathcal{P}}}$  for cycles  $C$  such that

$$\begin{cases} \sum_{(i,j) \in C} l_{ij} > 0, \\ \sum_{(i,j) \in C} v_{ij} < 1. \end{cases}$$

## Other components of the Branch-Cut-and-Price

- ▶ Automatic dual price smoothing **stabilization** [Pessoa et al., 2018]
- ▶ Ryan & Foster branching [Ryan and Foster, 1981]
- ▶ Multi-phase **strong branching** [Pecin et al., 2017]
- ▶ Strong **diving heuristic** with Limited Discrepancy Search [Sadykov et al., 2018]
  - ▶ 10 dives are performed
  - ▶ 10 candidates are evaluated before each fixing
  - ▶ Each time a set of items is fixed, we update the hard conflicts

# Structure of test instances



# Dimension of test instances

## 1386 instances

- ▶ Same flexibility (relative interval for the distance between consecutive tasks)
- ▶ Number of time periods  $\in \{20, 30, \dots, 110, 120\}$
- ▶ Number of chains  $\in \{3, 4, \dots, 9\}$ .
- ▶ Average number of items per chain  $\in \{5, 6, \dots, 10\}$ .
- ▶ Average number of items per bin  $\in \{2, 3, 4\}$ .
- ▶ As a result, number of items  $\in [15, 117]$  with  $\approx$  normal distribution.

# Main experiment results

Solved to optimality within 3 hours

<b>Method</b>	<b>% Solved</b>
BCP	69.5%
CPLEX 12.8	46.2%

On the set of instances solved by both methods, BCP is **9 times faster** on average

## Other experiment results (1)

Percentage of solved instance by number of chains

<b># of chains</b>	<b>% BCP</b>	<b>% CPLEX</b>
<b>3</b>	100.0%	93.4%
<b>4</b>	98.0%	75.3%
<b>5</b>	83.8%	58.1%
<b>6</b>	65.2%	35.4%
<b>7</b>	55.1%	26.8%
<b>8</b>	43.4%	17.7%
<b>9</b>	40.9%	16.7%

## Other experiment results (1)

Percentage of solved instances by number of periods

<b># of periods</b>	<b>% BCP</b>	<b>% CPLEX</b>
<b>20</b>	72%	67%
<b>30</b>	75%	63%
<b>40</b>	79%	67%
<b>50</b>	75%	53%
<b>60</b>	70%	48%
<b>70</b>	67%	41%
<b>80</b>	66%	34%
<b>90</b>	61%	33%
<b>100</b>	66%	37%
<b>110</b>	67%	35%
<b>120</b>	66%	31%

# Perspectives

## Ongoing work

- ▶ Support of Chvátal-Gomory rank-1 cuts
- ▶ Custom branch-and-bound algorithm for the pricing problem
- ▶ Tests on the instances of the special cases of the problem

## Research directions

- ▶ Limit on the number of bins per period
- ▶ Makespan objective
- ▶ (Flexible) Periodic Vehicle Routing

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