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To cite this version:


HAL Id: hal-02263716
https://inria.hal.science/hal-02263716v2
Submitted on 6 Aug 2019

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Adaptive Image Assimilation for 2D Velocity Reconstruction

Abstract ID: OS17-A018

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Introduction

• 4D-Var is formulated as a nonlinear least-squares problem combining all the available heterogeneous information (i.e., mathematical models, observation, and priori knowledge) for calibrating the state of dynamical equations.
• Insufficient estimation of background, error covariance matrix and scarce observation will lead to unreliable prediction.
• A regularizer based on structure dictionary sparsity would be a tool of choice to improve the performance.

Dictionary Learning and Regularization

Dictionary Learning

In the data-driven approach (CAI et al., 2014), a dictionary W is learned adaptively from dataset f,

$$\min_{\rho,W} \|\rho - Wf\|^2 + \eta \|\rho\|_0, \quad \text{s.t.} \quad W^TW = I$$

(1)

Remarks: The updated dictionary (Fig.5) contains more directional structures from the dataset (Fig.4) than fixed ones (Fig.1-3).

Figure 1 – Multi-level Haar wavelet
Figure 2 – Linear spline framelet
Figure 3 – Curvelet (at three scales)

Regulatization

In view of the fact that features in images contribute to optimizing the flow (TITAUD et al., 2010), we introduce a “vorticity structure dictionary” sparsity regularizer in the framework of 4D-Var for 2D velocity reconstruction.

In the case that the velocity field is indirectly accessible from the observation of passive tracers, the object function is expressed as

$$\min_{(u_0,v_0)} \sum \left| \partial_x C(t_i) - C^{\text{obs}}(t_i) \right|^2_{R^{-1}} + \frac{\alpha}{2} \left| (u_0,v_0) - (u_0, v_0)^0 \right|^2_{R^{-1}} + \frac{\mu}{2} \| W \zeta_0 \|^2_{\text{regularizer}}$$

(2)

$$\zeta_0 = \frac{u_0}{\partial_x} - \frac{v_0}{\partial_y}$$ relative vorticity, W: learned dictionary

Figure 4 – Data from a vorticity model
Figure 5 – Learned Dictionary from the data

Split Bregman Iteration for Adaptive 4D-Var

• Due to fast convergence, the split Bregman iteration is widely applied to solve the $l_2 + l_1$-norm non-smooth convex problem:

$$\min_{w,d} \|d| + R(w) + \frac{1}{2} \|d - \Phi(w)\|^2$$

(3)

• Regarding to the adaptive 4D-Var problem, we redefine variables

$$w := (u_0, v_0), \Phi(w) := W \zeta_0, R(w) := \frac{\alpha}{2} \sum \left| \partial_x C(t_i) - C^{\text{obs}}(t_i) \right|^2_{R^{-1}} + \frac{\mu}{2} \| W \zeta_0 \|^2_{\text{regularizer}}$$

The split Bregman iteration (GOLDSTEIN et al., 2009)

$$b^{k+1} = b^k + \alpha \left( \Phi(u^{k+1}) - d^{k+1} \right)$$

$$u^{k+1} = u^k + \lambda R(w^{k+1}) - \frac{\alpha}{\lambda} b^{k+1}$$

(4)

Numerical Implementation

Models: 2D shallow water equation and concentration transport equation

$$\frac{\partial u}{\partial t} + (f + \zeta) v = -\nu + \nabla \Phi$$

$$\frac{\partial v}{\partial t} + (f + \zeta) u = -\nu + \nabla \Phi$$

$$\frac{\partial h}{\partial t} + u \frac{\partial h}{\partial x} + v \frac{\partial h}{\partial y} = 0$$

$$\frac{\partial c}{\partial t} + u \frac{\partial c}{\partial x} + v \frac{\partial c}{\partial y} = 0$$

$$B = gh + \frac{1}{2} (u^2 + v^2); \text{the Bernoulli's potential, } g; \text{the gravity, } f = \beta y; \text{the Coriolis parameter on the } \beta \text{-plane}$$

Settings

• Test 1: Observation is generated by models and missing. Velocity is optimized via following different approaches, (First-order derivative smooth(FDS), Haar wavelet(Haar), discrete cosine framelet(DCT), linear spline framelet(Spline), Curvelet, learned dictionary(LDA) and the hybrid one(LDA+FDS))
• Test 2: 400 images (Middle: initial moment; Right: final moment) are acquired from Grenoble CORIOLIS platform (Left). Velocity is reconstructed by the proposed method.

Result

• Reconstruction errors comparison among seven approaches in test 1:

Figure 6 – Reconstruction errors comparison among seven approaches in test 1

• The analyzed (u,v) at initial (Left) and final (Right) moment by the proposed method in test 2:

Figure 7 – The analyzed (u,v) at initial (Left) and final (Right) moment by the proposed method in test 2

Conclusion

• Owing to the sparsity penalty and the directional structure in the learned dictionary, promising performance is shown on 2D velocity reconstruction, such as fast convergence and more consistency with the true state.
• In the real data test, the proposed approach achieves to reconstruct the anticycloonic secondary vortex.

Reference

