

HHO-DFN : a Hybrid High Order (HHO) method for the simulation of flow in large tridimensional Discrete Fracture Networks (DFN)

Florent Hédin, Géraldine Pichot, and Alexandre Ern

INRIA Paris, team SERENA

In fractured rocks, fluid flows mostly within a complex arrangement of fractures, classically modeled as a Discrete Fracture Network (DFN) [1]. In this model, the fractures are distributed in a three-dimensional domain and are modeled as ellipses whose position and orientation are evaluated from statistical laws given by geological studies. For large size production networks provided by external industrial partners, the network under study may contain millions of fractures generating millions of intersections (Figure 1, left). Solving flow within these large 3D DFNs requires robust and efficient numerical methods and software (Figure 1, right).

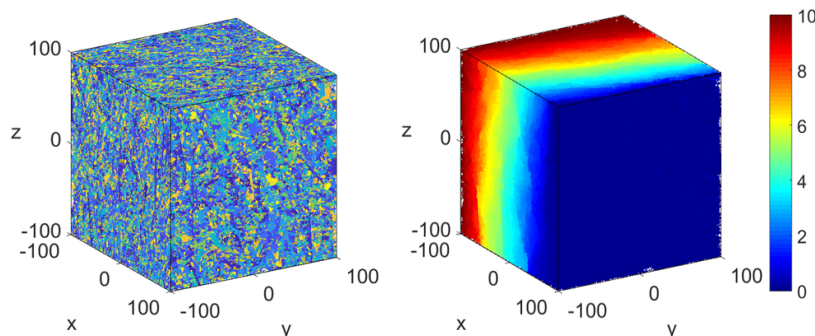


Figure 1: A DFN with 1,176,566 fractures and 2,410,537 of intersections. Fractures are modeled using ellipses (left), the hydraulic head (right) is computed with the NEF-Flow software, which implements the Mixed-Hybrid Finite Element Method for DFNs.

So far, the hydraulic head and velocity fields have been computed using the Mixed-Hybrid Finite Element (MHFEM) method, by the means of the software NEF-Flow [2]. As the flow in fractured rocks is highly channelled, a reduction of the computational cost is achieved by meshing the fractures independently, the fractures with the fine meshes are the ones that carry most of the flow. It yields non matching meshes at the intersections between fractures [3]. To further gain in flexibility in the mesh generation, in the management of non conforming meshes and in the number of degrees of freedom, we propose to solve the same problem with a HHO (Hybrid High Order) method[4]. HHO is closely related to Hybridizable Discontinuous Galerkin methods (HDG)[5], allowing general meshes (including polytopal cells and nonmatching interfaces) and arbitrary polynomial orders.

A HHO method for DFNs has been implemented in the C++ software library HHO-DFN. HHO capabilities are provided by the `Disk++` library [6], the meshes are produced by the `BLSURF_FRAC` software developed by the Inria's GAMMA3 team [7]. Test cases of increasing complexity will be shown to compare the results obtained with the two software, NEF-Flow and HHO-DFN, both in terms of performance and accuracy.

References

- [1] J. Erhel, J. de Dreuzy, and B. Poirriez. Flow simulation in three-dimensional discrete fracture networks. *SIAM Journal on Scientific Computing*, 31(4):2688–2705, 2009.
- [2] P. Laug and G. Pichot. Mesh Generation and Flow Simulation in Large Tridimensional Fracture Networks. <https://hal.inria.fr/hal-02102811>, 2019.
- [3] G. Pichot, J. Erhel, and J.-R. de Dreuzy. A generalized mixed hybrid mortar method for solving flow in stochastic discrete fracture networks. *SIAM Journal on Scientific Computing*, 34(1):B86–B105, 2012.
- [4] D.A Di Pietro, A. Ern, and S. Lemaire. An arbitrary-order and compact-stencil discretization of diffusion on general meshes based on local reconstruction operators. *Computational Methods in Applied Mathematics*, 14(4):461–472, 2014.
- [5] B. Cockburn, D.A. Di Pietro, and A. Ern. Bridging the hybrid high-order and hybridizable discontinuous galerkin methods. *ESAIM: M2AN*, 50(3):635–650, 2016.
- [6] M. Cicuttin, D.A. Di Pietro, and A. Ern. Implementation of discontinuous skeletal methods on arbitrary-dimensional, polytopal meshes using generic programming. *Journal of Computational and Applied Mathematics*, 344:852 – 874, 2018.
- [7] H. Borouchaki, P. Laug, and P. L. George. Parametric surface meshing using a combined advancing-front generalized-delaunay approach. *International Journal for Numerical Methods in Engineering*, 49(1-2):233–259, 2000.