



Finite element approximation of Maxwell's equations with unfitted meshes for borehole simulations

Théophile Chaumont-Frelet, Serge Nicaise, David Pardo

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Finite element approximation of Maxwell's equations with unfitted meshes for borehole simulations

T. Chaumont-Frelet¹ S. Nicaise² D. Pardo³

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²Univ. Valenciennes, LAMAV

³UPV/EHU, BCAM and Ikerbasque

ICIAM - July 17, 2019

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in complex nonlinear materials

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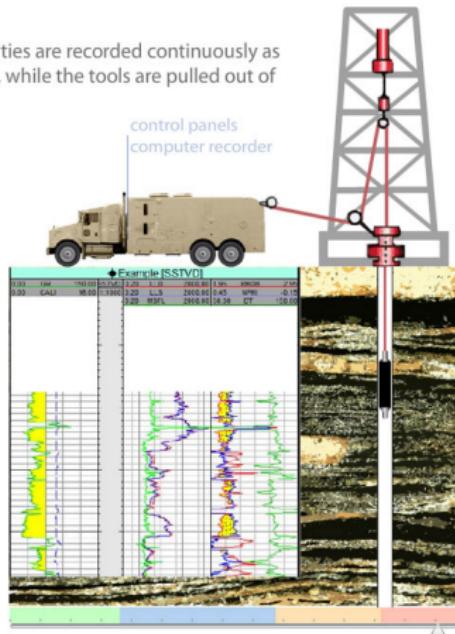
Geosteering



Guide the drilling trajectory using “real-time” magnetic measurements

Geosteering

The physical properties are recorded continuously as a function of depth, while the tools are pulled out of the well.



Reproduce the measurements using finite element discretizations

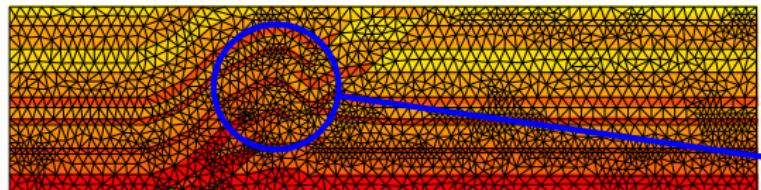
Non-fitting meshes (aka unfitted meshes)

Finite element discretizations are build on a mesh of the domain.

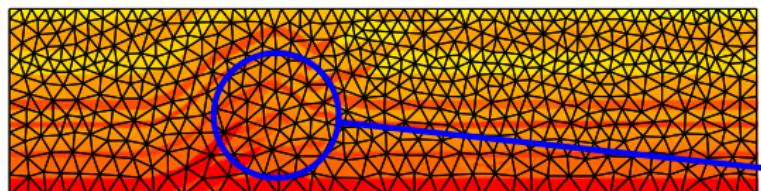
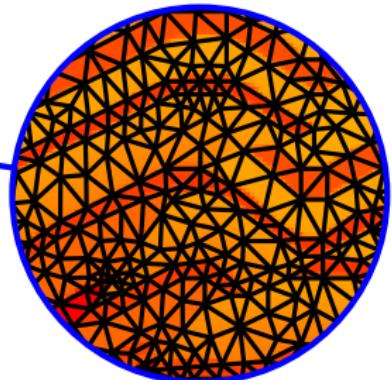
A mesh is fitting if the physical interfaces are aligned with faces.
The physical coefficients are constant (or smooth) inside each element.

A non-fitting mesh is generated independently of the physical coefficients.

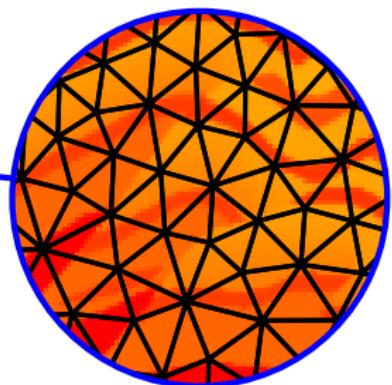
Fitting mesh vs non-fitting mesh



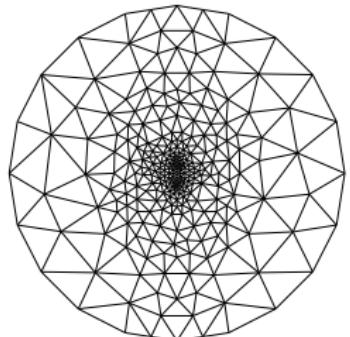
Fitting mesh



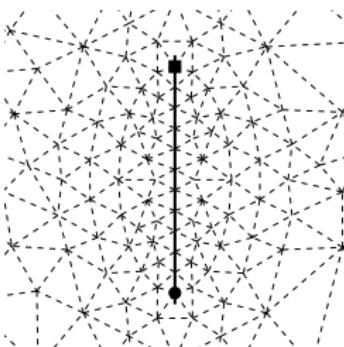
Non-fitting mesh



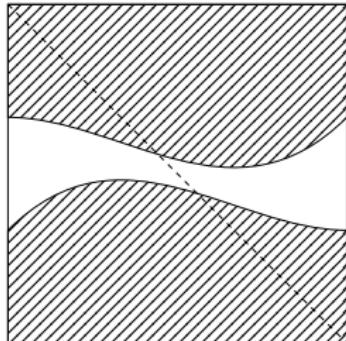
Non-fitting mesh for geosteering



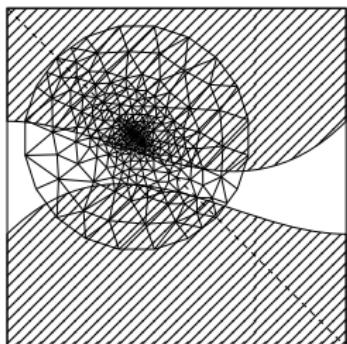
(a) Sliding mesh



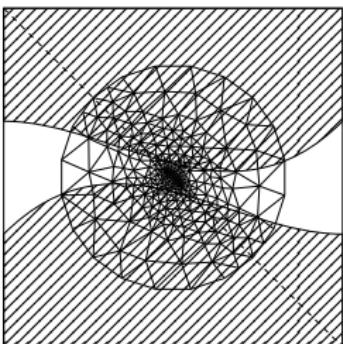
(b) Zoom around the logging tool



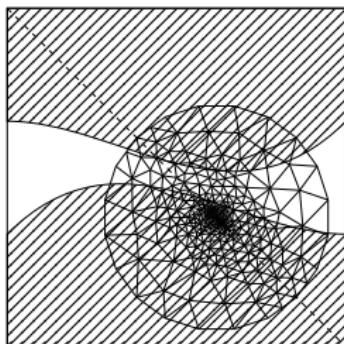
(c) Conductivity model and well trajectory



(d) Position 1



(e) Position 2



(f) Position 3

Pros and cons of non-fitting meshes

Main advantages of non-fitting meshes:

- simplified mesh generation (Cartesian grids)
- minimize remeshing
- larger elements

Possible drawbacks of non-fitting meshes:

- quadrature schemes to integrate the linear system coefficients
- possible accuracy loss

Pros and cons of non-fitting meshes

Main advantages of non-fitting meshes:

- simplified mesh generation (Cartesian grids)
- minimize remeshing
- larger elements

Possible drawbacks of non-fitting meshes:

- quadrature schemes to integrate the linear system coefficients
- **possible accuracy loss**

Outline

- 1 Maxwell's equations
- 2 The magnetic formulation
- 3 The electric formulation
- 4 Numerical results
- 5 Conclusions

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- 1** Maxwell's equations
- 2** The magnetic formulation
- 3** The electric formulation
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- 5** Conclusions

Maxwell's equations

Find $\mathbf{E}, \mathbf{H} : \Omega \rightarrow \mathbb{C}^3$ such that

$$\begin{cases} \nabla \times \mathbf{E} - i\omega\mu_0 \mathbf{H} = \mathbf{M}, \\ \nabla \times \mathbf{H} + \sigma \mathbf{E} = \mathbf{J}, \end{cases}$$

where

- \mathbf{M}, \mathbf{J} are given load terms (electric and/or magnetic dipoles),
- ω is the operating frequency,
- $\mu_0 = 4\pi \times 10^{-7} \text{ NA}^{-2}$ is the vacuum permeability,
- $\sigma : \Omega \rightarrow \mathbb{R}$ is the conductivity model.

The medium is **strongly dissipative** ($i\epsilon\omega + \sigma \simeq \sigma$).

The conductivity σ is piecewise constant.

The permeability μ_0 is **constant**.

We are mostly interested in **magnetic measurements**.

Regularity of the solution

In the following, we focus on the case where:

- Ω is convex,
- the medium is layered ($\sigma \simeq \sigma(z)$).

Magnetic field regularity

$$\textcolor{red}{H} \in \left(H^1(\Omega) \right)^3$$

Electric field regularity

$$\textcolor{red}{E}|_{\Omega_l} \in \left(H^1(\Omega_l) \right)^3 \quad \quad \textcolor{red}{E} \in \left(H^{1/2-\varepsilon}(\Omega) \right)^3$$

Approximation by polynomials (quick analysis)

Approximation of \mathbf{H}

$$\|\mathbf{H} - \pi_h \mathbf{H}\|_{0,\Omega} \simeq h$$

Approximation of \mathbf{E} with fitting meshes

$$\|\mathbf{E} - \pi_h \mathbf{E}\|_{0,\Omega} \simeq h$$

Approximation of \mathbf{E} with non-fitting meshes

$$\|\mathbf{E} - \pi_h \mathbf{E}\|_{0,\Omega} \simeq h^{1/2-\varepsilon}$$

The key question

Standard approximation theory (Céa's Lemma) yields:

Fitting meshes

$$\|\mathbf{E} - \mathbf{E}_h\|_{0,\Omega} + \|\mathbf{H} - \mathbf{H}_h\|_{0,\Omega} \simeq h$$

Non-fitting meshes

$$\|\mathbf{E} - \mathbf{E}_h\|_{0,\Omega} + \|\mathbf{H} - \mathbf{H}_h\|_{0,\Omega} \simeq h^{1/2}$$

Given that \mathbf{H} is **globally regular**, is the convergence rate of \mathbf{H}_h sharp?

Remark on the general case

In general, the regularity of \mathbf{E} depends on σ

Regularity of \mathbf{E}

$$\mathbf{E} \in \left(H^{\tau(\sigma)}(\Omega) \right)^3$$

where $0 < \tau(\sigma) < 1/2$.

Interpolation errors

$$\|\mathbf{H} - \pi_h \mathbf{H}\|_{0,\Omega} \simeq h \quad \|\mathbf{E} - \pi_h \mathbf{E}\|_{0,\Omega} \simeq h^{\tau(\sigma)}$$

Interpolation errors

$$\|\mathbf{E} - \pi_h \mathbf{E}\|_{0,\Omega} + \|\mathbf{H} - \pi_h \mathbf{H}\|_{0,\Omega} \simeq h^{\tau(\sigma)}$$

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The magnetic formulation

We recast Maxwell's equation into a second-order system:

$$\begin{cases} -i\omega\mu_0 \mathbf{H} + \nabla \times (\sigma^{-1} \nabla \times \mathbf{H}) &= \mathbf{M} \quad \text{in } \Omega, \\ (\sigma^{-1} \nabla \times \mathbf{H}) \times \mathbf{n} &= \mathbf{0} \quad \text{on } \partial\Omega. \end{cases}$$

We obtain $\mathbf{E} \simeq \nabla \times \mathbf{H}$ by post-processing.

Variational formulation

Find $\mathbf{H} \in H(\mathbf{curl}, \Omega)$ such that

$$b(\mathbf{H}, \mathbf{v}) = (\mathbf{M}, \mathbf{v})$$

for all $\mathbf{v} \in H(\mathbf{curl}, \Omega)$.

$$b(\mathbf{H}, \mathbf{v}) = -i\omega\mu_0(\mathbf{H}, \mathbf{v}) + (\sigma^{-1} \nabla \times \mathbf{H}, \nabla \times \mathbf{v})$$

Finite element discretization

We consider first-order Nédélec elements on a mesh $\mathcal{T}_h = \{K\}$:

$$\mathbf{V}_h = \left\{ \mathbf{v}_h \in H(\mathbf{curl}, \Omega) \mid \mathbf{v}_h|_K = \mathbf{a} \times \mathbf{x} + \mathbf{b}, \mathbf{a}, \mathbf{b} \in \mathbb{C}^3 \right\}.$$

Discrete variational formulation

Find $\mathbf{H}_h \in \mathbf{V}_h$ such that

$$b(\mathbf{H}_h, \mathbf{v}_h) = (\mathbf{M}, \mathbf{v}_h)$$

for all $\mathbf{v}_h \in \mathbf{V}_h$.

Error estimates

Assuming that $\mathbf{M} \in H(\mathbf{div}, \Omega)$:

Regularity

$$\mathbf{H} \in \left(H^1(\Omega)\right)^3 \quad \nabla \times \mathbf{H} \in \left(H^{1/2-\varepsilon}(\Omega)\right)^3$$

Interpolation error

$$\|\mathbf{H} - \boldsymbol{\pi}_h \mathbf{H}\|_{0,\Omega} \lesssim h \|\mathbf{M}\|_{\mathbf{div},\Omega} \quad \|\nabla \times (\mathbf{H} - \boldsymbol{\pi}_h \mathbf{H})\|_{0,\Omega} \lesssim h^{1/2-\varepsilon} \|\mathbf{M}\|_{0,\Omega}$$

Error estimate (Céa's Lemma)

$$\|\mathbf{H} - \mathbf{H}_h\|_{0,\Omega} + \|\nabla \times (\mathbf{H} - \mathbf{H}_h)\|_{0,\Omega} \lesssim h^{1/2-\varepsilon} \|\mathbf{M}\|_{\mathbf{div},\Omega}$$

Error estimates

We have

Interpolation error

$$\|\mathbf{H} - \pi_h \mathbf{H}\|_{0,\Omega} \lesssim h \|\mathbf{M}\|_{\mathbf{div},\Omega}$$

but only

Error estimate

$$\|\mathbf{H} - \mathbf{H}_h\|_{0,\Omega} \lesssim h^{1/2-\varepsilon} \|\mathbf{M}\|_{\mathbf{div},\Omega}$$

Can we improve this error estimate? Yes!

The Poisson equation

Consider the simpler problem

$$\begin{cases} \mathbf{u} - \Delta \mathbf{u} &= f \quad \text{in } \Omega, \\ \nabla \mathbf{u} \cdot \mathbf{n} &= 0 \quad \text{on } \partial\Omega, \end{cases}$$

where $f \in L^2(\Omega)$.

Using Lagrange elements, we are in the similar situation where:

Interpolation error

$$\|\mathbf{u} - \pi_h \mathbf{u}\|_{0,\Omega} \lesssim h^2 \|f\|_{0,\Omega}$$

Error estimate (Céa's lemma)

$$\|\mathbf{u} - \mathbf{u}_h\|_{0,\Omega} \lesssim h \|f\|_{0,\Omega}$$

Duality technique: the “Aubin-Nitsche trick”

Introduce ξ as the unique solution to

$$a(w, \xi) = (w, u - u_h), \quad \forall w \in H^1(\Omega).$$

Then, selecting $w = u - u_h$

$$\|u - u_h\|_{0,\Omega}^2 = a(u - u_h, \xi) = a(u - u_h, \xi - \pi_h \xi) \lesssim \|u - u_h\|_{1,\Omega} \|\xi - \pi_h \xi\|_{1,\Omega}.$$

We observe that ξ is solution to

$$\begin{cases} \xi - \Delta \xi &= u - u_h \quad \text{in } \Omega, \\ \nabla \xi \cdot n &= 0 \quad \text{on } \partial\Omega, \end{cases}$$

and $\xi \in H^2(\Omega)$ with

$$|\xi|_{2,\Omega} \lesssim \|u - u_h\|_{0,\Omega}$$

since $u - u_h \in L^2(\Omega)$.

Duality technique: the “Aubin-Nitsche trick”

Hence,

$$\begin{aligned}\|\mathbf{u} - \mathbf{u}_h\|_{0,\Omega}^2 &\lesssim \|\mathbf{u} - \mathbf{u}_h\|_{1,\Omega} \|\xi - \pi_h \xi\|_{1,\Omega} \\ &\lesssim h \|\mathbf{u} - \mathbf{u}_h\|_{1,\Omega} |\xi|_{2,\Omega} \\ &\lesssim h \|\mathbf{u} - \mathbf{u}_h\|_{1,\Omega} \|\mathbf{u} - \mathbf{u}_h\|_{0,\Omega}.\end{aligned}$$

Error estimate

$$\|\mathbf{u} - \mathbf{u}_h\|_{0,\Omega} \lesssim h \|\mathbf{u} - \mathbf{u}_h\|_{1,\Omega} \lesssim h^2 \|f\|_{0,\Omega}.$$

Application to Maxwell's equations

Direct application fails. Indeed, if we let ξ solve

$$b(w, \xi) = (w, H - H_h), \quad \forall w \in H(\mathbf{curl}, \Omega),$$

then $\xi \notin (H^1(\Omega))^3$ because $H - H_h \notin H(\mathbf{div}, \Omega)$.

The key idea is to introduce the Helmholtz-Hodge decomposition

$$H - H_h = \phi + \nabla p.$$

with $\nabla \cdot \phi = 0$.

We employ the “Aubin-Nitsche trick” to estimate $\|\phi\|_{0,\Omega}$.

Additional “orthogonality” arguments are used to estimate $\|\nabla p\|_{0,\Omega}$.

Details of the proof



T. Chaumont-Frelet, S. Nicaise and D. Pardo
SIAM J. Numerical Analysis 2018

For regular solution $\tau(\sigma) > 1/2$:



L. Zhong, S. Shu, G. Wittum and J. Xu
J. Computational Mathematics 2009

General case (done in the same time):



A. Ern and J.L. Guermond
Computer & Mathematics with Applications 2018

Error estimate

Error estimate

$$\|\mathbf{H} - \mathbf{H}_h\|_{0,\Omega} \lesssim h^{1-2\varepsilon} \|\mathbf{M}\|_{\mathbf{div},\Omega}$$

We can remove the “ -2ε ” if we employ “non-Sobolev” interpolation spaces.

The convergence rate is linear, as in the case of fitting meshes.

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The electric formulation

Another option is to consider the second-order system:

$$\begin{cases} -i\omega\mu_0\sigma \mathbf{E} + \nabla \times \nabla \times \mathbf{E} &= \mathbf{J} \quad \text{in } \Omega, \\ \mathbf{E} \times \mathbf{n} &= \mathbf{0} \quad \text{on } \partial\Omega, \end{cases}$$

and post-process $\mathbf{H} \simeq \nabla \times \mathbf{E}$.

Variational formulation

Find $\mathbf{E} \in H_0(\mathbf{curl}, \Omega)$ such that

$$b(\mathbf{E}, \mathbf{v}) = (\mathbf{J}, \mathbf{v})$$

for all $\mathbf{v} \in H_0(\mathbf{curl}, \Omega)$.

$$b(\mathbf{E}, \mathbf{v}) = -i\omega\mu_0(\sigma \mathbf{E}, \mathbf{v}) + (\nabla \times \mathbf{E}, \nabla \times \mathbf{v})$$

Finite element discretization

We consider first-order Nédélec elements on a mesh $\mathcal{T}_h = \{K\}$:

$$\mathbf{V}_h = \left\{ \mathbf{v}_h \in H_0(\mathbf{curl}, \Omega) \mid \mathbf{v}_h|_K = \mathbf{a} \times \mathbf{x} + \mathbf{b}, \mathbf{a}, \mathbf{b} \in \mathbb{C}^3 \right\}.$$

Discrete variational formulation

Find $\mathbf{E}_h \in \mathbf{V}_h$ such that

$$b(\mathbf{E}_h, \mathbf{v}_h) = (\mathbf{J}, \mathbf{v}_h)$$

for all $\mathbf{v}_h \in \mathbf{V}_h$.

Error estimate

Assuming that $\mathbf{J} \in H(\mathbf{div}, \Omega)$

Regularity

$$\mathbf{E} \in \left(H^{1/2-\varepsilon}(\Omega)\right)^3 \quad \nabla \times \mathbf{E} \in \left(H^1(\Omega)\right)^3$$

Interpolation error

$$\|\mathbf{E} - \boldsymbol{\pi}_h \mathbf{E}\|_{0,\Omega} \lesssim h^{1/2-\varepsilon} \|\mathbf{J}\|_{\mathbf{div},\Omega} \quad \|\nabla \times (\mathbf{E} - \boldsymbol{\pi}_h \mathbf{E})\|_{0,\Omega} \lesssim h \|\mathbf{J}\|_{0,\Omega}$$

Error estimate (Céa's Lemma)

$$\|\mathbf{E} - \mathbf{E}_h\|_{0,\Omega} + \|\nabla \times (\mathbf{E} - \mathbf{E}_h)\|_{0,\Omega} \lesssim h^{1/2-\varepsilon} \|\mathbf{J}\|_{\mathbf{div},\Omega}$$

Error estimate

We have

Interpolation error

$$\|\nabla \times (\mathbf{E} - \boldsymbol{\pi}_h \mathbf{E})\|_{0,\Omega} \lesssim h \|\mathbf{J}\|_{\text{div},\Omega}$$

but only

Error estimate

$$\|\nabla \times (\mathbf{E} - \mathbf{E}_h)\|_{0,\Omega} \lesssim h^{1/2-\varepsilon} \|\mathbf{J}\|_{\text{div},\Omega}$$

Can we improve this error estimate? Yes!

A modified “Aubin-Nitsche trick” (Step 1)

Let $\phi \in H_0(\mathbf{curl}, \Omega)$ solve

$$b(w, \phi) = (\nabla \times w, \nabla \times (\mathbf{E} - \mathbf{E}_h)), \quad \forall w \in H_0(\mathbf{curl}, \Omega),$$

so that

$$\|\nabla \times (\mathbf{E} - \mathbf{E}_h)\|_{0,\Omega}^2 = b(\mathbf{E} - \mathbf{E}_h, \phi) = b(\mathbf{E} - \mathbf{E}_h, \phi - \pi_h \phi).$$

Intermediate error estimate

$$\|\nabla \times (\mathbf{E} - \mathbf{E}_h)\|_{0,\Omega}^2 \lesssim h^{1/2-\varepsilon} (\|\phi - \pi_h \phi\|_{0,\Omega} + \|\nabla \times (\phi_h - \pi_h \phi)\|_{0,\Omega}) \|J\|_{\mathbf{div}, \Omega}.$$

It remains to estimate the interpolation errors in the right-hand side.

A modified “Aubin-Nitsche trick” (Step 2)

Since $\phi \in H_0(\mathbf{curl}, \Omega)$ is solution to

$$b(w, \phi) = (\nabla \times w, \nabla \times (E - E_h)), \quad \forall w \in H_0(\mathbf{curl}, \Omega),$$

we can show that $\phi \in \left(H^{1/2-\varepsilon}(\Omega)\right)^3$ with

$$|\phi|_{1/2-\varepsilon, \Omega} \lesssim \|\nabla \times (E - E_h)\|_{0, \Omega}.$$

First interpolation error estimate

$$\|\phi - \pi_h \phi\|_{0, \Omega} \lesssim h^{1/2-\varepsilon} \|\nabla \times (E - E_h)\|_{0, \Omega}$$

A modified “Aubin-Nitsche trick” (Step 3)

We have

$$\|\nabla \times \phi\|_{0,\Omega} \lesssim \|\nabla \times (\mathbf{E} - \mathbf{E}_h)\|_{0,\Omega},$$

but we cannot extend this result to

$$|\nabla \times \phi|_{s,\Omega}$$

for $s > 0$, because $\nabla \times (\mathbf{E} - \mathbf{E}_h) \notin H(\mathbf{curl}, \Omega)$.

We cannot estimate

$$\|\nabla \times (\phi - \pi_h \phi)\|_{0,\Omega}$$

directly.

A modified “Aubin-Nitsche trick” (Step 3)

We introduce $\psi = \phi - (\mathbf{E} - \mathbf{E}_h)$. We see that

$$\begin{aligned} b(\mathbf{v}, \psi) &= b(\mathbf{v}, \phi) - b(\mathbf{v}, \mathbf{E} - \mathbf{E}_h) \\ &= (\nabla \times \mathbf{v}, \nabla \times (\mathbf{E} - \mathbf{E}_h)) - b(\mathbf{v}, \mathbf{E} - \mathbf{E}_h) \\ &= -i\omega\mu_0(\mathbf{v}, \sigma(\mathbf{E} - \mathbf{E}_h)). \end{aligned}$$

We can show that $\nabla \times \psi \in (H^1(\Omega))^3$ with

$$|\nabla \times \psi|_{1,\Omega} \lesssim \|\mathbf{E} - \mathbf{E}_h\|_{0,\Omega}.$$

Intermediate interpolation result

$$\|\nabla \times (\psi - \pi_h \psi)\|_{0,\Omega} \lesssim h \|\mathbf{E} - \mathbf{E}_h\|_{0,\Omega} \lesssim h^{3/2-\varepsilon} \|J\|_{\text{div},\Omega}.$$

A modified “Aubin-Nitsche trick” (Step 3)

We have $\phi = E - E_h + \psi$, thus

$$\phi - \pi_h \phi = E - \pi_h E - \underbrace{(E_h - \pi_h E_h)}_{=0} + \psi - \pi_h \psi,$$

and

$$\|\nabla \times (\phi - \pi_h \phi)\|_{0,\Omega} \lesssim \|\nabla \times (E - \pi_h E)\|_{0,\Omega} + \|\nabla \times (\psi - \pi_h \psi)\|_{0,\Omega}.$$

Second interpolation error estimate

$$\|\nabla \times (\phi - \pi_h \phi)\|_{0,\Omega} \lesssim \left(h + h^{3/2-\varepsilon} \right) \|J\|_{\text{div},\Omega}.$$

A modified “Aubin-Nitsche trick” (Step 4)

We regroup the intermediate results:

$$\|\nabla \times (\mathbf{E} - \mathbf{E}_h)\|_{0,\Omega} \lesssim h^{1/2-\varepsilon} \left(h^{1/2-\varepsilon} + h + h^{3/2-\varepsilon} \right) \|\mathbf{J}\|_{\text{div},\Omega}.$$

Error estimate

$$\|\nabla \times (\mathbf{E} - \mathbf{E}_h)\|_{0,\Omega} \lesssim h^{1-2\varepsilon} \|\mathbf{J}\|_{\text{div},\Omega}$$

We can remove the “ -2ε ” if we employ “non-Sobolev” interpolation spaces.

The convergence rate is linear, as in the case of fitting meshes.

Surprisingly $\nabla \times \mathbf{E}_h$ converges faster than \mathbf{E}_h in $L^2(\Omega)$ -norm!

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2.5D Maxwell's equations

We assume that $\sigma(x, y, z) = \sigma(x, z)$.

$$\hat{\mathbf{H}}_a^\xi(x, z) = \int_{y=-\infty}^{+\infty} \mathbf{H}_a(x, y, z) e^{-i\xi y} dy, \quad \hat{\mathbf{H}}^\xi = (\hat{H}_x^\xi, \hat{H}_y^\xi)$$

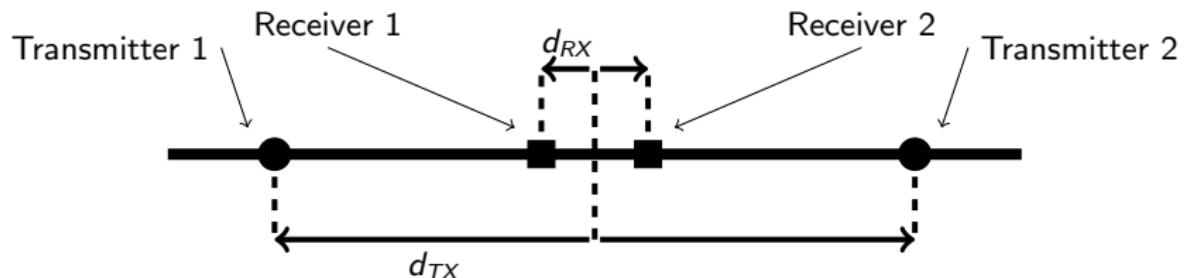
2.5D formulation

Find $(\hat{H}_y^\xi, \hat{\mathbf{H}}^\xi) \in H^1(\hat{\Omega}) \times H(\text{curl}, \hat{\Omega})$

$$\begin{cases} (i\omega\mu_0 + \xi^2\sigma^{-1})\hat{\mathbf{H}}^\xi + \text{curl} \left(\sigma^{-1} \text{curl } \hat{\mathbf{H}}^\xi \right) - i\xi\sigma^{-1}\nabla H_y^\xi &= \mathbf{M}^\xi, \\ i\omega\mu_0 H_y^\xi - \nabla \cdot (\sigma^{-1}\nabla \mathbf{H}_y^\xi) + i\xi\nabla \cdot (\sigma^{-1}\mathbf{H}^\xi) &= M_y^\xi. \end{cases}$$

$$\mathbf{H}_a(x, y, z) \simeq \sum_{j=1}^N w_j \hat{\mathbf{H}}_a(x, z) e^{i\xi_j y}$$

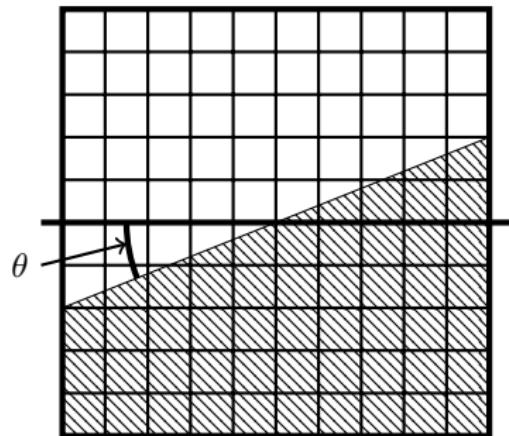
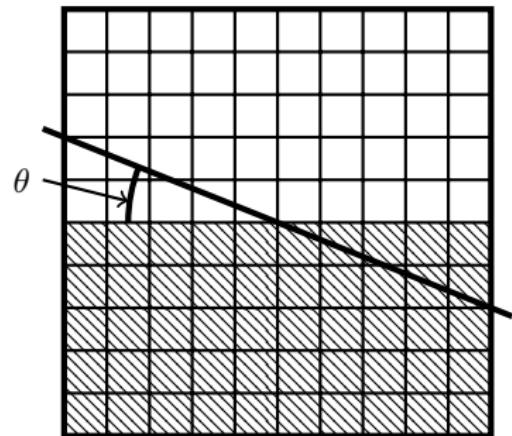
Logging tool



Logging tool
($d_{TX} = 0.6$ m, $d_{RX} = 0.1$ m)

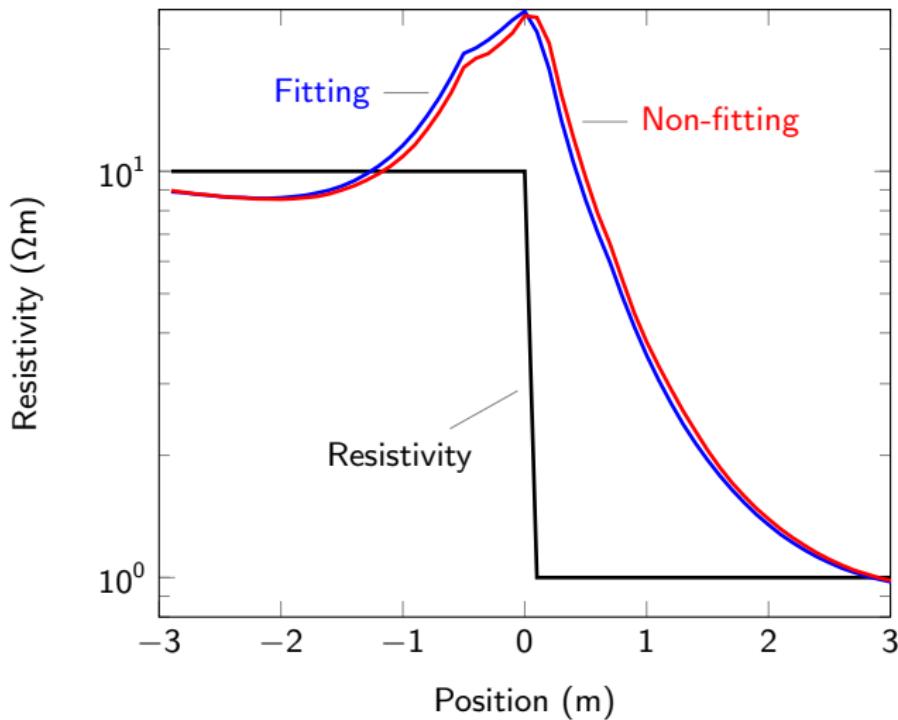
$$P_{log} = \frac{1}{2} \operatorname{Im} \left(\log \frac{\mathbf{H}_{zz}^{11}}{\mathbf{H}_{zz}^{12}} + \log \frac{\mathbf{H}_{zz}^{22}}{\mathbf{H}_{zz}^{21}} \right) \quad A_{log} = \frac{1}{2} \operatorname{Re} \left(\log \frac{\mathbf{H}_{zz}^{11}}{\mathbf{H}_{zz}^{12}} + \log \frac{\mathbf{H}_{zz}^{22}}{\mathbf{H}_{zz}^{21}} \right)$$

Benchmark

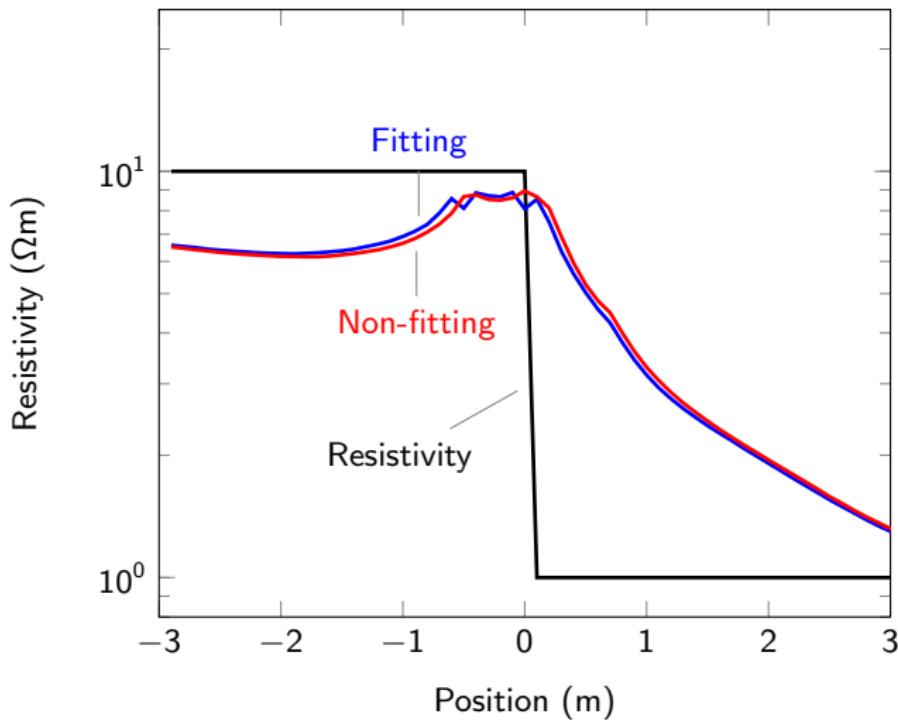


Fitting (left) and non-fitting (right) meshes

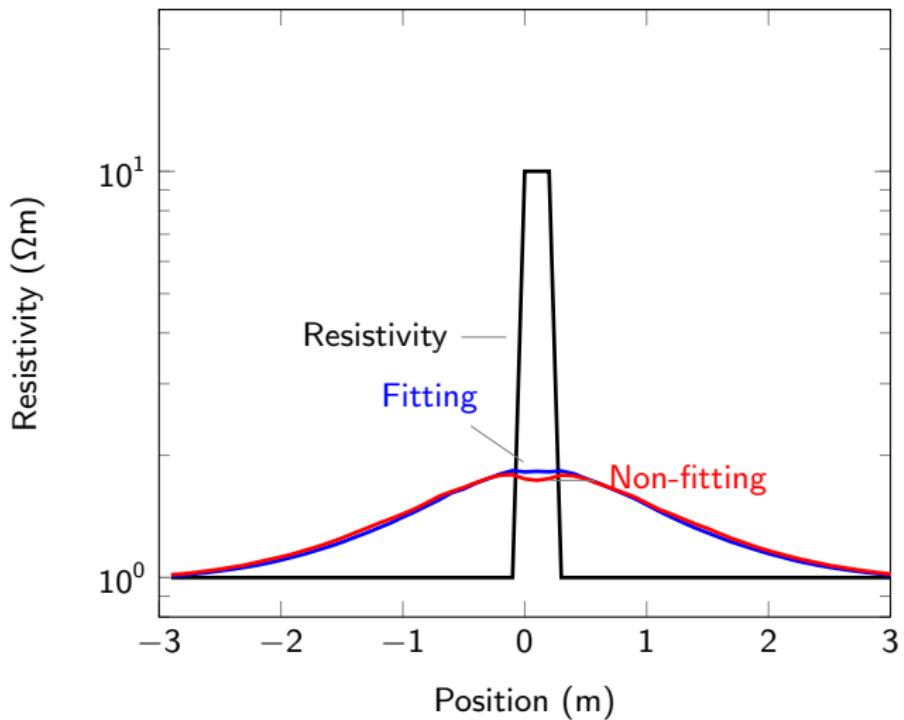
Two layers: Apparent resistivity (P-log)



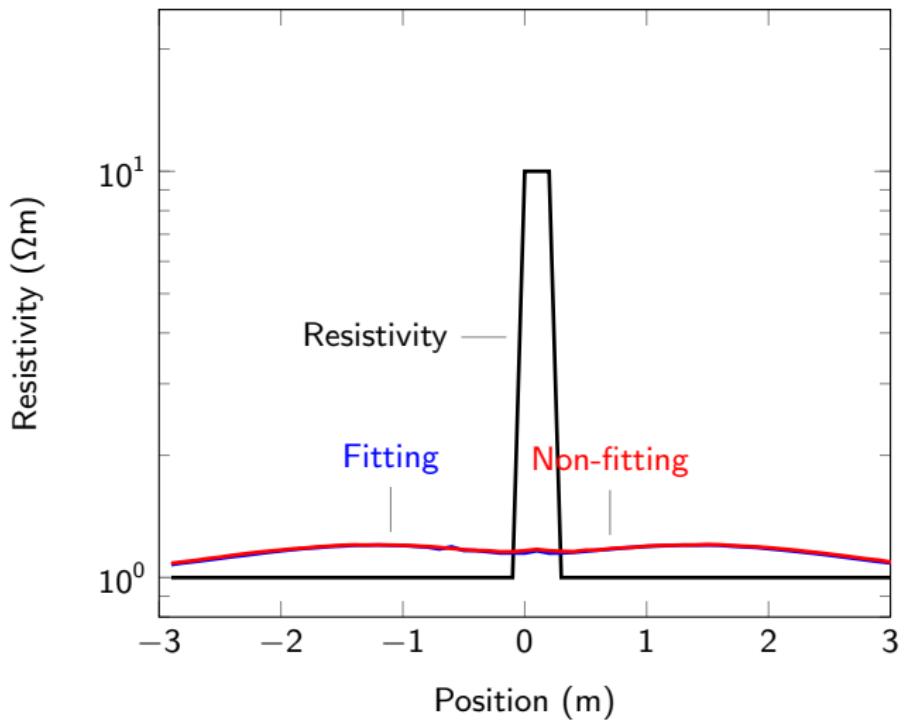
Two layers: Apparent resistivity (A-log)



Thin layer: Apparent resistivity (P-log)



Thin layer: Apparent resistivity (A-log)



Outline

- 1 Maxwell's equations
- 2 The magnetic formulation
- 3 The electric formulation
- 4 Numerical results
- 5 Conclusions

Main abstract results

Assume that $\mu = \mu_0$ is constant.

Assume that σ represents a layered media.

Assume that \mathbf{J} and \mathbf{M} are sufficiently smooth.

Assume that the linear system coefficients are exactly integrated.

Error estimates for fitting meshes

$$\|\mathbf{E} - \mathbf{E}_h\|_{L^2(\Omega)} = \mathcal{O}(h) \quad \|\mathbf{H} - \mathbf{H}_h\|_{L^2(\Omega)} = \mathcal{O}(h)$$

Error estimates for non-fitting meshes

$$\|\mathbf{E} - \mathbf{E}_h\|_{L^2(\Omega)} = \mathcal{O}(h^{1/2}) \quad \|\mathbf{H} - \mathbf{H}_h\|_{L^2(\Omega)} = \mathcal{O}(h)$$

Main abstract results

The use of non-fitting meshes deteriorates the convergence rate of E_h .

The convergence rate of H_h is preserved.



T. Chaumont-Frelet, S. Nicaise and D. Pardo
SIAM J. Numerical Analysis 2018

Non-fitting meshes can simulate tools based on magnetic field measurements.

Main numerical results

We have tested different quadrature schemes.

Non-fitting meshes provide accurate results with an appropriate scheme.



T. Chaumont-Frelet, D. Pardo and A. Rodríguez-Rozàs
Computational Geosciences 2018