

The operator approach to dynamic Strong Stackelberg Equilibria

Víctor Bucarey¹, Alain Jean-Marie², Eugenio Della Vecchia³, Fernando Ordóñez⁴

¹ Département d'Informatique, Université Libre de Bruxelles, vbucarey@gmail.com

² Inria, Université Côte d'Azur, alain.jean-marie@inria.fr

³ Departamento de Matemática, Universidad Nacional de Rosario, eugenio@fceia.unr.edu.ar

⁴ Departamento de Ingeniería Industrial, Universidad de Chile, fordon@dii.uchile.cl

The concept of Strong Stackelberg Equilibrium has received a renewed attention in the recent years, due notably to its application to security questions [1]. In Stackelberg games, it is assumed that one of the players, called the Leader, commits to a strategy. The other player, called the Follower, learns or observes this strategy, and reacts rationally to it. The game being with complete information, the Leader is able to predict the reaction of the follower, and is then able to optimize her strategy to maximize her own rewards. It may be that the follower's response is not unique, in which case this prediction of the leader is not possibly anymore. A *strong* Stackelberg Equilibrium (SSE) occurs when the Follower breaks such ties in favor of the Leader: she chooses the best option to for the Leader among the best ones for her.

Initially defined in static games, this concept has been extended to infinite-horizon, discounted dynamic games. Calculating dynamic SSE in such games within the set of general strategies appears to be very difficult in general [2], with the feature that stationary strategies are sub-optimal in general [3]. With the hope of reducing this complexity, as well memory storage for the strategies, many authors concentrate on *stationary feedback* policies. Even in this restricted class, the problem is still NP-Complete to compute in general [2] and several algorithms based on Mathematical Programming have been proposed in the literature for solving or approximating this problem [3].

In this paper, we study an alternative approach, inspired from the classical operator theory for Markov Decision Processes or Zero-Sum Nash Games. Our contribution is as follows: we first define a dynamic programming operator T corresponding to one-step strong Stackelberg games with a scrap value. Fixed-Point Equilibria (FPE) are defined as fixed points of T . The Value Iteration algorithm is defined as the iterations of T until convergence.

We illustrate through examples that FPE do not always exist. When they do, convergence of Value Iteration may or may not occur. When it does, the result is not necessarily a dynamic SSE. Nevertheless, we present sufficient conditions for a FPE to exist, and to coincide with a SSE.¹

References

- [1] D. Kar, T. Nguyen, F. Fang, M. Brown, A. Sinha, M. Tambe, and A. Jiang. Trends and applications in Stackelberg security games. In T. Başar and G. Zaccour, editors, *Handbook of Dynamic Game Theory*, chapter 28, pages 1223–1269. Springer, 2018.
- [2] J. Letchford, L. MacDermed, V. Conitzer, R. Parr, and C. L. Isbell. Computing optimal strategies to commit to in stochastic games. In *26th AAAI*, 2012.
- [3] Y. Vorobeychik and S. Singh. Computing Stackelberg equilibria in discounted stochastic games (corrected version). Retrieved online on Oct. 19, 2018, 2012.

¹Part of this research was realized during the SticAmSud project 16-STIC-10 DyGaMe.