# The fall of the reentry paradigm of cardiac fibrillation Guillaume Attuel, Hussein Yahia 

## To cite this version:

Guillaume Attuel, Hussein Yahia. The fall of the reentry paradigm of cardiac fibrillation. Second international Summer Institute on Network Physiology, Jul 2019, Como, Italy. , 2019. hal-02294891

HAL Id: hal-02294891
https://inria.hal.science/hal-02294891
Submitted on 23 Sep 2019

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Keywords: Atrial fibrillation, Random multifractal noise, Gap junction channel capacitance and kinetics, Branching process

## Abstract






 electric recording interpretation from the level of cell-to-cell coupling up to the level of neural modulations.

## Gray area

Is atrial fibrillation characterized only by spatio-temporal irregularity of local beat-to-beat cycles?


Figure 2: (Left) Nervous modulation affecting rhythm and action potential (Center) Artist representation of the cardiac tissue. (Right) Ion channels involved in action potential excitation
This work focuses on the conducting substrate.

## Main assumption

Cardiac pulse excitation and propagation

- Relaxation limit cycles: fast and slow time scales.
- Excitable threshold (saddle node bifurcation).
- Refractory state.
- Reaction diffusion with non diffusive inhibitor


Figure 5: Leff: Phase portrait with the two null-clines for the reactant $v$ and dynamically "slaved" inhibitor $w$. The stable-unstable pair of fixed
points is located at their intersection on the bottom left. Right: Resulting typical time trace of the propagating pulse.

## Pulse:

$$
\begin{aligned}
& \frac{\partial}{\partial t} v=\mu F(v, w)+\kappa \frac{\partial^{2}}{\partial x^{2}} v, \\
& \frac{\partial}{\partial t} w=G(v, w),
\end{aligned}
$$

- $(F, G)$ are acted upon by modulators such as a cholinergic hormone or an anti-arrhythmic drug.
- Cell-to-cell coupling affects $\kappa \frac{\partial^{2}}{\partial x^{2}} V$
- Gap junction channels are open pores made of connexins.
- They ensure homeostasis and electrical conduction: The connected substrate of cardiac cells is a syncytium.

What about the significance of large amplitude fluctuations?


Figure 4: Left: Probability distribution function of bipolar electrogram amplitudes. Right: Table of measured Fisher exponents over various areas
of the atria.


- Spatio-temporal irregularity versus self similar amplitude fluctuations: Weak versus Strong turbulence ("intensive" versus "extensive" chaos)?

What if gap junction channels close at wrong timings as pulse are crossing?


Figure 6: Left: Cardiac tissue microscopy exhibiting intercalated discs (arrows), where GICs reside. Right: Fluoroscopy
setting exhibiting connexins (Cx43 and Cx45).

The answer is that they charge capacitively


$$
\begin{array}{lclc}
\text { Capacitance } & \delta Q & = & C \delta v \\
\text { Conductance } J & = & -G \delta v \\
\text { Discharge } & C_{\perp} \partial_{t} v & = & \text { Pulse }-\delta(G \delta v)
\end{array}
$$

Zeroth order kinetics dependence on capacitive charge for the abnormally desynchronizing gap junction channels :
$\partial_{t} G \propto \delta Q$
An instability is triggered which leads to chaotic conductance and parallel electric field.

## Sketch of calculation steps

A quick linear analysis of the instability shows that the growth rate is

This implies that the abnormal current grows like $\delta_{i} J(s)=e^{\omega_{i} s} \delta_{j} J(0)$. The instability occurs at discrete times $t_{i}$ whenever $\omega_{i}>0$ and lasts typically $\tau_{i} \sim \tau_{r}$ where $\tau_{r} \approx 100 \mathrm{~ms}$ is the refractory period scale. The phase $\varphi$ of
the pulse train becomes random and the pulse train becomes random and singular at $t_{i}$. The number of back-scattered pulses along an initially excite pulse is $N(s)=\frac{1}{2 \pi} \int^{s}\|\delta \varphi\|$ at time $s$ (in Lagrangian coordinates following the initial pulse) and scales like $N(s) \sim \ln (s)$ because the phase is a (regularized) 2D random free field. To show this, we write a Lagrangian for
the pulse train scalar $\Psi$ in the form the pulse train scalar $\Psi$ in the form
$\mathcal{L}=\int \Psi^{*}\left(\partial_{t}+c \partial_{x}\right)\left(\partial_{t}-c \partial_{x}\right) \psi$,
where $c$ is the conduction velocity, then applying a Wick
rotation $t \rightarrow i t$. Thus, we have a random variable which behaves in law like:

where $\sigma_{a}$ is a random variable with probability distribution that of initial conditions $\delta J(0)$. This is sufficient to obtain the multifractal spectrum where the micro-canonical scaling exponents are defined as
$h=\frac{1}{N} \sum_{i}^{N} \omega_{i} \tau_{i}$.
The multifractal parameter $c_{2}$ is directly related to $\left\langle\delta v^{2}\right\rangle$, and is a coupling constant for the phase field, the probability distribution of which is
$p(\delta \varphi) \sim e^{\frac{1}{c_{2}} \int\|\delta \varphi\|^{2}}$

## Results

- Experimental and model energy time series exhibit a random multifractal noise nature above the beat-to-beat scale.


Figure 7: Time series, wavelet scalogram of the energy $E=\left(\partial_{t} \delta V\right)^{2}$ computed from the atria, with its maxima skeleton

## Revising the reentry paradigm

The paradigm of fibrillation assumes multiple reentry pathways for cardiac pulses take place. This paradigm is compatible with the highly irregular timely patterns.

- Random defects, which are the spiral wave cores, reset the pulse train phase origins locally.
- A statistical mixture of the phase resetting (hyperbolic) attractor ensues.
- The amplitude distribution is not strongly affected.
- It is well known that atrial fibrillation has a Fourier signature very close to white noise.
However, it fails to properly describe large amplitude fluctuations and multifractal spectra. A truly "infinite" dimensional (non-hyperbolic) attractor is required for log-normal scaling? What does a multifractal spectrum entails?

$$
\begin{aligned}
&\left\langle\delta_{a} V^{q}\right\rangle \sim \\
& a^{\tau(q)} \\
& \delta_{a} V \sim \prod_{i}^{N} \Omega_{i}=\exp \left(\sum_{i}^{N} \omega_{i} \ln (a)\right) \\
& h=\quad \frac{1}{N} \sum_{i}^{N} \omega_{i} \\
& p(h)=\exp _{i}(-N I(h)) \\
& D(h) \equiv I_{\text {convex }}(h)=\max _{q}(q h-\tau(q))
\end{aligned}
$$

- Statistical mixture of scaling $h$ at any scale $a$.
- Ultra-metric distance $N \sim \ln \left(\frac{a}{\epsilon}\right)$ (coarse graining $\epsilon$ ).
- Thermodynamical description by large deviations and Legendre transform.
- Canonical entropy (rate function) $I_{\text {convex }}$ related to Hausdorff dimensions $D(h)$ of $h$-singularity sets.

Theoretical answer (c.f. calculation steps section):

- Look for $\delta V \sim e^{\chi}$, with $\chi$ a real 2D "Gaussian free field"


## Conclusions

## In summary

- Evidence and modeling beyond paradigm of multiple reentries.
- In particular AF is a multifractal uncorrelated noise.
- Theory indicates an exponentiation of a "Gaussian free field".
- Model of abnormal capacitive $i_{g}$ builds up at gap junction channels.
The parallel electric potential gradient $\delta v$ drives the instability.
- $\left\langle\delta v^{2}\right\rangle$ determines the multifractal parameter $c_{2}$ as a coupling constant for the "free field".
- The free field is the phase of the pulse train
- Randomness of "volatility" comes from non causal tree nodes i.e. l+1D geometry.
Bipolar electrograms result from the convolution of local currents with the electrode $\Omega_{p}$

$$
\delta V(x)=\int \Omega_{p}\left(x-x^{\prime}\right)\left(i_{m}\left(x^{\prime}\right)-i_{g}\left(x^{\prime}\right)\right) \mathrm{d} x^{\prime}
$$

- The measured potential is modified from classic theory, involving only membrane currents $i_{m}$, by $i_{g}$ !
- Missing bits in the interpretation of data in all previous experiments with AF !
- In practice, all this implies a huge blunder made wit Fourier filtering for ECG Imaging (Cardiolnsight@ and Topera©)!
- $\chi$ is directly related to the phase of pulse trains. The exponentiation comes from the instability, which leads to phase defects at discrete times.
The hierarchical tree structure results from the logarithmic scaling of $\chi$


Figure 9: Instance of a spatio-temporal map of pulses in the l+iD model. The main feature to be noticed is the tree-like structure due to random back-scattered pulses along forward propagating pulses with a high death -
 Figure 10 : ECGI big blunder. Left: 2D snapshot of a simulation of pulses
in the $+1+2 \mathrm{D}$ model. The noisy nature is apparent upon visual inspection. Right: 2D snapshot of the phase of the Fourier filtered field ( 0 :red, $\pi$ :blue). ECG imaging assumes to look for multiple reentries and performs Fourier filtering in that respect. However, as it is demonstrated here, it results in artifacts because of the random phase, as "optical" vortices
which are here characterized by a rotational circulation everywhere, as opposed to a focal flow for the spiraling pulse train away from the core.

## Acknowledgment

G. A. dedicates this work to the late Alain Arneodo, to whom he was grateful, in memory of their many discussions and of his friendship.

## Some open questions

Phenomenology related to random branching process in l+1D modeling.
Properties of the parallel electric potential gradient $\delta V$ (in progress).

- Connections with depinning transition and avalanches.
- Higher dimensional network of cells in modeling.

Adding nervous modulations in modeling.

- Prove $i_{g}$ experimentally.

Find out new drugs acting on $i_{s}$.

- Electrical to structural remodeling (of substrate) in relation to altered transport and fastening of connexins (electro-hydro dynamics) ?


Figure 11: Structural remodeling of substrate : Depletion of connexin expression and fastening at gap junctions after long periods of atrial expression and
fibrillation.

