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# Cellular Automata and Discrete Complex Systems

25th IFIP WG 1.5 International Workshop, AUTOMATA 2019 Guadalajara, Mexico, June 26–28, 2019 Proceedings



Editors
Alonso Castillo-Ramirez 
University of Guadalajara
Guadalajara, Mexico

Pedro P. B. de Oliveira

Mackenzie Presbyterian University
Sao Paulo, Brazil

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#### **Preface**

This volume contains the full papers presented at the 25th International Workshop on Cellular Automata (CA) and Discrete Complex Systems (DCS), AUTOMATA 2019, held during June 26–28, 2019, at the University Center of Exact Sciences and Engineering, CUCEI, University of Guadalajara, México. The conference was the official annual event of the International Federation for Information Processing (IFIP), Working Group 5, on CA and DCS, of the Technical Committee 1, on Foundations of Computer Science.

AUTOMATA 2019 is part of an annual series of conferences established in 1995 as a collaboration forum between researchers in CA and DCS. Current topics of the conference include, but are not limited to, dynamical, topological, ergodic, and algebraic aspects of CA and DCS, algorithmic and complexity issues, emergent properties, formal languages, symbolic dynamics, tilings, models of parallelism and distributed systems, timing schemes, synchronous versus asynchronous models, phenomenological descriptions, scientific modeling, and practical applications.

AUTOMATA 2019 was the fourth time that the conference took place in a Latin American country, the last time having been in Santiago, Chile, in 2011. As a celebration of its Silver Jubilee, the event had a special session commemorating the date. We had three invited talks given by Pablo Arrighi, Hector Zenil, and Tullio Ceccherini-Silberstein; we sincerely thank them for accepting the invitation and their very interesting presentations. The abstracts of the invited talks are included here.

We received ten submissions as full papers for the conference. Each submission was reviewed by three members of the Program Committee. Based on these reviews and an open discussion, seven papers were accepted to be presented at the conference and to be included in the proceedings. We thank all authors for their contributions and hard work that made this event possible.

The conference program also involved short presentations of exploratory papers that are not included in these proceedings, and we wish to extend our thanks to the authors of the exploratory submissions.

We are indebted to the Steering Committee, Program Committee, and additional reviewers for their valuable help during the last months. We acknowledge the generous funding provided by the IFIP and the University of Guadalajara toward the organization of this event.

We are very grateful for the support of the local Organizing Committee and the authorities of CUCEI; in particular, our sincere thanks to Dr. Humberto Gutiérrez Pulido, Head of the Department of Mathematics, and to Dr. Ruth Padilla Muñoz, Rector of CUCEI. Finally, we acknowledge the excellent cooperation from the *Lecture Notes in Computer Science* team of Springer for their help in producing this volume in time for the conference.

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# Quantum Cellular Automata: Computability and Universality

#### P. Arrighi

Aix-Marseille University, Université de Toulon, CNRS, LIS, Marseille, and IXXI, Lyon, France pablo.arrighi@univ-amu.fr

This talk will provide an overview of the field of Quantum Cellular Automata (QCA). QCA consist in an array of identical finite-dimensional quantum systems. The whole array evolves in discrete time steps by iterating a linear operator G. Moreover this global evolution G is shift-invariant (it acts everywhere in the same way), causal (information cannot be transmitted faster than some fixed number of cells per time step), and unitary (the condition required by the postulate of evolutions in quantum theory, akin to reversibility). I will motivate their studies through the distinction between finite-dimensional quantum evolutions (unitary matrices, a.k.a quantum automata) and infinite-dimensional quantum evolutions (unitary operators, a.k.a operators). It turns out that basic questions surrounding computability and universality had only been solved for quantum automata, and not for quantum operators. We addressed the question of their computability in [1], building upon a structural theorem that actually decomposes the axiomatic version of OCA [9, 17] into an infinitely repeating quantum circuit of local gates [7, 8], which we will recall. We also addressed the question of the universality of quantum operators in a series of works [2–4] which we will summarize, and relate to the fascinating question of how much particle physics can be recast in terms of QCA [6, 10, 13, 18]. In all of these results, quantum information flows on a fixed, classical background: a rigid grid. Could the background itself be quantum? We will touch on this rather topical question [5, 11, 16].

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# Algorithmic Information Dynamics Reconstructs the Space-Time and Phase-Space Dynamics of Cellular Automata

Hector Zenil<sup>1,2,3</sup>, Narsis A. Kiani<sup>2,3</sup>, Jesper Tegnér<sup>3,4,5</sup>

Oxford Immune Algorithmics, Oxford, UK
Algorithmic Dynamics Lab, Karolinska Institute, Karolinska University Hospital, Stockholm, Sweden

hector.zenil@algorithmicnaturelab.org https://www.hectorzenil.net/

<sup>3</sup> Unit of Computational Medicine, Center for Molecular Medicine, Karolinska Institute, Stockholm, Sweden

<sup>4</sup> BESE Division, King Abdullah University of Science and Technology (KAUST), Kingdom of Saudi Arabia

<sup>5</sup> Algorithmic Nature Group, LABORES for the Natural and Digital Sciences, Paris, France

https://algorithmicnature.org/

Current widely used tools for causal inference are mostly based on classical statistics, including machine learning, and draw heavily upon statistical patterns in data, leading to inverse problems that involve estimating parameters such as initial conditions and boundary conditions on the basis of observed data. These inverse problems are often ill posed, not only because correlation does not imply causation, but also more importantly, because causation does not imply correlation, on which most of our current data-driven inference techniques are based. This means that in most, if not all physical and biological instances, it is not only difficult to infer stable states from noisy and chaotic data, but also that such inferences are impossible to make even in principle, when using traditional statistically-based tools to find true generating mechanisms. Here we show how the new theory of Algorithmic Information Dynamics [1, 2] can help reconstruct dynamical systems in application to one of the most studied discrete computational model, cellular automata. We will show how the theory of algorithmic probability can find reconstructions of space-time evolutions and phase-space land-scapes of these representative discrete dynamical systems.

At the core of Algorithmic Information Dynamics [1, 2], the algorithmic causal calculus that we introduced, is the quantification of the change of complexity of a system under natural or induced perturbations, particularly the direction (sign) and magnitude of the difference of algorithmic information approximations denoted by C between an object G, such a cellular automaton or a graph and its mutated version G', e.g. the flip of a cell bit (or a set of bits) or the removal of an edge e from G (denoted by  $G \setminus e = G'$ ). The difference  $|C(G) - C(G \setminus e)|$  is an estimation of the shared algorithmic

mutual information of G and  $G \setminus e$ . If e does not contribute to the description of G, then  $|C(G) - C(G \setminus e)| \le \log_2 |G|$ , where |G| is the uncompressed size of G, i.e. the difference will be very small and at most a function of the graph size, and thus C(G) and  $C(G \setminus e)$  have almost the same complexity. If, however,  $|C(G) - C(G \setminus e)| < log_2 |G|$ bits, then G and  $G \setminus e$  share at least n bits of algorithmic information in element e, and the removal of e results in a loss of information. In contrast, if  $C(G) - C(G \setminus e) > n$ , then e cannot be explained by G alone, nor is it algorithmically not contained/derived from G, and it is therefore a fundamental part of the description of G, with e as a generative causal mechanism in G, or else it is not part of G but has to be explained independently, e.g. as noise. Whether it is noise or part of the generating mechanism of G depends on the relative magnitude of n with respect to C(G) and to the original causal content of G itself. If G is random, then the effect of e will be small in either case, but if G is richly causal and has a very small generating program, then e as noise will have a greater impact on G than would removing e from an already short description of G. However, if  $|C(G) - C(G \setminus e)| \le \log_2 |G|$ , where |G| is e.g. the vertex count of a graph, or the runtime of a cellular automaton, G, then e is contained in the algorithmic description of G and can be recovered from G itself (e.g. by running the program from a previous step until it produces G with e from  $G \setminus e$ ).

We show how we can infer and reconstruct space-time evolutions by quantification of the disruptiveness of a perturbation. We can then extract the generating mechanism from the ordered time indices, from least to most disruptive and produce candidate generating models. Simpler rules have simpler hypotheses, with an almost perfect correspondence in row order. Some systems may look more disordered than others, but locally the relationship between single rows is mostly preserved (indicating local reversibility).

We show that the later in time a perturbation is injected into a dynamical system the less it contributes to the algorithmic information content of the overall space-time evolution. We then move from discrete 2D systems to the reconstruction of phase spaces and space-time evolutions of *N*-dimensional, continuous, chaotic, incomplete and even noisy dynamical systems.

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# The Garden of Eden Theorem: From Cellular Automata to Algebraic Dynamical Systems

#### Tullio Ceccherini-Silberstein

Dipartimento di Ingegneria, Università del Sannio, 82100 Benevento, Italy tullio.cs@sbai.uniroma1.it

The *Garden of Eden theorem* proved by Moore [5] and Myhill [6] in 1963 is a central result in the Theory of Cellular Automata and Symbolic Dynamics. Let A be a finite set, called the *alphabet*, and let G be a group, called the *universe*. The *configuration space* is  $A^G := \{x: G \to A\} \equiv \prod_{g \in G} A$  equipped with the *prodiscrete topology* (the product topology, where each factor A is discrete) and the G-action defined by  $[gx](h) = x(g^{-1}h)$  for all  $g, h \in G$  and  $x \in A^G$ , called the G-shift. A *cellular automaton* is a map  $\tau: A^G \to A^G$  which is *continuous* and G-equivariant (that is,  $\tau(gx) = g\tau(x)$  for all  $g \in G$  and  $x \in A^G$ ). Two configurations  $x, y \in A^G$  are said to be *almost equal* if there exists a finite subset  $F \subset G$  such that x(g) = y(g) for all  $g \in G \setminus F$ . Clearly, almost equality is an equivalence relation. One then says that a map  $\tau: A^G \to A^G$  is *pre-injective* if its restriction to each almost equality class is injective. We are now in position to state the Garden of Eden theorem of Moore and Myhill.

#### Theorem 1 (Garden of Eden theorem of Moore and Myhill (1963)).

Let A be a finite set, let  $G = \mathbb{Z}^d$  (the free Abelian group of rank  $d \ge 1$ ), and let  $\tau : A^G \to A^G$  be a cellular automaton. Then  $\tau$  is surjective if and only if it is pre-injective.

A group G is amenable if it admits a finitely additive left-invariant probability measure on the set  $\mathcal{P}(G)$  of all subsets of G, that is, a map  $\mu:\mathcal{P}(G)\to [0,1]$  such that  $\mu(A\cup B)=\mu(A)+\mu(B)-\mu(A\cap B)$ ,  $\mu(gA)=\mu(A)$  (here  $gA=\{ga:a\in A\}\subset G\}$  for all  $A,B\subset G$  and  $g\in G$ , and  $\mu(G)=1$ . The class of amenable group contains all finite groups, all Abelian groups, and more generally all solvable groups, and all finitely generated groups of sub-exponential growth, and it is closed under the operations of taking: subgroups, quotients, extensions, and direct limits. On the other hand, the free group  $\mathbb{F}_2$  of rank 2 (and therefore every group containing a non-Abelian free subgroup) is not amenable. In 1999 we proved the following:

### Theorem 2 (Garden of Eden theorem for amenable groups [3], see also [1]).

Let A be a finite set, let G be an amenable group, and let  $\tau: A^G \to A^G$  be a cellular automaton. Then  $\tau$  is surjective if and only if it is pre-injective.

Gromov [4] suggested that the Garden of Eden theorem could be extended to dynamical systems with a suitable hyperbolic flavor.

A dynamical system is a pair (X, G) where X is a compact metrizable space, called the *phase space* and G is a *countable* group acting on X by homeomorphisms. A continuous map  $\tau: X \to X$  which is G-equivariant (that is, commuting with the

action of G) is called an *endomorphism* of the dynamical system (X, G). One says that two points  $x, y \in X$  are *homoclinic* if for every  $\varepsilon > 0$  there exists a finite subset  $F \subset G$  such that  $d(gx, gy) < \varepsilon$  for all  $g \in G \setminus F$ . Homoclinicity is an equivalence relation: a map  $\tau : X \to X$  which is injective on each homoclinicity class is called *pre-injectice*. One says that (X, G) satisfies the Garden of Eden theorem if every endomorphism is surjective if and only if it is pre-injective.

Example 1. Let A be a finite set and let G be a countable group. Then  $(A^G, G)$ , where G acts on the configuration space  $A^G$  by the G-shift. Moreover  $x, y \in A^G$  are homoclinic if an only if they are almost equal. As a consequence, the two notions of pre-injectivity coincide. Finally, the endomorphisms of  $(A^G, G)$  are precisely the cellular automata.

Let now G be a countable group and denote by  $\mathbb{Z}[G]$  the *integral group ring* of G, that is, the ring consisting of all *finite* sums  $\sum_{g \in G} a_g \mathbf{g}$  with  $a_g \in \mathbb{Z}$ . Let  $f \in \mathbb{Z}[G]$  and denote by  $M_f := \mathbb{Z}[G]/f\mathbb{Z}[G]$  the quotient of  $\mathbb{Z}[G]$  by the principal ideal generated by f. Note that  $M_f$  is a  $\mathbb{Z}[G]$ -module. Its *Pontryagin dual*  $X_f := \widehat{M_f}$  is a compact metrizable Abelian group which is also a  $\mathbb{Z}[G]$ -module, and therefore, in particular, bears an action of G. The dynamical system  $(X_f, G)$  is called the *principal algebraic dynamical system* associated with the *polynomial*  $f \in \mathbb{Z}[G]$  (see [7]). In 2018 we proved the following:

#### Theorem 3 (Garden of Eden theorem for Harmonic Models [2]).

Let G be a countable Abelian group not virtually  $\mathbb{Z}^d$  for d=1,2. Suppose that  $f=\sum_g f_g \mathbf{g} \in \mathbb{Z}[G]$  satisfies the following conditions: (i)  $\sum_{g\in G} f_g = 0$ , (ii)  $f_g \leq 0$  for all  $g\in G\setminus\{1_G\}$ , (iii)  $f_g=f_{g^{-1}}$  for all  $g\in G$  (i.e., f is self-adjoint), (iv) supp  $(f):=\{g\in G: f_g\neq 0\}$ , the support of f, generates the group G, and (v) f is primitive, that is, there is no integer  $m\geq 2$  dividing all coefficients of f. Then  $(X_f,G)$  satisfies the Garden of Eden Theorem.

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