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An Eco-routing algorithm for HEVs under traffic conditions

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Abstract:

An extension of the bi-level optimization for the energy management of hybrid electric vehicles (HEVs) proposed in Le Rhun et al. (2019a) to the eco-routing problem is presented. Using the knowledge of traffic conditions over the entire road network, we search both the optimal path and state of charge trajectory. This problem results in finding the shortest path on a weighted graph whose nodes are (position, state of charge) pairs for the vehicle, the edge cost being evaluated thanks to the cost maps from optimization at the 'micro' level of a bi-level decomposition. The error due to the discretization of the state of charge is proven to be linear if the cost maps are Lipschitz. The classical A^* algorithm is used to solve the problem, with a heuristic based on a lower bound of the energy needed to complete the travel. The eco-routing method is validated by numerical simulations and compared to the fastest path on a synthetic road network.

Keywords: Eco-routing, Bi-level

1. INTRODUCTION

Road networks usually allow several paths to reach a destination given a starting position. While paths are traditionally chosen to minimize travel time, methods to minimize the fuel consumption have received increased attention in the recent years. This new criterion gives rise to the so-called eco-routing problem. The fuel savings between the optimized eco-routing and the path naturally chosen by the drivers can be important, up to 25%, see Ahn and Rakha (2007) and Ericsson et al. (2006). Besides, distance minimal paths have been shown to differ from the eco-routing paths, especially in congested traffic, see Barth et al. (2007). This indicates that eco-routing planning needs to take into account the traffic conditions.

Most methods propose to solve the eco-routing planning based on shortest path algorithms, using weighted graphs to represent the road network with edge costs corresponding to the consumption, see De Nunzio et al. (2017). Dijkstra or A^* algorithms can be used when edge costs are positive, or Ford-Bellman algorithm when edge costs can be negative.

In order to determine these edge costs, an estimator is needed. Consumption estimators, that take traffic conditions into account, are usually divided in two main categories: macroscopic models based on closed algebraic forms, see for instance Lighthill and Whitham (1955), and

microscopic models based on differential equations, see for instance Panwai and Dia (2005).

In the case of HEVs, the vehicle can use the electric motor to reduce the fuel consumption or recover energy thanks to regenerative braking. Therefore if costs are expressed in terms of energy, negative costs can appear. However, path-searching algorithms for negative costs tend to have a high computational cost. Thus it appears more efficient to express costs in terms of fuel consumption, which requires knowledge of the torque policy of the vehicle.

We propose a bi-level approach inspired by the one presented in Le Rhun et al. (2019a) in order to compute an optimal path adapted to the optimal torque policy of the vehicle following the traffic conditions.

The paper is organized as follows. Section 2 introduces the models for the hybrid vehicle, traffic conditions, and states the eco-routing problem of HEV resulting from the bi-level decomposition. Section 3 details the numerical errors due to the discretization. Section 4 presents the complexity of the algorithm used to solve the eco-routing problem. Section 5 discusses the numerical simulations performed using actual traffic data, mapped to a small synthetic road network.

2. MODELING THE ISSUE

2.1 Hybrid electric vehicle (HEV)

We consider a HEV with 'parallel' design (see e.g. Chau and Wong (2002)), where both the thermal engine and the electric motor can power the vehicle. Such HEVs can use the engine to recharge the battery, which allows for optimization of the global consumption along the travel.

Neglecting losses due to mechanical links, the torque of the engine T_e and motor T_m are linked through (1):

$$T_{prim}(v, a) = T_e + T_m R \quad (1)$$

with the expression of the torque at the primary shaft $T_{prim}(v, a)$ described in Appendix A, and R the reduction ratio between the engine and the motor.

Through experimental characterization, we model the consumption of the engine as a map $\hat{C}(\omega_e, T_e)$ depending on the rotation speed and torque request. We use the model presented in Appendix A to reformulate the consumption as a function of the electric motor torque T_m , i.e.

$$C(v, a, T_m) = \hat{C}(\omega_e(v, a), T_{prim}(v, a) - T_m R) \quad (2)$$

Likewise, we have a map for the electrical power \hat{P}_m required by the motor:

$$P_m(v, a, T_m) := \hat{P}_m(\omega_m(v, a), T_m), \quad (3)$$

with the convention that $P_m > 0$ for a discharge. Now we note C_{max} the maximum capacity of the battery and $SoC(t) \in [0, 1]$ its state of charge at time t . The dynamics of the state of charge then writes as:

$$S\dot{o}C(t) = \frac{1}{C_{max}} P_m(v(t), a(t), T_m(t)). \quad (4)$$

2.2 A road graph with traffic conditions

A road graph The consumption of a vehicle on a given portion of road is influenced by many parameters, that can be static such as the speed limit or the slope, or transient such as traffic and weather conditions. In the following, we consider a road network modeled as a graph where roads are the edges and intersections the nodes. The characteristics of the road portions are then attributes of the edges. In our case these attributes also include the traffic conditions, modeled as probability laws.

Probabilistic Traffic Model We use here the probabilistic traffic model introduced in Le Rhun et al. (2019b). The model is based on a subdivision of the roads into small *segments*, typically delimited by topological characteristics. We assume that the speed and acceleration of the vehicles on each segment are random variables $(\mathbf{V}(t), \mathbf{A}(t))$, constant over each time step $h_0 > 0$, with discrete i.i.d. distributions μ^s , called the *traffic distribution*. Fig. 1 shows an example of such a distribution.

We make the central assumption that drivers 'follow the traffic', meaning that their speed and acceleration coincide with the random variables $(\mathbf{V}(t), \mathbf{A}(t))$. In Le Rhun et al. (2019b) we established that this traffic model provides a statistically accurate estimate of the energy consumption of the hybrid vehicle.

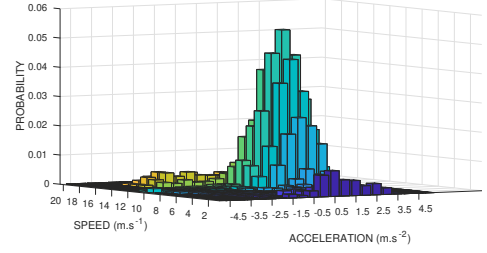


Fig. 1. Illustrative example of (speed,acceleration) distribution obtained from the traffic simulator SUMO.

2.3 A weighted state graph for hybrid electrical vehicle

The state graph In the case of ICE vehicles, the optimal path, i.e. the successive positions of the vehicle on the road graph, is sufficient to define the optimal strategy. In the case of a hybrid vehicle, the energy stored in the battery can be used to reduce the fuel consumption, adding a supplementary state, the state of charge (SoC) of the battery. Therefore the optimal consumption policy consists in the successive vehicle positions and states of charge of the battery.

Using the lexicographic product between graphs, see Fig.2, we define the 'state graph', denoted Γ , whose nodes have a form (N, SoC) , with N a node of the road graph and SoC a non-empty discrete set of state of charges values. Therefore an edge between (N_1, SoC_1) and (N_2, SoC_2) exists if and only if there is an edge between N_1 and N_2 .

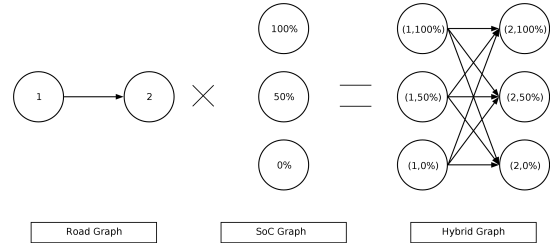


Fig. 2. Illustration of the lexicographic product

Finding the optimal path on the 'state graph' provides an approximation of the solution of the eco-routing problem for HEV.

Cost of an edge Since the consumption of the vehicle depends on the road and the traffic conditions, each edge has a specific expected cost. In order to obtain the edge cost between two nodes (N, SoC) and (N', SoC') , i.e. the expected consumption on the road segment $s_{NN'}$, the method proposed in Le Rhun et al. (2019a) is used. We recall in particular the formulation of the 'micro' problem for the road segment corresponding to the edge of the graph, with the penalization P on the final SoC. The resulting problem $(P_{micro}^{s_{NN'}})$ is

$$\min_{T_m} \mathbb{E} \left[\int_{t=0}^{t_f} C(\mathbf{V}(t), \mathbf{A}(t), T_m(t)) dt \right. \\ \left. + P_{s_{NN'}}(SoC_{s_{NN'}}(t_f), SoC') \right] \quad (5)$$

$$s.c \ \forall t, \dot{SoC}_{s_{NN'}}(t) = \frac{1}{C_{max}} P_m(\mathbf{V}(t), \mathbf{A}(t), T_m(t)) \quad (6)$$

$$\dot{D}_{s_{NN'}}(t) = \mathbf{V}(t) \quad (7)$$

$$T_m(t) \in [T_{min}, T_{max}] \quad (8)$$

$$SoC_{s_{NN'}}(t) \in [0, 1] \quad (9)$$

$$SoC_{s_{NN'}}(0) = SoC, \ D_{s_{NN'}}(0) = 0 \quad (10)$$

$$t_f = \min\{t, D_{s_{NN'}}(t) > L_{s_{NN'}}\} \quad (11)$$

The expected consumption, denoted $\nu_{s_{NN'}}(SoC, SoC')$, on a road segment $s_{NN'}$ with known traffic conditions is the value of the discrete version of problem ($\mathcal{P}_{micro}^{s_{NN'}}$).

2.4 Optimal path for HEV under traffic conditions

It is now possible to formulate an eco-routing problem for HEV as finding the optimal path on the 'state graph' Γ between a start node S and a destination node D . We denote by Γ_{SD} the set of paths between S and D in the 'state graph'. This optimal path problem consists in finding a target state of charge trajectory SoC^r . We introduce a_i (resp. b_i) the maximum charge (resp. discharge) to reduce the search space. With $a_i = 1$ and $b_i = 1$, we recover the discrete problem. The choice of a_i (resp. b_i) can be guided by the vehicle and segment characteristics. In the sequel, we assume:

$$a_i > 0, b_i > 0 \quad (12)$$

Therefore the eco-routing problem can be stated as follows:

$$\underset{\gamma \in \Gamma_{SD}}{\text{minimize}} \quad \sum_{i \in \gamma} \nu_{N_i N_{i+1}}(SoC_i^r, SoC_{i+1}^r) \quad (13)$$

$$s.t \quad \forall i \in \gamma, SoC_i^r \in [0, 1] \quad (14)$$

$$\forall i \in \gamma, SoC_{i+1}^r - SoC_i^r \in [-a_i, b_i] \quad (15)$$

3. DISCRETIZATION ERROR ANALYSIS

Since the SoC is discretized, it is useful to analyze the error between the discrete problem (P_h) and the continuous one (P) defined below. Indeed, let γ be a path on the road network, and consider the consumption minimization problem on this path. We define the criterion:

$$F^\gamma(SoC^r) := \sum_{i \in \gamma} \nu_{N_i N_{i+1}}(SoC_i^r, SoC_{i+1}^r) \quad (16)$$

Denote the continuous problem by (P^γ), and the discrete problem by (P_h^γ).

$$\underset{SoC^r}{\text{minimize}} \quad F^\gamma(SoC^r) \\ s.t \quad \forall i \in \gamma, SoC_i^r \in [0, 1] \quad (P^\gamma) \\ \forall i \in \gamma, SoC_{i+1}^r - SoC_i^r \in [-a_i, b_i]$$

$$\underset{SoC^r}{\text{minimize}} \quad F^\gamma(SoC^r) \\ s.t \quad \forall i \in \gamma, SoC_i^r \in \{0, h, \dots, 1\} \quad (P_h^\gamma) \\ \forall i \in \gamma, SoC_{i+1}^r - SoC_i^r \in [-a_i, b_i]$$

Denote the value of (P_h^γ) by V_h^γ , and the value of (P^γ) by V^γ . Let the eco-routing problem (P) be

$$\min_{\gamma} (V^\gamma) \quad (P)$$

and its discrete approximation (P_h) be

$$\min_{\gamma} (V_h^\gamma). \quad (P_h)$$

Theorem 1. Discretization Error

Assume that the criterion F^γ is Lipschitz continuous with constant L . Let S^* be a solution of (P^γ). Then there exists S_h feasible for (P_h^γ), such that $|S_h - S^*| = O(h)$ and

$$F^\gamma(S^*) \leq F^\gamma(S_h) \leq F^\gamma(S^*) + O(h). \quad (17)$$

Proof.

Let S^* an admissible solution for (P^γ). Let $\hat{S} = (1 - \varepsilon)S^*$.

Then $\hat{S} \in [0, 1]$ and also $\hat{S}_{i+1} - \hat{S}_i \in [-(1 - \varepsilon)a_i, (1 - \varepsilon)b_i]$.

Define the discretized grid $H = \{0, h, \dots, 1\}$, and a projector Π_h over H such that

$$\Pi_h(x) \in \underset{h \in H}{\text{argmin}} (|x - h|) \quad (18)$$

Let $\hat{S}^h := \Pi_h(\hat{S})$. Then $\hat{S}^h \in \{0, h, \dots, 1\}$ and

$$\hat{S}_{i+1}^h - \hat{S}_i^h \in [-(1 - \varepsilon)a_i - h, (1 - \varepsilon)b_i + h]. \quad (19)$$

Therefore \hat{S}_h is feasible for problem (P_h^γ) if and only if:

$$h - \varepsilon b_i \leq 0 \quad , \quad \varepsilon a_i - h \geq 0 \quad (20)$$

Thanks to assumption (12), we can take:

$$\varepsilon = \frac{h}{\min(\min_i(a_i), \min_i(b_i))} \quad (21)$$

Therefore, $\varepsilon = O(h)$. An admissible solution of (P_h^γ) is at $O(h)$ distance of an admissible solution of (P^γ). Since the objective function F^γ is Lipschitz continuous, the conclusion follows.

Corollary 2. Under the hypothesis of theorem 1, we have that

$$\text{val}(P) \leq \text{val}(P_h) \leq \text{val}(P) + O(h) \quad (22)$$

4. FIND THE SHORTEST PATH

The class of problems under consideration has directed graphs and non-negative costs. Therefore, we may use the classical Dijkstra's algorithm, see for instance Cormen (2009). Another possibility is to use the A^* algorithm, a generalization of Dijkstra's algorithm based on a heuristic estimate (called the heuristic distance) H of the distance of nodes to the destination node, allowing to decrease the number of visited nodes.

The A^* algorithm computes the optimal path if the heuristic distance H is *admissible* in the sense below, see Nilsson (1980).

Definition 3. Admissibility

A heuristic H is admissible if it never overestimates the real cost to reach the goal. In other terms,

$$\forall v \in \Gamma, H(v) \leq H^*(v) \quad (23)$$

where $H^*(v)$ is the minimum cost between node v and the goal node.

4.1 Complexity

The complexity of the A^* algorithm has been discussed for instance in Martelli (1977). In the worst case scenario, the destination node is explored after all the other nodes on the graph, and after that all the edges have been explored too. If n is the number of nodes and m the number of edges, the complexity is $O(n * operations + m * operations)$. The only operation which is not in $O(1)$ is the minimization over the nodes, that can be made in $O(\log(n))$. Then the complexity is $O(n * \log(n) + m)$ and we have that $m \leq n^2$. So the complexity of the A^* algorithm in the worst possible case is $O(n^2)$.

However, the number of nodes of the state graph depends of the discretization on the SoC^r as well as the number of edges. The number of nodes of the state graph is the product of the number of intersections n in the road network and the number of SoC divisions created. The number of edges of the state becomes m/h^2 . Then the complexity of the A^* algorithm according to the problem is $O(n/h \log(n/h) + m/h^2)$.

4.2 Heuristic distance

We propose here an admissible heuristic for the case of the hybrid vehicle eco-routing. We define H_c an estimate of the consumption to reach the physical destination

$$H_c := \eta_m \alpha_0 L \quad (24)$$

with L the travel length, η_m the maximum efficiency of the ICE to convert fuel to mechanical energy, α_0 defined in Appendix A, and m the mass of the vehicle.

Similarly, we define H_{SoC} as an estimate of the consumption to reach the desired state of charge

$$H_{SoC} := \eta_e C_{max} (SoC_f - SoC_{current}) \quad (25)$$

with η_e the maximum efficiency of the ICE to convert fuel to battery charge.

Finally, we must take into account the possibility of regenerative braking that can recover the kinetic energy of vehicle. Therefore, we define $H_{kinetic}$

$$H_{kinetic} := \frac{\eta_c m}{2} v_{max}^2 \quad (26)$$

with η_c a conversion factor of the kinetic energy into fuel according to the ICE and v_{max} the maximum speed. Since we want to reach a specific destination with a final state of charge, the heuristic is taken as the sum of these estimates. Additionally, we take into account that the fuel cannot be produced by the vehicle, giving the final expression:

$$H := \max(0, H_c + H_{SoC} - H_{kinetic}) \quad (27)$$

5. NUMERICAL SIMULATIONS

We now present numerical simulations for the eco-routing method. We study a simple road network comprised of a small ring with congested traffic with a mean speed of 40 km/h, enclosed in a larger ring with fluid traffic with a mean speed of 100 km/h. This can be considered as a very simplified model of a typical road network with ring roads around a city. For the sake of simplicity, each segment of the presented network has the same topological aspect. In particular, we are interested in comparing the solution

from the eco-routing, called ‘eco-path’ in the following, with the fastest path. We first study a specific travel with a fixed origin and destination, and then give some more general results on all possible travels on the network.

5.1 Study of a single travel

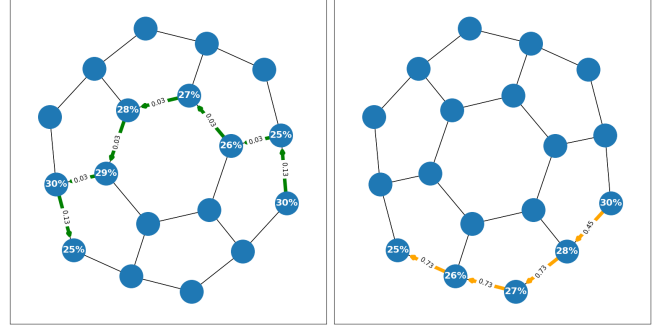


Fig. 3. Eco-path (left) and Fastest Path (right) - ($SoC_i = 30\%$, $SoC_f = 25\%$)

We show on Fig.3 the eco-path (green, left graph) and fastest path (orange, right graph) for a given travel with conditions ($SoC_i = 30\%$, $SoC_f = 25\%$). Keep in mind that the considered vehicle has a low battery capacity, see C_{max} in Table A.1, allowing for relatively significant SoC changes even on small road segments. The SoC values at the end of each road segment are indicated on the nodes, while the values from the ‘micro’ problems are displayed along each edge. In this particular example, the fastest path takes the 4 fast segments on the outer ring, while the eco-path uses the slower inner segments to reduce the consumption, with 7 segments in total.

Path	Value	Cons. (L)	SoC_f (%)	Time (s)	Dist. (km)
Eco	0.41	0.012	21	163	2.14
Fastest	2.66	0.038	4	44	1.22

Table 1. Comparison of Eco and Fastest Path - ($SoC_i = 30\%$, $SoC_f = 25\%$).

Table 1 summarizes for both paths the sum of values of the micro problems, consumption, final SoC, time and distance for a $SoC_f = 25\%$ constraints. The time is based on the average speed of each segment. The consumption and final SoC are recomputed by taking the average of 1000 resimulated travels using the local optimal policies determined by the eco-path, with i.i.d. sampling according to the traffic conditions for the (v, a) values. For this sample travel the eco-path consumption is one third of the fastest path, for a double distance and four times longer time. Note that the difference in terms of value is greater than the difference in consumption, which indicates that the eco-path has a better chance of following the reference SoC trajectory, since the value function of the micro problems is the sum of the consumption and penalty for the final SoC constraint at the end of the segment. This also shows in the final SoC value, with the eco-path being much closer to the prescribed $SoC_f = 25\%$, reaching 21% while the fastest path ends up at only 4%.

Fig. 4 and Table 2 show the same results for a final constraint $SoC_f = 20\%$. The consumption is still one

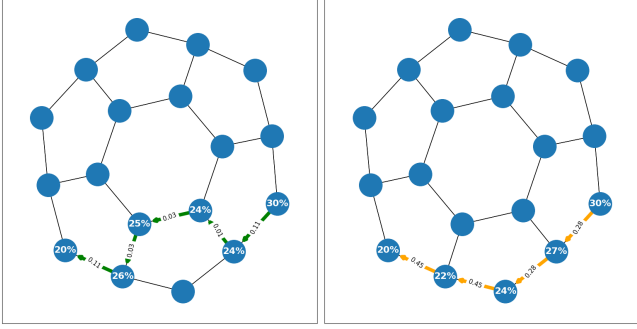


Fig. 4. Eco-path (left) and Fastest Path (right) - ($SoC_i = 30\%$, $SoC_f = 20\%$)

Path	Value	Cons. (L)	SoC_f (%)	Time (s)	Dist. (km)
Eco	0.28	0.013	19.7	106	1.53
Fastest	1.46	0.036	4	44	1.22

Table 2. Comparison of Eco and Fastest Path - ($SoC_i = 30\%$, $SoC_f = 20\%$).

third of the one of the fastest path, but now with a more moderate increase in travel distance and time. Also notice that the final SoC constraint is well satisfied in this case by the eco-path, reducing the gap between the resimulated consumption and the value of the eco-routing problem. All in all, we observe a classical trade-off, with the eco-routing having a lower consumption and better management of the state of charge, at the expense of choosing longer and slower paths.

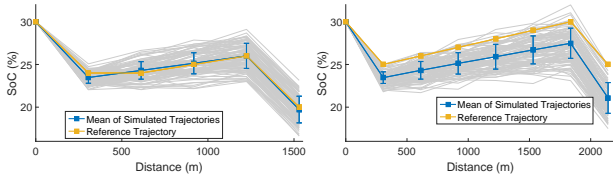


Fig. 5. Simulated and reference SoC trajectories for $\Delta SoC = -10\%$ (left) and $\Delta SoC = -5\%$ (right). Markers show the mean and vertical bars show $\pm\sigma$ of the simulated trajectories.

Now, we study the SoC a bit more in detail, with Fig.5 showing the evolution of the state of charge along the travel. Orange line is the reference SoC from the eco-routing solution, while blue line is the average (with standard deviation indicators) of the 1000 resimulations, with the first 100 resimulations also plotted in grey lines. We see that for the condition $\Delta SoC = -10\%$ (left), the reference SoC is very well matched by the set of resimulations, which means that the control decisions from the micro problems are able to satisfy the individual final SoC constraints at the end of each segment. In this case the penalty terms are typically close to zero, and the value function is close to the consumption. For the stricter condition $\Delta SoC = -5\%$ (right), we see that the micro solutions begin to have difficulties to reach the required SoC, which leads to penalty terms in the value function (as seen in Table 1 and Table 2) and a growing gap between the reference SoC from the eco-path and the actual SoC trajectory from the resimulated travels.

We look a bit more into the influence of the final SoC condition, by solving the previous test case for ΔSoC ranging from -10% to $+5\%$. We compare in particular the final SoC and consumption for the eco-path and the resimulated trajectories. Fig.6 shows the final SoC (eco-path reference and resimulations average with std indicators). The reference final SoC basically corresponds to the ΔSoC constraint (we recall that $SoC_i = 30\%$). As for the resimulated SoC, we see that for easier constraints such as 10% discharge, it coincides well with the reference, as already seen above in Fig.5. When the final SoC condition tightens, we see an increasing gap between the two curves, due to the fact that some of the micro solutions selected for the eco-path do not satisfy their prescribed SoCs. A way to reduce these gaps could be to use a smaller discretization for the initial and final SoC of the micro problems, enabling the routing algorithm to choose reference SoCs closer to the maximum feasible ΔSoC on the segments. Note also that for sufficiently long travels with more opportunities for recharging the battery, the gap may be compensated along the way, see the simulations of Le Rhun et al. (2019a).

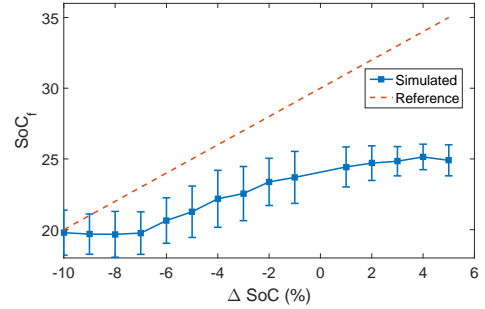


Fig. 6. Reference and resimulated SoC_f

On Fig. 7 we display the resimulated consumption for the eco and fastest paths. We observe a clear advantage of the eco-paths overall, with a consumption between one quarter and one third of the fastest path. The consumption for the fastest path tends to increase with respect to the ΔSoC , since the harder final SoC constraint requires an additional use of the engine. On the other hand, the consumption of the eco-path appears non-increasing, which is probably explained by the fact that the eco-path maintains a low consumption at the expense of an increasing violation of the final SoC constraints, see Fig. 6. This is related to the fact that the micro problems manage the reference SoC constraints thanks to the penalization term, which allows some trade-off between the consumption and the reference SoC constraints.

5.2 Study of all possible travels

While we previously focused on a single travel, we now perform some simulations while taking into account all possible travels on the road graph. According to the symmetry of the road network, we end up with a set of 54 travels which we solve for varying final SoC conditions. Fig.8 shows the average time and distance ratio between the eco-path and the fastest path (i.e. the average of the ratios for each individual travel).

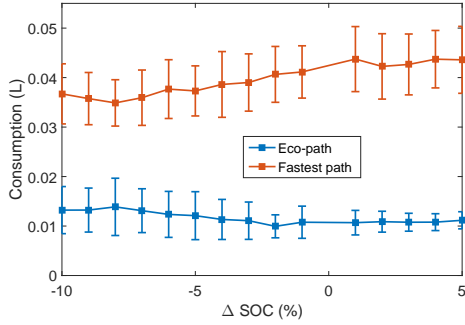


Fig. 7. Eco-path and Fastest path Consumption

For harder final SoC constraints, we notice a large difference between the two paths, with the eco-path being (on average) up to 7 times slower and 4 times longer than the fastest path. This behavior comes from the eco-path traveling repeatedly along segments that allow for recharging the battery, in order to meet the final constraint of a 5% charge. Note that this type of path seems to include cycles in the ‘physical’ road graph, but these are not cycles in the weighted graph augmented with the SoC values.

When the final SoC constraints are easier, the average time and distance of the eco-path and fastest path tend to be closer. Indeed, allowing an increased discharge of the battery will reduce the consumption of the fastest path, up to the point that it actually becomes identical to the eco-path.

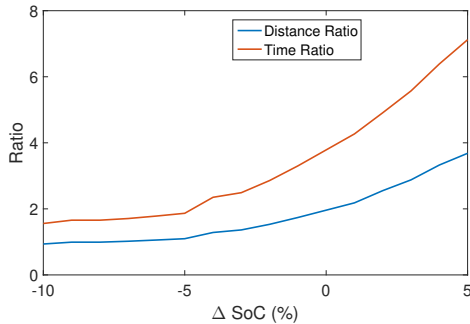


Fig. 8. Time and Distance Ratio

6. CONCLUSION

Simulations on a simple road network indicate that the eco-routing method computes optimal paths with a consumption significantly lower than the fastest path solutions. This reduced consumption comes with an expected trade-off in terms of travel distance and time, and considering an upper bound for time and/or distance would be a natural extension of the method, in order to obtain solutions that can fit the expectations of the drivers. We also observe that the accuracy of the reference SoC trajectory computed by the eco-routing tends to decrease for stricter final SoC conditions. Increasing the weight of the penalty term for the final SoC constraint would also improve this accuracy, however at the expense of a higher consumption overall. Another direction for improvement would be to use a finer SoC discretization for the initial and final conditions of the micro problems, as the error analysis has shown that

the values of the discretized problem approximate the one of the original problem up to the order of the discretization step size.

Appendix A. VEHICLE MODEL

Neglecting the slope effect, for a given speed v and acceleration a , we can express the torque and rotation speed at the wheel as:

$$T_w(v(t), a(t)) = (ma(t) + \alpha_2 v^2(t) + \alpha_1 v(t) + \alpha_0) r_w$$

$$\omega_w(v, a) = \frac{60v}{2\pi r_w}$$

with m the vehicle mass, r_w the wheel radius, and $\alpha_0, \alpha_1, \alpha_2$ defining a quadratic approximation of the road-load force. We note $G_R^i, G_{Eff}^i, Pa_{ratio}, Pa_{eff}$ the ratio and efficiency for the gears and powertrain, and R the motor/engine reduction ratio. Then we get the torque at the primary shaft T_{prim} and the rotation speed of the engine ω_e and motor ω_m :

$$T_{prim}(v, a) = \max \left(\frac{T_w(v, a)}{Pa_{ratio} Pa_{eff} G_R^i G_{Eff}^i}, T_{min} \right)$$

$$\omega_e(v, a) = \omega_w(v, a) Pa_{ratio} G_R^i G_{Eff}^i$$

$$\omega_m(v, a) = \omega_w(v, a) Pa_{ratio} G_R^i G_{Eff}^i R$$

Table A.1. Parameters used in simulations

m	r_w	α_0	α_1	α_2
1190kg	0.31725m	113.5	0.774	0.4212

i	1	2	3	4	5
G_R^i	3.416	1.809	1.281	0.975	0.767
G_{Eff}^i	1	1	1	1	1

Pa_{ratio}	Pa_{eff}	R	$C_{max}(C)$
59/13	0.95	3.3077	5335200

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