Adaptive Image Assimilation for 2D Velocity Reconstruction
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Introduction

• 4D-Var is formulated as a nonlinear least-squares problem combining all the available heterogeneous information (i.e., mathematical models, observation, and priori knowledge) for calibrating the state of dynamical equations.

• Insufficient estimation of background, error covariance matrix and scarce observation will lead to unreliable prediction.

• A regularizer based on structure dictionary sparsity would be a tool of choice to improve the performance.

Dictionary Learning and Regularization

Dictionary Learning

• In the data-driven approach (CAI et al., 2014), a dictionary $W$ is learned adaptively from dataset $f$.

$$\min_\rho \|\rho - Wf\|_2^2 + \eta \|\rho\|_0, \quad \text{s.t.} \quad W^TW = I \tag{1}$$

$\rho$: sparse coefficients, $\|\cdot\|_0$: number of nonzero elements

Remark: The updated dictionary (Fig.5) contains more directional structures from the dataset (Fig.4) than fixed ones (Fig.1-3).

Regularization

• In view of the fact that features in images contribute to optimizing the flow (TITAUD et al., 2010), we introduce a “vorticity structure dictionary” sparsity regularizer in the framework of 4D-Var for 2D velocity reconstruction.

• In the case that the velocity field is indirectly accessible from the observation of passive tracers, the object function is expressed as

$$\min \frac{\delta}{\delta \rho} \sum_{i=1}^{N} \|H_r C(t) - C^{\text{obs}}(t_i)\|^2_{R^{-1}} + \frac{\alpha}{2} \|\rho(t_0, y_0) - (u_0^0, v_0^0)\|^2_{R_0^{-1}} + \mu \|W \zeta\|_1 \tag{2}$$

$\zeta = \frac{\partial u}{\partial y} - \frac{\partial v}{\partial x}$: relative vorticity, $W$: learned dictionary

Numerical Implementation

• Models: 2D shallow water equation and concentration transport equation

$$\frac{\partial u}{\partial t} + (f + \zeta)\frac{\partial u}{\partial x} + \frac{\partial v}{\partial x} = -ru + v\Delta u$$

$$\frac{\partial v}{\partial t} + (f + \zeta)\frac{\partial v}{\partial x} + \frac{\partial u}{\partial x} = -rv + v\Delta v$$

$$\frac{\partial h}{\partial t} + \frac{\partial h u}{\partial x} + \frac{\partial h v}{\partial y} = -h \frac{\partial \zeta}{\partial x}$$

$$\frac{\partial C}{\partial t} + (u + f) \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = -\frac{\partial C}{\partial x}$$

$$\frac{\partial C}{\partial t} + (\alpha \beta + f) \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = -\frac{\partial C}{\partial x}$$

$$B = gh + \frac{1}{2}(a^2 v^2 + \beta^2 C^2)$$

Test 1: Observation is generated by models and missing. Velocity is optimized following different approaches. (First-order derivative smooth(FDS), Haar wavelet(Haar), discrete cosine framelet(DCT), linear spline framelet(Spline), Curvelet, learned dictionary(LDA) and the hybrid one(LDA+FDS))

Test 2: 400 images (Middle: initial moment; Right: final moment) are acquired from Grenoble CORIOLIS platform (Left). Velocity is reconstructed by the proposed method.

Result

• Reconstruction errors comparison among seven approaches in test 1:

• The analyzed $(u, v)$ at initial (Left) and final (Right) moment by the proposed method in test 2:

Conclusion

• Owing to the sparsity penalty and the directional structure in the learned dictionary, promising performance is shown on 2D velocity reconstruction, such as fast convergence and more consistency with the true state.

• In the real data test, the proposed approach achieves to reconstruct the anticyclonic secondary vortex.

Reference

