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Space-time Trefftz-DG methods on tent pitching meshes for elastoacoustic wave propagation

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The Inria logo is written in a white, elegant cursive script on a red rectangular background.

Space-time Trefftz-DG methods

on tent pitching meshes

for elastoacoustic wave propagation

Hélène Barucq¹, Henri Calandra², Julien Diaz¹, Elvira Shishenina^{1,2}

1. EPC Magique 3D, Inria, E2S-UPPA, CNRS 2. Total E&P

PROS AND CONS



Adapted to the complex geometries
High-order accuracy and hp-adaptivity
Explicit semi-discrete form
Conservation laws



Higher number of degrees of freedom,
compared to the methods with continuous approximation

E. TREFFTZ, 1926

Basis functions are **local solutions of the initial PDEs**

Frequency domain:

Farhat, Tezaur, Harari, Hetmaniuk (2003 - 2006), Gabard (2007), Badics (2014),
Hiptmair, Moiola, Perugia (2011 - 2016), Barucq, Bendali, Tordeux, Fares, Mattesi (2017)

Time domain:

Tsukerman, Egger, Kretschmar, Schnepf, Weiland (2014 - 2015), Wang, Tezaur, Farhat (2014),
Moiola, Perugia (2016 - 2018), Banjai, Georgoulis, Lijoka (2017),
Perugia, Schöberl, Stocker, Wintersteiger (2019)

EXPECTED ADVANTAGES AND DRAWBACKS



Higher order of convergence

Flexibility in the choice of basis functions

Low dispersion

Incorporation of propagation directions in the discrete space

Adaptivity and local space-time mesh refinement

EXPECTED ADVANTAGES AND DRAWBACKS



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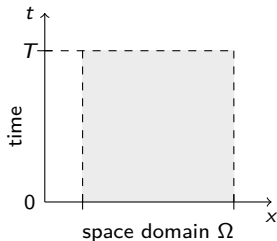


Huge (sparse) global space-time matrix

01

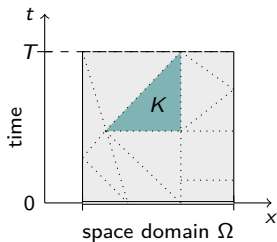
Mathematical Formulation

DOMAIN DESCRIPTION



$$\left\{ \begin{array}{ll} \frac{1}{c^2 \rho} \frac{\partial p}{\partial t} + \operatorname{div} \mathbf{v} = f & \text{in } \Omega \times [0, T], \\ \rho \frac{\partial \mathbf{v}}{\partial t} + \nabla p = 0 & \text{in } \Omega \times [0, T], \\ \mathbf{v}(\cdot, 0) = \mathbf{v}_0, \quad p(\cdot, 0) = p_0 & \text{in } \Omega, \\ \mathbf{v} \cdot \mathbf{n}^x = g_D & \text{in } \partial\Omega \times [0, T]. \end{array} \right.$$

SPACE-TIME DISCRETIZATION



$$\mathcal{F}_h = \bigcup_{K \in \mathcal{T}_h} \partial K \text{ - mesh skeleton}$$

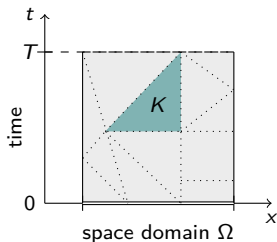
..... \mathcal{F}^I - internal

— \mathcal{F}^D - boundary

== \mathcal{F}^0 - initial time

-- \mathcal{F}^T - final time

SPACE-TIME DISCRETIZATION



$$\mathcal{F}_h = \bigcup_{K \in \mathcal{T}_h} \partial K - \text{mesh skeleton}$$

- \mathcal{F}^I - internal
- \mathcal{F}^D - boundary
- == \mathcal{F}^0 - initial time
- \mathcal{F}^T - final time

$$(\mathbf{v}_h, \mathbf{p}_h) \in \mathbf{V}^h(\mathcal{T}_h)^d \times V^h(\mathcal{T}_h) \equiv \mathbf{V}^h(\mathcal{T}_h)$$

$$(\omega, \mathbf{q}) \in V^h(\mathcal{T}_h)^d \times V^h(\mathcal{T}_h) \equiv \mathbf{V}^h(\mathcal{T}_h)$$

SPACE-TIME DISCRETIZATION

$$\begin{aligned}
& - \sum_K \int_K \mathbf{p}_h \left(\frac{1}{c^2 \rho} \frac{\partial \mathbf{q}}{\partial t} + \operatorname{div} \boldsymbol{\omega} \right) + \mathbf{v}_h \cdot \left(\rho \frac{\partial \boldsymbol{\omega}}{\partial t} + \nabla \mathbf{q} \right) \\
& + \sum_K \int_{\partial K} \left(\frac{1}{c^2 \rho} \check{\mathbf{p}}_h \mathbf{q} + \check{\mathbf{v}}_h \cdot \boldsymbol{\omega} \right) n_K^t + \left(\mathbf{q} \hat{\mathbf{v}}_h + \hat{\mathbf{v}}_h \boldsymbol{\omega} \right) \cdot \mathbf{n}_K^x = \sum_K \int_K f \mathbf{q}
\end{aligned}$$

SPACE-TIME DISCRETIZATION

$$\begin{aligned}
 & - \sum_K \int_K p_h \left(\frac{1}{c^2 \rho} \frac{\partial q}{\partial t} + \operatorname{div} \omega \right) + \mathbf{v}_h \cdot \left(\rho \frac{\partial \omega}{\partial t} + \nabla q \right) \\
 & + \sum_K \int_{\partial K} \left(\frac{1}{c^2 \rho} \check{p}_h q + \check{\mathbf{v}}_h \cdot \omega \right) n_K^t + \left(q \hat{\mathbf{v}}_h + \hat{\mathbf{v}}_h \omega \right) \cdot n_K^x = \sum_K \int_K f q
 \end{aligned}$$

SPACE-TIME DISCRETIZATION

$$\begin{aligned}
 & - \sum_K \int_K p_h \left(\frac{1}{c^2 \rho} \frac{\partial q}{\partial t} + \operatorname{div} \omega \right) + v_h \cdot \left(\rho \frac{\partial \omega}{\partial t} + \nabla q \right) \\
 & + \sum_K \int_{\partial K} \left(\frac{1}{c^2 \rho} \check{p}_h q + \check{v}_h \cdot \omega \right) n_K^t + \left(q \hat{v}_h + \hat{v}_h \omega \right) \cdot n_K^x = \sum_K \int_K f q
 \end{aligned}$$

TREFFTZ SPACE

$$\mathbf{T}(\mathcal{T}_h) \equiv \left\{ (\omega, q) \in \mathbf{V}^h(\mathcal{T}_h) \text{ s.t. } \frac{1}{c^2 \rho} \frac{\partial q}{\partial t} + \operatorname{div} \omega = \rho \frac{\partial \omega}{\partial t} + \nabla q = 0, \forall K \in \mathcal{T}_h \right\}$$

SPACE-TIME DISCRETIZATION

$$\begin{aligned}
 & - \sum_K \int_K p_h \left(\frac{1}{c^2 \rho} \frac{\partial q}{\partial t} + \operatorname{div} \omega \right) + \mathbf{v}_h \cdot \left(\rho \frac{\partial \omega}{\partial t} + \nabla q \right) \\
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 \end{aligned}$$

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SPACE-TIME DISCRETIZATION IN \mathcal{T}

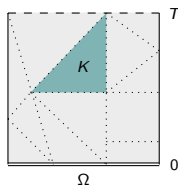
$$\sum_K \int_{\partial K} \left(\frac{1}{c^2 \rho} \check{p}_h q + \check{\mathbf{v}}_h \cdot \omega \right) n_K^t + \left(q \hat{\mathbf{v}}_h + \hat{\mathbf{v}}_h \omega \right) \cdot n_K^x = \sum_K \int_K f q$$

TREFFTZ-DG FORMULATION

$$\sum_K \int_{\partial K} \left(\frac{1}{c^2 \rho} \check{p}_h q + \check{v}_h \cdot \omega \right) n_K^t + \left(q \hat{v}_h + \hat{v}_h \omega \right) \cdot n_K^x = \sum_K \int_K f q$$

NUMERICAL FLUXES THROUGH THE ELEMENT FACES

$$\begin{array}{llll} \begin{pmatrix} \hat{v}_h \\ \hat{p}_h \end{pmatrix} & \equiv & \begin{pmatrix} \{v_h\} + \beta [p_h]_x \\ \{p_h\} + \alpha [v_h]_x \end{pmatrix} & \text{on } \mathcal{F}^I \quad \dots \\ \begin{pmatrix} \check{v}_h \\ \check{p}_h \end{pmatrix} & \equiv & \begin{pmatrix} \{v_h\} \\ \{p_h\} \end{pmatrix} & \text{on } \mathcal{F}^I \quad \dots \\ \begin{pmatrix} \hat{v}_h \cdot n_K^x \\ \hat{p}_h \end{pmatrix} & \equiv & \begin{pmatrix} g_D \\ p_h + \alpha (v_h \cdot n_K^x - g_D) \end{pmatrix} & \text{on } \mathcal{F}^D \quad - \\ \begin{pmatrix} \check{v}_h \\ \check{p}_h \end{pmatrix} & \equiv & \begin{pmatrix} v_h \\ p_h \end{pmatrix} & \text{on } \mathcal{F}^T \quad - - \\ \begin{pmatrix} \check{v}_h \\ \check{p}_h \end{pmatrix} & \equiv & \begin{pmatrix} v_0 \\ p_0 \end{pmatrix} & \text{on } \mathcal{F}^0 \quad = \end{array}$$



TREFFTZ-DG FORMULATION

Seek $(\mathbf{v}_h, p_h) \in \mathbf{V}^h(\mathcal{T}_h)$ s.t. $\forall (\omega, q) \in \mathbf{T}(\mathcal{T}_h)$ it holds true:

$$\mathcal{A}_{TDG}((\mathbf{v}_h, p_h); (\omega, q)) = \ell_{TDG}(\omega, q)$$

WELL POSEDNESS

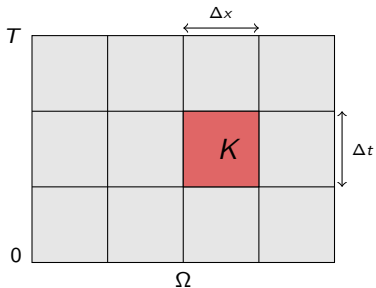
Trefftz-DG formulations for the first order Acoustic System, Elastodynamic System, and Elasto-Acoustic System are **well-posed***

* H. Barucq, H. Calandra, J. Diaz & Elvira Shishenina.
Space-Time Trefftz-Discontinuous Galerkin Approximation for Elasto-Acoustics. (2017)
<https://hal.inria.fr/hal-01614126/document>

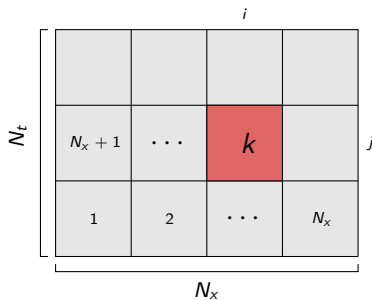
02

Implementation

MESH AND ELEMENT NUMBERING

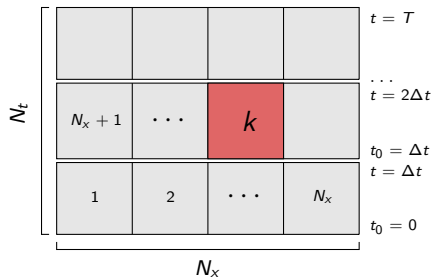


MESH AND ELEMENT NUMBERING

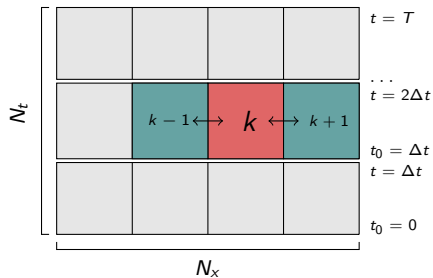


$$k = (j - 1) \times N_x + i$$

MESH AND ELEMENT NUMBERING



MESH AND ELEMENT NUMBERING



BILINEAR FORM

$$\mathcal{A}_{TDG} = \underbrace{\int_{\mathcal{F}^T} + \int_{\mathcal{F}^0}}_{\mathcal{A}_{TDG}^\Omega} + \underbrace{\int_{\mathcal{F}^I} + \int_{\mathcal{F}^D}}_{\mathcal{A}_{TDG}^I}$$

BILINEAR FORM

$$\mathcal{A}_{TDG} = \underbrace{\int_{\mathcal{F}^T} + \int_{\mathcal{F}^0}}_{\mathcal{A}_{TDG}^{\Omega}} + \underbrace{\int_{\mathcal{F}^I} + \int_{\mathcal{F}^D}}_{\mathcal{A}_{TDG}^I}$$

GLOBAL SPACE-TIME MATRIX

$$\mathbf{M} = \bar{\Delta}_x \mathbf{M}_{\Omega} + \bar{\Delta}_t \mathbf{M}_I$$

BILINEAR FORM

$$A_{TDG} = \underbrace{\int_{\mathcal{F}^T} + \int_{\mathcal{F}^0}}_{A_{TDG}^{\Omega}} + \underbrace{\int_{\mathcal{F}^I} + \int_{\mathcal{F}^D}}_{A_{TDG}^I}$$

GLOBAL SPACE-TIME MATRIX

$$\mathbf{M} = \bar{\Delta}_x M_{\Omega} + \bar{\Delta}_t M_I$$

M_{Ω} - block-diagonal

BILINEAR FORM

$$\mathcal{A}_{TDG} = \underbrace{\int_{\mathcal{F}^T} + \int_{\mathcal{F}^0}}_{\mathcal{A}_{TDG}^\Omega} + \underbrace{\int_{\mathcal{F}^I} + \int_{\mathcal{F}^D}}_{\mathcal{A}_{TDG}^I}$$

GLOBAL SPACE-TIME MATRIX

$$\mathbf{M} = \bar{\Delta}_x M_\Omega + \bar{\Delta}_t M_I$$

M_Ω - block-diagonal

M_I - sparse

GLOBAL MATRIX INVERSION

$$\mathbf{M}^{-1} \equiv \left[\bar{\Delta}_x M_\Omega + \bar{\Delta}_t M_I \right]^{-1}$$

GLOBAL MATRIX INVERSION

$$\begin{aligned}\mathbf{M}^{-1} &\equiv \left[\bar{\Delta}_x M_\Omega + \bar{\Delta}_t M_I \right]^{-1} \\ &= \\ &\left[I + \kappa (M_\Omega^{-1} M_I) \right]^{-1} \left[\bar{\Delta}_x M_\Omega \right]^{-1}\end{aligned}$$

GLOBAL MATRIX INVERSION

$$\begin{aligned} \mathbf{M}^{-1} &\equiv \left[\bar{\Delta}_x M_\Omega + \bar{\Delta}_t M_I \right]^{-1} \\ &= \\ &\left[I + \kappa (M_\Omega^{-1} M_I) \right]^{-1} \left[\bar{\Delta}_x M_\Omega \right]^{-1} \end{aligned}$$

TAYLOR EXPANSION (SMALL ENOUGH $\kappa = \frac{\bar{\Delta}_t}{\bar{\Delta}_x} \propto \frac{\Delta t}{\Delta x}$)

$$\mathbf{M}^{-1} = \left[\sum_{n=0}^{\infty} (-1)^n \kappa^n (M_\Omega^{-1} M_I)^n \right] \left[\bar{\Delta}_x M_\Omega \right]^{-1}$$

GLOBAL MATRIX INVERSION

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block-diagonal

GLOBAL MATRIX INVERSION

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$$\mathbf{M}^{-1} = \left[\sum_{n=0}^{\infty} (-1)^n \kappa^n (M_\Omega^{-1} M_I)^n \right] \left[\bar{\Delta}_x M_\Omega \right]^{-1}$$

block-diagonal x sparse

APPROXIMATE INVERSION ($\kappa = 10^{-2}$)

n	$\Delta_x = 10^{-2}$	$\Delta_x = 2 \cdot 10^{-2}$	$\Delta_x = 5 \cdot 10^{-2}$	$\Delta_x = 10^{-1}$
3	1.4166E-05	4.3741E-05	2.8780E-04	2.5772E-03
4	3.1623E-07	1.2656E-06	5.3868E-05	1.2674E-03
5	2.8903E-07	9.1744E-07	4.1029E-05	1.3010E-03

FULL INVERSION ($\kappa = 10^{-2}$)

n	$\Delta_x = 10^{-2}$	$\Delta_x = 2 \cdot 10^{-2}$	$\Delta_x = 5 \cdot 10^{-2}$	$\Delta_x = 10^{-1}$
·	2.2540E-07	8.9583E-07	5.5811E-05	1.3004E-03

APPROXIMATE INVERSION ($\kappa = 10^{-2}$)

n	$\Delta x = 10^{-2}$	$\Delta x = 2 \cdot 10^{-2}$	$\Delta x = 5 \cdot 10^{-2}$	$\Delta x = 10^{-1}$
3	1.4166E-05	4.3741E-05	2.8780E-04	2.5772E-03
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.	2.2540E-07	8.9583E-07	5.5811E-05	1.3004E-03

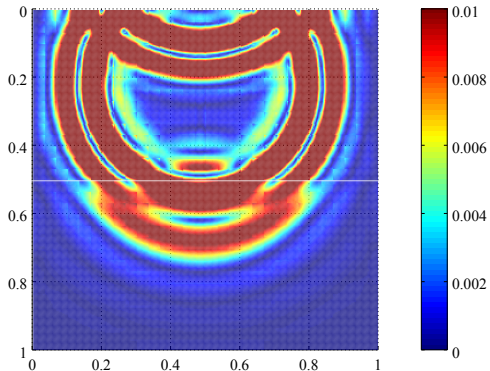
03

Numerical Results

2D ELASTO-ACOUSTIC SYSTEM

2D Elasto-acoustic system.

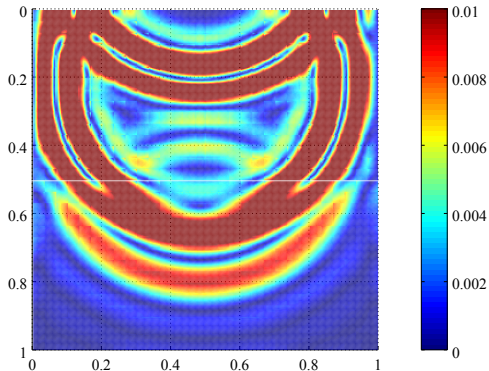
L^2 norm of numerical velocity v



2D ELASTO-ACOUSTIC SYSTEM

2D Elasto-acoustic system.

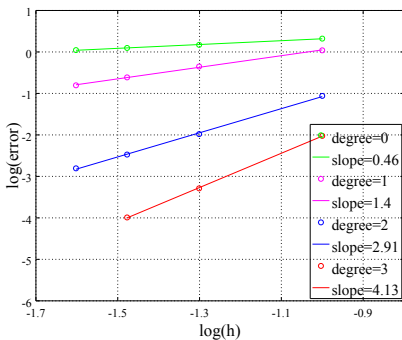
L^2 norm of numerical velocity v



2D ACOUSTIC AND ELASTODYNAMIC SYSTEMS

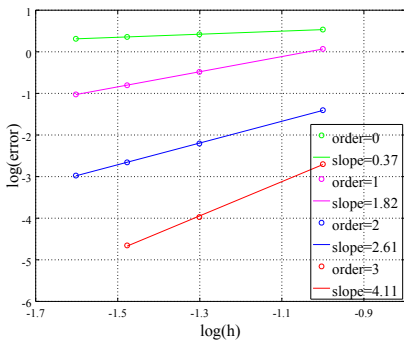
2D Acoustic system.

Convergence of velocity v in function of cell size



2D Elastodynamic system.

Convergence of velocity v in function of cell size



04

Tent-Pitching Meshes

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A meshing algorithm for space time discontinuous Galerkin method. (2000)

J. Erickson, D. Guoy, J.M. Sullivan & A. Üngör.

Building Space-Time Meshes over Arbitrary Spatial Domains. (2005)

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A tent pitching scheme motivated by Friedrichs theory.(2015)

J. Gopalakrishnan, J. Schöberl, and C. Wintersteiger.

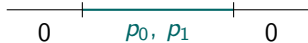
Mapped tent pitching schemes for hyperbolic systems. (2017)

I. Perugia, J. Schöberl, P. Stocker and C. Wintersteiger.

Tent pitching and Trefftz-DG method for the acoustic wave equation.. (Preprint, 2019)

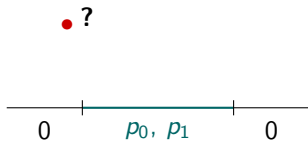
1D WAVE EQUATION

$$\begin{cases} \frac{1}{c_F^2} \frac{\partial^2 p}{\partial t^2} - \frac{\partial^2 p}{\partial x^2} = 0, \\ p(\cdot, 0) = p_0, \\ \frac{\partial p}{\partial t}(\cdot, 0) = p_1. \end{cases}$$



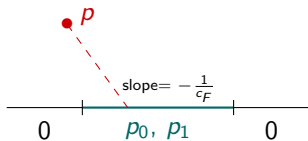
1D WAVE EQUATION

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1D WAVE EQUATION

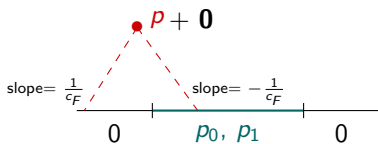
$$\left\{ \begin{array}{l} \frac{1}{c_F^2} \frac{\partial^2 p}{\partial t^2} - \frac{\partial^2 p}{\partial x^2} = 0, \\ p(\cdot, 0) = p_0, \\ \frac{\partial p}{\partial t}(\cdot, 0) = p_1. \end{array} \right.$$



$$p(x, t) = \frac{1}{2} (p_0(x - c_F t) + p_0(x + c_F t)) + \frac{1}{2c_F} \int_{x - c_F t}^{x + c_F t} p_1(s) ds$$

1D WAVE EQUATION

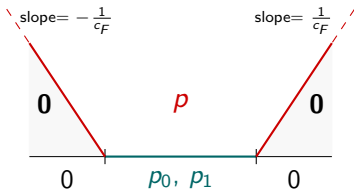
$$\left\{ \begin{array}{l} \frac{1}{c_F^2} \frac{\partial^2 p}{\partial t^2} - \frac{\partial^2 p}{\partial x^2} = 0, \\ p(\cdot, 0) = p_0, \\ \frac{\partial p}{\partial t}(\cdot, 0) = p_1. \end{array} \right.$$



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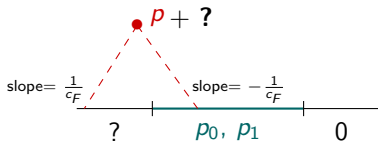
$$\left\{ \begin{array}{l} \frac{1}{c_F^2} \frac{\partial^2 p}{\partial t^2} - \frac{\partial^2 p}{\partial x^2} = 0, \\ p(\cdot, 0) = p_0, \\ \frac{\partial p}{\partial t}(\cdot, 0) = p_1. \end{array} \right.$$



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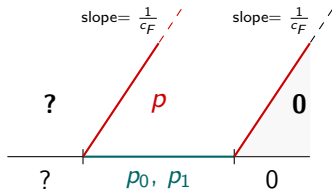
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MESH GENERATION

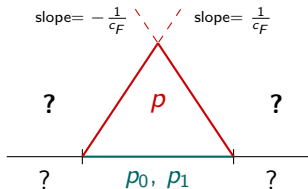
$$\begin{cases} \frac{1}{c_F^2} \frac{\partial^2 p}{\partial t^2} - \frac{\partial^2 p}{\partial x^2} = 0, \\ p(\cdot, 0) = p_0, \\ \frac{\partial p}{\partial t}(\cdot, 0) = p_1. \end{cases}$$



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MESH GENERATION

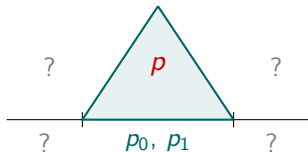
$$\left\{ \begin{array}{l} \frac{1}{c_F^2} \frac{\partial^2 p}{\partial t^2} - \frac{\partial^2 p}{\partial x^2} = 0, \\ p(\cdot, 0) = p_0, \\ \frac{\partial p}{\partial t}(\cdot, 0) = p_1. \end{array} \right.$$



$$p(x, t) = \frac{1}{2} (p_0(x - c_F t) + p_0(x + c_F t)) + \frac{1}{2c_F} \int_{x - c_F t}^{x + c_F t} p_1(s) ds$$

MESH GENERATION

$$\left\{ \begin{array}{l} \frac{1}{c_F^2} \frac{\partial^2 p}{\partial t^2} - \frac{\partial^2 p}{\partial x^2} = 0, \\ p(\cdot, 0) = p_0, \\ \frac{\partial p}{\partial t}(\cdot, 0) = p_1. \end{array} \right.$$

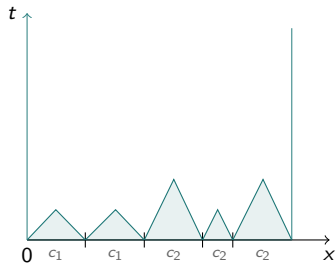


$$p(x, t) = \frac{1}{2} (p_0(x - c_F t) + p_0(x + c_F t)) + \frac{1}{2c_F} \int_{x - c_F t}^{x + c_F t} p_1(s) ds$$

MESH GENERATION

1st step

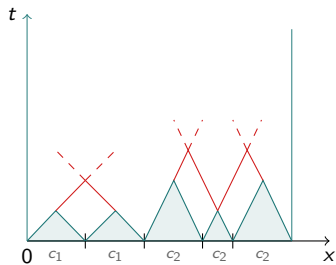
initial condition



MESH GENERATION

2nd step

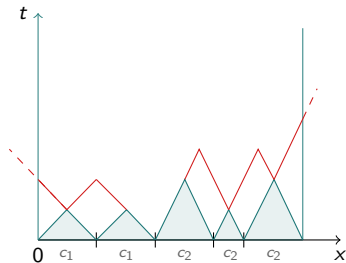
initial condition



MESH GENERATION

2nd step

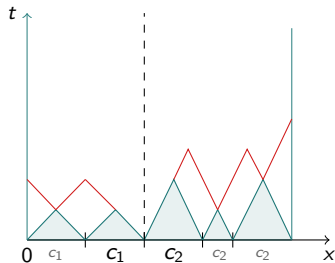
initial condition
boundary condition



MESH GENERATION

2nd step

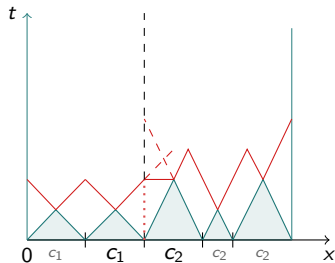
initial condition
boundary condition
heterogeneity?



MESH GENERATION

2nd step

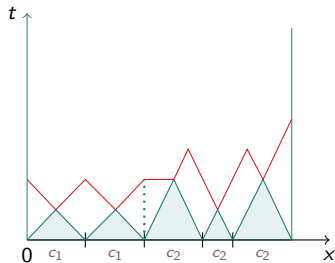
initial condition
boundary condition
heterogeneity?



MESH GENERATION

2nd step

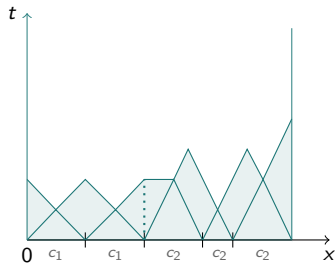
initial condition
boundary condition
heterogeneity



MESH GENERATION

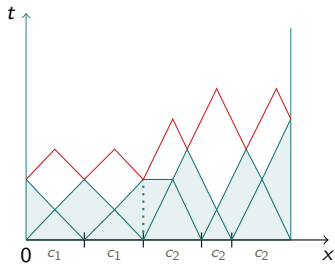
2nd step

initial condition
boundary condition
heterogeneity



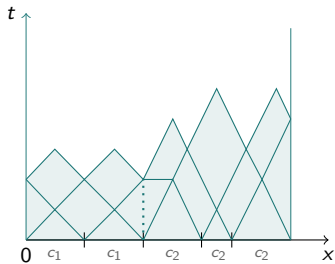
MESH GENERATION

and so on..



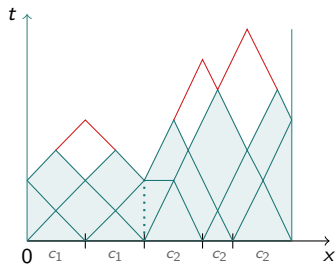
MESH GENERATION

and so on..



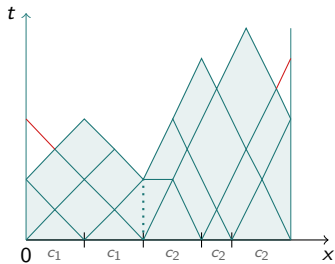
MESH GENERATION

and so on..



MESH GENERATION

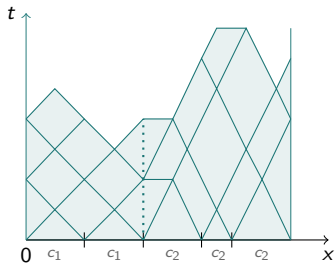
and so on..



MESH GENERATION

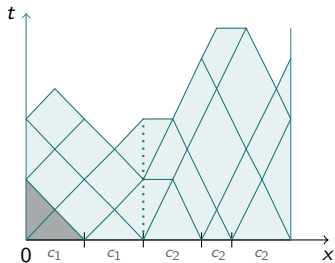
and so on..

$$c_F |n_K^x| / |n_K^t| \leq 1$$



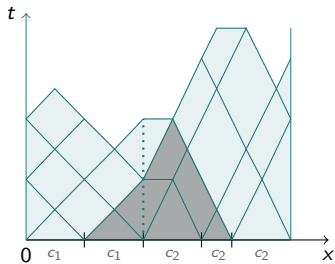
MESH GENERATION

mesh is divided in
space-time patches



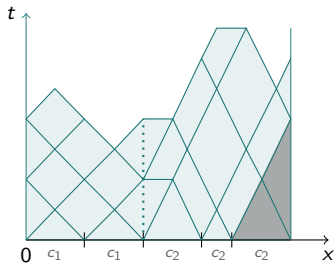
MESH GENERATION

mesh is divided in
space-time patches



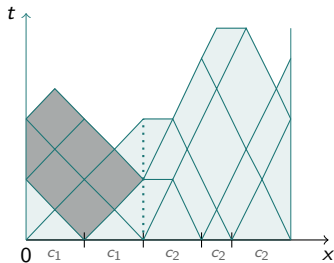
MESH GENERATION

mesh is divided in
space-time patches



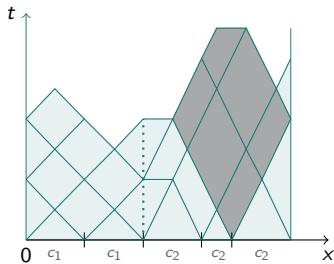
MESH GENERATION

mesh is divided in
space-time patches



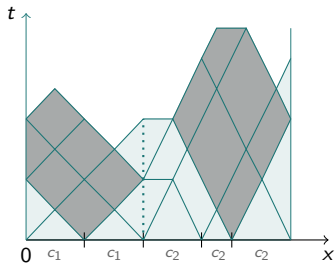
MESH GENERATION

mesh is divided in
space-time patches



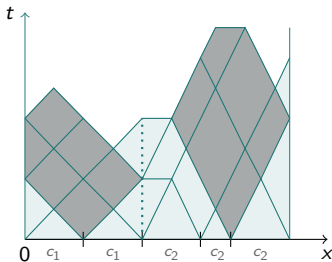
MESH GENERATION

mesh is divided in
space-time patches



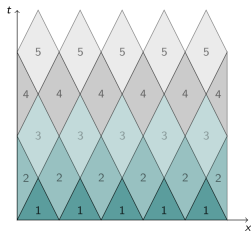
MESH GENERATION

mesh is divided in
space-time patches

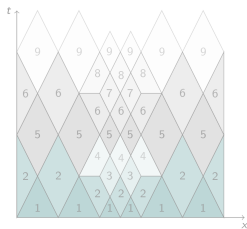


can be computed independently!

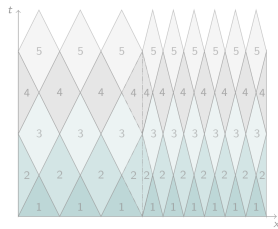
1D TENT PITCHING MESHES



Uniform tent mesh
(homogeneous medium)

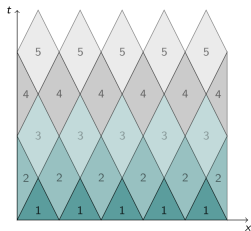


Non-uniform tent mesh
(homogeneous medium)

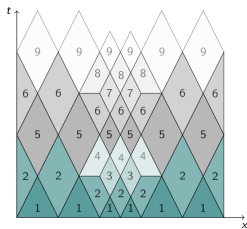


Non-uniform tent mesh
(heterogeneous medium)

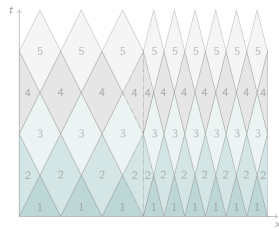
1D TENT PITCHING MESHES



Uniform tent mesh
(homogeneous medium)

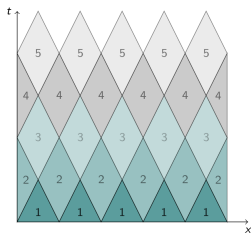


Non-uniform tent mesh
(homogeneous medium)

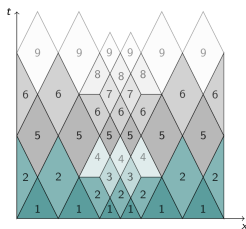


Non-uniform tent mesh
(heterogeneous medium)

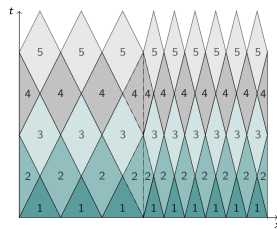
1D TENT PITCHING MESHES



Uniform tent mesh
(homogeneous medium)

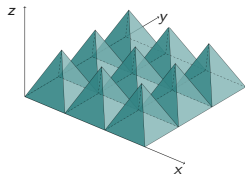


Non-uniform tent mesh
(homogeneous medium)

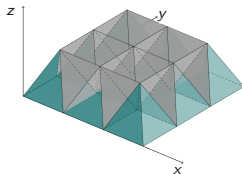


Non-uniform tent mesh
(heterogeneous medium)

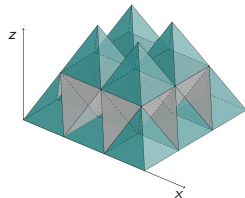
2D TENT PITCHING MESHES



1st step



2nd step



3rd step

STABILITY CONDITIONS

$$c_F |n_K^x| / |n_K^t| \leq 1$$

- TDG acoustic formulation

$$V_P |n_K^x| / |n_K^t| \leq 1$$

- TDG elastodynamic formulation

3D+TIME

Tent Pitcher &
Space-time DG



explicit scheme
local time-stepping

4D integrals

3D+TIME

Tent Pitcher &
Space-time DG



explicit scheme
local time-stepping

4D integrals

Space-time
Trefftz-DG



global
sparse matrix

3D integrals

3D+TIME

Tent Pitcher &
Space-time DG



Space-time
Trefftz-DG



explicit scheme
local time-stepping

4D integrals



**global
sparse matrix**

3D integrals

3D+TIME

Tent Pitcher &
Space-time Trefftz-DG

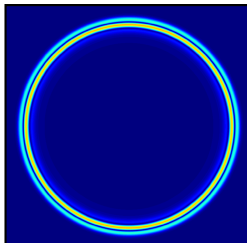


explicit scheme
local time-stepping

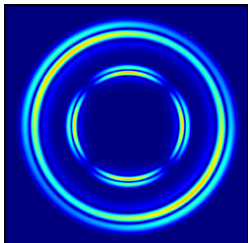
3D integrals

PROPAGATION OF NUMERICAL VELOCITY

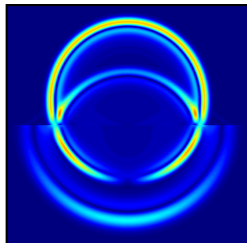
2D Acoustic domain.



2D Elastodynamic domain.

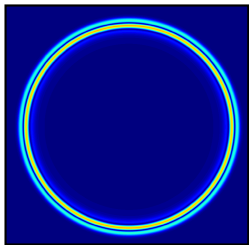


2D Elasto-acoustic domain.



PROPAGATION OF NUMERICAL VELOCITY

2D Acoustic domain.



P-wave

2D Elastodynamic domain.

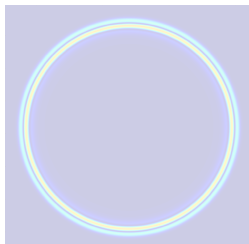


2D Elasto-acoustic domain.

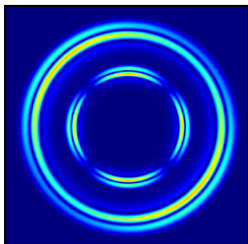


PROPAGATION OF NUMERICAL VELOCITY

2D Acoustic domain.



2D Elastodynamic domain.



P, S-waves

2D Elasto-acoustic domain.



PROPAGATION OF NUMERICAL VELOCITY

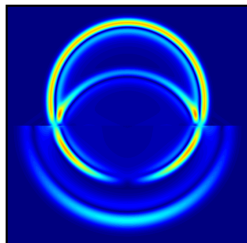
2D Acoustic domain.



2D Elastodynamic domain.



2D Elasto-acoustic domain.



incident, reflected
P, S-waves,
P, S head waves

PROPAGATION OF NUMERICAL VELOCITY

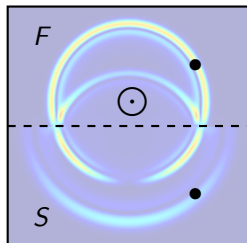
2D Acoustic domain.



2D Elastodynamic domain.



2D Elasto-acoustic domain.

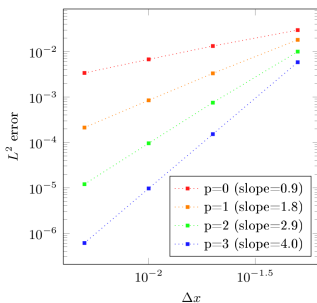


- - source
- - receiver

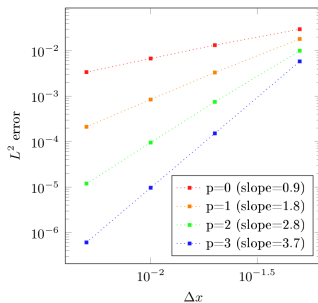
CONVERGENCE CURVES

Convergence of velocity v_F (left) and v_S (right) in space and time as a function of cell size $\Delta x = \Delta y = \min\{V_P, c_F\}\sqrt{2}\Delta t$ (point-source case).

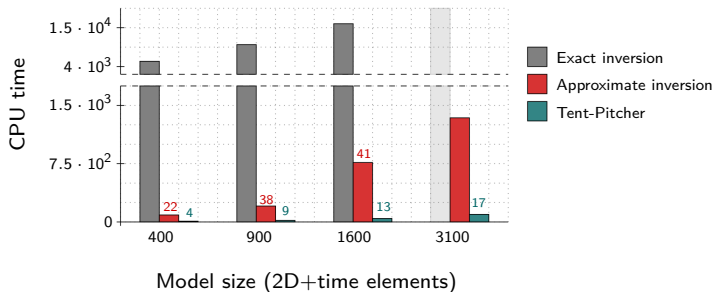
$$\alpha_1 = \beta_1 = 0.5$$



$$\alpha_1 = \beta_1 = 0.5$$



SPEEDUP FOR DIFFERENT MODEL SIZES



05

Conclusion

ON-GOING WORK AND PERSPECTIVES

Analytical study of convergence

4D Mesh generation ¹

Absorbing boundary conditions

Implicit-explicit coupling

A posteriori error estimates, hp-adaptivity

Space-time parallelization

¹ K. Voronin.

A parallel mesh generator in 3D/4D. (2018)