A brief survey of 20th century logical notations

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We give here the logical notations used in a certain number of works of logic. Most of 20th century's work of historical importance is covered here. Although the list contains secondary work, the overview given here is not necessarily representative of the whole of contemporary logic.

An empty cell means that either there is no symbol corresponding to a concept, or that I didn't find any by skimming through the book. (Some symbols are rarely used and I may have missed them.) One must not confuse an empty cell with the mention "none." The latter means that the concept exists but that no symbol is associated to it. This is sometimes the case with "and." Some authors write pq for $p \wedge q$ (like one often writes ab for $a \times b$).

Note also that the symbol representations in this document are, in general, approximations of the symbols actually used. The latter can be of a different size, larger in width or in height, bolder or lighter, etc.

Many symbols stem from Peano who inspired himself from Gabelsberger's shorthand alphabet.

Certain works have two dates:

- the first date between parentheses represents a likely but unchecked date; this can be the case for a first edition of a book of which I only consulted a later edition; it is therefore possible that the notations changed between the editions;

- the second date between square brackets corresponds to the date where the notation has been observed; for instance, "Carnap (1934) [1951]" means that I observed the notation in a 1951 edition (actually the third printing of "The logical syntax of language") but the original German publication was in 1934.

The table gives an idea of the introduction dates of the various symbols. However, one should be careful not to draw too hasty conclusions. For instance, it can seem that Schönfinkel introduced "&", "→" and "(Ex)" in 1924, but actually the author claims to borrow the symbols used by Hilbert in his classes. However, it might be the first published use of these symbols.

Only object language symbols are shown. Some authors also define other symbols for the metalanguage. For instance, Skolem in 1928 has $\rightarrow$ for the consequence relation between formulæ and $\rightleftarrows$ for their equivalence.

Concept / Author	not	or	and	if/then	for all	exists	iff
Frege (1879)	⊢ A			$\begin{array}{c}B\\A\end{array}$	x $\Phi(x)$		
Peirce (1885)	$\overline{x}$			$\prec$	Π_i	Σ_j	
Peano (1889)	$-$	$\cup$	$\cap$	$\supset$	$\supset_x$		
Schröder (1890) (to check)	$\overline{a}$	$+$	$\cdot$	€	Π_i	Σ_i	
Peano (ca. 1895) [1901]	$-$	$\cup$	$\cap$	$\supset$	$\supset_x$	$\exists x$	
Hilbert (1904)	$\overline{a}$	o.	u.	$\mid$			
Russell (1908)	$\sim$	$\vee$	$\cdot$	$\supset$	(x)	$\exists x$	$\equiv$
Russell &Whitehead (1910–1913)	$\sim$	$\vee$	$\cdot$	$\supset$	(x)	$(\exists x)$	$\equiv$
Löwenheim (1915)	$\overline{a}$	$+$	$\cdot$	€	Π_i	Σ_i	
Hilbert (1917/18)	$\overline{X}$	$\times$/none	$+$	$\rightarrow$	(x)		$=$
Skolem (1920)	$\overline{a}$	$+$	$\cdot$		Π_i	Σ_i	
Post (1921)	$\sim$	$\vee$	$\cdot$	$\supset$			$\equiv$
Tarski (1923)	$\sim$	$\vee$		$\supset$			
Schönfinkel (1924)	$\overline{a}$	$\vee$	&	$\rightarrow$	(x)	(Ex)	$\sim$
Ramsey (1925)	$\sim$	$\vee$	$\cdot$	$\supset$	(x)	$(\exists x)$	
Hilbert (1925)	$\overline{A}$	$\vee$	&	$\rightarrow$	(x)	(Ex)	
Kolmogorov (1925)	$\overline{A}$			$\rightarrow$	(x)	(Ex)	
Hilbert (1927)	$\overline{A}$	$\vee$	&	$\rightarrow$	(x)	(Ex)	
Skolem (1928)	$\overline{a}$	$+$	$\cdot$		Π_i	Σ_i	
Herbrand (1928)	$\sim$	$\vee$	&	$\supset$	(x)	$(\exists x)$	$\equiv$
Hilbert & Ackermann (1928) [1950]	$\overline{A}$	$\vee$	&	$\rightarrow$	(x)	(Ex)	$\sim$
Łukasiewicz (1929)	Np	Apq	Kpq	Cpq	Px	Sx	Epq
Heyting (1929)	$\neg$	$\vee$	$\wedge$	$\supset$	(x)	(Ex)	$\supset\subset$
Gödel (1930)	$\overline{p}$	$\vee$	&	$\rightarrow$	(x)	(Ex)	$\sim$
Heyting (1931)	$\neg p$	$\vee$					
Gödel (1931)	$\overline{p}$ (also $\sim$)	$\vee$	&	$\rightarrow$ (also $\supset$)	(x) (also $x\Pi$)	(Ex)	$\equiv$
Herbrand (1931)	$\sim$	$\vee$	$\times$	$\rightarrow$	(x)	(Ex)	$\equiv$
Gentzen (1934)	$\neg$	$\vee$	&	$\supset$	$\forall x$	$\exists x$	$\supset\subset$
Hilbert & Bernays (1934–39)	$\overline{A}$	$\vee$	&	$\rightarrow$	(x)	(Ex)	$\sim$
Carnap (1934) [1951]	$\sim$	$\vee$	$\cdot$	$\supset$	(x)	$(\exists x)$	$\equiv$
Tarski (1940)	$\sim$	$\vee$	$\wedge$	$\rightarrow$	$\underset{x}{A}$	$\underset{x}{E}$	$\leftrightarrow$
Quine (ML, 1940)	$\sim$	$\vee$	$\cdot$	$\supset$	(x)	$(\exists x)$	$\equiv$
Henkin (1949)	$\sim$			$\supset$	(x)	$(\exists x)$	
Henkin (1950)	$\sim$	$\vee$	none	$\supset$	(x)	$(\exists x)$	
Kleene (IM, 1952)	$\neg$	$\vee$	&	$\supset$	$\forall x$	$\exists x$	$\sim$
Carnap (1954)	$\sim$	$\vee$	$\cdot$	$\supset$	(x)	$(\exists x)$	$\equiv$
Church (1956)	$\sim$	$\vee$	none	$\supset$	(x)	$(\exists x)$	$\equiv$
Suppes (IL, 1957)	$-p$	$\vee$	&	$\rightarrow$	(x)	$(\exists x)$	$\leftrightarrow$
Davis (C&U, 1958) [1982]	$-A$		&	$\supset$	(x)	$(\exists x)$	
Bernays (AST, 1958) [1968]	$\overline{p}$	$\vee$	&	$\rightarrow$	(x)	(Ex)	$\leftrightarrow$
Suppes (AST, 1960)	$-p$	$\vee$	&	$\rightarrow$	$\forall x$	$\exists x$	$\leftrightarrow$
Quine (STL, 1963)	$\sim$			$\supset$	(x)	$(\exists x)$	$\equiv$
Mendelson (1964) [1968]	$\sim$	$\vee$	$\wedge$	$\supset$	(x)	(Ex)	$\equiv$
Kleene (LM, 1967) [1971]	$\neg$	$\vee$	&	$\supset$	$\forall x$	$\exists x$	$\sim$
Smullyan (1968)	$\sim$	$\vee$	$\wedge$	$\supset$	$(\forall x)$	$(\exists x)$	$\leftrightarrow$
Blanché (1968)	$\sim$	$\vee$	$\cdot$	$\supset$	(x)	$(\exists x)$	$\equiv$
Putnam (1971)	$\sim$	$\vee$	$\wedge$	$\supset$	(x)	$(\exists x)$	$\equiv$
Boolos & Jeffrey (1974) [1996]	$-p$	$\vee$	&	$\rightarrow$	$\forall x$	$\exists x$	$\leftrightarrow$
Copi (SL, 1979)	$\sim$	$\vee$	$\cdot$	$\supset$	(x)	$(\exists x)$	$\equiv$
Quine (1982)	$-p$	$\vee$	none	$\rightarrow$	$\forall x$	$\exists x$	$\leftrightarrow$
Vax (1982)	$\sim$	$\vee$	$\wedge$	$\Rightarrow$	$\forall x$	$\exists x$	$\Leftrightarrow$
Rivenc (1989)	$\neg$	$\vee$	$\wedge$	$\rightarrow$	$\forall x$	$\exists x$	$\leftrightarrow$
Gochet & Gribomont (1990)	$\neg$	$\vee$	$\wedge$	$\supset$	$\forall x$	$\exists x$	$\equiv$
Lalement (1990)	$\neg$	$\vee$	$\wedge$	$\Rightarrow$	$\forall x$	$\exists x$	$\Leftrightarrow$
Ruyer (1990)	$\neg$	$\vee$	$\wedge$	$\rightarrow$	$\forall x$	$\exists x$	$\leftrightarrow$
Smullyan (1992)	$\sim$	$\vee$	$\wedge$	$\supset$	$\forall x$	$\exists x$	$\equiv$
Largeault (1992)	$\neg$	$\vee$	&	$\rightarrow$	$\forall x$	$\exists x$	$\leftrightarrow$
Cori & Lascar (1993)	$\neg$	$\vee$	$\wedge$	$\Rightarrow$	$\forall x$	$\exists x$	$\Leftrightarrow$
Largeault (1993)	$\neg$	$\vee$	&	$\rightarrow$	$\forall x$	$\exists x$	$\leftrightarrow$
Chazal (1996)	$\neg$	$\vee$	$\wedge$	$\supset$	$\forall x$	$\exists x$	$\bigcirc$
Gauthier (1997)	$\sim$		$\wedge$	$\supset$	$\forall x$	$\exists x$	
Leroux (1998)	$\neg$	$\vee$	$\wedge$	$\supset$	$\forall x$	$\exists x$	$\equiv$
Encyclop. Britannica (1999)	$\sim$	$\vee$	none/$\cdot$	$\supset$	$(\forall x)$	$(\exists x)$	$\equiv$

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