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Improved energy-aware strategies for periodic real-time tasks under reliability constraints

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Abstract—This paper revisits the real-time scheduling problem recently introduced by Haque, Aydin and Zhu (2017). In this challenging problem, task redundancy ensures a given level of reliability while incurring a significant energy cost. By carefully setting processing frequencies, allocating tasks to processors and ordering task executions, we improve on the previous state-of-the-art approach with an average gain in energy of 20%. Furthermore, we establish the first complexity results for specific instances of the problem.

Index Terms—Real-time systems, energy-aware systems, reliability, mapping, scheduling

I. INTRODUCTION

This work revisits recent results presented by Haque, Aydin and Zhu [1]. We call [1] the reference paper throughout the text. The reference paper deals with the following optimization problem: given a set of independent real-time tasks subject to (possibly different) periodic deadlines, how to execute them on a parallel platform and match all deadlines while minimizing the expected energy consumption? The problem is complicated by the need to enforce some reliability threshold, which is a standard constraint in real-time systems. In Section II below, we provide all details about this optimization problem. We do not provide any further motivation for this study; instead, we refer the reader to the reference paper. We simply point out that energy consumption and reliability have become extremely important and challenging problems in real-time systems.

The paper is organized as follows. Section II provides a detailed description of the optimization problem, and of the three-step approach used in the reference paper to solve it: (i) for each task, compute a set of replicas and their frequencies; (ii) map and statically schedule all replicas onto the platform; (iii) dynamically update the schedule based on actual completion times (instead of worst-case ones) and observed successful executions. These three steps are described in Sections II-B to II-D. Section III provides an example explaining the limitation of the scheduling strategy proposed in [1]. Then Section IV outlines new propositions for each step, as well as a new complexity result that establishes the combinatorial nature of the scheduling step when the mapping is given. Section IV-D summarizes new heuristics, while Section IV-E deals with complexity results. Section V is devoted to a comprehensive experimental comparison of the results of the reference paper against those obtained with our improved approach†. Section VI presents related work, with the aim of complementing the related work already covered in the reference paper: we focus on recent works that quote the reference paper and briefly discuss their contributions. Finally, Section VII gives concluding remarks and hints for future work.

II. PREVIOUS APPROACH

In this section, we present the optimization problem in full details, and we describe the approach of the reference paper. Key notations are summarized in Table I.

A. Optimization problem

The inputs to the optimization problem are a set of real-time tasks, a set of processors and a reliability target:

Tasks – There are \( n \) periodic real-time tasks \( \tau_1, \tau_2, \ldots, \tau_n \). Task \( \tau_i \) has worst-case execution time (WCET) \( c_i \) under the maximum available frequency \( f_{\text{max}} \). Tasks actually complete execution earlier than their estimated WCET: execution times are assumed to be data-dependent and randomly sampled from some probability distribution whose support is upper bounded by the WCET. Task \( \tau_i \) generates a sequence of instances with period \( p_i \). The \( k \)-th instance of task \( \tau_i \) is released at date \( (k - 1)p_i \) and its deadline is \( kp_i \). The whole input pattern repeats every hyperperiod of length \( L := \text{lcm}(p_1, \ldots, p_n) \). Each task \( \tau_i \) has \( \frac{L}{p_i} \) instances within the hyperperiod.

Processors \( \mathcal{P} \) – The platform consists of \( M \) homogeneous processors, with same set \( F \) of frequencies ranging from \( f_{\text{min}} \) to \( f_{\text{max}} \). Without loss of generality, we normalize the frequencies to enforce \( f_{\text{max}} = 1 \). At frequency \( f_j \), a processor needs up to \( c_i f_j \) seconds, which is equal to \( \frac{c_i}{f_j} \) in the reference paper, to complete an instance of task \( \tau_i \). The utilization \( u_i \) of task \( \tau_i \) at frequency \( f_j \) is defined as \( u_i f_j = \frac{c_i f_j}{p_i} \). The utilization of a processor is the sum of the utilizations of all tasks that are assigned to it.

†The algorithms are implemented in C++ and R and the related code, data and analysis are available at https://doi.org/10.6084/m9.figshare.9778319.v1.
Fault model – One considers transient faults, modeled by an exponential distribution with average arrival rate \( \lambda \). It has been proved that the fault rate \( \lambda \) increases when the frequency is scaled down to save energy using DVFS [2]. Let \( \lambda_0 \) denote the fault rate at frequency \( f_{\text{max}} \). Then the fault rate at frequency \( f_i \) is \( \lambda(f_i) = \lambda_0 \times \exp^{\frac{d}{f_{\text{min}}} - 1} \), where \( d \) is the sensitivity factor; \( f_{\text{min}} \) is the lowest frequency we compute the number of replicas needed, the reliability threshold \( \tau \) number of instances of task \( \tau_i \) in the hyperperiod \( f_j \) frequency \( f_j \in F = \{f_{\text{min}}, \ldots, f_{\text{max}} = 1\} \).\( c_{\tau_i f_j} \) WCET for task \( \tau_i \) at frequency \( f_j \) \( u_{\tau_i f_j} \) utilization of task \( \tau_i \) at frequency \( f_j \) \( J_{\text{opt}}(i) \) most energy-efficient frequency for task \( \tau_i \) \( R_i(f_j) \) reliability of one instance of task \( \tau_i \) under \( f_j \) \( R_i \) target reliability threshold for one instance of task \( \tau_i \) \( k_{i,f_j} \) min. replica number for task \( \tau_i \) under \( f_j \) to meet \( R_i \) \( E(\tau_i, f_j, k_{i,f_j}) \) energy cost for task \( \tau_i \) with \( k_{i,f_j} \) replicas under \( f_j \)

Given a frequency \( f_j \), we start by computing the number \( k_{i,f_j} \) of copies that are needed for (each instance of) \( \tau_i \). The reliability using a single task instance (a single replica) is \( R_i(f_j) = \exp^{-\lambda(f_j)\tau_i f_j} \). If \( R_i(f_j) \geq R_i \), the reliability threshold is enforced, and we need a single replica, hence \( k_{i,f_j} = 1 \). Otherwise, using a total of \( r \) replicas, the reliability increases to \( R_i^r(f_j) = 1 - (1 - R_i(f_j))^r \) (the task fails only if all \( r \) copies fail), and we take the minimum value \( r \) such that \( R_i^r(f_j) \geq R_i \). This leads to \( k_{i,f_j} = r = \left\lfloor \log(1 - R_i) / \log(1 - R_i(f_j))^r) \right\rfloor \) with \( R_i = \sqrt[1 - \omega(1 - R_i(f_{\text{max}})^{h_i})] \) (see Section II-A). The reference paper maps different copies onto different processors, so necessarily \( k_{i,f_j} \leq M \). If no value of \( k_{i,f_j} \) can be found, frequency \( f_j \) cannot be used, and a higher frequency must be selected.

Once we have determined the number \( k_{i,f_j} \) of copies of \( \tau_i \) at frequency \( f_j \) (with \( 1 \leq k_{i,f_j} \leq M \)), we compute the corresponding energy cost. The reference paper adopts a conservative strategy and sums up the energy cost of all copies. This is pessimistic because as soon as a copy is successful, the remaining copies are interrupted (if already started) or simply cancelled (if not started). The energy cost of a copy is estimated as the power times the worst-case execution time \( c_{i,f_j} = c_i / f_j \), which is an upper bound. As for the power \( P(f_j) \) at frequency \( f_j \), we use

\[
P(f_j) = P_{\text{static}} + P_{\text{dyn}}(f_j) = P_{\text{static}} + (P_{\text{indep}} + C \times f_j^3)
\]

where \( P_{\text{static}} \) (the static power), \( P_{\text{indep}} \) (the frequency-independent part of dynamic power) and \( C \) (the effective switching capacitance) are system-dependent constants. The energy cost \( E(\tau_i, f_j, 1) \) for one copy of task \( \tau_i \) at \( f_j \) is then:

\[
E(\tau_i, f_j, 1) = P(f_j) \times c_{i,f_j}
\]

The final energy cost with \( k_{i,f_j} \) copies is estimated as \( E(\tau_i, f_j, k_{i,f_j}) = k_{i,f_j} \times E(\tau_i, f_j, 1) \). The total CPU time is then estimated as \( S(\tau_i, f_j, k_{i,f_j}) = k_{i,f_j} \times c_{i,f_j} \).

Furthermore, each processor always consumes static power when idle (this consumption can be eliminated only by a complete shutdown). Hence, we account for static power whenever the mapping and scheduling phases described below leave processors idle.

### Table I: Key Notations

<table>
<thead>
<tr>
<th>Notation</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( c_i )</td>
<td>WCET for task ( \tau_i ) under max. available frequency</td>
</tr>
<tr>
<td>( p_i )</td>
<td>WCET period for task ( \tau_i )</td>
</tr>
<tr>
<td>( h_i )</td>
<td>number of instances of task ( \tau_i ) in the hyperperiod</td>
</tr>
<tr>
<td>( f_j )</td>
<td>frequency ( f_j \in F = {f_{\text{min}}, \ldots, f_{\text{max}} = 1} )</td>
</tr>
<tr>
<td>( c_{\tau_i f_j} )</td>
<td>WCET for task ( \tau_i ) at frequency ( f_j )</td>
</tr>
<tr>
<td>( u_{\tau_i f_j} )</td>
<td>utilization of task ( \tau_i ) at frequency ( f_j )</td>
</tr>
<tr>
<td>( J_{\text{opt}}(i) )</td>
<td>most energy-efficient frequency for task ( \tau_i )</td>
</tr>
<tr>
<td>( R_i(f_j) )</td>
<td>reliability of one instance of task ( \tau_i ) under ( f_j )</td>
</tr>
<tr>
<td>( R_i )</td>
<td>target reliability threshold for one instance of task ( \tau_i )</td>
</tr>
<tr>
<td>( w_i )</td>
<td>uniform probability of failure scaling factor</td>
</tr>
<tr>
<td>( k_{i,f_j} )</td>
<td>min. replica number for task ( \tau_i ) under ( f_j ) to meet ( R_i )</td>
</tr>
<tr>
<td>( E(\tau_i, f_j, k_{i,f_j}) )</td>
<td>energy cost for task ( \tau_i ) with ( k_{i,f_j} ) replicas under ( f_j )</td>
</tr>
</tbody>
</table>

B. Replica sets

In the reference paper, the first step is to construct a table with all information needed. For each task \( \tau_i \) and each possible frequency we compute the number of replicas needed, the corresponding energy cost and the CPU time.
### C. Mapping and static schedule

The first step provides an initial configuration as input to the second step, the mapping and static scheduling onto processors. The initial configuration consists of an assigned frequency and number of replicas for each task instance. Given a configuration, the mapping builds a schedule for an hyperperiod of length $L = \text{lcm}(p_i)$ as follows:

- sort the tasks by decreasing total CPU time; renumber them to have

\[
S(\tau_1, f_{opt(1)}, k_{1,f_{opt(1)}}) \geq S(\tau_2, f_{opt(2)}, k_{2,f_{opt(2)}}) \geq \ldots
\]

where $f_{opt(i)}$ is the frequency leading to the lowest energy consumption to meet the reliability threshold;

- for $i$ ranging from 1 to $n$, successively map all $k_{i,f_{opt(i)}}$ copies of $\tau_i$ onto $k_{i,f_{opt(i)}}$ different processors, using the First-Fit Decreasing (FFD) bin packing heuristic [4]. When mapping all the $\frac{L}{p_i}$ instances of a given task copy on a processor (bin), we use the standard Earliest Deadline First (EDF) scheduling heuristic [5]. EDF tells us that a given processor (bin) is a fit for that copy if and only if the utilization of that processor does not exceed 1. Recall that the utilization of a processor is the sum of the utilizations of all task instances assigned to it.

Hence, for a given task, the mapping finds the first processor whose utilization makes it a fit for the first task copy (and all its instances). Then it finds the next processor whose utilization makes it a fit for the second task copy (and all its instances), and so on. If the mapping succeeds, we have built a static schedule for the hyperperiod. But it may well be the case that it is impossible to find a processor onto which to map a given task copy in the procedure, because all processor utilizations are too high to accommodate that copy. Then the reference paper proposes to enter an iterative procedure as follows:

- Change the initial configuration into the one where every task copy executes at maximal speed $f_{max} = 1$. This requires to fetch the values $k_{i,f_{max}}$ from the table and reordering the tasks by decreasing total CPU time $S(\tau_1, f_{max}, k_{1,f_{max}})$.
- Apply the mapping heuristic (FFD mapping and EDF schedule) to the new configuration.

Now, if the latter mapping fails again, there is no solution, resource utilization is too high. However, if the mapping succeeds, its energy cost may be very high. The reference paper proposes a refinement scheme where some tasks are relaxed, meaning that their frequency is decreased down to its predecessor (going from their current value $f_j$ down to $f_{j-1}$). Initially, all tasks have frequency $f_{max}$, and some tasks are greedily selected for relaxation. If relaxing a task $\tau_i$ fails to lead to a successful mapping, or if we have reached its energy-optimal frequency $f_{opt(i)}$, then $\tau_i$ is marked as ineligible. The scheme stops when all tasks become ineligible. The reference paper uses three different greedy criteria to pick up the next task to be relaxed among eligible tasks, Largest Energy First (LEF), Largest Power First (LPF), and Largest Utilization First (LUF). We refer to the reference paper for details. We have implemented the first two variants, LEF and LPF, because they are shown to outperform LUF in the reference paper.

### D. Dynamic schedule

The static schedule, also called canonical schedule, is based upon the WCET of each task and EDF. It is never executed exactly as such, because the actual execution time of a task instance will be shorter than its WCET. Still, it is used as the baseline to guide dynamic updates. From the canonical schedule, each processor has an assignment list made up with all task instances that it has to execute during the hyperperiod.

Let us follow the operation of a given processor $P$. For simplicity, assume that $P$ computes the full EDF schedule for all tasks in its assignment list, during the entire hyperperiod. The reference paper uses a data structure, called Canonical Execution Queue (CEQ), to avoid the high cost of computing the static schedule, while preserving the same outcome. Recall that the EDF schedule uses the WCET of each task, and preemption, so that a given task instance may well be split into several chunks. Hence $P$ has an ordered list of chunks together with their starting and finish times in its EDF schedule.

Let $t$ be the starting time of the next chunk $\chi$, from task instance $\tau$, to be processed by $P$ in its EDF schedule (initially, $t = 0$). In the canonical schedule, this chunk executes in the interval $[t_s, t_f]$, where $t_s$ is the starting time and $t_f$ the finish time on $P$. Let $k$ be the number of copies of $\tau$ in the canonical schedule, distributed over $k$ different processors. A major idea of the reference paper is to differentiate the action of $P$ depending upon whether its own copy of $\tau$ is the first copy to start execution among the $k$ processors. So if the chunk $\chi$ is indeed the first chunk of any copy of $\tau$ to start execution, then $P$ promotes its copy of $\tau$ to the status of primary replica, and all the chunks of $\tau$ as primary chunks. $P$ signals the other $k - 1$ processors that their copies (and their copy chunks) are secondary replicas (or chunks). Otherwise, the chunk $\chi$ has already been marked either as primary (if it is not the first chunk of $\tau$, but $\tau$ has been marked primary on $P$ previously) or as secondary (if some other processor has signalled $P$ previously). The action of $P$ is the following:

- if $\chi$ is a primary chunk, then $P$ starts its execution immediately, using the frequency given by the canonical schedule. This execution will last for a duration of $t_f - t_s$ seconds in the worst case;
- if $\chi$ is a secondary replica, then $P$ executes it at frequency $f_{max}$, and using ALAP (As Late As Possible) scheduling, as further detailed below.

The rationale for executing secondary chunks at highest frequency $f_{max}$ is that it allows for a minimal execution time, hence a maximal delay for ALAP scheduling. When delaying secondary chunks, we hope that the primary copy will complete before secondary copies actually start, hence will be cancelled whenever the primary copy succeeds. At least, this ALAP strategy should minimize overlap between primary and secondary copies, hence minimize redundant work. We point out that the choice for primary/secondary replicas is done
dynamically, by the first processor to start a task instance in the actual execution of the hyperperiod. It may well be the case that two different instances of the same task have not the same primary processor.

There remains to explain in full details how secondary chunks are scheduled on $P$. In fact, all secondary chunks are delayed according to the finish time of $\tau$ in the canonical schedule. Recall that the canonical schedule provides an execution interval $[t_s, t_f]$ for every chunk of $\tau$. Assume there are $m$ chunks. For notational convenience, let $[t_s(i), t_f(i)]$, $1 \leq i \leq m$, denote the $m$ execution intervals in the canonical schedule. The sum of these $m$ interval lengths $t_f(i) - t_s(i)$, $1 \leq i \leq m$, is equal to $c_l f$, if $\tau$ is an instance of task $\tau_1$, and if $f$ is its frequency from the canonical schedule. Since we have decided to execute $\tau$ at frequency $f_{\text{max}} = 1$, we only need $c_1$ seconds, in the worst-case, to execute $\tau$, and $c_1 \leq c_l f$. For instance, if $f = 0.5$ and $c_l = \frac{3}{2}$, we only need (at most) half the time planned in the canonical schedule. The reference paper uses backfilling and reserves a total of $c_1$ seconds in the dynamic schedule, starting from the last interval and going backwards to the first interval, allocating slots until $c_1$ seconds are reserved. Then $P$ will execute $\tau$ greedily using these intervals from the beginning until completion of $\tau$. Finally, all chunks of $\tau$ will be scheduled across $n$ new intervals $[t'_s(i), t'_f(i)]$, where $1 \leq i \leq n$ and $n \leq m$, because it is very likely $\tau$ will finish before its WCET. Here are two examples with $c_1 = 20$, $f = 0.5$, $c_l f = \frac{3}{2}$, and $m = 3$:

- the intervals in the canonical schedule are $[5, 35]$, $[40, 46]$ and $[50, 54]$. We reserve 20 seconds out of the 40 available in these three intervals by keeping the third interval entirely (4 seconds), then keeping the second interval entirely (6 seconds) and then keeping the last 10 seconds of the first interval. Then $P$ will use these reserved slots ($[25, 35]$, $[40, 46]$ and $[50, 54]$) to execute its copy of $\tau$ at frequency $f_{\text{max}}$, starting from time 25.
- the intervals in the canonical schedule are $[10, 18]$, $[40, 46]$ and $[50, 76]$. We reserve 20 seconds out of the 40 available in the three intervals by just keeping the last fraction $[56, 76]$ of the third interval (20 seconds), and leaving the first and second intervals empty.

In both cases, $P$ will consume the slots from the beginning of the first reserved interval, until consuming all time units needed to finish $\tau$, say at time $t'$ ($t' \leq t_f(n)$). Once the copy of $\tau$ on the processor $P$ is successfully executed, all other copies (chunks) of $\tau$ are removed from the EDF list on other processors. Then $P$ will start processing the next chunk $c_l t'$ in its list right at time $t'$ if $c_l t'$ is primary, otherwise, the chunks are delayed using the same mechanism.

To summarize, consecutive primary replicas are scheduled ASAP (As Soon As Possible), while secondary replicas are scheduled ALAP, which aims at reducing the overlap between copies over different processors, so as to minimize energy consumption.

\begin{figure}
\centering
\begin{tabular}{c}
\hline
$P$ & $P_1$ & $P_2$ \\
\hline
$\tau_2$ & 1 & 2 & 1 & 2 & 1 \\
$\tau_1$ & 2 & 1 & 2 & 1 & 1 \\
\hline
\end{tabular}
\caption{Canonical Execution Queue (CEQ) with two tasks $\tau_1$ and $\tau_2$ ($c_1 = c_2 = 0.5$, $p_1 = 2$ and $p_2 = 3$) on each processor, at frequency $f = 0.5$.}
\end{figure}

III. MOTIVATIONAL EXAMPLE

We illustrate one limitation of the strategy in [1] with a simple example with two tasks $\tau_1$ and $\tau_2$ and two processors $P_1$ and $P_2$. Assume that: (i) for WCET, $c_1 = c_2 = 0.5$ at maximum frequency $f_{\text{max}} = 1$; (ii) for periods $p_1 = 2$ and $p_2 = 3$; (iii) $\tau_1$ and $\tau_2$ have been scheduled to run at frequency $f = 0.5$, which takes $c_1 f = c_2 f = \frac{3}{2} = 1.5$ time unit; and (iv) the reliability threshold is met with one replica per processor for each task. For simplicity, also assume that: (i) each instance of $\tau_1$ is a primary replica on $P_1$ and secondary on $P_2$, while each instance of $\tau_2$ is a primary replica on $P_2$ and secondary on $P_1$; and (ii) there is no difference between actual execution times and the WCETs.

Figure 1 shows the CEQ, the Canonical Execution Queue, for the example (it is the same for both processors). Figure 2 shows the execution on both processors with a CEQ-based strategy as described in Section II-D. Recall that for secondary replicas, the strategy delays their executions according to CEQ and runs them at $f_{\text{max}}$. In the example, there is a complete overlap between each primary and each secondary replica due to the constraint given by the CEQ. This is wasteful because the secondary replica can never be canceled whenever the primary replica succeeds. However, it would be possible to avoid any overlapping by advancing each primary replica while delaying each secondary replica. In the example, this simply means interchanging $\tau_1$ and $\tau_2$ on $P_2$. Without overlap, the executions of all secondary replicas at maximum frequency will be cancelled most of the time, leading to a substantial energy gain.
IV. NEW STRATEGIES

We identify several possible reasons why the approach in the reference paper may be sub-optimal:

- All optimizations in the dynamic schedule aim at reducing overlap among replicas, so as to avoid redundant work. In their final schedule, the primary replica of task $\tau_i$ is executed at assigned frequency $f_j$, but the frequency of all $k_i, f_j - 1$ secondary replicas is increased to $f_{max}$. However, energy consumption is estimated with all replicas at frequency $f_j$. We revisit the estimate for energy consumption and use a different formula, with one copy executing at $f_j$, and the rest at $f_{max}$: this is expected to be closer to actual execution scenarios;

- The mapping uses the First-Fit Decreasing (FFD) bin-packing heuristic, which is likely to create imbalance across processors. Instead, we use the Worst-Fit Decreasing (WFD) bin-packing heuristic [6], which selects the least-loaded processor that is a fit for the current task copy. WFD has been shown to reduce imbalance in a related framework [7]; we use it with a similar motive;

- The mapping maps all copies of a task before proceeding to the next task. Instead, we map the first copy of each task, and then the second copy of each task (whenever it exists), and so on. This layered approach is expected to: (1) evenly map primary replicas onto processors; (2) decrease the overlap among copies of the same task as long as EDF priority constraints do not call for a full reordering of the tasks during (or after) the mapping. For instance, assume for simplicity that all tasks have the same period, so that the schedule is not constrained by EDF. Then, the mapping of the reference paper will place all replicas of the first task at the beginning of the assignment list of the processors, with possibly more primary replicas on the first processors, while it is better to delay all copies but one and insert first copies of other tasks instead.

- For the dynamic schedule, we keep the idea of running one primary replica of an instance of task $\tau_i$ at assigned frequency $f_j$, and to execute all other replicas at frequency $f_{max}$, but we implement several novel aggressive strategies to reorder and delay chunks, as described in Section IV-C.

We outline these modifications step by step below, in Sections IV-A, IV-B and IV-C, which are the respective counterparts, with the same title, of Sections II-B, II-C and II-D. Section IV-D summarizes our new heuristics. Note that we have both online and offline scheduling heuristics. For online scheduling strategies, which are able to determine dynamically the primary copy and secondary copies on the fly, all copies are mapped onto processors using frequency $f_j$; this is because we must reserve enough room for each copy in the mapping. In other words, we assume that each copy is a primary copy in the mapping, and some of them become secondary only during execution. On the contrary, for offline scheduling strategies, we have decided primary copy and secondary copies during the mapping phase, which means we map one replica at $f_j$ and the rest at $f_{max}$. In both scenarios, recall that we use different formulas to estimate the energy cost in both cases. Finally, Section IV-E is devoted to complexity results.

A. Replica sets

For task $\tau_i$ at frequency $f_j$, the reference paper determines the number of copies $k_{i, f_j}$ needed to match the reliability threshold $R_i$ and estimates the energy cost as $k_{i, f_j} E(\tau_i, f_j, 1)$ where $E(\tau_i, f_j, 1)$ is the energy cost for a single copy. Instead, we propose to estimate the energy cost as:

$$E(\tau_i, f_j, k_{i, f_j}) = E(\tau_i, f_j, 1) + (k_{i, f_j} - 1)E(\tau_i, f_{max}, 1)$$

(1)

because secondary replicas are actually executed at $f_{max}$ in the reference paper. Equation (1) is in accordance with the pessimistic scenario where no replica is cancelled.

Note that since we do not use the same estimation formula for the energy cost as in the reference paper, we may find a different frequency $f_{opt(i)}$ to be used for the mapping step. Again, only one copy (the primary copy) will actually be executed at frequency $f_{opt(i)}$, while all remaining copies (the secondaries) will be executed at frequency $f_{max}$. For each task $\tau_i$ and a given primary frequency $f_j$, we determine the minimum number of replicas $r_i$ such that the reliability threshold $R_i$ for each instance is met. Recall that the reliability using a single task instance (a single replica) is $R_i(f_j) = \exp^{-\lambda(f_j) t_i f_j}$. If $R_i(f_j) \geq R_i$, the reliability threshold is enforced, and we need a single replica, hence, $k_{i, f_j} = 1$. Otherwise, using $r$ additional replicas at $f_{max}$, the reliability increases to $1-\omega(1-R_i(f_j))^{r+1}$, instead of $1-\omega(1-R_i(f_j))^r$, and we take the minimal value of $r$ such that this reliability exceeds $R_i = \sqrt[\lambda]{\omega(1-R_i(f_{max})^{\frac{1}{\lambda}})}$. The new value of $r$ may be smaller than before, because each replica is more reliable. Of course, the new value of $r$ leads to a new value of $k_{i, f_j} = r + 1$. We use this new value of $k_{i, f_j}$ to compute the energy cost. Note that the reference paper assumed that task WCETs are inversely proportional to the assigned frequency where $c_{i, f_j} = \frac{\omega}{f_j}$, but in reality, the interplay between task execution time and frequency is complicated [8]. To be closer to real cases, we let WCETs obey a randomized speedup function similar to Amdhal’s law: specifically for task $\tau_i$ at frequency $f_j$, $c_{i, f_j} = s_i c_j + (1-s_i) \frac{1}{f_j}$, where the sequential fraction $s_i$ is drawn uniformly and randomly in the interval $[0.1, 0.3]$.

We retain the frequency $f_{opt(i)}$ that minimizes Equation (1), and we use $k_{i, f_{opt(i)}}$ copies of task $\tau_i$ all running at frequency $f_{opt(i)}$ as input to the mapping phase for all online scheduling strategies. On the contrary, for offline scheduling strategies, we use one copy at frequency $f_{opt(i)}$ and $k_{i, f_{opt(i)}} - 1$ copies at $f_{max}$ as we explained before. Note that we try through all possible frequencies between $f_{min}$ and $f_{max} = 1$, but the final number of valid frequency levels may be smaller than the number of available frequency levels. There are several
possibilities: 1) We should not consider lower frequency levels that makes the task \( r_i \) miss its deadline; 2) As we decrease the frequency level, the number of required replicas may increase or remain the same, and the energy consumption may not be strictly decreasing; 3) A lower frequency may introduce more overlap that can not be avoided than at a higher frequency, so we enforce that \( c_i, f_i + c_i, f_{\max} \leq p_i \).

### B. Mapping and static schedule

We map each replica to a processor while ensuring that no two replicas of the same task are assigned to the same processor. The mapping is done for a whole hyperperiod, with the following constraint for each task: when the first iteration (in case the period differs from the hyperperiod) of a replica is assigned to a given processor, all the other iterations of the same replica will be assigned to the same processor. It allows a simple feasibility check based upon the utilization of all the replicas assigned to the processor.

Recall that any success of a primary replica leads to the immediate cancellation of the secondary replicas, a crucial source of energy saving. The objective of the proposed mapping is thus to avoid overlapping between the execution of the primary and secondary replicas for each task: the primary must be terminated as soon as possible, while the secondaries must be delayed as much as possible. To this end, the mapping strategy ventilates primaries on all processors, in order to minimize conflict among several primary executions. Moreover, it allocates the secondaries while leaving idle time on all processors. This slack can then be used at execution time to delay the execution of the secondaries.

#### Algorithm 1: Mapping (WFD layer by layer)

**Input:** The WCETs \( c_i, f_i \), the period \( p_i \), the assigned frequency \( f_i \), the number of secondary replicas \( r_i \), the number of instances \( h_i \)

**Output:** An allocation of all replicas on the processors \( \sigma_m \)

1. **begin**
2. execute WFD on the primaries considering non-increasing order of \( c_i, f_i \times h_i \)
3. if will do online scheduling afterwards then
   4. \( f_i \leftarrow f_i \)
   5. else
   6. \( f_i \leftarrow f_{\max} \)
7. execute WFD on the secondaries layer-by-layer considering non-increasing order of \( c_i, f_i \times h_i \) while ensuring that no processor executes more than one replica of each task
8. if \( \exists m, \sum_{r_i \in \sigma_m} c_i, f_i \leq p_i \) then
   9. return not feasible
10. else
11. return \( \sigma_m \)

As shown in Algorithm 1, given a list of primary and secondary replicas for each task, and their execution times, we first execute the Worst-Fit Decreasing (WFD) allocation on the primaries ordered by their total execution time (Line 2). For all secondaries, we assign the same frequency as the primary if preparing for online scheduling afterwards. We assign them the maximum frequency \( f_{\max} \) otherwise (Lines 3-6). Then we execute WFD on the secondaries layer-by-layer and ordered by their total execution time (Line 7), which means we consider the first secondary of all tasks, then the second and so on (as long as it exists). If we successfully map all replicas onto the processors, and the sum of the utilization of each replica on each processor is less than or equal to one (see Section II-C), then we return this allocation. Otherwise, we could not find a feasible mapping with this replica setting (Lines 8-11). We observed that the competitor strategy FFD in the reference paper tends to compact all replicas onto the minimum number of processors, while WFD spreads replicas among all available processors, which may give a higher static energy cost. To reduce the influence of static power, we first run WFD with the number of processors used by FFD. If WFD is not able to find a feasible mapping, then we increase the number of processors by one up to the total available number.

#### Algorithm 2: Replication setting

**Input:** A set of tasks with cost \( c_i \) and reliability requirement \( R_i \)

**Output:** A set of minimum frequency \( f_i \) and number of secondary replicas \( r_i \)

1. **begin**
   2. /* start with all primaries at energy-optimal frequency */
   3. for \( i \in [1, \ldots, n] \) do
      4. \( f_i \leftarrow f_{\text{opt}(i)} \)
      5. /* reliability requirement */
      6. \( r_i \leftarrow \left\lceil \frac{\log(1 - R_i f_i)}{\log(1 - R_i f_{\max})} \right\rceil \)
   7. map the tasks to the processors with Algorithm 1
   8. if feasible then
      9. return \( \{f_i, r_i\} \)
      10. /* reset all primaries at \( f_{\max} \) */
      11. for \( i \in [1, \ldots, n] \) do
         12. \( f_i \leftarrow f_{\max} \)
      13. /* reliability requirement */
      14. \( r_i \leftarrow \left\lceil \frac{\log(1 - R_i f_i)}{\log(1 - R_i f_{\max})} \right\rceil \)
      15. return \( \{f_i, r_i\} \)
   16. else do
   17. /* enter the relaxing phase */
   18. while any primary frequency can be decreased do
      19. select task \( i \) with LEF or LPF criteria (cf. Section II-C)
      20. \( f_i \leftarrow f_i - 1 \)
      21. \( r_i \leftarrow \left\lceil \frac{\log(1 - R_i f_i)}{\log(1 - R_i f_{\max})} \right\rceil \)
      22. map the tasks to the processors with Algorithm 1
      23. restore \( f_i \) and mark that task \( i \) can not be further decreased
      24. if mapping is not feasible
      25. return \( \{f_i, r_i\} \)

We use this allocation mechanism to determine the number of replicas and their frequencies for each task (see Algorithm 2). As described in Section IV-A, we already know for each task, at each frequency level, how many replicas are needed to meet the reliability threshold. If we can find a feasible mapping with each task at its energy-optimal frequency \( f_{\text{opt}(i)} \), then we return this optimal setting (Lines 2-7). Otherwise, we check the other end, all tasks run at \( f_{\max} \)
that takes the shortest time possible. If it is still impossible to map all replicas, then there does not exist a feasible mapping (Lines 8-13). If there exists a feasible mapping, we will enter the relaxing phase that decreases each primary frequency \( f_i \) iteratively until it is no longer possible (Lines 14-19). For the sake of readability, at Line 16, it is written that we decrease by one step the considered frequency while, in fact, we only consider valid frequencies as explained at the end of Section IV-A. Finally, we return the solution with a frequency level and number of secondary replicas for each task (Line 20).

C. Dynamic schedule

For the scheduling phase, it is important to start primary replicas as soon as possible, and to delay secondary replicas as much as possible to minimize the overlapping, while still meeting all deadlines. As explained in Section II-D, the reference paper uses the canonical schedule to compute the maximum delay for secondary replicas. Our improvements rely on the following techniques:

- Consider a scheduling interval defined by two consecutive deadlines in the global schedule. Inside the interval, task chunks to be executed are ordered by the EDF policy. We observe that we can freely reorder the chunks without missing any deadline, by definition of an interval. It means that in each interval, we should reorder to execute all primary replicas first, and then secondary replicas.

- It is possible to use only a fraction \( \alpha \) of each scheduling interval, where \( \alpha \) is the utilization. Here is why: at the mapping phase, as long as the total utilization of replicas that are mapped onto the processor is less than or equal to one, then we are able to find a valid scheduling using EDF. Assume we have mapped three tasks \( t_i, t_j, t_k \) onto processor \( p \), and that the utilization is \( \alpha = \frac{c_{i} / p_i}{t_k} + \frac{c_{j} / p_j}{t_k} + \frac{c_{k} / p_k}{t_k} \). Either we keep the mapping and have a fraction \( \frac{1}{1 - \alpha} \) of the interval where \( p \) is idle, or we slow down the execution time of all three tasks by a factor \( \alpha \), then we will have a new utilization \( \beta = \frac{c_{i} / p_i}{p_i \alpha} + \frac{c_{j} / p_j}{p_j \alpha} + \frac{c_{k} / p_k}{p_k \alpha} = 1 \), which also gives us a feasible mapping without any idle time. This idea can be used in two ways:

  1) Schedule while keeping a fraction \( 1 - \alpha \) of idleness in each interval. Then, each primary replica is pushed to be beginning of the interval, while secondaries are pushed back to the end of the interval, with idleness in between.

  2) Scale the WCET of all tasks by \( \frac{1}{\alpha} \), which also gives a valid canonical schedule, but with longer worst case expected execution time for all tasks. This gives a better reference to further delay the start time of secondary replicas.

- Because we have delayed the start time of secondary replicas, there are some idle slots in the schedule. We take advantage of these idle slots by pre-fetching other primary replica chunks in the availability list: those primaries have been released but were scheduled later because they have lower EDF priority than the current secondary replicas.

D. Heuristics

Based on the above ideas, we propose several new scheduling heuristics which improve upon EDF_PAPER, the adaptive dynamic scheduling (and the most efficient) heuristic of the reference paper.

EDF_PAPER_PF is an adaptive online scheduling that simply adds the pre-fetching mechanism to EDF_PAPER.

EDF_PAPER_PF_Utility is an online scheduling heuristic where we refine EDF_PAPER_PF by using the utilization of each processor. We scale the worst case execution time of all replicas of a given processor by a factor \( \frac{1}{\alpha} \), where \( \alpha \) is the utilization of that processor.

EDF_IDLE_CEQ is an offline scheduling that builds the EDF schedule for the whole hyperperiod. In each interval defined by two consecutive deadlines, we only use (on each processor) a fraction of the interval defined by the static utilization. It consists in the following steps:

1) consider at each interval the EDF schedule with the constraint of keeping a fraction \( 1 - \alpha \) of idle time. For secondaries, we refer to the canonical schedule to delay their start time without missing any deadline;

2) start the primary replicas and put aside the secondary replicas in a waiting list to be executed at the end of the interval. Note that for each secondary, as it is impossible to know its actual execution time before its execution, we need to reserve the space for its WCET and to finish execution within the interval;

3) fill in the idle period by inserting other primary replicas that are available;

4) finish the execution of the interval with the secondary replicas in the waiting list, with their actual execution time.

EDF_IDLE_CEQ_ONLINE is the online version of EDF_IDLE_CEQ. It has two major advantages compared to the offline version: (1) we can dynamically decide the primary copy of each task instance, which gives us the flexibility to speed up replicas on the fly; (2) as long as we finish one replica successfully, we can safely cancel other replicas of the same task instance earlier than in static schedules, which gives us more flexibility to adjust the schedule afterwards. Moreover, all algorithms have to reserve for the secondaries some time slots corresponding to their WCET. As their actual execution times are usually shorter, this dynamically frees some time slots that the online schedule uses to prefetch available primary replica chunks.

E. Complexity analysis

This section is devoted to the proof of several complexity results for the scheduling phase. The global optimization problem is obviously NP-hard, since it is a generalization of the makespan minimization problem with a fixed number of parallel processors [9]. However, the complexity of the sole scheduling phase is open: if the number of replicas has already been decided for each task, and if the frequency and assigned processor of each replica has also been decided,
the scheduling phase aims at minimizing the expected energy consumption. We state a lower bound for this scheduling problem in Section IV-E1, and we assess the complexity of achieving this lower bound: in Section IV-E2, we show that the instance with identical WCETs is polynomial, while in Section IV-E3, we show the instance with different WCETs is NP-complete in the strong sense.

1) Lower bound: Consider the following instance of the scheduling phase:

- All tasks have the same period $p$, hence, there is a single instance of each task in the hyperperiod of length $L = p$.
- Hence, EDF constraints do not apply, and each task is scheduled without preemption (as a single chunk)
- $F = 1$: there is a unique frequency $f_{\text{max}} = 1$
- For $1 \leq i \leq n$, task $\tau_i$ has $k_i$ replicas, including itself.

The $j$-th replica, with $1 \leq j \leq k_i$, is mapped onto processor $P_{\text{alloc}(i,j)}$, where $1 \leq \text{alloc}(i,j) \leq M$. For each task $\tau_i$, replicas are mapped onto different processors: $\text{alloc}(i,j_1) \neq \text{alloc}(i,j_2)$ for $1 \leq j_1 < j_2 \leq k_i$.

- The WCET of any replica of task $\tau_i$ is $R_i = R_i(f_{\text{max}})$, and its consumed energy is $E_i = E(\tau_i, f_{\text{max}}, 1)$.
- $P_{\text{static}} = 0$, meaning that no energy is spent when a processor is idle

Thus, each of the $M$ processors has a list of assigned replicas to execute. It can choose any ordering because all tasks have the same period; hence, all deadlines will be enforced, regardless of the ordering. We further assume that the mapping is valid, which translates on each processor $P_q$, $1 \leq q \leq M$, by the condition:

$$\sum_{1 \leq i \leq n, 1 \leq j \leq k_i, \text{alloc}(i,j) = q} WCET_i \leq p$$

**Proposition 1.** A lower bound on the total expected energy consumed is

$$E_{LB} = \sum_{i=1}^{n} \left( E_i \sum_{j=1}^{k_i} (1 - R_i)^{j-1} \right)$$  \hspace{1cm} (2)

The bound $E_{LB}$ is met if and only if the scheduling achieves no overlap between any two replicas of the same task.

**Proof.** For each task, we need to execute the replica which is scheduled in first position. If this replica fails, with probability $1 - R_i$, we need to execute the replica which is scheduled in second position. If both replicas fail, we need to execute the replica which is scheduled in third position, and so on. This directly leads to the lower bound $E_{LB}$.

Now if any two replicas of the same task, say $\tau_i$, do overlap, then with some non-zero probability, both replicas will execute, and the consumed energy will be strictly higher than the contribution of $\tau_i$ to Equation (2). This concludes the proof.

2) Identical costs:

<table>
<thead>
<tr>
<th>$P_0$</th>
<th>$R_1$</th>
<th>$R_2$</th>
<th>$R_3$</th>
<th>$R_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_{1,1}$</td>
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<td>$F_{1,2,2}$</td>
<td>$F_{1,3,2}$</td>
<td></td>
</tr>
<tr>
<td>$P_{1,2}$</td>
<td>$F_{2,1,1}$</td>
<td>$R_1$</td>
<td>$F_{2,1,2}$</td>
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<tr>
<td>$P_{1,3}$</td>
<td>$F_{2,2,1}$</td>
<td>$R_2$</td>
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<tr>
<td>$P_{2,1}$</td>
<td>$F_{2,3,1}$</td>
<td>$R_3$</td>
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<td>$P_{3,1}$</td>
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<td>$P_{3,2}$</td>
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<td>$P_{3,3}$</td>
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<tr>
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<td>$R_1$</td>
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<td>$P_{4,3}$</td>
<td>$F_{4,3,1}$</td>
<td>$R_3$</td>
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</tbody>
</table>

![Figure 3: Scheduling for a solution of $\mathcal{I}$ of 3-Partition, with $m = 4$, $B = 20$, and $(a_1, \ldots, a_{12}) = (4, 3, 8, 9, 7, 6, 11, 6, 7, 10, 4, 5)$. On processor $P_0$ the digits are the sizes of the $A_i$'s.](image)

**Proposition 2.** When all tasks have the same WCET ($c_i = c$ for $1 \leq i \leq n$), one can build a schedule meeting the lower bound of Equation (2) in polynomial time.

**Proof.** We construct a bipartite graph with tasks on the left (with $n$ vertices) and processors on the right (with $M$ vertices). Task $\tau_i$ has $k_i$ edges, connecting each of its replicas to the assigned processor. Hence we have $K = \sum_{i=1}^{n} k_i$ edges. Let $\delta$ be the maximum degree of a vertex in the graph. According to Konig's edge coloring theorem, one can find a collection of $\delta$ perfect matchings that cover all edges in the graph, in time $O(\delta K)$ [10]. Since $\delta K \leq (n+M)^3$, this is indeed polynomial in the problem size. These matchings directly lead to a schedule with minimal makespan $\delta c$. By construction, this schedule is guaranteed overlap-free. 

3) Arbitrary costs:

**Proposition 3.** When tasks have different WCET, determining whether the lower bound of Equation (2) can be met, is a problem which is NP-complete in the strong sense.

**Proof.** Let $\text{NOOVERLAP}$ denote the problem with different WCETs. $\text{NOOVERLAP}$ clearly belongs to the class NP: a certificate can be the description of the schedule with start and end times for each replica, and one can check in quadratic time that no two replicas of the same task overlap. We establish completeness in the strong sense through a reduction from 3-Partition [9]. Let $\mathcal{I}$ be an instance of 3-Partition. $\mathcal{I}$ comprises $3m$ integers, $a_1, \ldots, a_{3m}$ such that $\sum_{i=1}^{3m} a_i = mB$ and $\frac{B}{4} < a_i < \frac{B}{2}$ for all $i$. The question is: can we partition the $a_i$'s into $m$ subsets $S_1, \ldots, S_m$ such that each subset has total size $B$: $\sum_{j \in S_i} a_j = B$? The size of $\mathcal{I}$ is $\Omega(m \log B)$. From the instance $\mathcal{I}$ of 3-Partition, we build an instance $\mathcal{J}$ of $\text{NOOVERLAP}$: this instance contains three types of tasks: some replicated tasks (the $R_i$'s), some filling tasks which constrain
the replicated tasks (the \( F_{i,j} \)'s), and the tasks corresponding to the integers in instance \( I \) (the \( A_i \)'s). Specifically:

- There are \( 1 + m(m-1) \) processors denoted \( P_0 \) and \( P_{i,j} \) with \( 1 \leq i \leq m \) and \( 1 \leq j \leq m-1 \).
- All tasks have the same period \( p = (2m - 1)B \).
- There are \( m-1 \) replicated tasks of size \( B \), \( R_1, \ldots, R_{m-1} \).
- For any \( i \), \( 1 \leq i \leq m-1 \), there is one replica of \( R_i \) on processor \( P_0 \), and 1 on each of the processors \( P_{k,i} \), \( 1 \leq k \leq m \). Thus, each \( R_i \) is replicated \( m+1 \) times.
- Two tasks \( F_{i,j,1} \) and \( F_{i,j,2} \) are mapped on each processor \( P_{i,j} \), \( 1 \leq i \leq m-1, 1 \leq j \leq m \). \( F_{i,j,1} \) is of size \( 2(i-1)B \) and \( F_{i,j,2} \) is of size \( 2(m-i)B \). The total load of processor \( P_{i,j} \) is \( B + 2(i-1)B + 2(m-i)B = (2m-1)B \) and there is no slack on that processor. Note that, to ease the writing, we have kept in our description a null size task on processors \( P_{1,1} \) and \( P_{i,m} \).
- In addition to one replica of each of the tasks \( R_1, \ldots, R_{m-1} \), \( 3m \) tasks \( A_1, \ldots, A_{3m} \) are mapped to processor \( P_0 \), where task \( A_i \) has size \( a_i \). Therefore, the total load of processor \( P_0 \) is \( (m-1)B + \sum_{i=1}^{3m} a_i = (2m-1)B \) and there is no slack on that processor either.

Instance \( J \) contains \( 1+m(m-1) \) processors and \( (m-1)(m+1) + 2m(m-1) + 3m = 3m^2 + 3m - 1 \) replicas. All tasks have size \( O(mB) \). Hence, the size of \( J \) is polynomial in the size of \( I \). We now prove that if \( I \) has a solution, then \( J \) has a solution. Let \( S_1, \ldots, S_m \) be the solution of \( I \). Then we schedule the tasks of \( J \) as follows and as illustrated by Figure 3:

- For any \( i \), \( 1 \leq i \leq m-1 \), and any \( j \), \( 1 \leq j \leq m \), on processor \( P_{i,j} \), task \( F_{i,j,1} \) is executed during the interval \([0,2(i-1)B] \), a replica of \( R_j \) during the interval \([2(i-1)B,(2i-1)B] \), and \( F_{i,j,2} \) during the interval \([(2i-1)B,(2m-1)B] \).
- For any \( i \), \( 1 \leq i \leq m-1 \), a replica of \( R_i \) is executed on \( P_0 \) during the interval \([(2i-1)B,2iB] \).
- For any \( j \), \( 1 \leq j \leq m \), the tasks corresponding to the \( j \)-th partition of \( I \), i.e., the tasks \( A_k \) such that \( k \in S_j \), are executed on \( P_0 \) during the time interval \([2(j-1)B,(2j-1)B] \).

One can easily check that this schedule is valid and that two replicas of a task \( R_i \) are never executed simultaneously. Therefore there exists a schedule without overlap for \( J \) if there exists a solution for \( I \).

Let us now assume that there exists a valid schedule for \( J \), i.e., a schedule without any overlap. Let us consider any \( i \), \( 1 \leq i \leq m-1 \). Processor \( P_{i,1} \) (respectively \( P_{i,m} \)) contains a replica \( R_i \) and a task of size \( (2m-2)B \) (we do not care about the null-size task). Therefore, the replica \( R_i \) is executed on \( P_{i,1} \) (resp. \( P_{i,m} \)) either during the interval \([0,B] \) or during \([(2m-2)B,(2m-1)B] \). Then, for any \( j \), \( 1 < j \leq \frac{m}{2} \), processor \( P_{i,j} \) (respectively \( P_{i,m-j+1} \)) contains a replica \( R_i \), a task of size \( (2i-1)B \) and one of task \( 2(m-i)B \). From what precedes, there is already a replica of \( R_i \) executed during the time interval \([0,B] \) and one during \([(2m-2)B,(2m-1)B] \). Therefore, the replica \( R_i \) is executed on \( P_{i,j} \) (resp. \( P_{i,m-j+1} \)) either during the interval \([(2i-1)B,2iB] \) or during \([(2m-i)B,(2m-i+1)B] \). Overall, on processor \( P_0 \), the replica \( R_i \) must be executed during one of the intervals \([(2j-1)B,(2j)B] \), for \( 1 \leq j \leq m-1 \), because at all the other instants, there is already one \( R_i \) replica being executed on one other processor, and because the schedule is without any overlap. However, there are \( m-1 \) such intervals and \( m-1 \) such replicas. Therefore, the tasks \( A_i \)'s must be executed during the intervals \([(2j-2)B,(2j-1)B] \), for \( 1 \leq j \leq m \). This is a set of \( m \) intervals each of size \( B \). Because the schedule is valid, all the \( A_i \)'s are executed during these intervals. Let \( S_j \), \( 1 \leq j \leq m \), be the set of the indices of the \( A_i \)'s executed during \([(2j-2)B,(2j-1)B] \). Then the subsets \( S_j \) define a solution to \( I \).

\[ \Box \]

V. PERFORMANCE EVALUATION

In this section, we present the simulation results to evaluate the performance of our whole strategy compared to the best solution proposed in the reference paper [1]. In Section V-A, we describe the parameters and settings used during the experimental campaign. We present the results in Section V-B.

A. Experimental methodology

We designed a discrete event simulator, which is publicly available at [11]. For each data point, we considered 2000 data sets with 20 tasks. Task periods are randomly generated between 10ms and 100ms. The utilization of each task is generated randomly using the UUnifast scheme [12], with the total utilization \( U_{tot} \) as input. The set of frequencies is taken from a real microprocessors [13] which has \( F = 5 \) frequency levels including 1.0, 0.8, 0.6, 0.4 and 0.15. We also validate our methods on two other real frequencies sets [14], [8]. Following [1], in the rest of the section, we assume that the transient fault arrival rate at \( f_{max} \) is \( \lambda_0 = 10^{-6} \) and the system sensitivity factor \( d = 4 \). The static power and the frequency-independent part of the dynamic power are set to 5% and 15% respectively of the maximum frequency-dependent power consumption with \( C = 1 \). We use the ratio \( BC/WC \) of the best-case (BC) over worst-case (WC) execution time, to model workload variability. The actual execution time of each task is determined according to a uniform distribution between BC and WC. To compare the strategies under all parameter settings, we covered all of the variables from [1]. For the rest of the section, we keep the number of cores at 8, vary the value of \( U_{tot} \) from 1.5 to 3.5 to study the impact of system load, vary \( BC/WC \) from 0.2 to 1.0 to show the influence of workload variability, and vary the probability of failure scaling factor \( w \) from \( 10^{-5} \) to \( 10^{-1} \) to evaluate the impact of target reliability. Due to space limitations, we only report here a subset of our simulations results. For instance, we consider heuristic LPF (cf. Section II-C) for choosing the candidate task for relaxation, which is shown to be the best heuristic in [1]. However, a full set of results supplemented with two additional real frequencies sets and a larger range for \( s_i, s_i \in [0.1,0.5] \) can be found in [15].
B. Results

We use the same baseline scheme as the reference paper\(^2\), i.e., classic EDF scheduling with First-Fit Decreasing mapping and replica sets from [1] (where it is called the static scheme). Figures 4 and 5 present the energy consumption of our strategies and of the best performance strategy from the reference paper, divided by the energy consumption of the baseline. Therefore, the lower the better and data points below the \(y = 1\) line denote cases in which these strategies outperform the static scheme (i.e., achieve a lower energy consumption). These figures represent the trends with various parameters. See [15] for the full set of results.

Each subfigure shows results for a different combination of mapping and replica settings. Each line of subfigures is for a different replica sets (either from the reference paper or our new set) while each column is for a different mapping (FFD and WFD). For example, the bottom right plot presents energy savings of several scheduling approaches (different line colors) with WFD mapping and our replica setting. We report in these figures the number of seeds (out of 2000 in total) that could find a feasible solution for each setting. These numbers are reported in black above the horizontal axis in each figure. Note that WFD tends to find fewer or the same number of feasible solutions than FFD. This is because FFD tries to pack more tasks onto processors while WFD tends to spread tasks onto less loaded processors. Moreover, we plot the lower bound for online scheduling and offline scheduling (different line styles) in black lines, by which we could know the maximum energy saving that can be achieved without any overlapping and failure. We can notice that when we apply our replica setting, the lower bound of the offline scheduling is sometimes slightly lower than that of the online scheduling. The reason is that during the mapping phase, in the online scheduling, we assign all secondaries the same frequency as the primary, which makes it more difficult to find a feasible mapping than in the offline scheduling that assigns secondaries the maximum frequency. Hence, the offline scheduling may pick a lower frequency (see Section IV-B).

A clear observation is that our scheduling heuristics outperform EDF\(_\text{Paper}\) under almost all combinations of settings. In particular, EDF\(_\text{Idle_CEQ}_{\text{Online}}\) always achieves the best performance. EDF\(_\text{Idle_CEQ}_{\text{Online}}\) could save up to 25\% with respect to EDF\(_\text{Paper}\) (see Fig. 4c). It should be noted that to clearly see the different capabilities of the scheduling heuristics in reducing the overlapping, we need more than one replica for each task. As we decrease the value of \(w\), more replicas are needed to meet the reliability target. In Figure 4, we keep the value of \(BC/WC\) at 1 and increase the system load. We observe that the energy savings are closer to the lower bound at lower utilization, as we can find a feasible partitioning without overlapping for the minimum energy configurations. In Figure 5, for a fixed system load \((U_{\text{tot}} = 2.5)\), the higher the \(BC/WC\) ratio,

\(^2\)The authors of [1] have not provided their source code to us; we did our best to ensure a fair assessment and comparison of results.

the lower the workload variability, and the further from the lower bound. This is because as \(BC/WC\) increases, jobs have larger execution times and there is a higher chance of overlap between the primary and the secondary replicas.

Another common trend is that, in the majority of cases, WFD and our new replica setting help save energy. By considering our replica setting, we could find a lower \(f_{\text{opt}(t)}\). This is because our replica setting plans for one replica at \(f_1\) and \(x\) replicas at \(f_{\text{max}}\) while the reference setting plans to run all replicas at \(f_1\). By considering WFD, we usually: 1) achieve the highest energy savings, 2) improve the average performance of scheduling heuristics. For example, in subfigure 4a, from the two plots in the second row (with the same replica set but with different mappings), we can see that more scheduling heuristics achieve the lower bound with WFD than with FFD (with whom only EDF\(_\text{Idle_CEQ}_{\text{Online}}\) did). In subfigure 4d, we could see that applying the replica sets and mapping of the reference paper (top left plot), the best energy savings are achieved by EDF\(_\text{Idle_CEQ}_{\text{Online}}\) (around 37.5\%), while the other heuristics save less that 25\%. With our replica sets and mapping (bottom right), our scheduling heuristics achieve more than 40\% energy savings. Even the EDF\(_\text{Paper}\) saves up to 25\%. Thus, our solutions outperform those of the reference paper. These improvements are representative of the trends that can be observed for all considered graphs, but suffer from some exceptions. For example, in Fig. 4a, when \(w = 10^{-5}\), applying our replica set gives a higher lower bound which means we will lose even without any overlapping in the schedule. Overall, our task mapping and replica setting never achieve significantly bad performance, and most of the time achieves the best performance. The average improvement of our method over [1], computed across all our experiments, can be estimated to be of the order of 20\%.

VI. RELATED WORK

Liu and Layland first introduced the Earliest Deadline First (EDF) and the Rate Monotonic (RM) scheduling policies for real-time systems and provided the utilization bounds for both policies in 1973 [5]. Since then, the real-time scheduling problem has been extensively studied. A significant amount of work has aimed at developing new scheduling algorithms under different assumptions, including the well-known energy management technology Dynamic Voltage and Frequency Scaling (DVFS). We refer to the original article [1] for the related work. Here, we only cover relevant work citing paper [1].

In [16], Taherin et al. propose an approach for energy management that is only applied on “low-criticality tasks in low-criticality mode to preserve the original reliability of the system”. This approach cannot guarantee that a reliability threshold is met. Keeping the maximum power consumption below the chip thermal design power, Ansari et al. [17] have proposed a peak power management approach to meet thermal design power in fault-tolerant system. However, in the scheduling task graph, all tasks have the same period/deadline. Cao et al. in [18] proposed an affinity-driven modeling and scheduling
approach to makespan optimization. They optimize average peak temperature and makespan, but not energy minimization. To improve quality in real-time system, Cao et al. in [19] proposed QoS-adaptive approximate real-time computation optimization. Approximate results of tasks are allowed: each task is composed of a mandatory part and an optional one that refines the result of the mandatory task. However, failures are not considered. Zhou et al. consider in [20] both transient and permanent faults. They try to improve soft-error reliability while satisfying a lifetime reliability constraint, but do not attempt to minimize energy consumption.

VII. CONCLUSION

In this work, we have revisited the challenging problem presented by Haque, Aydin and Zhu in [1], namely minimizing the expected energy consumption of a set of preemptive periodic real-time tasks, executing on a parallel platform where processors are subject to transient failures. Replication is used to enforce all deadlines, as well as the reliability threshold. We have improved the approach of [1] as follows. First, we use a different formula to estimate the energy consumption, which is supposed to be closer to actual execution scenarios. Secondly, in the mapping phase, we apply a layered WFD strategy, which is expected to be helpful for load-balancing, and for decreasing the overlap among copies of the same task. Finally, we implement several novel scheduling strategies, which introduce the idea of reordering chunks in between deadlines and of taking advantage of the utilization of the processor. Moreover, we have established that the sole problem of scheduling tasks with different WCETs, knowing the number of replicas, frequency and assigned processor for each task, is NP-complete in the strong sense.

We have evaluated the improvement of our strategies with a discrete event simulator (made publicly available). Extensive
experiments conducted for various range of parameters have shown that: 1) Our new replica sets and WFD help finding a lower frequency, which gives good pre-condition for further energy savings; 2) our scheduling heuristics significantly outperform EDF. More specifically, EDF_IDLE_CEQ and EDF_IDLE_CEQ_ONLINE have the best performance under a wide range of scenarios, with an average gain in energy of 20%.

Future work will aim at extending the algorithms to periodic workflows instead of independent task sets. The dependencies between nodes will dramatically complicate the problem. Another interesting direction is to deal with the same problem with independent tasks, but targeting heterogeneous multicore systems.

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