



Efficient high-order discrete geometric model from CAD

François Guibault, Houman Borouchaki and
Patrick Laug

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Context

High-order FEM analyses in complex domains would benefit from a correspondingly high-order representation of domain boundaries.

CAD systems are not well suited for direct integration in mesh generation and analysis processes:

- Parametrization dependencies,
- Computational cost of CAD requests,
- Parallelization difficulties,
- Platform portability and licencing issues.

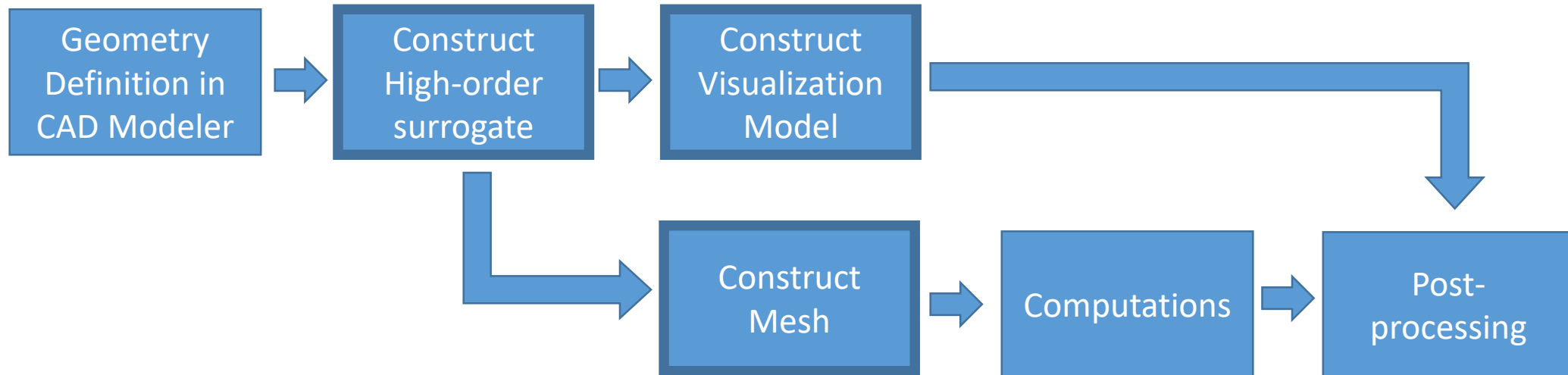
Need for a CAD-system independent high-order representation of computational domains.

Objectives

1. Construct a high-order surrogate geometric representation using piecewise polynomials of a user-specified degree.
2. Ensure that the resulting representation is suitable for mesh generation and visualization purposes:
 - Provide discrete primitives to graphic systems,
 - Guarantee conformity between patches for meshing,
 - Enforce user-specified geometric tolerances,
 - Reduce required number of elements.
3. Isolate subsequent meshing, computing and visualization processes from the original CAD representation.

Constraints

1. Rely on curve and surface evaluation of CAD entities only (no derivatives required).
2. Cope with ill-defined parameterization of CAD entities.
3. Accept definitions of CAD entities from several CAD systems and in several standard formats: STEP, IGES, etc.



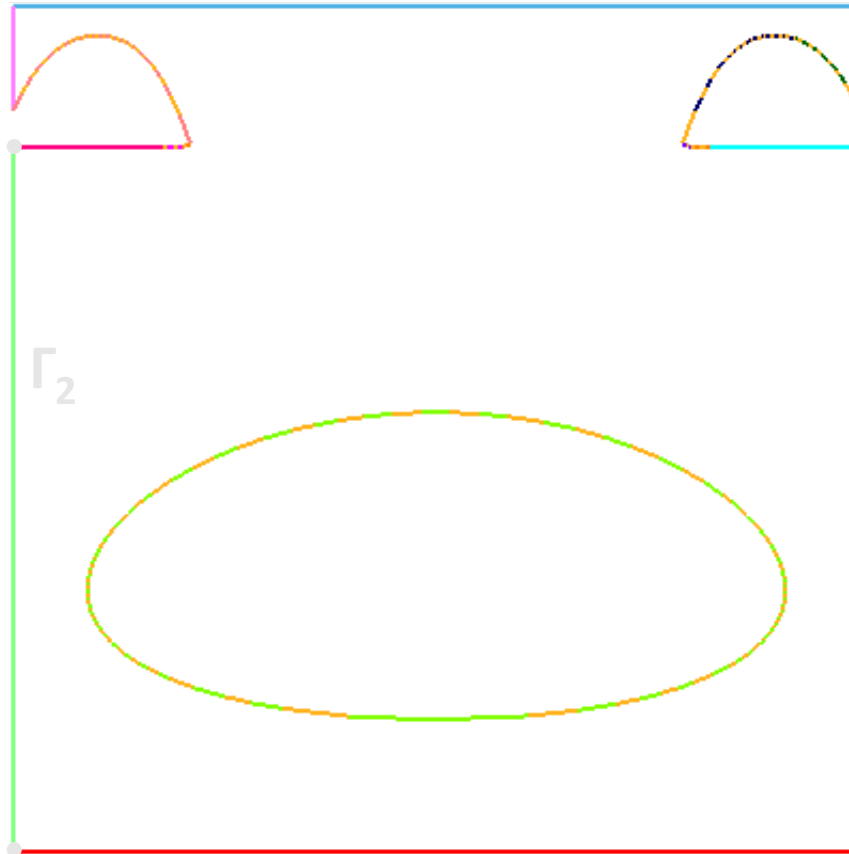
1. B-Rep Geometry

- **CAD representation**
 - An object is defined from its boundary = collection of surface patches (faces)
 - Each patch is defined by a parametric domain and a continuous parametric function (typically NURBS) $\sigma : \mathbb{R}^2 \rightarrow \mathbb{R}^3$

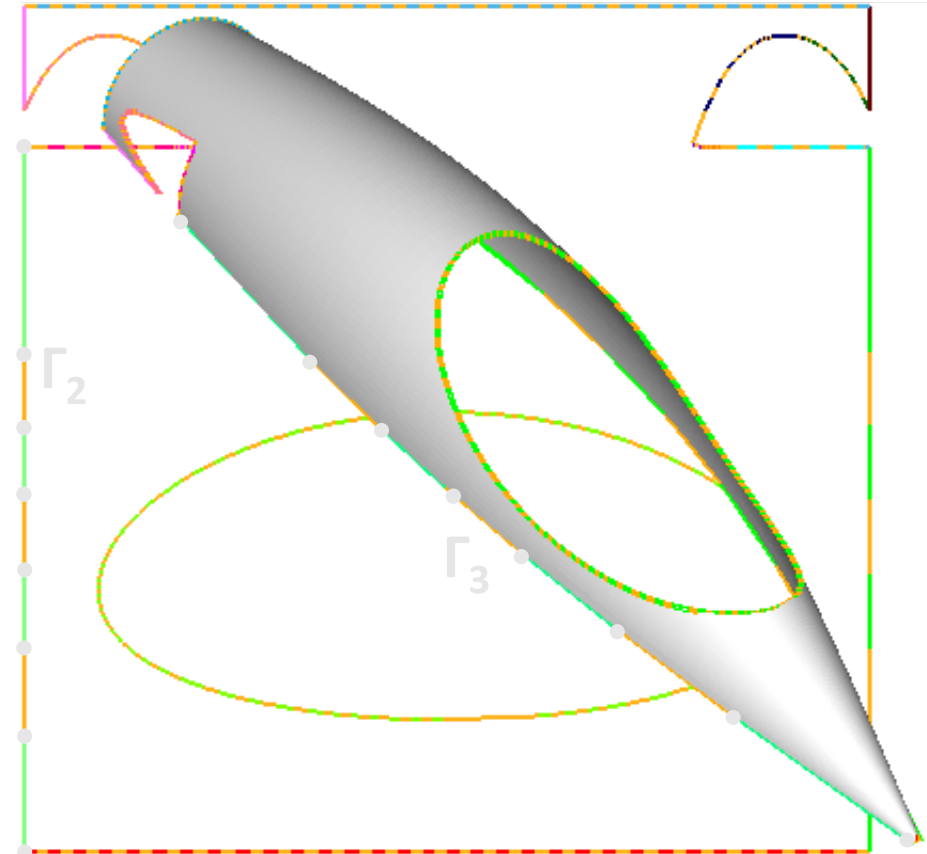
2. Discrete representation (construction)

- Curve $\omega : \mathbb{R} \rightarrow \mathbb{R}^2 \quad t \mapsto (u,v)$
- Surface $\sigma : \mathbb{R}^2 \rightarrow \mathbb{R}^3 \quad (u,v) \mapsto \sigma(u,v)$
- We want to **build a “triangulation”** that accurately reflects the geometry of the surface (proximity and regularity)
- Iso-parametric curves (for $u = u_i$ or $v = v_j$) generally give an accurate description of the geometry \Rightarrow we will use them as privileged directions for edges of the triangulation

2.1. Make a discrete model of curves in 2D and 3D



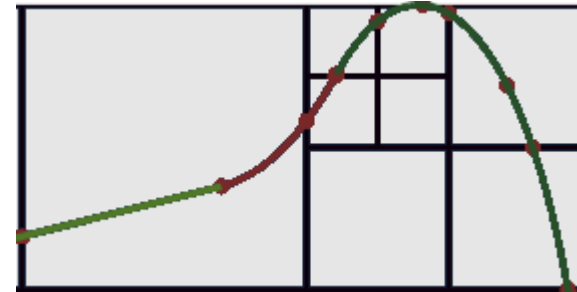
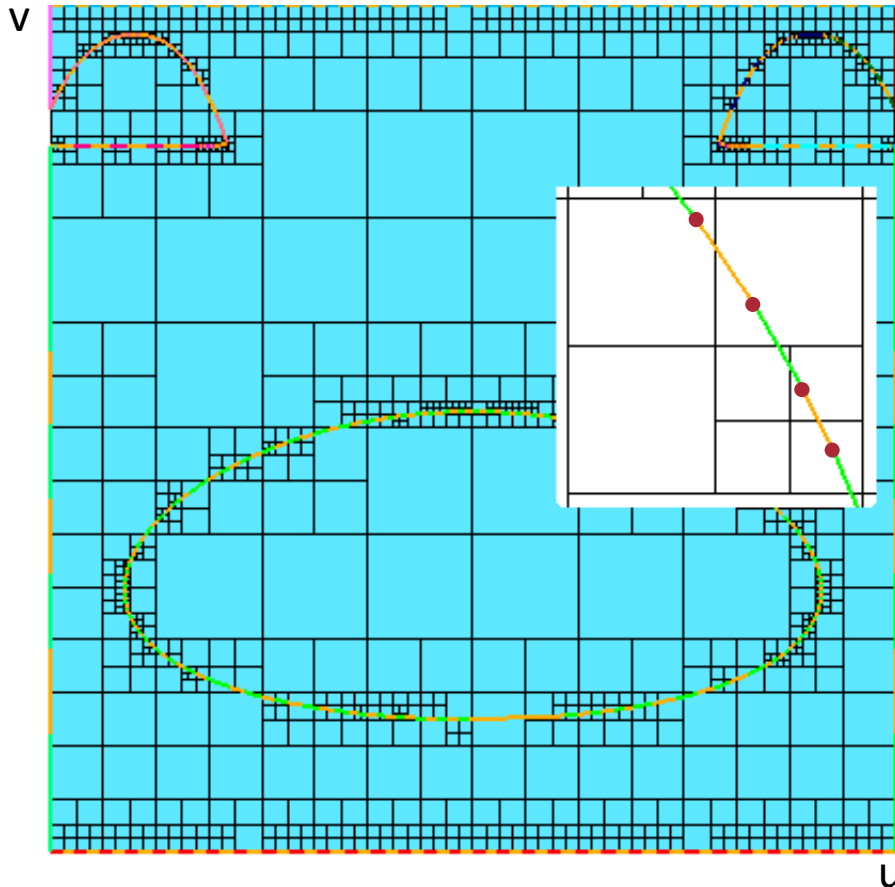
In 2D: geometric support for
 $\Gamma_2 = \omega([a,b]) \quad \omega : \mathbb{R} \rightarrow \mathbb{R}^2$



In 3D: geometric support for
 $\Gamma_3 = \sigma(\Gamma_2) \quad \sigma : \mathbb{R}^2 \rightarrow \mathbb{R}^3$

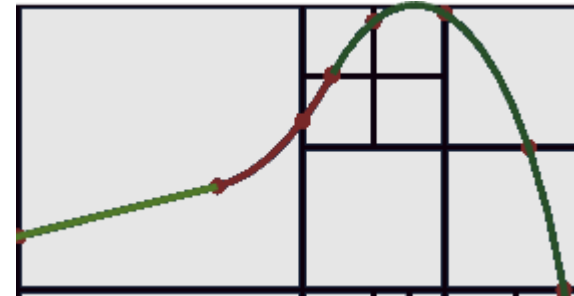
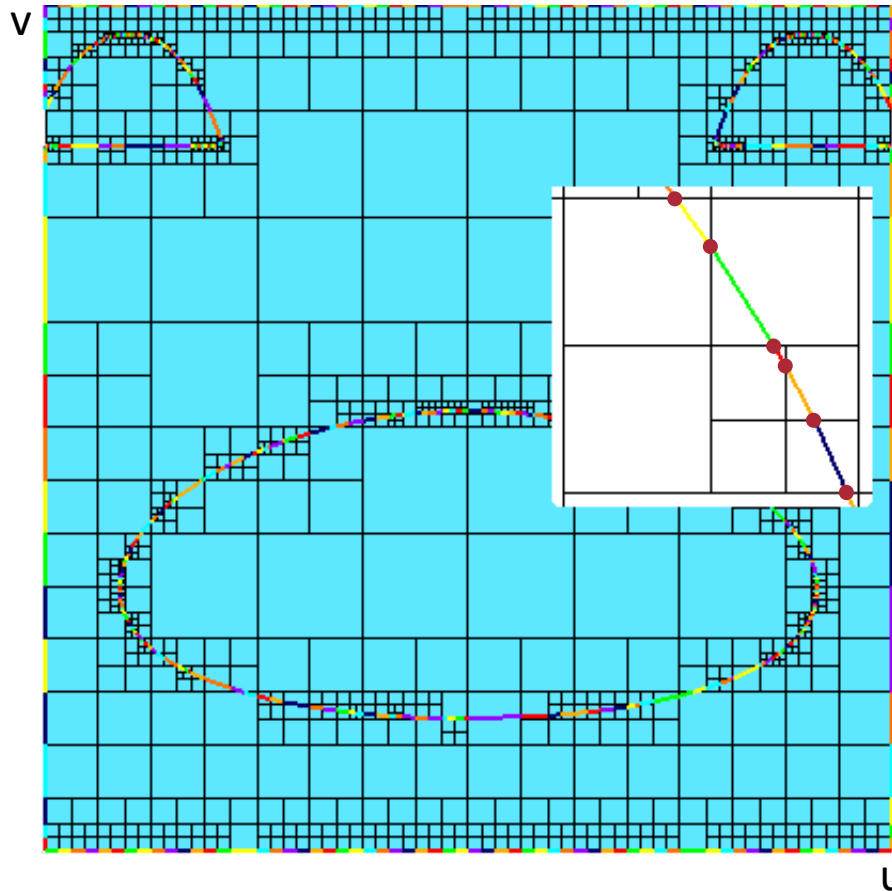
2.2. Build a quadtree from the previous vertices

- The interior of each leaf contains at most one vertex
- Iso-parametric curves (privileged directions) appear here!



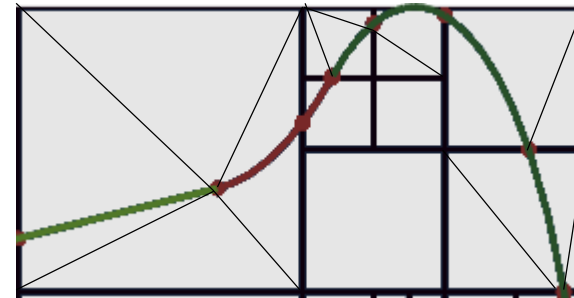
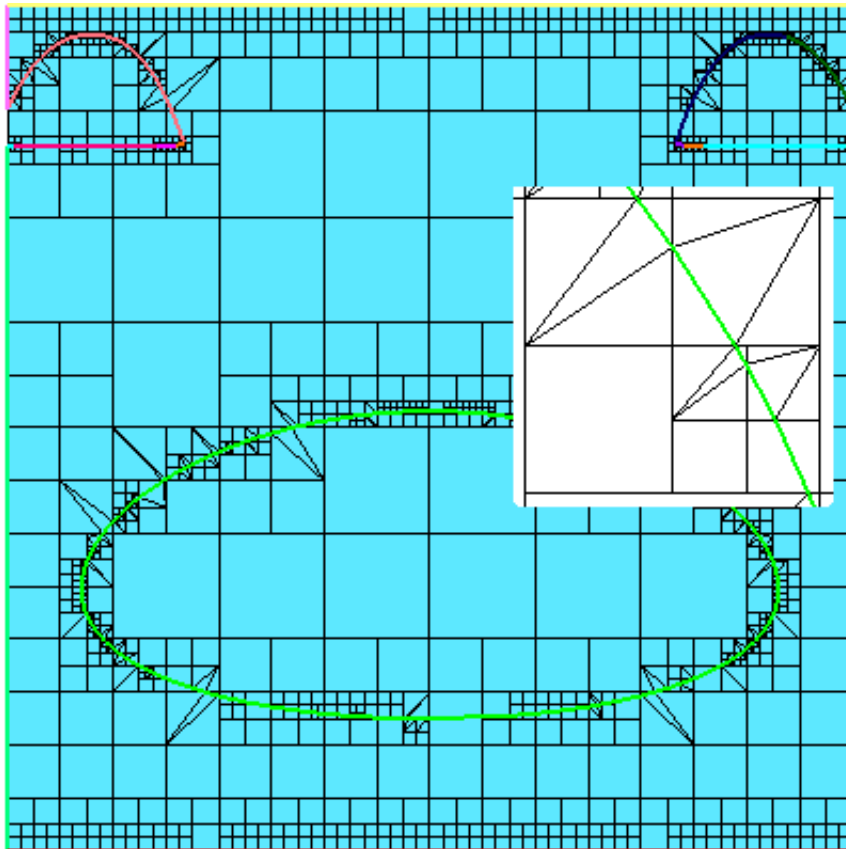
2.3. Insert and merge edges into the quadtree

- Generally one *trace* per leaf



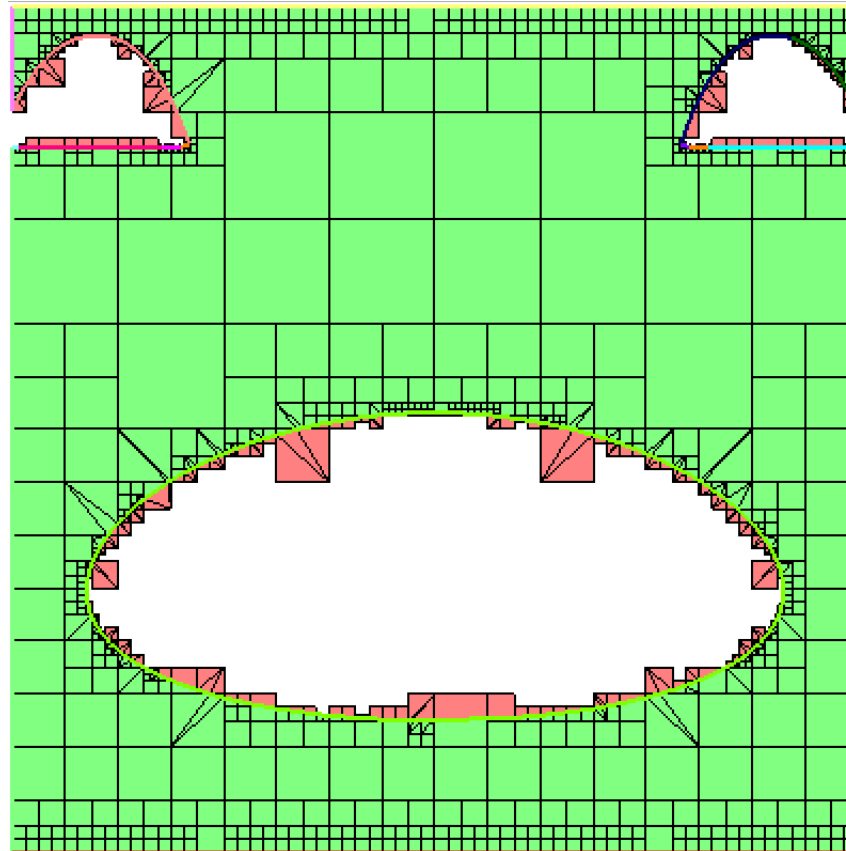
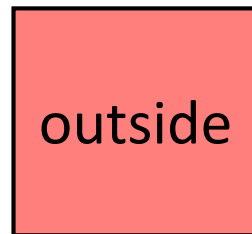
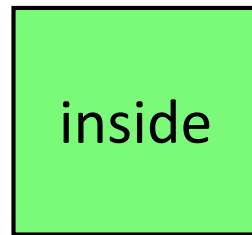
2.4. Build a leaf-triangulation

- Constrained “triangulations”: in fact **triangles and quadrilaterals**
- If there is an interior vertex, it is “starred”
- Otherwise, simple general algorithm (with particular cases for 0, 1 or 2 traces crossing the leaf)



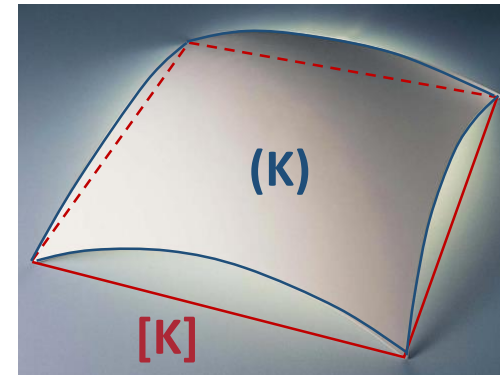
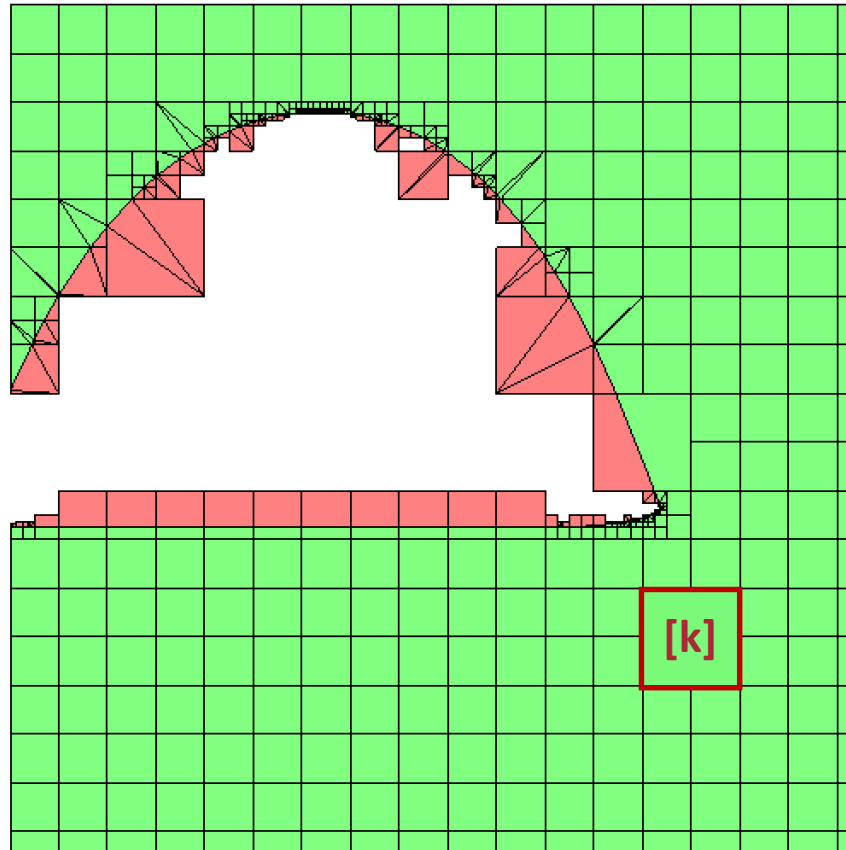
2.5. Identify all the elements of the leaf-triangulation

- Seed touching the contour of the unit square: inside or outside
- Grow by adjacency between elements or leaves



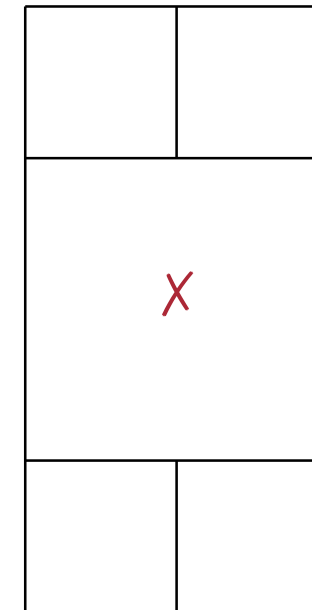
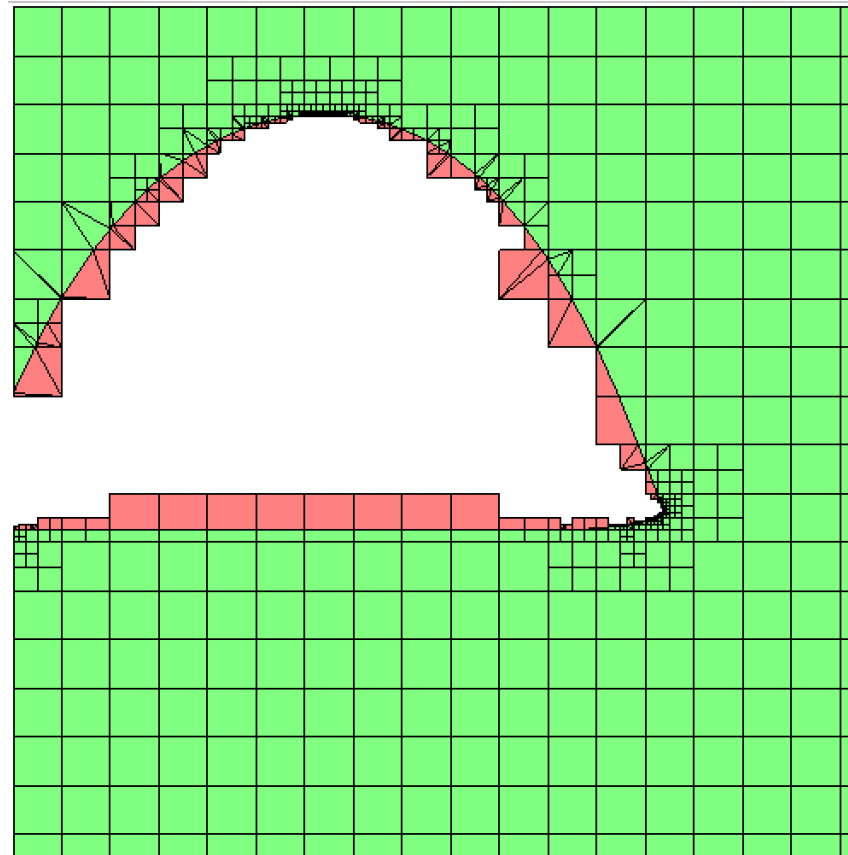
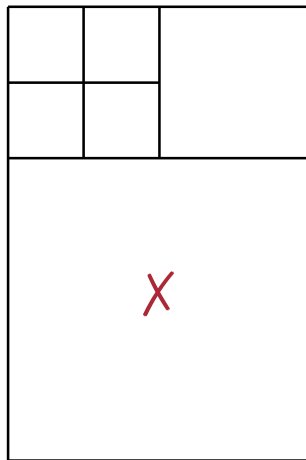
2.6. Make a surface geometric support

- $[k]$ element in 2D, $[K]$ element in 3D images of vertices, $(K) = \sigma([k])$
- Proximity $d_0((K), [K])$ where d_0 is a distance



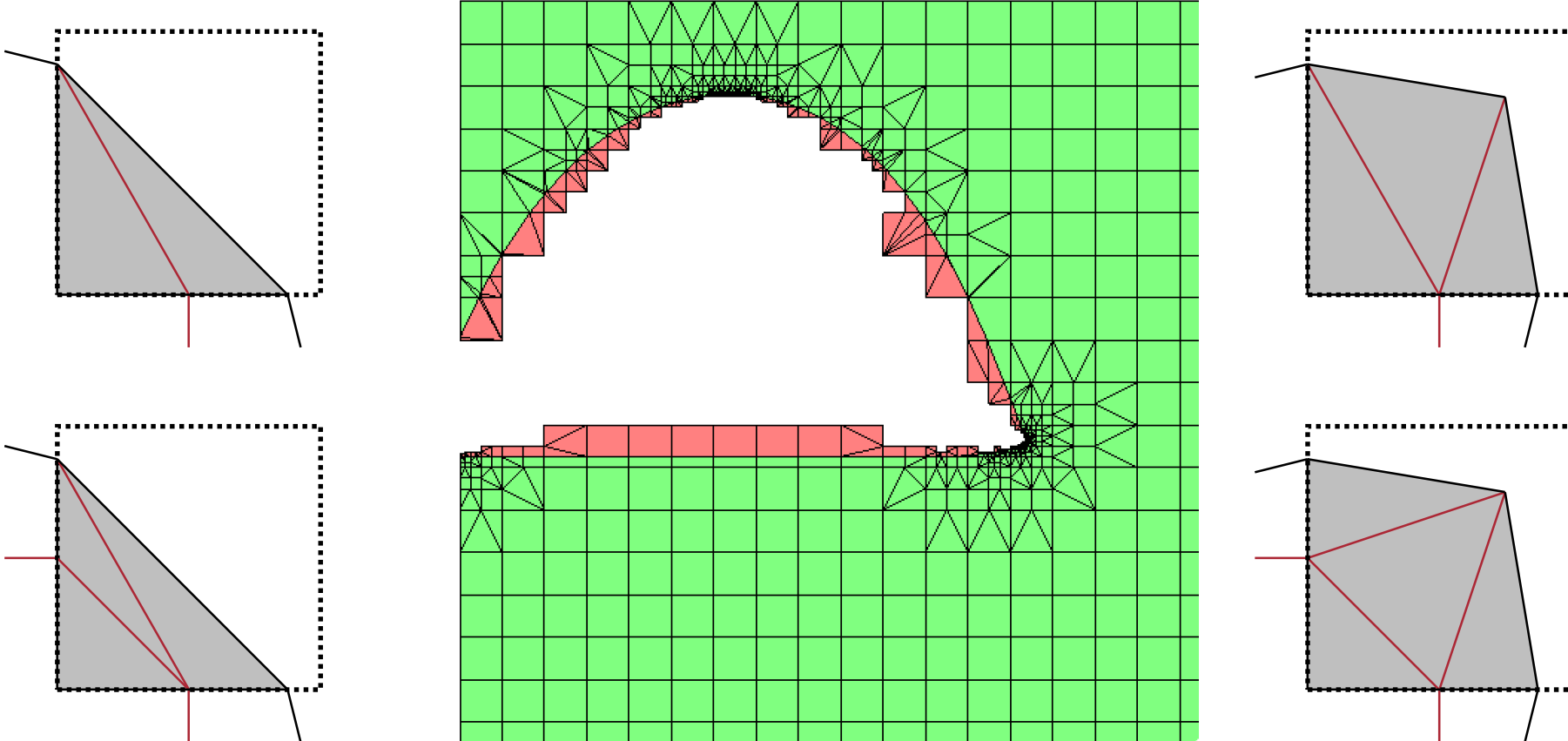
2.7. Balance the quadtree

- Impose 2:1 rule between neighboring leaves (in fact a stricter rule)
- This makes the next step of building a conformal domain-triangulation easier

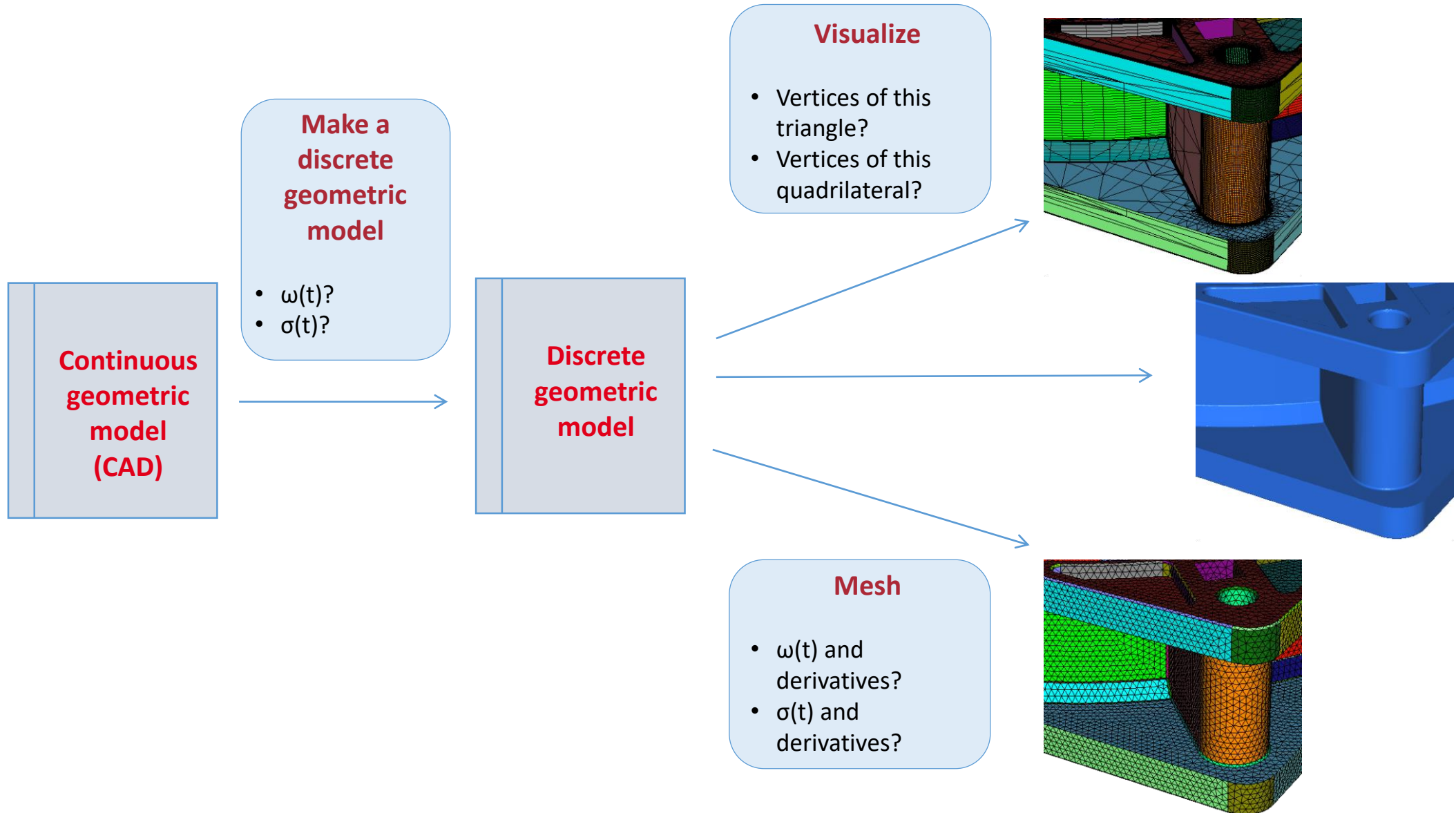


2.8. Make a conformal domain-triangulation

- $[k]$ is a triangle or a quadrilateral
- Only 0, 1, or 2 hanging points on 2 adjacent edges



Geometric model, visualization, meshing

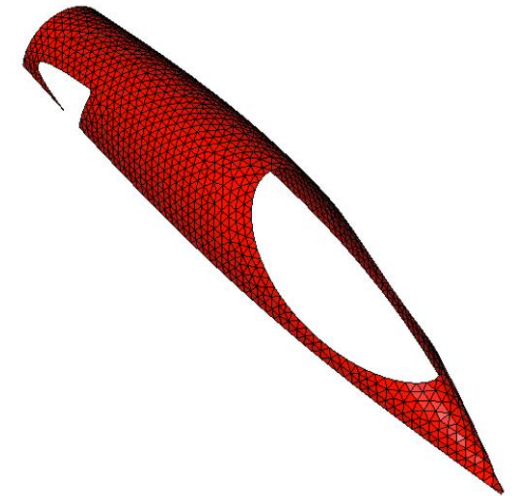
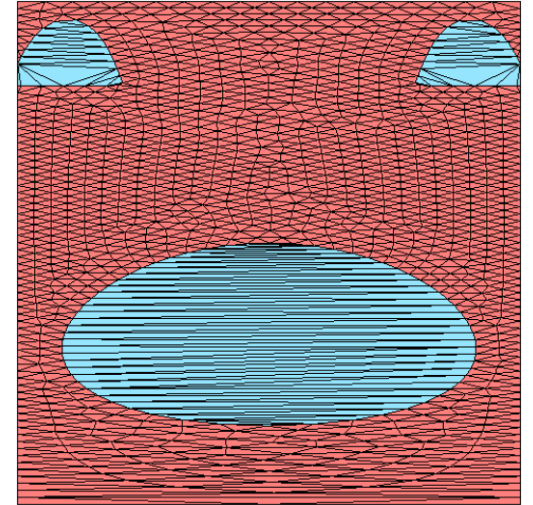


3. Surface meshing

- Idea: mesh parametric domain Ω w.r.t induced metric in Ω

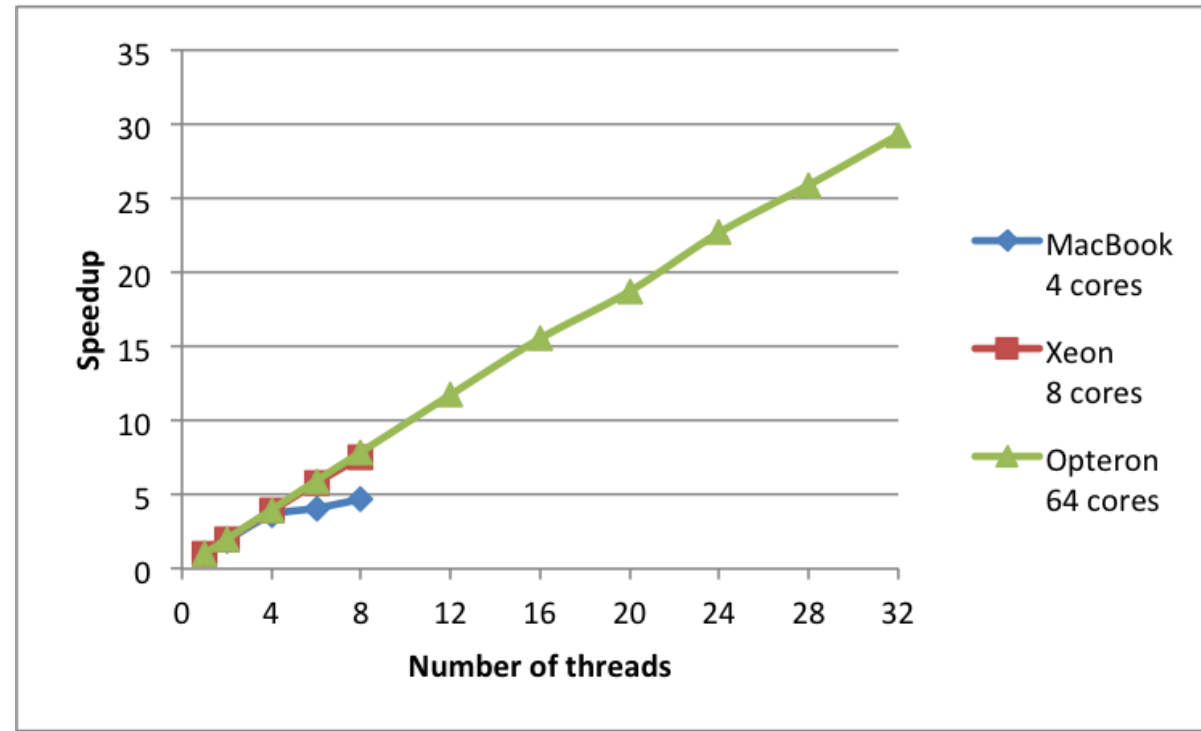
$$M_2 = \begin{pmatrix} S_u^T \\ S_v^T \end{pmatrix} M_3 \begin{pmatrix} S_u & S_v \end{pmatrix} \quad M_2 = \frac{1}{h^2(P)} \begin{pmatrix} \langle S_u, S_u \rangle & \langle S_u, S_v \rangle \\ \langle S_v, S_u \rangle & \langle S_v, S_v \rangle \end{pmatrix} = \frac{1}{h^2(P)} M_S$$

- For each **surface** patch, $\sigma(u,v)$ is approximated by a piecewise polynomial function
- Similarly for each **curve**, $\omega(t)$ is approximated by a piecewise polynomial function
- First order derivatives of σ ($\sigma_u(u,v)$, $\sigma_v(u,v)$ and $M_\sigma(u,v)$) and ω ($\omega_t(t)$), are approximated by polynomial functions
- Second order derivatives of σ and ω are also approximated by polynomial functions, for curvature dependent meshes



4. Examples

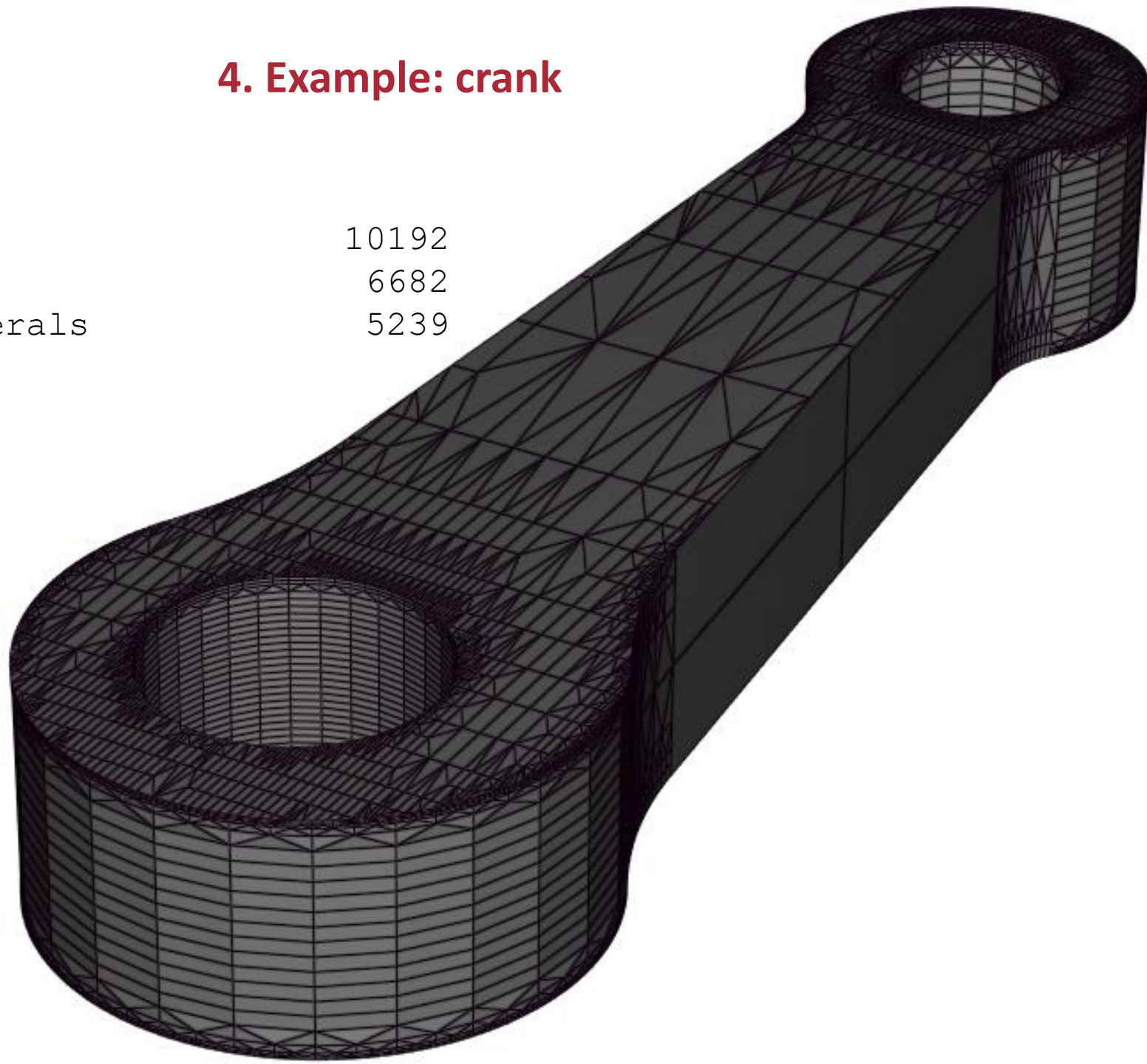
- Two examples: crank and aircraft
- Parallel meshing: almost linear scaling



4. Example: crank

Degree 1

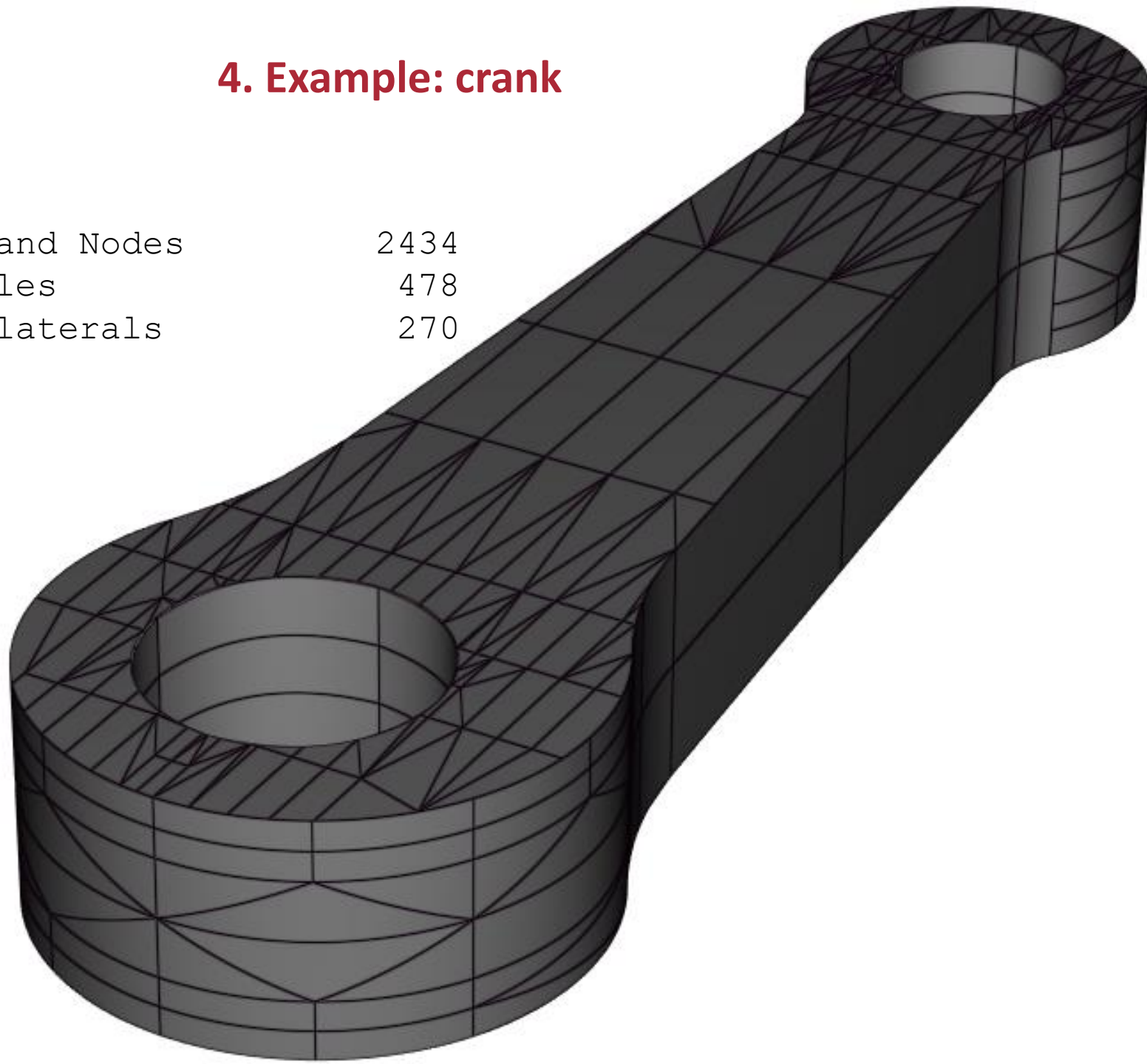
Number of Vertices	10192
Number of Triangles	6682
Number of Quadrilaterals	5239



4. Example: crank

Degree 2

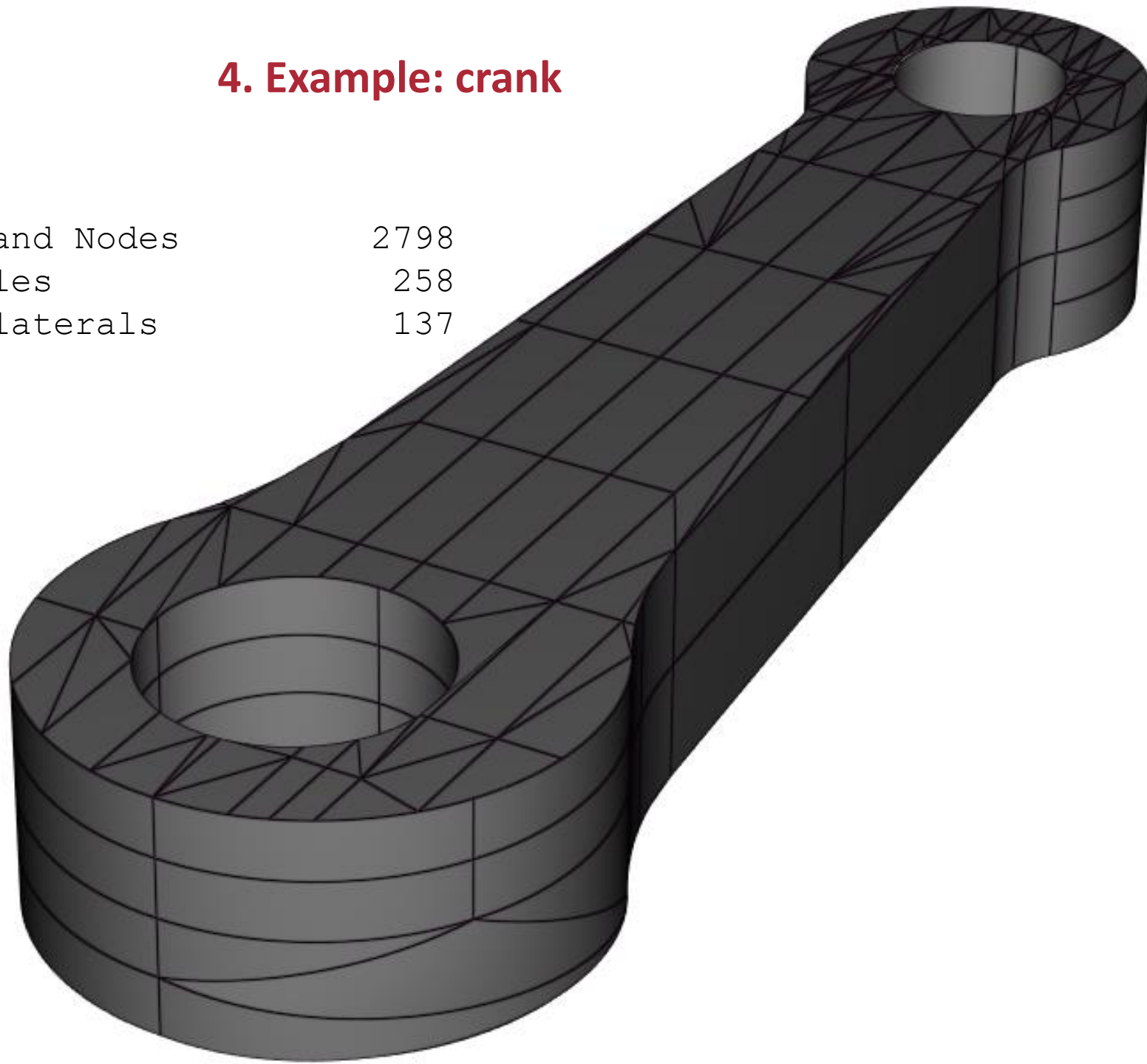
Number of Vertices and Nodes	2434
Number of P2-Triangles	478
Number of Q2-Quadrilaterals	270



4. Example: crank

Degree 3

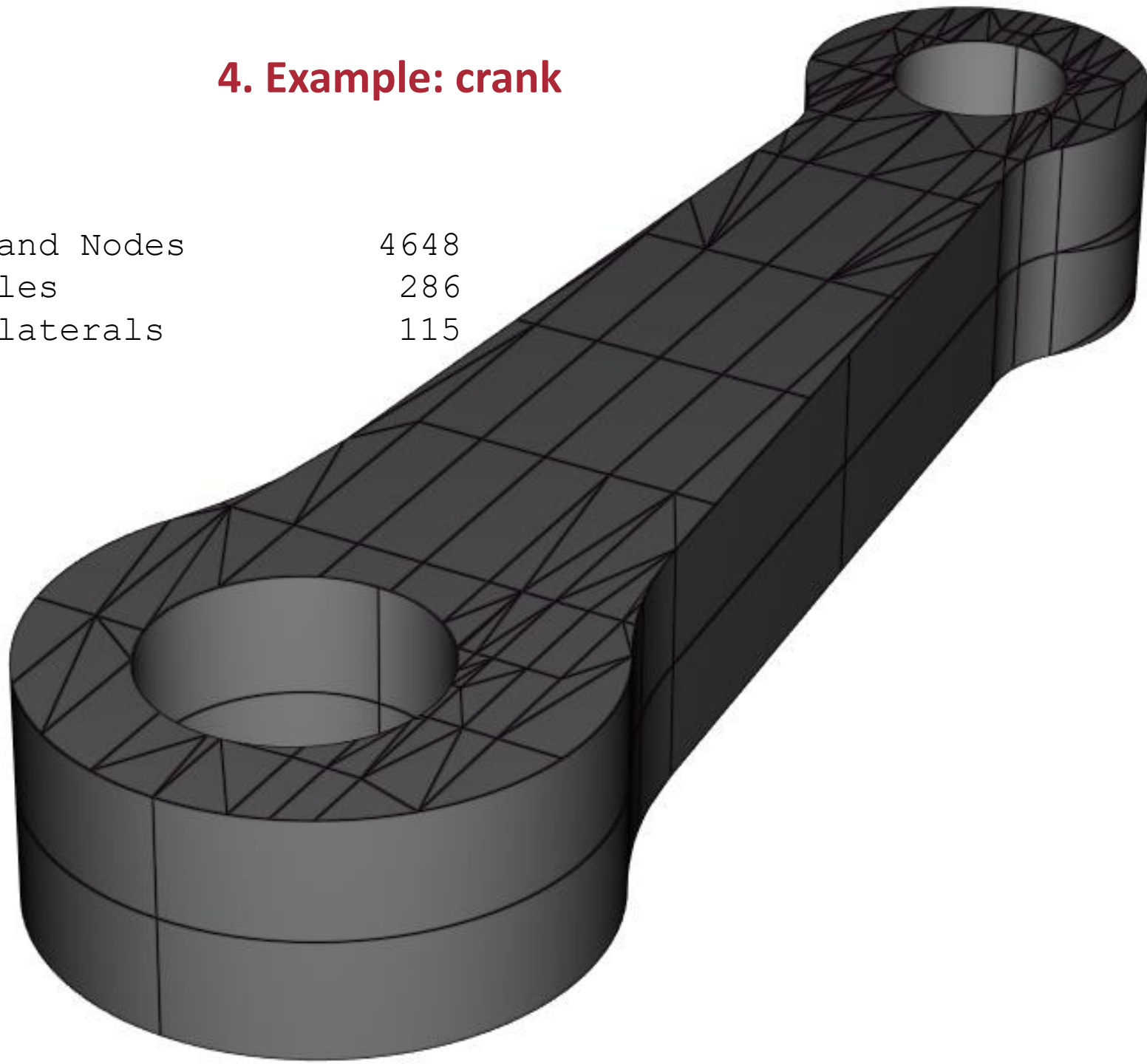
Number of Vertices and Nodes	2798
Number of P3-Triangles	258
Number of Q3-Quadrilaterals	137



4. Example: crank

Degree 4

Number of Vertices and Nodes	4648
Number of P4-Triangles	286
Number of Q4-Quadrilaterals	115



5. Conclusions

- New method for building a **high-order discrete CAD model**
- Most elements are aligned with iso-parametric curves \Rightarrow accurate description of the surface geometry
- Fast and robust \Rightarrow **visualization**
- Also **meshing** for numerical simulations: efficiency, non-degenerate and piecewise constant derivatives, parallelism
- Work in progress:
 - Complete distance evaluation
 - Parallelization of high-order meshing

THANK YOU !

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