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Encoding high-cardinality string categorical variables

Patricio Cerda and Gaël Varoquaux

Abstract—Statistical analysis usually requires a vector representation of categorical variables, using for instance *one-hot encoding*. This encoding strategy is not practical when the number of different categories grows, as it creates high-dimensional feature vectors. Additionally, the corresponding entries in the raw data are often represented as strings, that have additional information not captured by one-hot encoding.

Here, we seek low-dimensional vectorial encoding of string categorical variables with high-cardinality. Ideally, these should *i)* be scalable to a very large number of categories, *ii)* be interpretable to the end user, and *iii)* facilitate statistical analysis. We introduce two new encoding approaches for string categories: a *Gamma-Poisson matrix factorization* on character-level substring counts, and a *min-hash encoder*, based on min-wise random permutations for fast approximation of the Jaccard similarity between strings. Beyond the well-known properties of min-hash collisions, we establish properties of these representations showing that they are useful to capture inclusions. Both approaches are scalable and are suitable for streaming settings. Extensive experiments on real and simulated data show that these encoding methods improve prediction performance for real-life supervised-learning problems with high-cardinality string categorical variables and works as well as standard approaches with clean, low-cardinality ones. We recommend the following: *i)* if scalability is the main concern, the min-hash encoder is the best option as it does not require any fitting to the data; *ii)* if interpretability is important, the Gamma-Poisson factorization is a good alternative, as it can be interpreted as one-hot encoding, giving each encoding dimension a feature name that summarizes the substrings captured. Both models remove the need for hand-crafting features and data cleaning of string columns in databases and can be used for feature engineering in online autoML settings.

1 Introduction

Tabular datasets often contain columns with string entries. However, fitting statistical models on such data generally requires a numerical representation of all entries, which calls for building an encoding, or vector representation of the entries. Considering string entries as nominal -unordered- categories is convenient for their statistical analysis. In such situations, categories are supposed to be mutually exclusive and unrelated, with a fixed known set of possible values. Yet, in many real-world datasets, the entries of string columns are not standardized in a small number of categories. This poses challenges for statistical learning. First, the set of all possible categories may be huge and not known a priori, as the number of different strings in the column can indefinitely increase with the number of samples. Second, categories may not be unrelated: they often carry some morphological or semantical links.

The classic approach to encode categorical variables in a statistical analysis is *one-hot encoding*: creating vectors that agree with the general intuition of nominal categories: orthogonal and equidistant [1]. For high-cardinality categories, one-hot encoding leads to feature vectors of high dimensionality. This is especially problematic in big data settings, which can lead to a very large number of categories, posing computational and statistical problems.

Data engineering practices typically tackle these issues with data-cleaning techniques [2], [3]. In particular, deduplication tries to merge different variants of the same entity [4], [5]. A related concept is that of *normalization*, used in databases and text processing to put entries in canonical

forms. However, data cleaning or normalization often requires human intervention, and are the major hurdles to data analysis¹. To avoid the cleaning step, *Similarity encoding* [6] relaxes one-hot encoding by using *string similarities* [7]. Hence, it addresses the problem of related categories and has been shown to improve statistical analysis upon one-hot encoding [6]. Yet, it does not tackle the problem of high cardinality, and the data analyst much resort to heuristics such as choosing a subset of the training categories [6].

Here, we seek encoding approaches for statistical analysis on string categorical data that is suited to a very large number of categories without any human intervention: avoiding data cleaning, feature engineering, or architecture search. Our goals are: i) to provide feature vectors of limited dimensionality without any cleaning step, even for very large datasets; ii) to improve statistical analysis tasks such as supervised learning; and iii) to preserve the intuitions behind categories: entries can be arranged in natural groups that can be easily interpreted. We study two novel encoding methods that both address scalability and statistical performance: a min-hash encoder, based on locality-sensitive hashing (LSH) [8], and a low-rank model of co-occurrences in character n-grams: a Gamma-Poisson matrix factorization, suited to counting statistics. Both models scale linearly with the number of samples and are suitable for statistical analysis in streaming settings. Moreover, we show that the Gamma-Poisson factorization model enables interpretability with a sparse encoding that expresses the entries of the data as linear combinations of a small number of latent categories, built from their substring information.

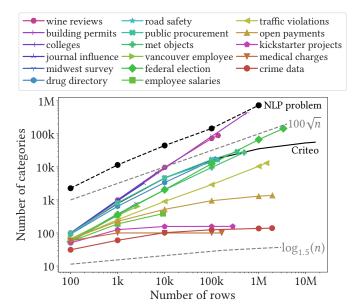


Fig. 1: Number of categories versus number of samples. In general, a higher number of samples implies a higher number of categories for the respective variable. In general, the cardinality of categories grows slower than words in a typical NLP problem (Wikipedia articles in this case).

This interpretability is very important: opaque and black-box machine learning models have limited adoption in real-world data-science applications. Often, practitioners resort to manual data cleaning to regain interpretability of the models.

Finally, we demonstrate on 17 real-life datasets that our encoding methods improve supervised learning on non curated data without the need for dataset-specific choices. As such, these encodings provide a scalable and automated replacement to data cleaning or feature engineering, and restore the benefits of a low-dimensional categorical encoding, as one-hot encoding.

The paper is organized as follows. Section 2 states the problem in detail and the prior art on creating feature vectors from categorical variables. Section 3 introduces and studies our two encoding approaches. In section 4, we present our experimental study with an emphasis on interpretation and on statistical learning on 17 non-curated datasets. Section 5 discusses these results, after which appendices provide information on the datasets and the experiments to facilitate the reproduction of our findings.

2 PROBLEM SETTING AND PRIOR ART

The academic literature in statistics often considers datasets that contain only categorical variables with a low cardinality, as datasets² in the UCI repository [9]. In such settings, the popular *one-hot encoding* is a suitable solution for supervised learning [1]. It models categories as mutually exclusive and, as categories are known a priory, new categories are not expected to appear in the test set. With enough data, supervised learning can then be used to link each category to a target variable.

TABLE 1: Examples of high-cardinality categorical variables.

Count	Non Proprietary Name	Employee Position Title
1736	alcohol	Police Aide
1089	ethyl alcohol	Master Police Officer
556	isopropyl alcohol	Mechanic Technician II
16	polyvinyl alcohol	Police Officer III
12	isopropyl alcohol swab	Senior Architect
12	62% ethyl alcohol	Senior Engineer Technician
6	alcohol 68%	Social Worker III
6	alcohol denat	Bus Operator
5	dehydrated alcohol	(h) Como catagorias in the

(a) Count for some of the categories containing the word *alco-hol* in the *Drug Directory* dataset. The dataset contains more than 120k samples.

(b) Some categories in the *Employee Salaries* dataset. For 10 000 employees, there are almost 400 different occupations. Yet, they share relevant substrings.

2.1 High-cardinality categorical variables

However, in many real-world problems, the number of different string entries in a column is very large, often growing with the number of observations (Figure 1). Consider for instance the *Drug Directory* dataset³: one of the variables is a categorical column with *non proprietary names* of drugs. As entries in this column have not been normalized, many different entries are likely related: they share a common ingredient such as "alcohol" (see Table 1a). Another example is the *Employee Salaries* dataset⁴. Here, a relevant variable is the *position title* of employees. As shown in Table 1b, here there is also overlap in the different occupations.

High-cardinality categorical variables may arise from variability in their string representations, such as abbreviations, special characters, or typos⁵. Such non-normalized data often contains very rare categories. Yet, these categories tend to have common morphological information. Indeed, the number of unique entries grows less fast with the size of the data than the number of words in natural language (Figure 1). In both examples above, drug names and position titles of employees, there is an implicit taxonomy. Crafting feature engineering or data-cleaning rules can recover a small number of relevant categories. However it is time consuming and often needs domain expertise.

Notation

We write sets of elements with capital curly fonts, as \mathcal{X} . Elements of a vector space are written in bold \mathbf{x} with the i-th entry denoted by x_i , and matrices are in capital and bold \mathbf{X} , with x_{ij} the entry on the i-th row and j-th column.

Let C be a categorical variable such that $dom(C) \subseteq \mathcal{S}$, the set of finite length strings. We call *categories* the elements of dom(C). Let $s_i \in \mathcal{S}, i=1...n$, be the category corresponding to the i-th sample of a dataset. For statistical learning, we want to find an encoding function enc: $\mathcal{S} \to \mathbb{R}^d$, such as $enc(s_i) = \mathbf{x}_i$. We call \mathbf{x}_i the *feature map* of s_i . Table 2 contain a summary of the main variables used in the next sections.

²See for example, the Adult dataset (https://archive.ics.uci.edu/ml/datasets/adult)

 $^{^3}$ Product listing data for all unfinished, unapproved drugs. Source: U.S. Food and Drug Administration (FDA)

⁴Annual salary information for employees of the Montgomery County, MD, U.S.A. Source: https://data.montgomerycountymd.gov/

⁵A taxonomy of different sources of *dirty* categorical variables can be found on [10], and a formal description of data quality problems is proposed by [11].

TABLE 2: Summary of notations

Symbol	Definition
S	Set of all finite-length strings.
$\mathcal{G}(s)\subseteq\mathcal{S}$	Set of all consecutive n-grams in $s \in \mathcal{S}$.
$\mathcal{V} = \bigcup_{i=1}^n \mathcal{G}(s_i)$	Vocabulary of n-grams in the train set.
C	Categorical variable.
n	Number of samples.
d	Dimension of the categorical encoder.
$m = \mathcal{V} $	Cardinality of the vocabulary.
$\mathbf{F} \in \mathbb{R}^{n imes m}$	Count matrix of n-grams.
$\mathbf{X} \in \mathbb{R}^{n imes d}$	Feature matrix of \tilde{C} .
$\operatorname{sim}: \mathcal{S} \times \mathcal{S} \to [0, 1]$	String similarity.
$h_k: \mathcal{S} \to [0,1]$	Hash function with salt value equal to k .
$Z_k: \mathcal{S} \to [0,1]$	Min-hash function with salt value equal to k .

2.2 One-hot encoding, limitations and extensions

2.2.1 Shortcomings of one-hot encoding

From a statistical-analysis standpoint, the multiplication of entries that express related information is challenging for two reasons. First, it dilutes the information: learning on rare categories is hard. Second, with one-hot encoding, representing these as separate categories creates high-dimension feature vectors. This high dimensionality entails large computational and memory costs; it increases the complexity of the associated learning problem, resulting in a poor statistical estimation. Dimensionality reduction of the one-hot encoded matrix can help with this issue, but at the risk of loosing information.

Moreover, encoding all unique entries with orthogonal vectors discards the overlap information visible in the string representations. Also, one-hot encoding cannot assign a feature vector to new categories that may appear in the testing set, even if its representation is close to one in the training set. Heuristics, such as assigning the zero vector to new categories, create collisions if more than one new category appears. As a result, one-hot encoding is ill suited to online learning settings: if new categories arrive, the entire encoding of the dataset has to be recomputed and the dimensionality of the feature vector becomes unbounded.

2.2.2 Similarity encoding for string categorical variables

For categorical variables represented by strings, *similarity encoding* extends one-hot encoding by taking into account a measure of string similarity between pairs of categories [6].

Let $s_i \in \mathcal{S}, i=1...n$, the category corresponding to the i-th sample of a given training dataset. Given a string similarity $\sin(s_i, s_j)$, similarity encoding builds a feature map $\mathbf{x}^{\sin} \in \mathbb{R}^k$ as:

$$\mathbf{x}_i^{\text{sim}} \stackrel{\text{def}}{=} [\text{sim}(s_i, s^{(1)}), \text{sim}(s_i, s^{(2)}), \dots, \text{sim}(s_i, s^{(k)})] \in \mathbb{R}^k,$$
(1)

where $\{s^{(l)}, l=1\ldots k\}\subseteq \mathcal{S}$ is the set of all unique categories in the training set—or a subset of prototype categories chosen heuristically⁶. With the previous definition, one-hot encoding corresponds to taking the discrete string similarity:

$$\operatorname{sim}_{\text{one-hot}}(s_i, s_i) = \mathbb{1}[s_i = s_i], \tag{2}$$

where $\mathbb{I}[\cdot]$ is the indicator function.

⁶In this work, we use as dimensionality reduction technique the k-means strategy explained in [6].

Empirical work on databases with categorical columns containing non-normalized entries showed that similarity encoding with a continuous string similarity brings significant benefits upon one-hot encoding [6]. Indeed, it relates rare categories to similar, more frequent ones. In columns with typos or morphological variants of the same information, a simple string similarity is often enough to capture additional information. Similarity encoding outperforms a bag-of-n-grams representation of the input string, as well as methods that encode high-cardinality categorical variables without capturing information in the strings representations [6], such as *target encoding* [12] or *hash encoding* [13].

A variety of string similarities can be considered for similarity encoding, but [6] found that a good performer was a similarity based on n-grams of consecutive characters. This n-gram similarity is based on splitting the two strings to compare in their character n-grams and calculating the Jaccard coefficient between these two sets [14]:

$$\operatorname{sim}_{\operatorname{n-gram}}(s_i, s_j) = J(\mathcal{G}(s_i), \mathcal{G}(s_j)) = \frac{|\mathcal{G}(s_i) \cap \mathcal{G}(s_j)|}{|\mathcal{G}(s_i) \cup \mathcal{G}(s_j)|}$$
(3)

where G(s), is the set of consecutive character n-grams for the string s.

Beyond the use of string similarity, an important aspect of similarity encoding is that it is a prototype method, using as prototypes a subset of the categories in the train set.

2.3 Related solutions for encoding string categories

2.3.1 Bag of n-grams

A simple way to capture morphology in a string is to characterize it by the count of its character n-grams. This is sometimes called a *bag-of-n-grams* characterization of strings. Such representation has been shown to be efficient for spelling correction [14] or for named-entity recognition [15].

For high-cardinality categorical variables, the number of different n-grams tends to increase with the number of samples. Yet, this number increases slowly, slower than in a typical NLP problem (see Figure 2). Indeed, categorical variables have less entropy than free text: they are usually repeated, often have subparts in common, and refer to a particular, more restrictive subject.

Representing strings by character-level n-grams is related to vectorizing texts by their tokens or words. Common practice uses *term-frequency inverse-document-frequency (tf-idf)* reweighting: dividing a token's count in a sample by its count in the whole document. Dimensionality reduction by a singular value decomposition (SVD) on this matrix leads to a simple topic extraction, latent semantic analysis (LSA) [16]. A related but more scalable solution for dimensionality reduction are random projections, which give low-dimensional approximation of Euclidean distances [17], [18].

2.3.2 Word embeddings

If the string entries are common words, an approach to represent them as vectors is to leverage word embeddings developed in natural language processing [19], [20]. Euclidean similarity of these vectors captures related semantic meaning in words. Multiple words can be represented as a weighted sum of their vectors, or with more complex approaches [21]. To cater for out-of-vocabulary strings,

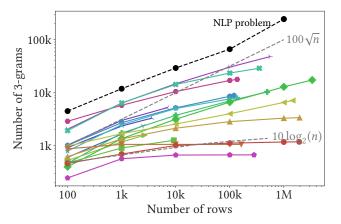


Fig. 2: Number of 3-gram versus number of samples (colors as in Figure 1). The number of different n-grams tends to increase slower than in a typical NLP problem (Wikipedia articles in this case).

FastText [22] considers subword information of words, *i.e.*, character-level n-grams. Hence, it can encode strings even in the presence of typos. Word vectors computed on very large corpora are available for download. These have captured fine semantic links between words. However, to analyze a given database, the danger of such approach is that the semantic of categories may differ from that in the pretrained model. These encodings do not adapt to the information specific in the data at hand. Moreover, they cannot be trained directly on the categorical variables for two reasons: categories usually lack of enough context as they are composed of short strings; and the number of samples in some datasets is not enough to properly train a neural network.

3 SCALABLE ENCODING OF STRING CATEGORIES

We now describe two novel approaches for categorical encoding of string variables. Both are based on the characterlevel structure of categories. The first one, that we call minhash encoding, comes from the document indexation literature, and in particular from the idea of locality-sensitive hashing (LSH) [8]. The method is stateless and it has been shown to approximate the Jaccard coefficient between two strings [23]. The second one is the Gamma-Poisson factorization [24], a matrix factorization technique—originally used in the probabilistic topic modeling literature—that assumes a Poisson distribution on the n-gram counts of categories, with a Gamma prior on the activations. An online algorithm of the factorization matrix allows to scale the method with a linear complexity on the number of samples. Both methods capture the morphological similarity of categories in a reduced dimensionality.

3.1 Min-hash encoding

3.1.1 Background: min-hash

Locality-sensitive hashing (LSH) [8] has been extensively used for approximate nearest neighbor search as an efficient way of finding similar objects (documents, pictures, etc.) in high-dimensional settings. One of the most famous functions in the LSH family is the min-hash function [23], [25],

originally designed to retrieve similar documents in terms of the Jaccard coefficient of the word counts of documents (see [26], chapter 3, for a primer).

Let \mathcal{X}^* be a totally ordered set and π a random permutation of the order in \mathcal{X} . For any non-empty $\mathcal{X} \subseteq \mathcal{X}^*$ with finite cardinality, the min-hash function $Z(\mathcal{X})$ can be defined as:

$$Z(\mathcal{X}) \stackrel{\text{def}}{=} \min_{x \in \mathcal{X}} \pi(x) \tag{4}$$

Note that $Z(\mathcal{X})$ can be also seen as a random variable. As shown in [23], for $\mathcal{X}, \mathcal{Y} \subseteq \mathcal{X}^*$, the min-hash function has the following property:

$$\mathbb{P}\left(Z(\mathcal{X}) = Z(\mathcal{Y})\right) = \frac{|\mathcal{X} \cap \mathcal{Y}|}{|\mathcal{X} \cup \mathcal{Y}|} = J(\mathcal{X}, \mathcal{Y}) \tag{5}$$

Where J is the Jaccard coefficient between the two sets. For a controlled approximation, several random permutations can be taken, which defines a min-hash signature. For d permutations π_i drawn i.i.d., Equation 5 leads to:

$$\sum_{j=1}^{d} \mathbb{1}[Z_j(\mathcal{X}) = Z_j(\mathcal{Y})] \sim \mathcal{B}(d, J(\mathcal{X}, \mathcal{Y})). \tag{6}$$

where \mathcal{B} denotes the Binomial distribution. Dividing the above quantity by d thus gives a consistent estimate of the Jaccard coefficient $J(\mathcal{X}, \mathcal{Y})^7$.

Without loss of generality, we can consider the case of \mathcal{X}^{\star} being equal to the real interval [0,1], so now for any $x \in [0,1]$, $\pi_i(x) \sim \mathcal{U}(0,1)$.

Proposition 3.1. Marginal distribution. *If* $\pi(x) \sim \mathcal{U}(0,1)$, and $\mathcal{X} \subset [0,1]$ such that $|\mathcal{X}| = k$, then $Z(\mathcal{X}) \sim Dir(k,1)$.

Proof. It comes directly from considering that
$$\mathbb{P}(Z(\mathcal{X}) \leq z) = 1 - \mathbb{P}(Z(\mathcal{X}) > z) = 1 - \prod_{i=1}^{k} \mathbb{P}(\pi(x_i) > z) = 1 - (1-z)^k$$
. \square

Now that we know the distribution of the min-hash random variable, we will show how each dimension of a minhash signature maps inclusion of sets to simple arithmetic comparisons.

Proposition 3.2. Inclusion. Let $\mathcal{X}, \mathcal{Y} \subset [0, 1]$ such that $|\mathcal{X}| = k_x$ and $|\mathcal{Y}| = k_y$.

(i) If
$$\mathcal{X} \subset \mathcal{Y}$$
, then $Z(\mathcal{Y}) \leq Z(\mathcal{X})$.

(ii)
$$\mathbb{P}\Big(Z(\mathcal{Y}) \leq Z(\mathcal{X}) \ \middle| \ \mathcal{X} \cap \mathcal{Y} = \emptyset\Big) = k_y/(k_x + k_y)$$

Proof. (i) is trivial and (ii) comes directly from Prop. 3.1:

$$\mathbb{P}\left(Z(\mathcal{Y}) - Z(\mathcal{X}) \le 0 \mid \mathcal{X} \cap \mathcal{Y} = \emptyset\right)$$

$$= \int_0^1 \int_0^x f_{Z(\mathcal{Y})}(y) f_{Z(\mathcal{X})}(y) \, dy \, dx$$

$$= \int_0^1 \left(1 - (1 - x)^{k_y}\right) k_x (1 - x)^{k_x - 1} dx$$

$$= k_y / (k_x + k_y)$$

⁷Variations of the min-hash algorithm, as the min-max hash method [27] have proven to reduce the variance of the Jaccard similarity approximation.

At this point, we do not know anything about the case when $\mathcal{X} \not\subseteq \mathcal{Y}$, so for a fixed $Z(\mathcal{X})$, we can not ensure that any set with lower min-hash value has \mathcal{X} as inclusion. The following theorem allows us to define regions in the vector space generated by the min-hash signature that, with high probability, are associated to inclusion rules.

Theorem 3.1. Identifiability of inclusion rules.

Let $\mathcal{X}, \mathcal{Y} \subset [0,1]$ be two finite sets such that $|\mathcal{X}| = k_x$ and $|\mathcal{Y}| = k_y$. $\forall \epsilon > 0$, if $d \ge \lceil -\log(\epsilon)/\log(1 + \frac{k_x}{k_y}) \rceil$, then

$$\mathcal{X} \not\subseteq \mathcal{Y} \Rightarrow \mathbb{P}\left(\sum_{j=1}^{d} \mathbb{1}[Z_j(\mathcal{Y}) \le Z_j(\mathcal{X})] = d\right) \le \epsilon.$$
 (7)

Proof. First, notice that:

$$\mathcal{X} \not\subseteq \mathcal{Y} \iff \exists k \in \mathbb{N}, 0 \leq k < k_x \text{ such that } |\mathcal{X} \cap \mathcal{Y}| = k$$

Then, defining $\mathcal{Y}' \stackrel{\text{def}}{=} \mathcal{Y} \setminus (\mathcal{X} \cap \mathcal{Y})$, with $|\mathcal{Y}'| = k_y - k$:

$$\mathbb{P}\left(Z(\mathcal{Y}) \leq Z(\mathcal{X}) \mid \mathcal{X} \subseteq \mathcal{Y}\right) = \mathbb{P}\left(Z(\mathcal{Y}') \leq Z(\mathcal{X}) \mid \mathcal{X} \cap \mathcal{Y}' = \emptyset\right)$$

$$= (k_y - k)/(k_x + k_y - k)$$

$$\leq k_y/(k_x + k_y)$$

$$= \mathbb{P}\left(Z(\mathcal{Y}) \leq Z(\mathcal{X}) \mid \mathcal{X} \cap \mathcal{Y} = \emptyset\right)$$

Finally:

$$\mathbb{P}\left(\sum_{j=1}^{d} \mathbb{1}[Z_{j}(\mathcal{Y}) \leq Z_{j}(\mathcal{X})] = d \mid \mathcal{X} \not\subseteq \mathcal{Y}\right) \\
= \mathbb{P}\left(Z(\mathcal{Y}) \leq Z(\mathcal{X}) \mid \mathcal{X} \not\subseteq \mathcal{Y}\right)^{d} \\
\leq \mathbb{P}\left(Z(\mathcal{X}) \leq Z(\mathcal{Y}) \mid \mathcal{X} \cap \mathcal{Y} = \emptyset\right)^{d} = (k_{y}/(k_{x} + k_{y}))^{d}$$

Theorem 3.1 tells us that taking enough random permutations, ensures that when $\forall j, Z_j(\mathcal{Y}) \leq Z_j(\mathcal{X})$ the probability that $\mathcal{X} \not\subseteq \mathcal{Y}$ is small. This result is very important, as it shows a global property of the min-hash representation when using several random permutations, going beyond the well-known properties of collisions in the min-hash signature.

3.1.2 The min-hash encoder

A practical way to build a computationally efficient implementation of min-hash is to use a hash function with different salt numbers instead of random permutations. Indeed, hash functions can be built with suitable *i.i.d.* random-process properties [25]. Thus, the min-hash function can be constructed as follows:

$$Z_j(\mathcal{X}) = \min_{x \in \mathcal{X}} h_j(x), \tag{8}$$

where h_j is a hash function⁸ on \mathcal{X}^* with salt value j.

For the specific problem of categorical data, we are interested in a fast approximation of $J(\mathcal{G}(s_i),\mathcal{G}(s_j))$, where $\mathcal{G}(s)$ is the set of all consecutive character n-grams for the string s. We define the min-hash encoder as:

$$\mathbf{x}^{\text{min-hash}}(s) \stackrel{\text{def}}{=} [Z_1(\mathcal{G}(s)), \dots, Z_d(\mathcal{G}(s))] \in \mathbb{R}^d,$$
 (9)

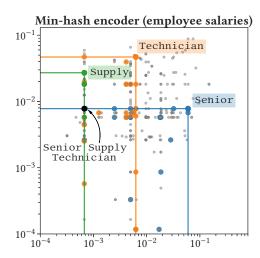


Fig. 3: The min-hash encoder transforms containment into inequality operations. Color dots are categories that contain the corresponding colored substrings and gray dots are categories that do not contain any of them. The category Senior Supply Technician (black dot) is in the intersection of the three containment regions.

Considering the hash functions as random processes, Equation 6 implies that this encoder has the following property:

$$\frac{1}{d} \mathbb{E}\left[\|\mathbf{x}^{\min\text{-hash}}(s_i) - \mathbf{x}^{\min\text{-hash}}(s_j)\|_{\ell_0} \right] = J(\mathcal{G}(s_i), \mathcal{G}(s_j))$$
(10)

Proposition 3.2 tells us that the min-hash encoder transforms, the inclusion relation of strings into an order relation in the feature space. This is especially relevant for learning tree-based models, as theorem 3.1 proofs that by performing a reduced number of splits in the min-hash dimensions, the space can be divided between the elements that contain and do not contain a given substring s_i . As an example, Figure 3 shows this global property of the min-hash encoder for the case of the employe salaries dataset with $d{=}2$. The substrings Senior, Supply and Technician are all included in the category Senior Supply Technician , and as consequence, the position for this category in the encoding space will be always in the bottom-left region generated by the substrings.

Finally, this encoder is specially suitable for very large scale settings, as it is very fast to compute and completely stateless. A stateless encoding is very useful in distributed computing setting: different workers can then process data simultaneously without exchanging information. Its drawback is that, as it relies on hashing, the encoding cannot easily be inverted and interpreted in terms of the original string entries.

3.2 Gamma-Poisson factorization

To facilitate interpretation, we now introduce an encoding approach that estimates latent categories generating the string entries.

3.2.1 Model

П

We use a generative model of strings from latent categories. For this, we rely on the Gamma-Poisson model [24], a matrix factorization technique well-suited to counting statistics

⁸Here we use a 32bit version of the MurmurHash3 function [28].

counting statistics. The idea was originally developed for finding low-dimensional representations, known as topics, of documents given its word count representation. As the string entries we consider are much shorter than text documents and can contain typos, we rely on their substring string: we represent each observation by its count vector of character-level structure of n-grams. Each observation, a string entry described by its count vector $\mathbf{f} \in \mathbb{N}^m$, is modeled as a linear combination of d unknown prototypes or topics, $\mathbf{\Lambda} \in \mathbb{R}^{d \times m}$:

$$\mathbf{f} \approx \mathbf{x} \, \mathbf{\Lambda},$$
 (11)

Here, $\mathbf{x} \in \mathbb{R}^d$ are the activations that decompose the observation \mathbf{f} in the prototypes $\mathbf{\Lambda}$ in the count space. As we will see later, these prototypes can be seen as latent categories.

Given a training dataset with n samples, the model estimates the unknown prototypes Λ by factorizing the data's bag-of-n-grams representation $\mathbf{F} \in \mathbb{N}^{n \times m}$, where m is the number of different n-grams in the data:

$$\mathbf{F} \approx \mathbf{X} \mathbf{\Lambda}, \quad \text{with } \mathbf{X} \in \mathbb{R}^{n \times d}, \mathbf{\Lambda} \in \mathbb{R}^{d \times m}$$
 (12)

As **f** is a count vector, it is natural to consider a Poisson distribution for each of its elements:

$$p(f_j|(\mathbf{x}\,\mathbf{\Lambda})_j) = \frac{1}{f_j!}(\mathbf{x}\,\mathbf{\Lambda})_j^{f_j} e^{-(\mathbf{x}\,\mathbf{\Lambda})_j}, \quad j = 1, ..., m. \quad (13)$$

For a prior on the elements of $\mathbf{x} \in \mathbb{R}^d$, we use a Gamma distribution, as it is the conjugate prior of the Poisson distribution:

$$p(x_i) = \frac{x_i^{\alpha_i - 1} e^{-x_i/\beta_i}}{\beta_i^{\alpha_i} \Gamma(\alpha_i)}, \quad i = 1, ..., d.$$
 (14)

Where α , $\beta \in \mathbb{R}^d$ are the shape and scale parameters of the Gamma distribution for each one of the d topics.

3.2.2 Estimation strategy

To fit the model to the input data, we maximize the likelihood \mathcal{L} of the model, denoted by:

$$\mathcal{L} = \prod_{i=1}^{m} \frac{(\mathbf{x}\,\mathbf{\Lambda})_{j}^{f_{j}} e^{-(\mathbf{x}\,\mathbf{\Lambda})_{j}}}{f_{j}!} \prod_{i=1}^{d} \frac{x_{i}^{\alpha_{i}-1} e^{-x_{i}/\beta_{i}}}{\beta_{i}^{\alpha_{i}} \Gamma(\alpha_{i})}$$
(15)

Maximizing the log-likelihood with respect to the parameters gives:

$$\frac{\partial}{\partial \Lambda_{ij}} \log \mathcal{L} = \frac{f_j}{(\mathbf{x} \, \mathbf{\Lambda})_j} x_i - x_i \tag{16}$$

$$\frac{\partial}{\partial x_i} \log \mathcal{L} = \sum_{i=1}^m \frac{f_j}{(\mathbf{x} \mathbf{\Lambda})_j} \Lambda_{ij} - \Lambda_{ij} + \frac{\alpha_i - 1}{x_i} - \frac{1}{\beta_i}$$
 (17)

As explained in [24], these expressions are analogous to solving the following non-negative matrix factorization (NMF) with the generalized Kullback-Leibler divergence⁹ as loss:

$$\begin{pmatrix} \mathbf{F} \\ \operatorname{diag}(\boldsymbol{\beta})^{-1} \end{pmatrix} = \mathbf{X} \begin{pmatrix} \boldsymbol{\Lambda} \\ \operatorname{diag}(\boldsymbol{\alpha}) - I_d \end{pmatrix}$$
(18)

In other words, the Gamma-Poisson model can be interpreted as a constrained non-negative matrix factorization in which the generalized Kullback-Leibler divergence is minimized between ${\bf F}$ and ${\bf X}{\bf \Lambda}$, subject to a Gamma prior

in the distribution of the elements of X. The Gamma prior induces sparsity in the activations x of the model.

To solve the NMF problem above, [29] proposes the following recurrences:

$$\Lambda_{ij} \leftarrow \Lambda_{ij} \left(\sum_{\ell=1}^{n} \frac{f_{\ell j}}{(\mathbf{X}\boldsymbol{\Lambda})_{\ell j}} x_{\ell i} \right) \left(\sum_{\ell=1}^{n} x_{\ell i} \right)^{-1}$$

$$x_{\ell i} \leftarrow x_{\ell i} \left(\sum_{j=1}^{m} \frac{f_{\ell j}}{(\mathbf{X}\boldsymbol{\Lambda})_{\ell j}} \Lambda_{ij} + \frac{\alpha_{i} - 1}{x_{\ell i}} \right) \left(\sum_{j=1}^{m} \Lambda_{ij} + \beta_{i}^{-1} \right)^{-1}$$

$$(20)$$

As **F** is a sparse matrix, the summations above need to be computed only on the non-zero elements of **F**. This fact considerably decreases the computational cost of the algorithm.

Following [30], we present an online (or streaming) version of the Gamma-Poisson (algorithm 1) for the Generalized KL divergence. The basic idea of the algorithm is to exploit the fact that in the recursion for Λ , the summations are done with respect to the training samples. Instead of computing the numerator and denominator in the entire training set at each update, one can update this values only with mini-batches of data, which considerably decreases the memory usage and time of the computations.

For better computational performance, we adapt the implementation of this solver to the specificities of our problem—factorizing substring counts across entries of a categorical variable. We improve the efficiency of the algorithm by saving a dictionary of the activations for each category in the convergence of the previous mini-batches (algorithm 1, line 4) and use them as an initial guess for the same category in a future mini-batch. This is a warm restart and is especially important in the case of categorical variables because for most datasets, the number of unique categories is much lower than the number of samples.

The hyper-parameters of the algorithm and its initialization can affect convergence. One important parameter is ρ , the discount factor for the previous iterations of the topic matrix $\Lambda^{(t)}$ (algorithm 1, line 9-10). Figure 11 in the appendix shows that choosing ρ =0.95 gives a good compromise between stability of the convergence and data fitting in term of the Generalized KL divergence. With respect to the initialization of the topic matrix $\Lambda^{(0)}$, a good option is to choose the centroids of a k-means clustering (Figure 12) in a hashed version of the n-gram count matrix \mathbf{F} (in order to speed-up the k-means algorithm) and then project back to the n-gram space with a nearest neighbors algorithm. In the case of a streaming setting, the same approach can be used in a subset of the data.

3.2.3 Inferring feature names

An encoding strategy with dimensions that have individual interpretation is easily understandable by humans and facilitates the full statistical analysis. A straightforward strategy for interpretation of the Gamma Poisson encoder is to describe each encoding dimension by the features of the string entries that it captures. For this, one alternative is to track the feature maps corresponding to each input category, and assign labels based on the input categories that activate the most in a given dimensionality. Another option is to

⁹In the sense of the NMF literature. See for instance [29].

Algorithm 1: Online Gamma-Poisson factorization

```
Input : F, \Lambda^{(0)}, \alpha, \beta, \rho, q, \eta, \epsilon
         Output: X, \Lambda
  1 while \|\mathbf{\Lambda}^{(t)} - \mathbf{\Lambda}^{(t-1)}\|_F > \eta do
                        draw \mathbf{f}_t from the training set \mathbf{F}.
  2
                        while \|\mathbf{x}_t - \mathbf{x}_t^{old}\|_2 > \epsilon \ \mathbf{do}
  3
  4
                      \left| \begin{bmatrix} \mathbf{x}_t \left( \frac{\mathbf{f}_t}{\mathbf{x}_t \boldsymbol{\Lambda}^{(t)}} \right) \boldsymbol{\Lambda}^{(t)\mathsf{T}} + \boldsymbol{\alpha} - 1 \end{bmatrix} . \left[ \mathbf{1} \, \boldsymbol{\Lambda}^{(t)\mathsf{T}} + \boldsymbol{\beta}^{-1} \right]^{\cdot - 1} \right. end
  5
                      egin{aligned} \mathbf{	ilde{A}}_t \leftarrow \mathbf{\Lambda}^{(t)}. \left[ \mathbf{x}_t^\mathsf{T} \left( rac{\mathbf{f}_t}{\mathbf{x}_t \mathbf{\Lambda}^{(t)}} 
ight) 
ight] \ \mathbf{	ilde{B}}_t \leftarrow \mathbf{x}_t^\mathsf{T} \mathbf{1} \end{aligned}
  7
                       if t \equiv 0 \mod q, then
  8
                                   \begin{aligned} \mathbf{A}^{(t)} &\leftarrow \rho \, \mathbf{A}^{(t-q)} + \sum_{s=t-q+1}^t \tilde{\mathbf{A}}^{(s)} \\ \mathbf{B}^{(t)} &\leftarrow \rho \, \mathbf{B}^{(t-q)} + \sum_{s=t-q+1}^t \tilde{\mathbf{B}}^{(s)} \\ \mathbf{\Lambda}^{(t)} &\leftarrow \mathbf{A}^{(t)} . / \, \mathbf{B}^{(t)} \end{aligned}
10
11
                       end
12
                       t \leftarrow t + 1
13
14 end
```

apply the same strategy, but for substrings, such as words contained in the input categories. In the experiments, we follow the second approach as a lot of datasets are composed of entries that are overlapped, hence individual words carry more information for interpretability than the entire strings.

This method can be applied to any encoder, but it is expected to work well if the encodings are sparse and composed of only positive values with a meaningful magnitude. The Gamma prior of the Gamma-Poisson factorization ensures these properties.

4 EXPERIMENTAL STUDY

We now study the different encoding methods in terms of interpretability and for supervised-learning performance. For this purpose, we use three different types of data: simulated categorical data, and real data with curated and non-curated categories.

We benchmark the following strategies: one-hot, tf-idf, fastText [31], similarity encoding [6], and the Gamma-Poisson factorization on character n-grams. For all the strategies based on a n-gram representation, we use the set of 2-4 character grams 10 . For a fair comparison across encoding strategies, we used the same dimensionality d in all approaches. To set the dimensionality of one-hot encoding, tf-idf and fastText, we used a truncated SVD (implemented efficiently following [32]). Note that dimensionality reduction improves one-hot encoding with tree-based learners for data with rare categories [6]. For similarity encoding, we selected prototypes with a k-means strategy, following [6], as it gives slightly better prediction results than the most frequent categories 11 .

4.1 Real-life datasets with string categories

4.1.1 Datasets with high-cardinality categories

In order to evaluate the different encoding strategies presented in the previous sections, we collected 17 real-world datasets containing a prediction task and at least one relevant high-cardinality categorical variable as feature¹². Table 3 shows a quick description of the datasets and the corresponding categorical variables (see Appendix C.1.1 for a description of datasets and the related learning tasks).

Table 3 also details the source of high-cardinality for the datasets: multi-label, typos, description and multi-language. We call multi-label the situation where a single column contains multiple information shared by several entries, e.g., supply technician, where supply denotes the type of activity, and technician denotes the rank of the employee (as opposed, e.g., to manager). The source typos refers to entries having small morphological variations, as midwest and mid-west. Description are categorical entries that are composed of a short free-text description, so they are closer to a typical NLP problem, although they are usually constrained to a very particular subject, so they frequently contain very recurrent informative words and some entries might be repeated. Finally, multi-language are datasets in which the categorical variable contains more that one language across the different entries.

4.1.2 Datasets with curated strings

We also evaluate the behavior of encoders when the categorical variables have already been curated: usually, standardization of entries generates low cardinality categorical variables. For this, we collected seven of such datasets (see Appendix C.1.2).

4.2 Recovering latent categories

4.2.1 Recovery on simulated data

Table 3 shows that the most common scenario for high cardinality string variables is the *multi-label* categories. The second most common problem is the presence of *typos* (or any source of morphological variation of the same idea). To analyze these two cases in a more controlled setting, we created two simulated categorical variables. Table 4 show examples of the categories we generated, taking as a base 8 "ground truth" categories of animals: chicken, eagle, giraffe, horse, leopard, lion, tiger and turtle.

The multi-label data was created by concatenating k+2 ground truth categories, with k following a Poisson distribution—hence, all entries contain at least two labels. For the generation of data with typos, we added 10% of typos to the original ground truth categories by randomly replacing one character by another one (x, y, or z).

To measure the ability of an encoder to recover a feature matrix close to a one-hot encoding matrix of ground-truth categories in these simulated settings, we use the Normalized Mutual Information (NMI) as metric. Given two random variables X_1 and X_2 , the NMI is defined as:

$$NMI = 2 \frac{I(X_1; X_2)}{H(X_1) + H(X_2)}$$
 (21)

 12 If a dataset has more than one categorical variable, only one selected variable was encoded with the proposed approaches, while the rest of them were one-hot encoded.

 $^{^{10}}$ In addition to the word as tokens, pretrained versions of fastText also use the set of 3-6 character n-grams.

¹¹An implementation of these strategies can be found on https://dirty-cat.github.io. We do not test the *random projections* strategy for similarity encoding as it is not scalable.

TABLE 3: Non-curated datasets. Description for the corresponding high-cardinality categorical variable.

Dataset	#samples	#categories	#categories per 1000 samples	Gini coefficient	Mean category length (#chars)	Source of high cardinality	
Crime Data	1.5M	135	64.5	0.85	30.6	Multi-label	
Medical Charges	163k	100	99.9	0.23	41.1	Multi-label	
Kickstarter Projects	281k	158	123.8	0.64	11.0	Multi-label	
Employee Salaries	9.2k	385	186.3	0.79	24.9	Multi-label	
Open Payments	2.0M	1.4k	231.9	0.90	24.7	Multi-label	
Traffic Violations	1.2M	11.3k	243.5	0.97	62.1	Typos; Description	
Vancouver Employees	2.6k	640	341.8	0.67	21.5	Multi-label	
Federal Election	3.3M	145.3k	361.7	0.76	13.0	Typos; Multi-label	
Midwest Survey	2.8k	844	371.9	0.67	15.0	Typos	
Met Objects	469k	26.8k	386.1	0.88	12.2	Typos; Multi-label	
Drug Directory	120k	17.1k	641.9	0.81	31.3	Multi-label	
Road Safety	139k	15.8k	790.1	0.65	29.0	Multi-label	
Public Procurement	352k	28.9k	804.6	0.82	46.8	Multi-label; Multi-language	
Journal Influence	3.6k	3.2k	956.9	0.10	30.0	Multi-label; Multi-language	
Building Permits	554k	430.6k	940.0	0.48	94.0	Typos; Description	
Wine Reviews	138k	89.1k	997.7	0.23	245.0	Description	
Colleges	7.8k	6.9k	998.0	0.02	32.1	Multi-label	

TABLE 4: Examples of simulated categorical variables.

Туре	Example categories
Ground truth	<pre>chicken; eagle; giraffe; horse; leopard; lion; tiger; turtle.</pre>
Multi-label	lion chicken; horse eagle lion; tiger leopard giraffe turtle.
Typos (10%)	<pre>chxcken; eazle; gixaffe; gizaffe; hoyse; lexpard; lezpard; lixn; tiyer; tuxtle.</pre>

Where $I(\cdot;\cdot)$ is the mutual information and $H(\cdot)$ the entropy. To apply this metric to the feature matrix \mathbf{X} generated by the encoding of all ground truth categories, we consider that \mathbf{X} , after rescaling 13 , can be seen as a two dimensional probability distribution. For a encoders that can produce encoding matrices with negative values, we take the elementwise absolute value of \mathbf{X} . The NMI is a classic measure of correspondences between clustering results [33]. Beyond its information-theoretical interpretation, an appealing property is that it is invariant to order permutations. The NMI of any permutation of the identity matrix is equal to 1 and the NMI of any constant matrix is equal to 0. Thus, the NMI in this case is interpreted as a recovering metric of a one-hot encoded matrix of latent, ground truth, categories.

Table 5 shows the NMI values for both simulated datasets. The Gamma-Poisson factorization obtains the highest values in both multi-label and typo settings and for different dimensionalities of the encoders. The best recovery

TABLE 5: **Recovery of latent categories**: Normalized mutual information for different encoders.

Encoder	Multilabel			Typos			
Encouer	k=6	k=8	k = 10	k=6	k=8	k = 10	
Tf-idf + SVD	0.16	0.18	0.17	0.17	0.17	0.17	
FastText + SVD	0.08	0.09	0.09	0.08	0.08	0.09	
Similarity Encoder	0.32	0.25	0.24	0.72	0.82	0.78	
Min-hash Encoder	0.14	0.15	0.13	0.14	0.15	0.13	
Gamma-Poisson	0.76	0.82	0.79	0.78	0.83	0.80	

TABLE 6: Recovering true categories in low-cardinality case. Normalized mutual information for different encoders (d=30)

Dataset (cardinality)	Gamma- Poisson	Similarity Encoding	Tf-idf + SVD	FastText + SVD	
Adult (5)	0.75	0.71	0.54	0.19	
California Housing (5)	0.46	0.51	0.56	0.20	
Dating Profiles (18)	0.52	0.24	0.25	0.12	
House Prices (15)	0.83	0.24	0.31	0.11	

for each setting is obtained when the dimensionality of the encoder is equal to the number of ground-truth categories, *i.e.*, k=8.

4.2.2 Results for real data

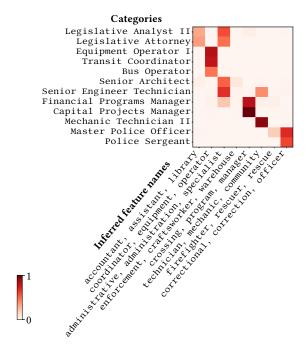
For curated data, the cardinality is usually low. We nevertheless perform the encoding using a default choice of d=30, to gauge how well turn-key generic encoding represent these curated strings. Table 6 shows the NMI values for the different curated datasets (see the Appendix C.1.2 for a description of the datasets), measuring how much the generated encoding resembles a one-hot encoding on the curated categories. Despite the fact that it is used with a dimensionality larger than the cardinality of the curated category, Gamma-Poisson factorization has the highest recovery performance in 3 out of 4 datasets.

These experiments measuring the ability to recover latent categories suggest that Gamma-Poisson factorization can safely be understood as encoding by loadings on hidden categories. To validate this intuition, Figure 4 shows such encodings in the case of the simulated data as well as the real-world non-curated Employees Salaries dataset.

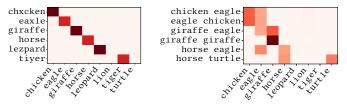
4.3 Encoding for supervised learning

To study how the encoders perform for statistical analysis, we now turn to measuring prediction accuracy in supervised-learning tasks.

 $^{^{13}}An~\ell_1$ normalization of the rows , followed by an ℓ_1 element-wise normalization of the entire encoding matrix.



(a) Employee Salaries dataset (Occupation)



(b) Simulated typos (c) Simulated multi-label categories

Fig. 4: The Gamma-Poisson factorization gives positive and sparse representations that are easily interpretable. Examples of encoding vectors (d=8) for a real dataset (a) and for simulated data (b and c) obtained with a Gamma-Poisson factorization. The x-axis shows the activations for each dimension with their respective inferred feature names. Figure 15 in the Appendix shows the obtained feature names for other encoders.

4.3.1 Experiment settings

We use gradient boosted trees, as implemented in XGBoost [34]. Note that trees can be implemented on categorical variables¹⁴. However, this encounter the same problems as one-hot encoding: the number of comparisons grows with the number of categories. Hence, the best trees approaches for categorical data use target encoding to impose an order on categories [35]. We also investigated other supervised-learning: linear models, neural networks, and kernel machines with RBG and polynomial kernels. However, even with significant hyper-parameter tuning, they under-performed XGBoost on our tabular datasets. The good performance of gradient-boosted trees is consistent with previous reports of systematic benchmarks [36].

Depending on the dataset, the learning task can be either

TABLE 7: Wilcoxon test p-values for different encoders between SVD and Gaussian random projections as a dimensionality reduction technique

Encoder SVD v/s Random projection (p-va					
Tf-idf	0.003				
FastText	0.001				
One-hot	0.492				

regression, binary or multiclass classification 15 . As datasets get different prediction scores, we visualize encoders' performance with prediction accuracy results scaled in a relative score. It is a dataset-specific scaling of the original score, in order to bring performance across datasets in the same range. In other words, for a given dataset i:

relative
$$\operatorname{score}_{j}^{i} = 100 \frac{\operatorname{score}_{j}^{i} - \min_{j} \operatorname{score}_{j}^{i}}{\max_{j} \operatorname{score}_{j}^{i} - \min_{j} \operatorname{score}_{j}^{i}}$$
 (22)

where $score_j^i$ is the the prediction score for the dataset i with the configuration $j \in \mathcal{J}$, the set of all trained models—in terms of dimensionality, type of encoder and cross-validation split. The relative score is figure-specific and is only indented to be used as a visual comparison of classifiers' performance across multiple datasets. A higher relative score means better results.

For a proper statistical comparison of encoders, we use a ranking test across multiple datasets [37]. Note that in this framework each dataset represents a single sample, and not the cross-validation splits which are not mutually independent. To do so, for a particular dataset, encoders were ranked according to the median score value over cross-validation splits. At the end, the relevant statistics is the average ranking across datasets. This statistic is computed for all encoders at a fixed dimensionality d. This test compares the average ranking using a F-distribution. If the null hypothesis is rejected, we use a Nemenyi post-hoc test to verify whether the difference in performance across pairs of encoders is significant [37].

To do pairwise comparison between two encoders, we use a pairwise Wilcoxon signed rank test and we provide the corresponding p-values given the null hypothesis that the two encoders are equally performing across different datasets.

4.3.2 Prediction with non-curated data

Figure 5 shows the results of the prediction benchmark with 17 datasets. It compares encoders in terms of the relative score of Equation 22. All n-gram based encoders clearly improve upon one-hot encoding, at all dimensions. Minhash seems to have a slightly better prediction performance for both dimensionalities in terms of the median relative score across categories, despite of being the only studied method that does not require a data fit.

Results of the Nemenyi ranking test for both dimensionalities are similar. It confirms the superior performance of n-gram-based methods in comparison to one-hot encoding.

¹⁴XGBoost does not support categorical features. The recommended option is to use one-hot encoding (https://xgboost.readthedocs.io).

 $^{^{15}\}mbox{Depending}$ on the type of prediction task, we used different scores to evaluate the performance of the supervised learning problem: for regression, we used the R^2 score; for binary classification, the average precision; and for multiclass classification, the accuracy score.

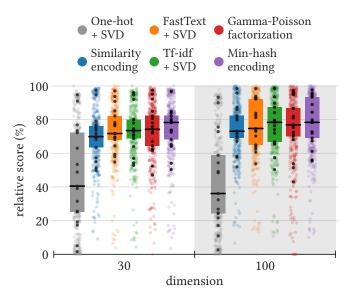


Fig. 5: Encoding with subword information performs significantly better than one-hot. Classifier: XGBoost. Comparison of encoders in terms of a relative score (the prediction score on the particular dataset, rescaled with respect to the global maximum and minimum score values across dimensions). Color dots indicate the scores for each crossvalidation fold, black dots the median score across folds for a dataset, the black line indicates the median score and the box gives the interquartile range. For one-hot encoding, bagof-n-grams and fastText, the dimensionality was fixed by using a truncated SVD.

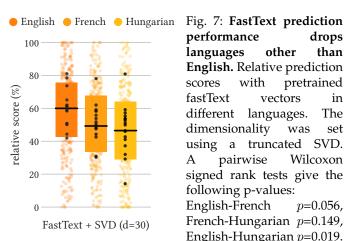


Fig. 6: Relative score

performance drops languages than other **English.** Relative prediction scores with pretrained fastText vectors in different languages. The dimensionality was set using a truncated SVD. pairwise Wilcoxon Α signed rank tests give the following p-values:

English-French p=0.056French-Hungarian p=0.149, English-Hungarian p=0.019.

In Figure 5, we compare our two encoding methods with techniques that use a dimensionality reduction based on a SVD. An stateless alternative to the SVD is to use random projections instead [38]. Table 7 uses a Wilcoxon signed rank test to verify the prediction performance of both approaches. For tf-idf and Fastext, the SVD is significantly superior to random projections. On the contrary, there is no statistical difference for one-hot, even though the performance is slightly better for SVD (p-value equal to 0.492).

Another relevant result is shown in Figure 7, were we compare the prediction results for Fastext pretrained in different languages (English, French et Hungarian). English

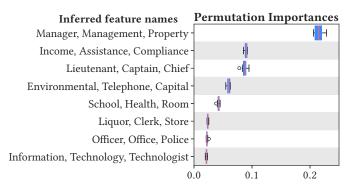


Fig. 8: Gamma-Poisson gives interpretable feature names. Permutation importances for the categorical variable Employee Position Title in the Employee Salaries dataset. Here we show the 8 most important latent topics from a total of 30. The overall feature importances for every feature in the dataset are shown in Figure 16 in the appendix.

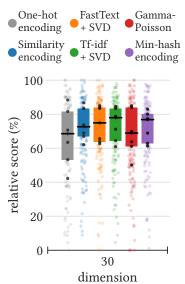


Fig. 9: All encoders perform well for lowcardinality datasets (classifier: XGBoost). The relative denotes score of the percentage the corresponding score value with respect to the minimum and maximum scores by dataset. Color dots indicate the scores for each cross-validation fold, black dots the median score across folds, black line indicates median score and the box gives the interquartile range.

has the best performance between the three languages, and its superiority is statistically significant at 5% when compared with Hungarian. This tells us the semantics learned by English helps to improve prediction.

[WIP] DISCUSSION

One-hot encoding is not well suited for a column of a table containing categories represented with many different strings. A character n-gram count vector can represent strings well. Yet, it dilutes the notion of categories, because it creates extremely high-dimensional vectors. Indeed, strings that represent categories, even when normalized, tend to have many n-grams in common. A good encoding should capture string similarity between entries and reflect it in a lower dimensional encoding.

We have studied several encoding approaches to capture the structural similarities of string categorical variables in tables. We use a low-rank approximation of the multivariate distribution of n-gram with a Gamma-Poisson factorization, which leads to a form of non-negative matrix factorization adapted to count data. We also show that the min-hash

encoding have a very good prediction performance, even though is the only method that does not require any data fit.

Experiments on 17 real-word dataset show that all encoding techniques that describe well the similarities between entries in the data lead to a good prediction. However, the encodings created by the Gamma-Poisson factorization are much more interpretable: the model estimates latent categories from the data, and gives a sparse, nonnegative encoding on these. Character-level vectors embeddings from natural language processing give competitive prediction performance only if they are followed by a dimensionality reduction technique that is fit to the data, as the SVD. In this sense, it may be preferable to fully capture the semantics from the data, as with the Gamma-Poisson factorization.

6 CONCLUSION

This work shows how to overcome the limitations of onehot encoding for high-cardinality string categories, as with non-normalized entries. Two aspects are important for an encoding to give a good prediction in supervised learning settings: a character-level representation of the entries, and capturing well the similarity between the different string entries. The min-hash encoder seems to have a slightly superior prediction performance (with gradient boosted trees) and is the more scalable method, as it is the only one that does not require any fit to the data. The Gamma-Poisson factorization gives a good encoding approach: it can be implemented online with an efficient solver; it leads to good prediction; and it finds interpretable latent categories thanks to substrings repeated across the entries. As such, it gives a readily-usable replacement to one-hot encoding for highcardinality string categorical variables: it enables a statistical analysis on reduced-dimension encoding that still captures meaningful categorical information. It avoids one of the time-consuming steps of a data-science study: normalizing entries of databases via various forms of data cleaning such as deduplication or standardizing rules.

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REFERENCES

- [1] J. Cohen, P. Cohen, S. G. West, and L. S. Aiken, *Applied multiple regression/correlation analysis for the behavioral sciences*. Routledge, 2013.
- [2] D. Pyle, Data preparation for data mining. Morgan Kaufmann, 1999, vol. 1.
- [3] E. Rahm and H. H. Do, "Data cleaning: Problems and current approaches," *IEEE Data Engineering Bulletin*, vol. 23, no. 4, pp. 3– 13, 2000.
- [4] W. E. Winkler, "Overview of record linkage and current research directions," in *Bureau of the Census*. Citeseer, 2006.
- [5] A. K. Elmagarmid, P. G. Ipeirotis, and V. S. Verykios, "Duplicate record detection: A survey," *IEEE Transactions on knowledge and data engineering*, vol. 19, no. 1, pp. 1–16, 2007.
- [6] P. Cerda, G. Varoquaux, and B. Kégl, "Similarity encoding for learning with dirty categorical variables," Machine Learning, Jun 2018. [Online]. Available: https://doi.org/10.1007/s10994-018-5724-2

- [7] W. H. Gomaa and A. A. Fahmy, "A survey of text similarity approaches," *International Journal of Computer Applications*, vol. 68, no. 13, pp. 13–18, 2013.
- [8] A. Gionis, P. Indyk, R. Motwani et al., "Similarity search in high dimensions via hashing," in Vldb, vol. 99, no. 6, 1999, pp. 518–529.
- [9] D. Dheeru and E. Karra Taniskidou, "UCI machine learning repository," 2017. [Online]. Available: http://archive.ics.uci.edu
- [10] W. Kim, B.-J. Choi, E.-K. Hong, S.-K. Kim, and D. Lee, "A tax-onomy of dirty data," *Data mining and knowledge discovery*, vol. 7, no. 1, pp. 81–99, 2003.
- [11] P. Oliveira, F. Rodrigues, and P. R. Henriques, "A formal definition of data quality problems," in *Proceedings of the 2005 International Conference on Information Quality (MIT IQ Conference)*, 2005.
- [12] D. Micci-Barreca, "A preprocessing scheme for high-cardinality categorical attributes in classification and prediction problems," ACM SIGKDD Explorations Newsletter, vol. 3, no. 1, pp. 27–32, 2001.
- [13] K. Weinberger, A. Dasgupta, J. Langford, A. Smola, and J. Attenberg, "Feature hashing for large scale multitask learning," in *Proceedings of the 26th Annual International Conference on Machine Learning*. ACM, 2009, pp. 1113–1120.
- [14] R. C. Angell, G. E. Freund, and P. Willett, "Automatic spelling correction using a trigram similarity measure," *Information Processing & Management*, vol. 19, no. 4, pp. 255–261, 1983.
- [15] D. Klein, J. Smarr, H. Nguyen, and C. D. Manning, "Named entity recognition with character-level models," in *Proceedings of* the seventh conference on Natural language learning at HLT-NAACL 2003-Volume 4. Association for Computational Linguistics, 2003, pp. 180–183.
- [16] T. K. Landauer, P. W. Foltz, and D. Laham, "An introduction to latent semantic analysis," *Discourse processes*, vol. 25, no. 2-3, pp. 259–284, 1998.
- [17] W. B. Johnson and J. Lindenstrauss, "Extensions of lipschitz mappings into a hilbert space," Contemporary mathematics, vol. 26, no. 189-206, p. 1, 1984.
- [18] D. Achlioptas, "Database-friendly random projections: Johnson-lindenstrauss with binary coins," Journal of computer and System Sciences, vol. 66, no. 4, pp. 671–687, 2003.
- [19] J. Pennington, R. Socher, and C. Manning, "Glove: Global vectors for word representation," in EMNLP, 2014, pp. 1532–1543.
- [20] T. Mikolov, K. Chen, G. Corrado, and J. Dean, "Efficient estimation of word representations in vector space," in *ICLR Workshop Papers*, 2013.
- [21] S. Arora, Y. Liang, and T. Ma, "A simple but tough-to-beat baseline for sentence embeddings," 2016.
- [22] P. Bojanowski, E. Grave, A. Joulin, and T. Mikolov, "Enriching word vectors with subword information," *Transactions of the Association of Computational Linguistics*, vol. 5, no. 1, pp. 135–146, 2017.
- [23] A. Z. Broder, "On the resemblance and containment of documents," in *Proceedings. Compression and Complexity of SEQUENCES* 1997 (Cat. No. 97TB100171). IEEE, 1997, pp. 21–29.
- [24] J. Canny, "Gap: A factor model for discrete data," in Proceedings of the 27th Annual International ACM SIGIR Conference on Research and Development in Information Retrieval, ser. SIGIR '04. New York, NY, USA: ACM, 2004, pp. 122–129. [Online]. Available: http://doi.acm.org/10.1145/1008992.1009016
- [25] A. Z. Broder, M. Charikar, A. M. Frieze, and M. Mitzenmacher, "Min-wise independent permutations," *Journal of Computer and System Sciences*, vol. 60, no. 3, pp. 630–659, 2000.
- [26] J. Leskovec, A. Rajaraman, and J. D. Ullman, Mining of massive datasets. Cambridge university press, 2014.
- [27] J. Ji, J. Li, S. Yan, Q. Tian, and B. Zhang, "Min-max hash for jaccard similarity," in 2013 IEEE 13th International Conference on Data Mining. IEEE, 2013, pp. 301–309.
- [28] A. Appleby, "Murmurhash3 http://code. google. com/p/smhasher/wiki," 2014.
- [29] D. D. Lee and H. S. Seung, "Algorithms for non-negative matrix factorization," in *Advances in Neural Information Processing Systems* 13, T. K. Leen, T. G. Dietterich, and V. Tresp, Eds. MIT Press, 2001, pp. 556–562. [Online]. Available: http://papers.nips.cc/paper/ 1861-algorithms-for-non-negative-matrix-factorization.pdf
- [30] A. Lefevre, F. Bach, and C. Févotte, "Online algorithms for non-negative matrix factorization with the itakura-saito divergence," in *Applications of Signal Processing to Audio and Acoustics (WASPAA)*. IEEE, 2011, p. 313.
- [31] T. Mikolov, E. Grave, P. Bojanowski, C. Puhrsch, and A. Joulin, "Advances in pre-training distributed word representations,"

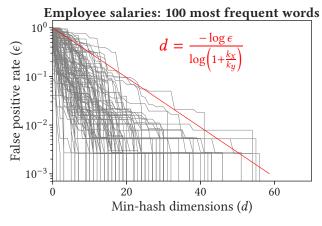


Fig. 10: Number of dimensions required to identify inclusions. Grey lines are the proportion of false positives obtained for the 100 most frequent words in the employee salaries dataset (H_0 corresponds to identifying categories that do not contain the given word). The red line represents the theoretical minimum dimensionality required to obtain a desired false positive rate (with $k_x/k_y=0.125$, the inverse of the maximum number of words per category), as shown in Theorem 3.1.

- in International Conference on Language Resources and Evaluation (LREC), 2018.
- [32] N. Halko, P.-G. Martinsson, and J. Tropp, "Finding structure with randomness: Probabilistic algorithms for constructing approximate matrix decompositions," SIAM review, vol. 53, p. 217, 2011.
- [33] N. X. Vinh, J. Epps, and J. Bailey, "Information theoretic measures for clusterings comparison: Variants, properties, normalization and correction for chance," *Journal of Machine Learning Research*, vol. 11, no. Oct, pp. 2837–2854, 2010.
- [34] T. Chen and C. Guestrin, "XGBoost: A scalable tree boosting system," in SIGKDD, 2016, pp. 785–794.
- [35] L. Prokhorenkova, G. Gusev, A. Vorobev, A. Dorogush, and A. Gulin, "Catboost: unbiased boosting with categorical features," in *Neural Information Processing Systems*, 2018, p. 6639.
- [36] R. S. Olson, W. La Cava, Z. Mustahsan, A. Varik, and J. H. Moore, "Data-driven advice for applying machine learning to bioinformatics problems," arXiv preprint arXiv:1708.05070, 2017.
- [37] J. Demšar, "Statistical comparisons of classifiers over multiple data sets," *Journal of Machine learning research*, vol. 7, no. Jan, pp. 1–30, 2006.
- [38] A. Rahimi and B. Recht, "Random features for large-scale kernel machines," in *Advances in neural information processing systems*, 2008, p. 1177.
- [39] F. Pedregosa, G. Varoquaux, A. Gramfort, V. Michel, B. Thirion, O. Grisel, M. Blondel, P. Prettenhofer, R. Weiss, V. Dubourg, J. Vanderplas, A. Passos, D. Cournapeau, M. Brucher, M. Perrot, and E. Duchesnay, "Scikit-learn: Machine learning in Python," *Journal of Machine Learning Research*, vol. 12, pp. 2825–2830, 2011.

APPENDIX A ALGORITHMIC CONSIDERATIONS

A.1 Min-hash encoder

A.2 Gamma-Poisson factorization

Algorithm 1 requires some input parameters and initializations that can affect convergence. One important parameter is ρ , the discount factor for the fitting in the past. Figure 11 shows that choosing ρ =.95 gives the best compromise between stability of the convergence and data fitting in terms of the Generalized KL divergence.

TABLE 8: Median scores by dataset for XGBoost (d=30).

Datasets	One-hot + SVD	Similarity encoder	Tfldf + SVD	FastText + SVD	Gamma Poisson	Min-hash encoder
building permits	0.244	0.505	0.550	0.544	0.570	0.566
colleges	0.499	0.532	0.537	0.530	0.524	0.527
crime data	0.443	0.445	0.445	0.446	0.445	0.446
drug directory	0.971	0.979	0.980	0.982	0.980	0.981
employee salaries	0.880	0.905	0.892	0.901	0.906	0.900
federal election	0.135	0.141	0.144	0.151	0.146	0.146
journal influence	0.019	0.138	0.164	0.221	0.118	0.133
kickstarter projects	0.879	0.879	0.880	0.880	0.879	0.880
medical charge	0.904	0.905	0.905	0.904	0.904	0.904
met objects	0.771	0.790	0.789	0.796	0.791	0.794
midwest survey	0.575	0.635	0.646	0.636	0.651	0.653
public procurement	0.678	0.677	0.678	0.678	0.674	0.678
road safety	0.553	0.562	0.562	0.560	0.563	0.566
traffic violations	0.782	0.789	0.789	0.790	0.792	0.793
vancouver employee	0.395	0.550	0.530	0.509	0.556	0.568
wine reviews	0.439	0.671	0.724	0.657	0.724	0.679

With respect to the initialization of the topic matrix $\Lambda^{(0)}$, the best option is to choose the centroids of a k-means clustering (Figure 12) in a hashed version of the n-gram count matrix ${\bf F}$ in a reduced dimensionality (in order to speed-up convergence of the k-means algorithm) and then project back to the n-gram space with a nearest neighbors algorithm.

APPENDIX B ADDITIONAL RESULTS

APPENDIX C REPRODUCIBILITY

C.1 Dataset Description

C.1.1 Non-curated datasets

Building Permits¹⁶ (sample size: 554k). Permits issued by the Chicago Department of Buildings since 2006. Target (regression): *Estimated Cost*. Categorical variable: *Work Description* (cardinality: 430k).

Colleges¹⁷ (7.8k). Information about U.S. colleges and schools. Target (regression): *Percent Pell Grant*. Cat. var.: *School Name* (6.9k).

Crime Data¹⁸ (1.5M). Incidents of crime in the City of Los Angeles since 2010. Target (regression): *Victim Age*. Categorical variable: *Crime Code Description* (135).

Drug Directory¹⁹ (120k). Product listing data submitted to the U.S. FDA for all unfinished, unapproved drugs. Target (multiclass): *Product Type Name*. Categorical var.: *Non Proprietary Name* (17k).

Employee Salaries²⁰ (9.2k). Salary information for employees of the Montgomery County, MD. Target (regression): *Current Annual Salary*. Categorical variable: *Employee Position Title* (385).

¹⁶https://www.kaggle.com/chicago/chicago-building-permits

¹⁷https://beachpartyserver.azurewebsites.net/VueBigData/ DataFiles/Colleges.txt

¹⁸https://data.lacity.org/A-Safe-City/

Crime-Data-from-2010-to-Present/y8tr-7khq

¹⁹https://www.fda.gov/Drugs/InformationOnDrugs/ucm142438. htm

²⁰https://catalog.data.gov/dataset/employee-salaries-2016

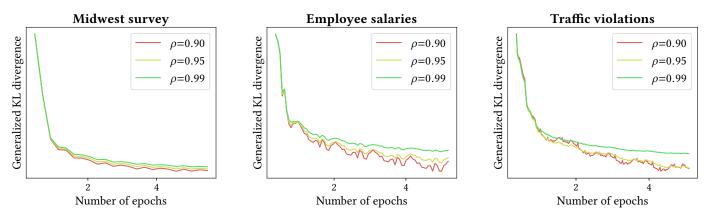


Fig. 11: Convergence for different ρ values (discount factor) for the Gamma-Poisson model.

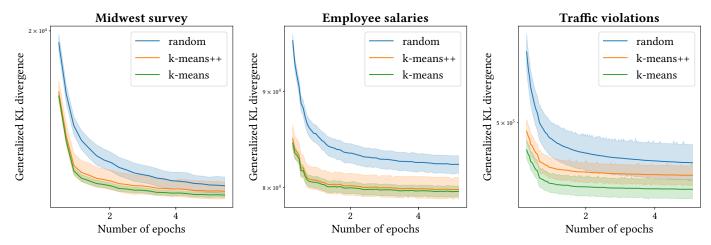


Fig. 12: Convergence for different initializations for the Gamma-Poisson model. In all experiments, the value $\rho = 0.95$ is used, as it gives a good trade-off between convergence and stability of the solution across the number of epochs.

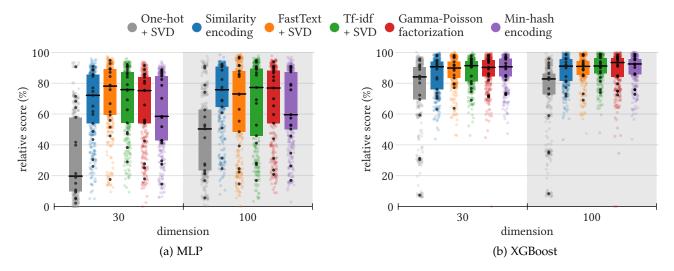


Fig. 13: Encoders that consider subword information perform significantly better than one-hot. Classifier: XGBoost. Comparison of encoders in terms of a relative score (the prediction score on the particular dataset, rescaled with respect to the global maximum and minimum score values across dimensions). Color dots indicate the scores for each cross-validation fold, black dots the median score across folds for a dataset, the black line indicates the median score and the box gives the interquartile range. For one-hot encoding, bag-of-n-grams and fastText, the dimensionality was fixed by using a truncated SVD.

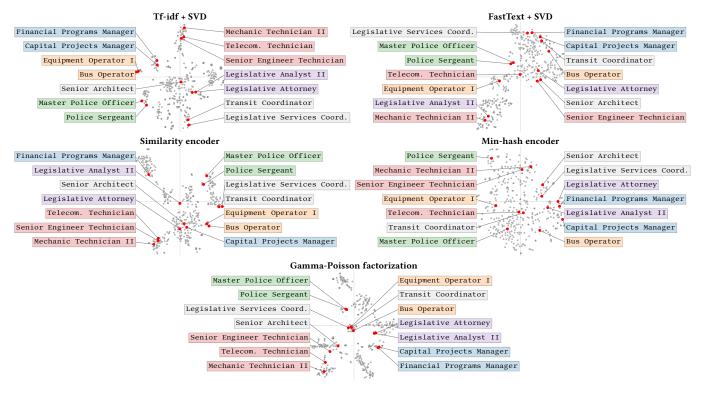


Fig. 14: Projection with t-SNE for different encoding methods for the categorical variable *Employee Position Title* in the *Employee Salaries* dataset. The original dimensionality for each encoding is d=10. Gray dots represent the rest of observed categories in the dataset.

Federal Election²¹ (3.3M). Campaign finance data for the 2011-2012 US election cycle. Target (regression): *Transaction Amount*. Categorical variable: *Memo Text* (17k).

Journal Influence²² (3.6k). Scientific journals and the respective influence scores. Target (regression): *Average Cites per Paper*. Categorical variable: *Journal Name* (3.1k).

Kickstarter Projects²³ (281k). More than 300,000 projects from the Kickstarters platform https://www.kickstarter.com. Target (binary): *State*. Categorical variable: *Category* (158).

Medical Charges²⁴ (163k). Inpatient discharges for Medicare beneficiaries for more than 3,000 U.S. hospitals. Target (regression): *Average Total Payments*. Categorical var.: *Medical Procedure* (100).

Met Objects²⁵ (469k). Information on artworks objects of the Metropolitan Museum of Art's collection. Target (binary): *Department*. Categorical variable: *Object Name* (26k).

Midwest Survey²⁶ (2.8k). Survey to know if people self-identify as Midwesterners. Target (multiclass): *Census Region* (10 classes). Categorical var.: *What would you call the part of the country you live in now?* (844).

Open Payments²⁷ (2M). Payments given by healthcare

manufacturing companies to medical doctors or hospitals (year 2013). Target (binary): *Status* (if the payment was made under a research protocol). Categorical var.: *Company name* (1.4k).

Public Procurement²⁸ (352k). Public procurement data for the European Economic Area, Switzerland, and the Macedonia. Target (regression): *Award Value Euro*. Categorical var.: *CAE Name* (29k).

Road Safety²⁹ (139k). Circumstances of personal injury of road accidents in Great Britain from 1979. Target (binary): *Sex of Driver*. Categorical variable: *Car Model* (16k).

Traffic Violations³⁰ (1.2M). Traffic information from electronic violations issued in the Montgomery County, MD. Target (multiclass): *Violation type* (4 classes). Categorical var.: *Description* (11k).

Vancouver Employee³¹(2.6k). Remuneration and expenses for employees earning over \$75,000 per year. Target (regression): *Remuneration*. Categorical variable: *Title* (640).

Wine Reviews³² (138k). Wine reviews scrapped from WineEnthusiast with variety, location, winery, price, and description. Target (regression): *Points*. Categorical variable: *Description* (89k).

²¹https://classic.fec.gov/finance/disclosure/ftpdet.shtml

²²https://github.com/FlourishOA/Data

²³https://www.kaggle.com/kemical/kickstarter-projects

²⁴https://www.cms.gov/Research-Statistics-Data-and-Systems/ Statistics-Trends-and-Reports/Medicare-Provider-Charge-Data/ Inpatient.html

²⁵https://github.com/metmuseum/openaccess

²⁶https://github.com/fivethirtyeight/data/tree/master/ region-survey

^{^27}https://openpaymentsdata.cms.gov

²⁸https://data.europa.eu/euodp/en/data/dataset/ted-csv

²⁹https://data.gov.uk/dataset/road-accidents-safety-data

³⁰https://catalog.data.gov/dataset/traffic-violations-56dda

³¹https://data.vancouver.ca/datacatalogue/

employeeRemunerationExpensesOver75k.htm

³²https://www.kaggle.com/zynicide/wine-reviews/home

C.1.2 Curated datasets

Adult³³ (sample size: 32k). Description. Target (binary): *Income*. Categorical variable: *Occupation* (cardinality: 15).

Cacao Flavors³⁴ (1.7k). Description. Target (multiclass): *Bean Type*. Categorical variable: *Broad Bean Origin* (97).

California Housing³⁵ (nsamples). Description. Target (): *name*. Categorical variable: *name* (cardinality).

Dating Profiles³⁶ (nsamples). Description. Target (): *name*. Categorical variable: *name* (cardinality).

House Prices³⁷ (nsamples). Description. Target (): *name*. Categorical variable: *name* (cardinality).

House Sales³⁸ (nsamples). Description. Target (): *name*. Categorical variable: *name* (cardinality).

Intrusion Detection³⁹ (nsamples). Description. Target (): *name*. Categorical variable: *name* (cardinality).

C.2 Learning pipeline

Sample size: Datasets' size range from a couple of thousand to several million samples. To reduce computation time on the learning step, the number of samples was limited to 100k for large datasets.

Data preprocessing: We removed rows with missing values in the target or in any explanatory variable other than the selected categorical variable, for which we replaced missing entries by the string 'nan'. The only additional preprocessing for the categorical variable was to transform all entries to lower case.

Cross-validation: For every dataset, we made 20 random splits of the data, with one third of samples for testing at each time. In the case of binary classification, we performed stratified randomization.

Performance metrics: Depending on the type of prediction task, we used different scores to evaluate the performance of the supervised learning problem: for regression, we used the \mathbb{R}^2 score; for binary classification, the average precision; and for multi-class classification, the accuracy score.

Parametrization of classifiers: We used the scikitlearn [39] for most of the data processing. For all the experiments, we used the scikit-learn compatible implementations of XGBoost [34], with a grid search on the learning_rate (0.05, 0.1, 0.3) and max_depth (3, 6, 9) parameters. All datasets and encoders use the same parametrization.

Dimensionality reduction: We used the scikit-learn implementations of TruncatedSVD and GaussianRandomProjection, with the default parametrization in both cases.

C.3 Online Resources

Experiments are available in Python code at https://github.com/pcerda/tkde2019. Implementations and examples on learning with string categories can be found at http://

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dirty-cat.github.io. The available encoders are compatible with the scikit-learn's API.

³³https://archive.ics.uci.edu/ml/datasets/adult

³⁴https://www.kaggle.com/rtatman/chocolate-bar-ratings

³⁵https://github.com/ageron/handson-ml/tree/master/datasets/housing

³⁷https://www.kaggle.com/c/house-prices-advanced-regression-techniques

³⁸https://www.kaggle.com/harlfoxem/housesalesprediction

³⁹https://archive.ics.uci.edu/ml/datasets/KDD+Cup+1999+Data

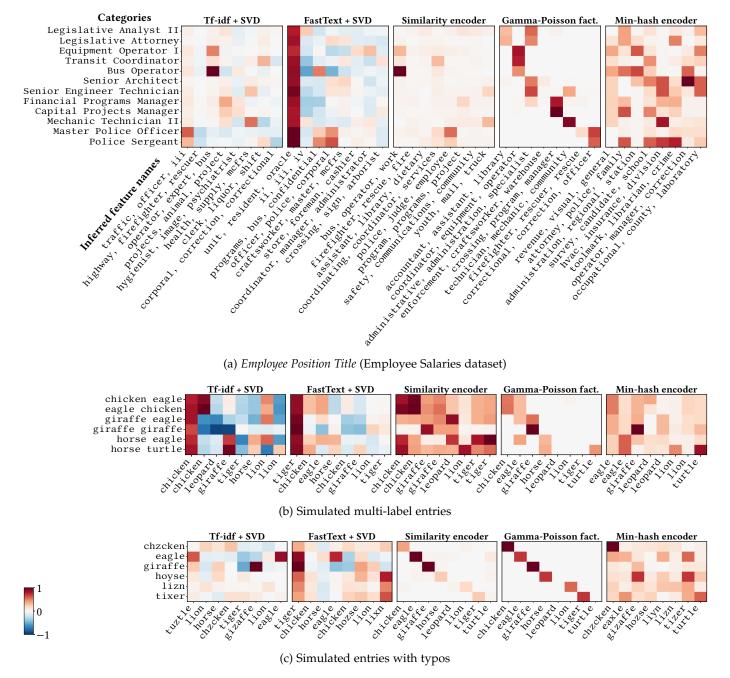
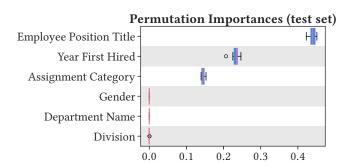


Fig. 15: The Gamma-Poisson factorization gives positive and sparse representations that are easily interpretable. Encoding vectors (d=8) for simulated (a and b) and a real dataset (c) obtained with different encoding methods for some categories (x-axis). The *y*-axis shows the activations with their respective inferred feature names.



 $Fig.\ 16: \ {\bf Overall\ permutation\ importances\ for\ every\ feature} \\ \ in\ the\ Employee\ Salaries\ dataset.$