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# Space-time Trefftz-DG methods on tent pitching meshes for elastoacoustic wave propagation

Hélène Barucq, Henri Calandra, Julien Diaz, Elvira Shishenina

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The Inria logo is written in a white, elegant cursive script on a red rectangular background.

Space-time Trefftz-DG methods

on tent pitching meshes

for elastoacoustic wave propagation

Hélène Barucq<sup>1</sup>, Henri Calandra<sup>2</sup>, Julien Diaz<sup>1</sup>, Elvira Shishenina<sup>1,2</sup>

1. EPC Magique 3D, Inria, E2S-UPPA, CNRS 2. Total E&P

## PROS AND CONS



Adapted to the complex geometries  
High-order accuracy and hp-adaptivity  
Explicit semi-discrete form  
Conservation laws



Higher number of degrees of freedom,  
compared to the methods with continuous approximation

E. TREFFTZ, 1926

Basis functions are **local solutions of the initial PDEs**

### Frequency domain:

Farhat, Tezaur, Harari, Hetmaniuk (2003 - 2006), Gabard (2007), Badics (2014),  
Hiptmair, Moiola, Perugia (2011 - 2016), Barucq, Bendali, Tordeux, Fares, Mattesi (2017)

### Time domain:

Tsukerman, Egger, Kretschmar, Schnepf, Weiland (2014 - 2015), Wang, Tezaur, Farhat (2014),  
Moiola, Perugia (2016 - 2018), Banjai, Georgoulis, Lijoka (2017),  
Perugia, Schöberl, Stocker, Wintersteiger (2019)

### EXPECTED ADVANTAGES AND DRAWBACKS



Higher order of convergence

Flexibility in the choice of basis functions

Low dispersion

Incorporation of propagation directions in the discrete space

Adaptivity and local space-time mesh refinement

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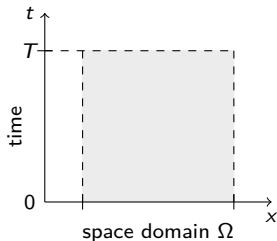


Huge (sparse) global space-time matrix

# 01

## Mathematical Formulation

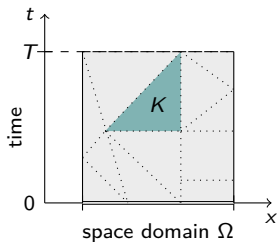
## DOMAIN DESCRIPTION



$$\left\{ \begin{array}{ll} \frac{1}{c^2 \rho} \frac{\partial p}{\partial t} + \operatorname{div} \mathbf{v} = f & \text{in } \Omega \times [0, T], \\ \rho \frac{\partial \mathbf{v}}{\partial t} + \nabla p = 0 & \text{in } \Omega \times [0, T], \\ \mathbf{v}(\cdot, 0) = \mathbf{v}_0, \quad p(\cdot, 0) = p_0 & \text{in } \Omega, \\ \mathbf{v} \cdot \mathbf{n}^x = g_D & \text{in } \partial\Omega \times [0, T]. \end{array} \right.$$



## SPACE-TIME DISCRETIZATION



$$\mathcal{F}_h = \bigcup_{K \in \mathcal{T}_h} \partial K \text{ - mesh skeleton}$$

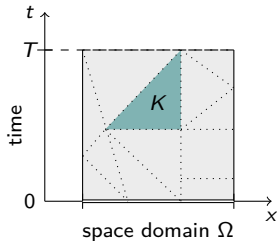
.....  $\mathcal{F}^I$  - internal

—  $\mathcal{F}^D$  - boundary

==  $\mathcal{F}^0$  - initial time

--  $\mathcal{F}^T$  - final time

## SPACE-TIME DISCRETIZATION



$$\mathcal{F}_h = \bigcup_{K \in \mathcal{T}_h} \partial K - \text{mesh skeleton}$$

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--  $\mathcal{F}^T$  - final time

$$(\mathbf{v}_h, \mathbf{p}_h) \in \mathbf{V}^h(\mathcal{T}_h)^d \times V^h(\mathcal{T}_h) \equiv \mathbf{V}^h(\mathcal{T}_h)$$

$$(\omega, \mathbf{q}) \in V^h(\mathcal{T}_h)^d \times V^h(\mathcal{T}_h) \equiv \mathbf{V}^h(\mathcal{T}_h)$$

## SPACE-TIME DISCRETIZATION

$$\begin{aligned}
 & - \sum_K \int_K \mathbf{p}_h \left( \frac{1}{c^2 \rho} \frac{\partial \mathbf{q}}{\partial t} + \operatorname{div} \boldsymbol{\omega} \right) + \mathbf{v}_h \cdot \left( \rho \frac{\partial \boldsymbol{\omega}}{\partial t} + \nabla \mathbf{q} \right) \\
 & + \sum_K \int_{\partial K} \left( \frac{1}{c^2 \rho} \check{\mathbf{p}}_h \mathbf{q} + \check{\mathbf{v}}_h \cdot \boldsymbol{\omega} \right) n_K^t + \left( \mathbf{q} \hat{\mathbf{v}}_h + \hat{\mathbf{v}}_h \boldsymbol{\omega} \right) \cdot \mathbf{n}_K^x = \sum_K \int_K f \mathbf{q}
 \end{aligned}$$

## SPACE-TIME DISCRETIZATION

$$\begin{aligned}
 & - \sum_K \int_K p_h \left( \frac{1}{c^2 \rho} \frac{\partial q}{\partial t} + \operatorname{div} \omega \right) + \mathbf{v}_h \cdot \left( \rho \frac{\partial \omega}{\partial t} + \nabla q \right) \\
 & + \sum_K \int_{\partial K} \left( \frac{1}{c^2 \rho} \check{p}_h q + \check{\mathbf{v}}_h \cdot \omega \right) n_K^t + \left( q \hat{\mathbf{v}}_h + \hat{\mathbf{v}}_h \omega \right) \cdot n_K^x = \sum_K \int_K f q
 \end{aligned}$$

## SPACE-TIME DISCRETIZATION

$$\begin{aligned}
 & - \sum_K \int_K \rho_h \left( \frac{1}{c^2 \rho} \frac{\partial q}{\partial t} + \operatorname{div} \omega \right) + \mathbf{v}_h \cdot \left( \rho \frac{\partial \omega}{\partial t} + \nabla q \right) \\
 & + \sum_K \int_{\partial K} \left( \frac{1}{c^2 \rho} \check{\rho}_h q + \check{\mathbf{v}}_h \cdot \omega \right) n_K^t + \left( q \hat{\mathbf{v}}_h + \hat{\mathbf{v}}_h \omega \right) \cdot n_K^x = \sum_K \int_K f q
 \end{aligned}$$

## TREFFTZ SPACE

$$\mathbf{T}(\mathcal{T}_h) \equiv \left\{ (\omega, q) \in \mathbf{V}^h(\mathcal{T}_h) \text{ s.t. } \frac{1}{c^2 \rho} \frac{\partial q}{\partial t} + \operatorname{div} \omega = \rho \frac{\partial \omega}{\partial t} + \nabla q = 0, \forall K \in \mathcal{T}_h \right\}$$

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SPACE-TIME DISCRETIZATION IN  $\mathcal{T}$ 

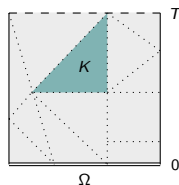
$$\sum_K \int_{\partial K} \left( \frac{1}{c^2 \rho} \check{p}_h q + \check{\mathbf{v}}_h \cdot \omega \right) n_K^t + \left( q \hat{\mathbf{v}}_h + \hat{\mathbf{v}}_h \omega \right) \cdot n_K^x = \sum_K \int_K f q$$

## TREFFTZ-DG FORMULATION

$$\sum_K \int_{\partial K} \left( \frac{1}{c^2 \rho} \check{p}_h q + \check{v}_h \cdot \omega \right) n_K^t + \left( q \hat{v}_h + \hat{v}_h \omega \right) \cdot n_K^x = \sum_K \int_K f q$$

## NUMERICAL FLUXES THROUGH THE ELEMENT FACES

$$\begin{array}{llll} \begin{pmatrix} \hat{v}_h \\ \hat{p}_h \end{pmatrix} & \equiv & \begin{pmatrix} \{v_h\} + \beta [p_h]_x \\ \{p_h\} + \alpha [v_h]_x \end{pmatrix} & \text{on } \mathcal{F}^I \quad \dots \\ \begin{pmatrix} \check{v}_h \\ \check{p}_h \end{pmatrix} & \equiv & \begin{pmatrix} \{v_h\} \\ \{p_h\} \end{pmatrix} & \text{on } \mathcal{F}^I \quad \dots \\ \begin{pmatrix} \hat{v}_h \cdot n_K^x \\ \hat{p}_h \end{pmatrix} & \equiv & \begin{pmatrix} g_D \\ p_h + \alpha (v_h \cdot n_K^x - g_D) \end{pmatrix} & \text{on } \mathcal{F}^D \quad - \\ \begin{pmatrix} \check{v}_h \\ \check{p}_h \end{pmatrix} & \equiv & \begin{pmatrix} v_h \\ p_h \end{pmatrix} & \text{on } \mathcal{F}^T \quad - - \\ \begin{pmatrix} \check{v}_h \\ \check{p}_h \end{pmatrix} & \equiv & \begin{pmatrix} v_0 \\ p_0 \end{pmatrix} & \text{on } \mathcal{F}^0 \quad = \end{array}$$



## TREFFTZ-DG FORMULATION

Seek  $(\mathbf{v}_h, p_h) \in \mathbf{V}^h(\mathcal{T}_h)$  s.t.  $\forall (\omega, q) \in \mathbf{T}(\mathcal{T}_h)$  it holds true:

$$\mathcal{A}_{TDG}((\mathbf{v}_h, p_h); (\omega, q)) = \ell_{TDG}(\omega, q)$$



## WELL POSEDNESS

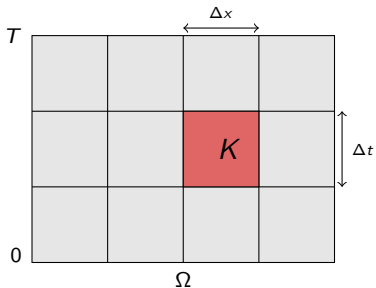
Trefftz-DG formulations for the first order Acoustic System, Elastodynamic System, and Elasto-Acoustic System are **well-posed**\*

\* H. Barucq, H. Calandra, J. Diaz & Elvira Shishenina.  
**Space-Time Trefftz-Discontinuous Galerkin Approximation for Elasto-Acoustics.** (2017)  
<https://hal.inria.fr/hal-01614126/document>

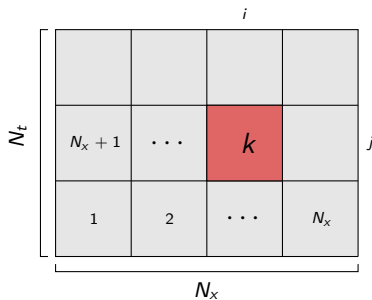
# 02

## Implementation

## MESH AND ELEMENT NUMBERING

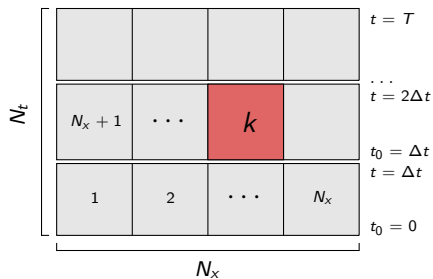


## MESH AND ELEMENT NUMBERING

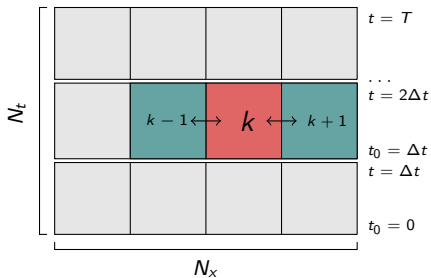


$$k = (j - 1) \times N_x + i$$

## MESH AND ELEMENT NUMBERING



## MESH AND ELEMENT NUMBERING



## BILINEAR FORM

$$\mathcal{A}_{TDG} = \underbrace{\int_{\mathcal{F}^T} + \int_{\mathcal{F}^0}}_{\mathcal{A}_{TDG}^{\Omega}} + \underbrace{\int_{\mathcal{F}^I} + \int_{\mathcal{F}^D}}_{\mathcal{A}_{TDG}^I}$$

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## GLOBAL SPACE-TIME MATRIX

$$\mathbf{M} = \bar{\Delta}_x \mathbf{M}_{\Omega} + \bar{\Delta}_t \mathbf{M}_I$$



## BILINEAR FORM

$$A_{TDG} = \underbrace{\int_{\mathcal{F}^T} + \int_{\mathcal{F}^0}}_{A_{TDG}^{\Omega}} + \underbrace{\int_{\mathcal{F}^I} + \int_{\mathcal{F}^D}}_{A_{TDG}^I}$$

## GLOBAL SPACE-TIME MATRIX

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$M_{\Omega}$  - block-diagonal

## BILINEAR FORM

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## GLOBAL SPACE-TIME MATRIX

$$\mathbf{M} = \bar{\Delta}_x M_\Omega + \bar{\Delta}_t M_I$$

$M_\Omega$  - block-diagonal

$M_I$  - sparse

### GLOBAL MATRIX INVERSION

$$\mathbf{M}^{-1} \equiv \left[ \bar{\Delta}_x M_\Omega + \bar{\Delta}_t M_I \right]^{-1}$$

## GLOBAL MATRIX INVERSION

$$\begin{aligned}\mathbf{M}^{-1} &\equiv \left[ \bar{\Delta}_x M_\Omega + \bar{\Delta}_t M_I \right]^{-1} \\ &= \\ &\left[ I + \kappa (M_\Omega^{-1} M_I) \right]^{-1} \left[ \bar{\Delta}_x M_\Omega \right]^{-1}\end{aligned}$$

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TAYLOR EXPANSION (SMALL ENOUGH  $\kappa = \frac{\bar{\Delta}_t}{\bar{\Delta}_x} \propto \frac{\Delta t}{\Delta x}$ )

$$\mathbf{M}^{-1} = \left[ \sum_{n=0}^{\infty} (-1)^n \kappa^n (M_\Omega^{-1} M_I)^n \right] \left[ \bar{\Delta}_x M_\Omega \right]^{-1}$$

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block-diagonal

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block-diagonal x sparse

APPROXIMATE INVERSION ( $\kappa = 10^{-2}$ )

$n$	$\Delta_x = 10^{-2}$	$\Delta_x = 2 \cdot 10^{-2}$	$\Delta_x = 5 \cdot 10^{-2}$	$\Delta_x = 10^{-1}$
3	1.4166E-05	4.3741E-05	2.8780E-04	2.5772E-03
4	3.1623E-07	1.2656E-06	5.3868E-05	1.2674E-03
5	2.8903E-07	9.1744E-07	4.1029E-05	1.3010E-03

FULL INVERSION ( $\kappa = 10^{-2}$ )

$n$	$\Delta_x = 10^{-2}$	$\Delta_x = 2 \cdot 10^{-2}$	$\Delta_x = 5 \cdot 10^{-2}$	$\Delta_x = 10^{-1}$
·	2.2540E-07	8.9583E-07	5.5811E-05	1.3004E-03



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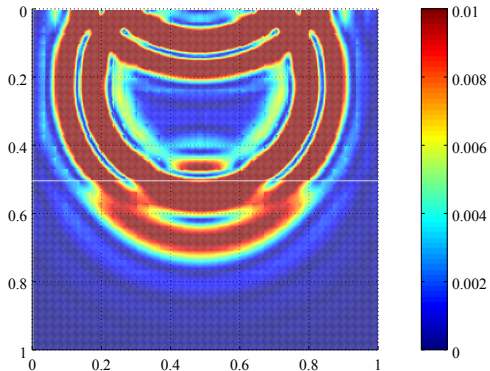
# 03

## Numerical Results

## 2D ELASTO-ACOUSTIC SYSTEM

2D Elasto-acoustic system.

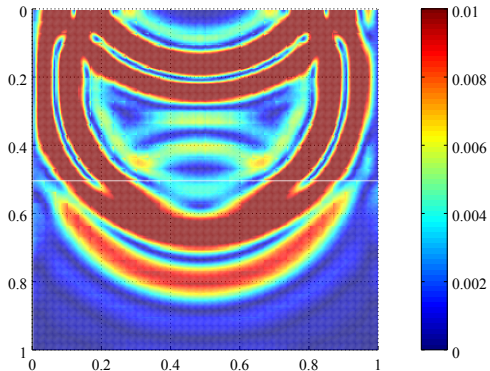
$L^2$  norm of numerical velocity  $v$



## 2D ELASTO-ACOUSTIC SYSTEM

2D Elasto-acoustic system.

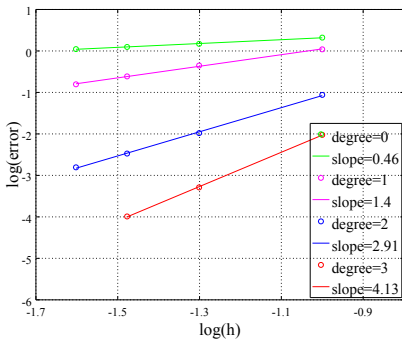
$L^2$  norm of numerical velocity  $v$



## 2D ACOUSTIC AND ELASTODYNAMIC SYSTEMS

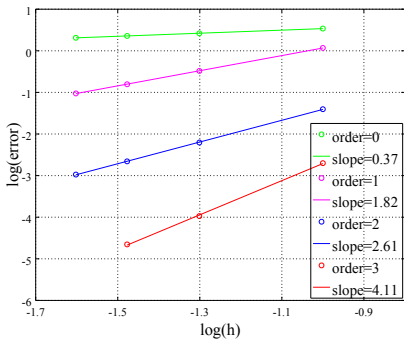
2D Acoustic system.

Convergence of velocity  $v$  in function of cell size



2D Elastodynamic system.

Convergence of velocity  $v$  in function of cell size



# 04

## Tent-Pitching Meshes

## BIBLIOGRAPHY

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**A meshing algorithm for space time discontinuous Galerkin method.** (2000)

J. Erickson, D. Guoy, J.M. Sullivan & A. Üngör.

**Building Space-Time Meshes over Arbitrary Spatial Domains.** (2005)

J. Gopalakrishnan, P. Monk, and P. Sepúlveda.

**A tent pitching scheme motivated by Friedrichs theory.**(2015)

J. Gopalakrishnan, J. Schöberl, and C. Wintersteiger.

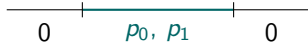
**Mapped tent pitching schemes for hyperbolic systems.** (2017)

I. Perugia, J. Schöberl, P. Stocker and C. Wintersteiger.

**Tent pitching and Trefftz-DG method for the acoustic wave equation..** (Preprint, 2019)

## 1D WAVE EQUATION

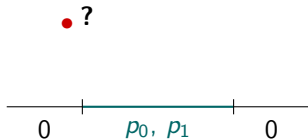
$$\begin{cases} \frac{1}{c_F^2} \frac{\partial^2 p}{\partial t^2} - \frac{\partial^2 p}{\partial x^2} = 0, \\ p(\cdot, 0) = p_0, \\ \frac{\partial p}{\partial t}(\cdot, 0) = p_1. \end{cases}$$





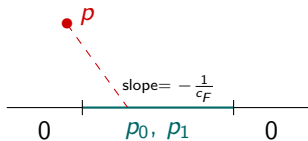
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## 1D WAVE EQUATION

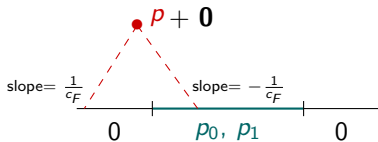
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$$p(x, t) = \frac{1}{2} (p_0(x - c_F t) + p_0(x + c_F t)) + \frac{1}{2c_F} \int_{x - c_F t}^{x + c_F t} p_1(s) ds$$

## 1D WAVE EQUATION

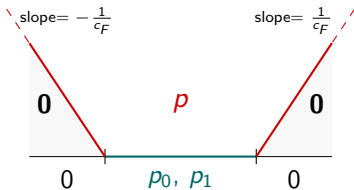
$$\left\{ \begin{array}{l} \frac{1}{c_F^2} \frac{\partial^2 p}{\partial t^2} - \frac{\partial^2 p}{\partial x^2} = 0, \\ p(\cdot, 0) = p_0, \\ \frac{\partial p}{\partial t}(\cdot, 0) = p_1. \end{array} \right.$$



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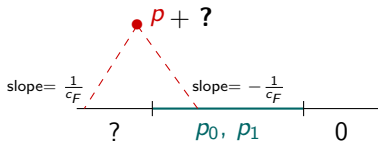
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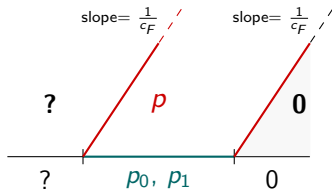
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$$p(x, t) = \frac{1}{2} (p_0(x - c_F t) + p_0(x + c_F t)) + \frac{1}{2c_F} \int_{x - c_F t}^{x + c_F t} p_1(s) ds$$

## MESH GENERATION

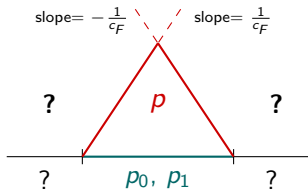
$$\left\{ \begin{array}{l} \frac{1}{c_F^2} \frac{\partial^2 p}{\partial t^2} - \frac{\partial^2 p}{\partial x^2} = 0, \\ p(\cdot, 0) = p_0, \\ \frac{\partial p}{\partial t}(\cdot, 0) = p_1. \end{array} \right.$$



$$p(x, t) = \frac{1}{2} (p_0(x - c_F t) + p_0(x + c_F t)) + \frac{1}{2c_F} \int_{x - c_F t}^{x + c_F t} p_1(s) ds$$

## MESH GENERATION

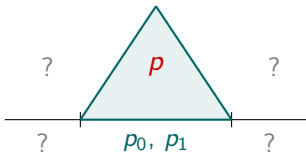
$$\left\{ \begin{array}{l} \frac{1}{c_F^2} \frac{\partial^2 p}{\partial t^2} - \frac{\partial^2 p}{\partial x^2} = 0, \\ p(\cdot, 0) = p_0, \\ \frac{\partial p}{\partial t}(\cdot, 0) = p_1. \end{array} \right.$$



$$p(x, t) = \frac{1}{2} (p_0(x - c_F t) + p_0(x + c_F t)) + \frac{1}{2c_F} \int_{x - c_F t}^{x + c_F t} p_1(s) ds$$

## MESH GENERATION

$$\left\{ \begin{array}{l} \frac{1}{c_F^2} \frac{\partial^2 p}{\partial t^2} - \frac{\partial^2 p}{\partial x^2} = 0, \\ p(\cdot, 0) = p_0, \\ \frac{\partial p}{\partial t}(\cdot, 0) = p_1. \end{array} \right.$$



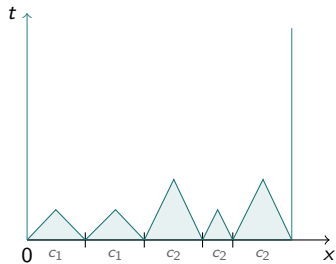
$$p(x, t) = \frac{1}{2} (p_0(x - c_F t) + p_0(x + c_F t)) + \frac{1}{2c_F} \int_{x - c_F t}^{x + c_F t} p_1(s) ds$$



## MESH GENERATION

**1<sup>st</sup> step**

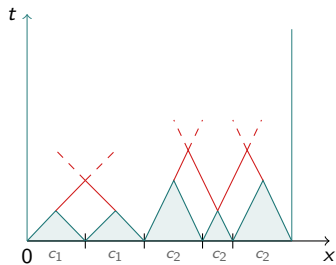
initial condition



## MESH GENERATION

2<sup>nd</sup> step

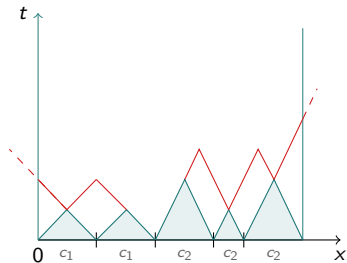
initial condition



## MESH GENERATION

2<sup>nd</sup> step

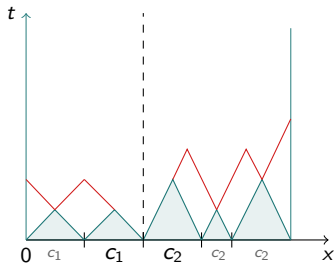
initial condition  
boundary condition



## MESH GENERATION

2<sup>nd</sup> step

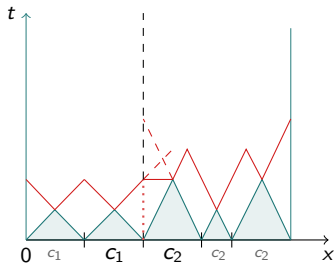
initial condition  
boundary condition  
**heterogeneity?**



## MESH GENERATION

2<sup>nd</sup> step

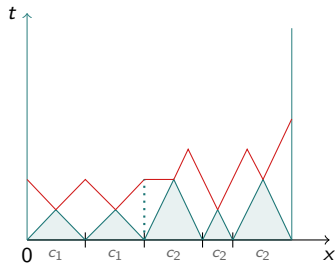
initial condition  
boundary condition  
**heterogeneity?**



## MESH GENERATION

### 2<sup>nd</sup> step

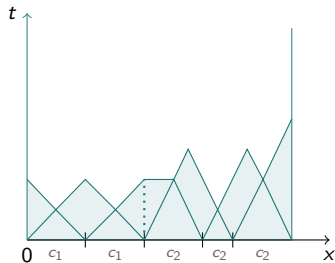
initial condition  
boundary condition  
heterogeneity



## MESH GENERATION

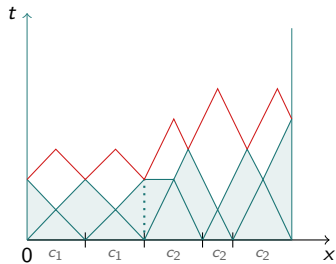
### 2<sup>nd</sup> step

initial condition  
boundary condition  
heterogeneity



## MESH GENERATION

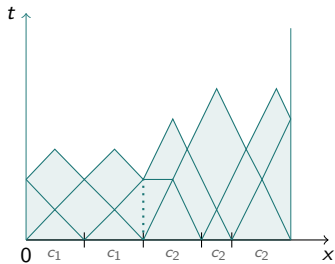
and so on..





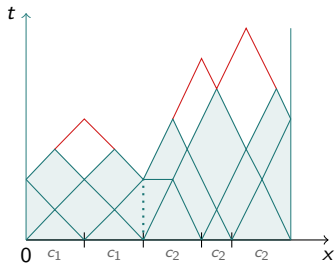
## MESH GENERATION

and so on..



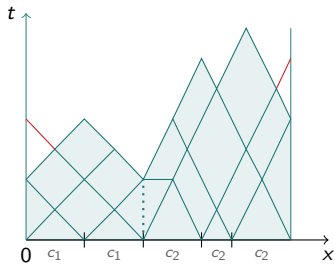
## MESH GENERATION

and so on..



## MESH GENERATION

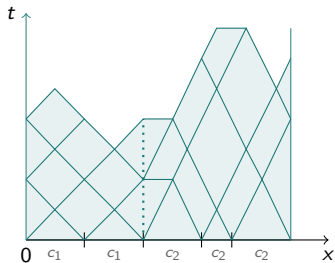
and so on..



## MESH GENERATION

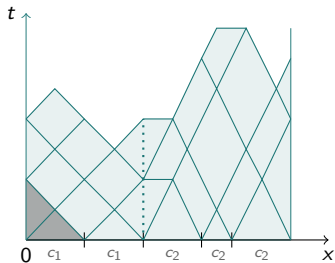
and so on..

$$c_F |n_K^x| / |n_K^t| \leq 1$$



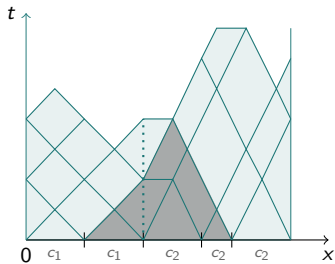
## MESH GENERATION

mesh is divided in  
**space-time patches**



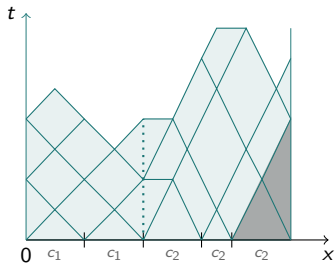
## MESH GENERATION

mesh is divided in  
**space-time patches**



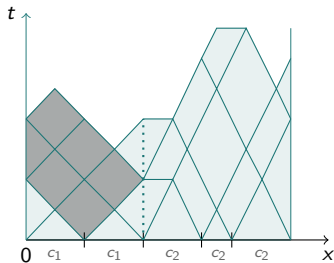
## MESH GENERATION

mesh is divided in  
**space-time patches**



## MESH GENERATION

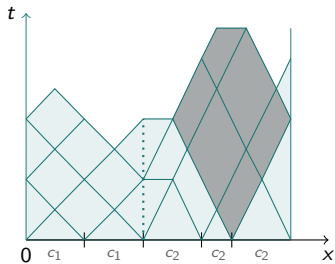
mesh is divided in  
**space-time patches**





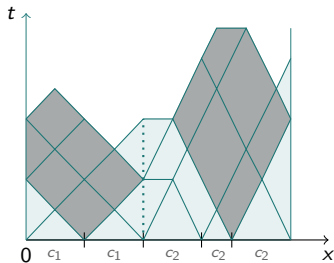
## MESH GENERATION

mesh is divided in  
**space-time patches**



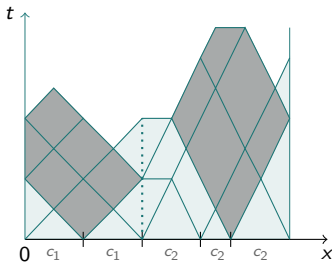
## MESH GENERATION

mesh is divided in  
**space-time patches**



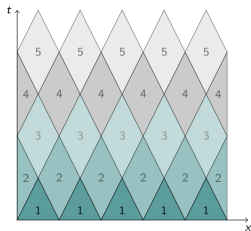
## MESH GENERATION

mesh is divided in  
**space-time patches**

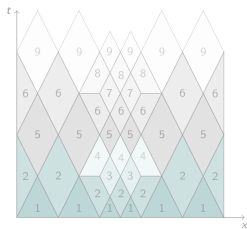


**can be computed independently!**

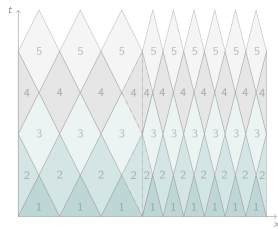
## 1D TENT PITCHING MESHES



Uniform tent mesh  
(homogeneous medium)

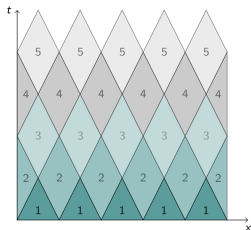


Non-uniform tent mesh  
(homogeneous medium)

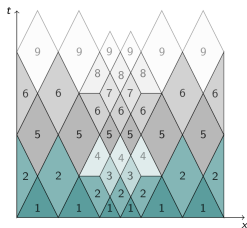


Non-uniform tent mesh  
(heterogeneous medium)

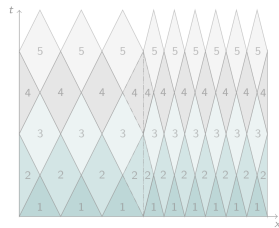
## 1D TENT PITCHING MESHES



Uniform tent mesh  
(homogeneous medium)

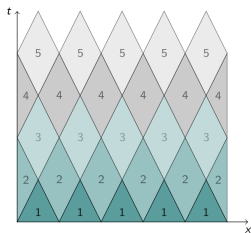


Non-uniform tent mesh  
(homogeneous medium)

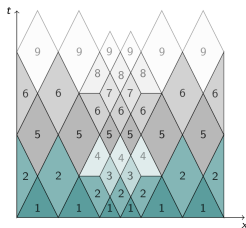


Non-uniform tent mesh  
(heterogeneous medium)

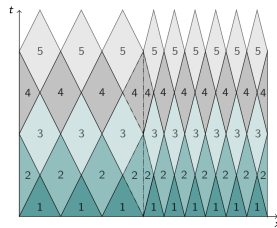
## 1D TENT PITCHING MESHES



Uniform tent mesh  
(homogeneous medium)

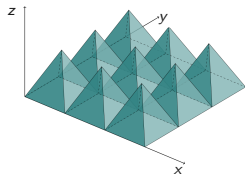


Non-uniform tent mesh  
(homogeneous medium)

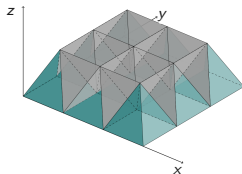


Non-uniform tent mesh  
(heterogeneous medium)

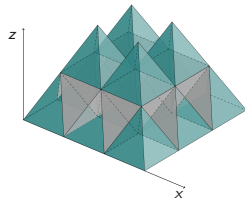
## 2D TENT PITCHING MESHES



1<sup>st</sup> step



2<sup>nd</sup> step



3<sup>rd</sup> step

### STABILITY CONDITIONS

$$c_F |n_K^x| / |n_K^t| \leq 1$$

- TDG acoustic formulation

$$V_P |n_K^x| / |n_K^t| \leq 1$$

- TDG elastodynamic formulation



3D+TIME

Tent Pitcher &  
Space-time DG



**explicit scheme**  
**local time-stepping**

**4D integrals**

3D+TIME

Tent Pitcher &  
Space-time DG



**explicit scheme**  
**local time-stepping**

**4D integrals**

Space-time  
Trefftz-DG



**global**  
**sparse matrix**

**3D integrals**

3D+TIME

Tent Pitcher &  
Space-time DG



Space-time  
Trefftz-DG



explicit scheme  
local time-stepping

**4D integrals**



**global  
sparse matrix**

**3D integrals**

3D+TIME

Tent Pitcher &  
Space-time Trefftz-DG

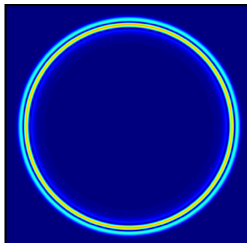


**explicit scheme**  
**local time-stepping**

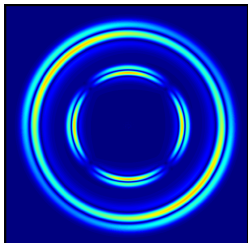
**3D integrals**

## PROPAGATION OF NUMERICAL VELOCITY

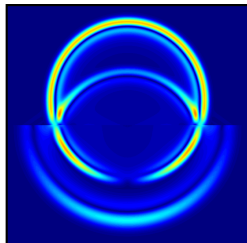
2D Acoustic domain.



2D Elastodynamic domain.

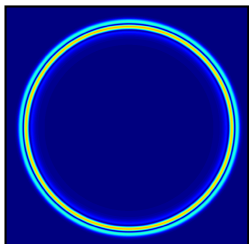


2D Elasto-acoustic domain.



## PROPAGATION OF NUMERICAL VELOCITY

2D Acoustic domain.

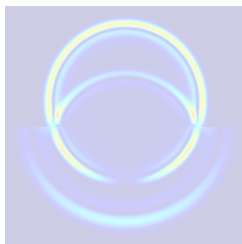


P-wave

2D Elastodynamic domain.



2D Elasto-acoustic domain.

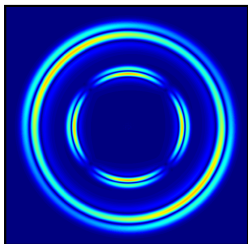


## PROPAGATION OF NUMERICAL VELOCITY

2D Acoustic domain.



2D Elastodynamic domain.



2D Elasto-acoustic domain.



P, S-waves

## PROPAGATION OF NUMERICAL VELOCITY

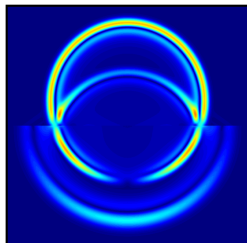
2D Acoustic domain.



2D Elastodynamic domain.



2D Elasto-acoustic domain.



incident, reflected  
P, S-waves,  
P, S head waves



## PROPAGATION OF NUMERICAL VELOCITY

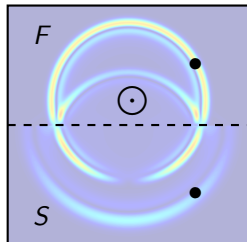
2D Acoustic domain.



2D Elastodynamic domain.



2D Elasto-acoustic domain.

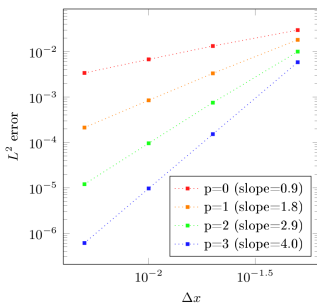


- - source
- - receiver

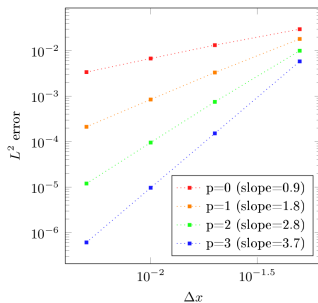
## CONVERGENCE CURVES

Convergence of velocity  $v_F$  (left) and  $v_S$  (right) in space and time as a function of cell size  $\Delta x = \Delta y = \min\{V_P, c_F\}\sqrt{2}\Delta t$  (point-source case).

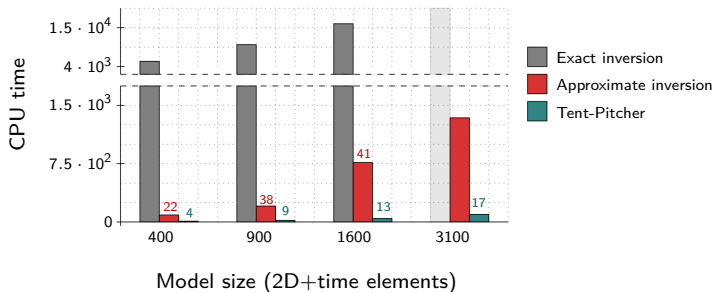
$$\alpha_1 = \beta_1 = 0.5$$



$$\alpha_1 = \beta_1 = 0.5$$



## SPEEDUP FOR DIFFERENT MODEL SIZES



# 05

## Conclusion

### ON-GOING WORK AND PERSPECTIVES

Analytical study of convergence

4D Mesh generation <sup>1</sup>

Absorbing boundary conditions

Implicit-explicit coupling

A posteriori error estimates, hp-adaptivity

Space-time parallelization

<sup>1</sup> K. Voronin.

A parallel mesh generator in 3D/4D. (2018)