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QUANTIFYING THE UNCERTAINTIES INTRODUCED BY DIMENSION REDUCTION IN FLUID DYNAMICS

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CONTENT

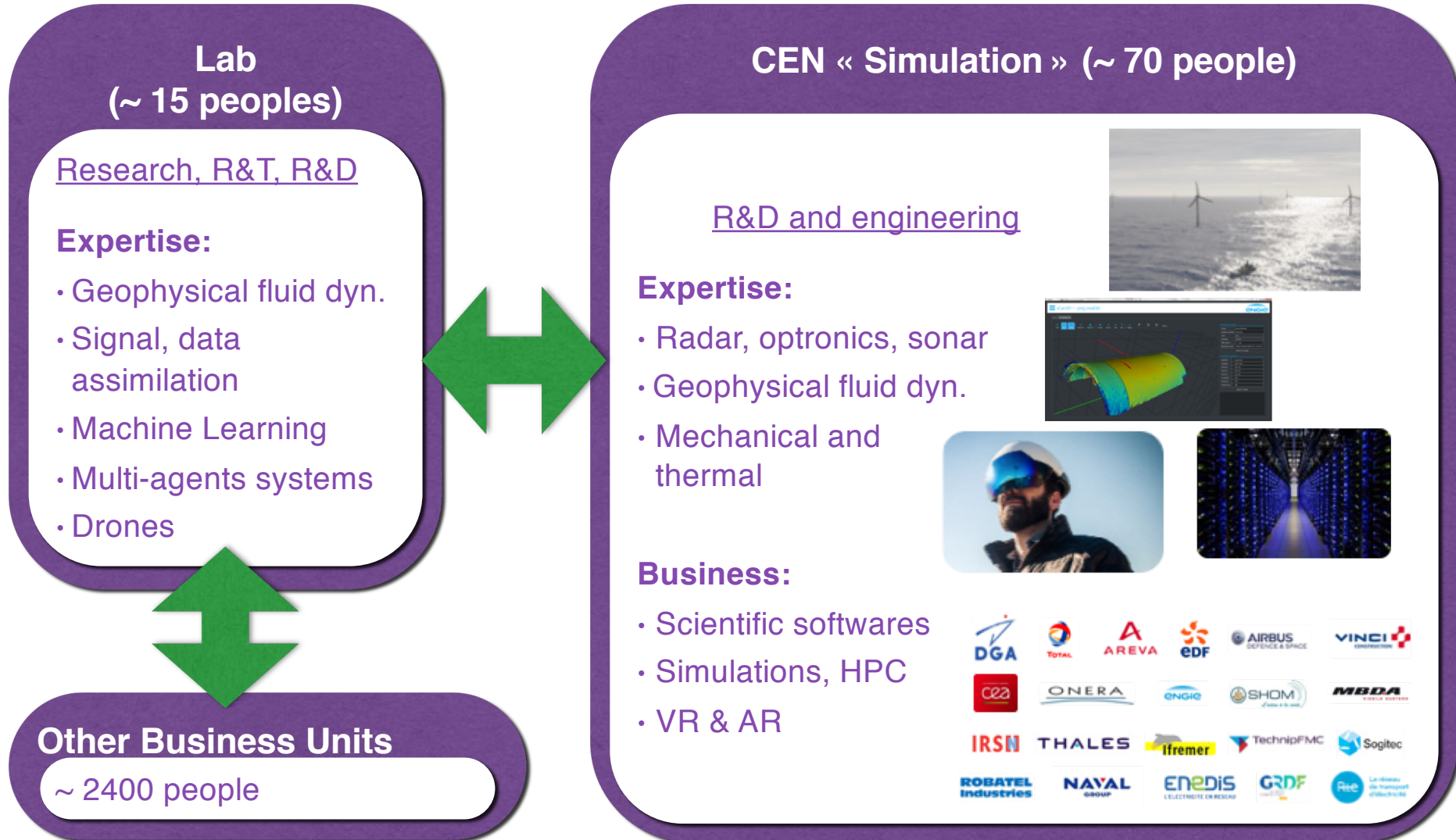
1. Context : observer for wind turbine application
2. Physics, data & reduced order model (ROM)
3. Simulation, measurements & data assimilation
4. Reduced order model under location uncertainty
5. Results



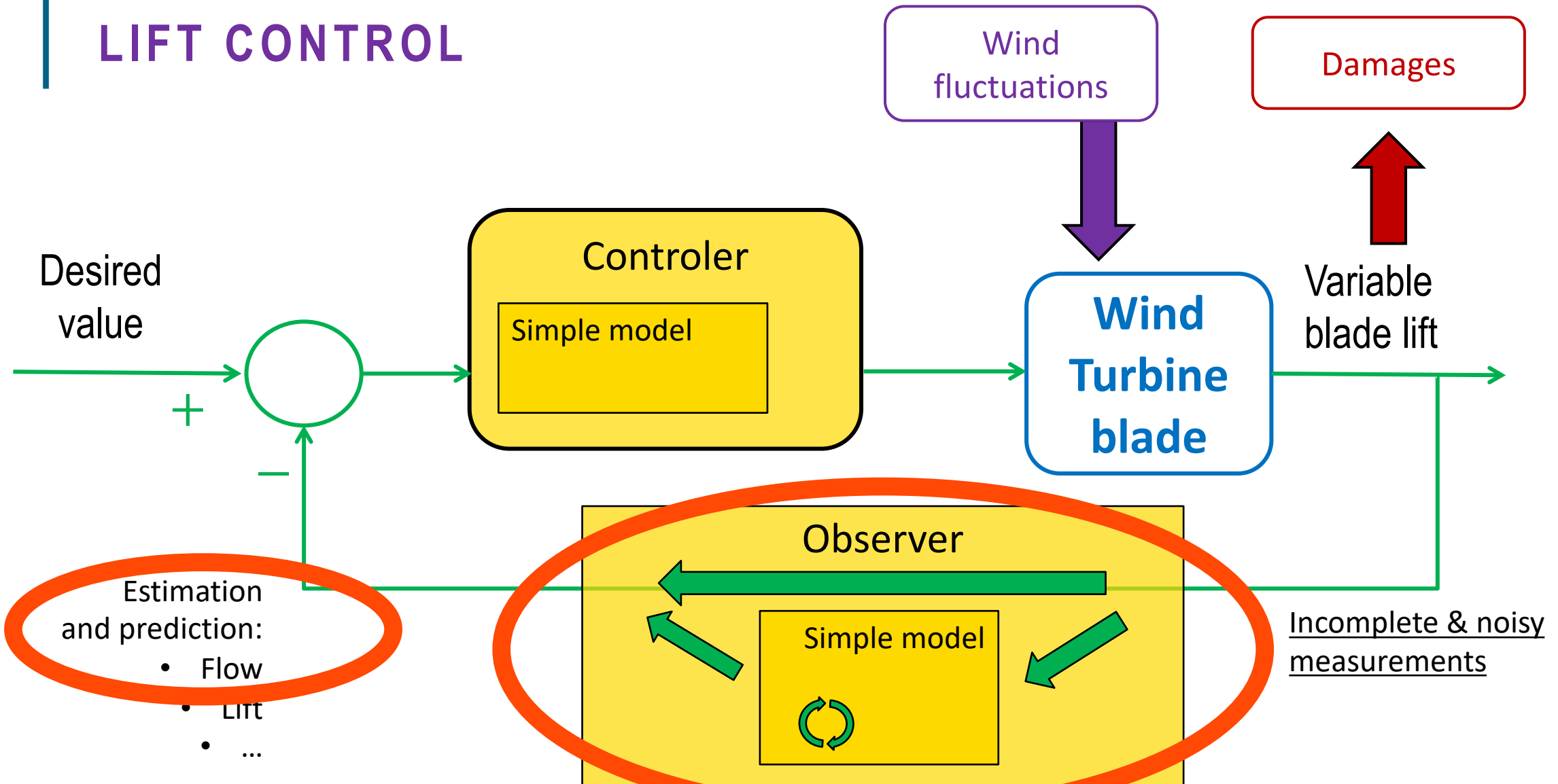
PART I

CONTEXT :
OBSERVER FOR WIND
TURBINE APPLICATIONS

SCALIAN



WIND TURBINE BLADE LIFT CONTROL





PART II

PHYSICS, DATA & REDUCED ORDER MODEL

REDUCED ORDER MODEL (ROM)

Solution of an PDE with the form:

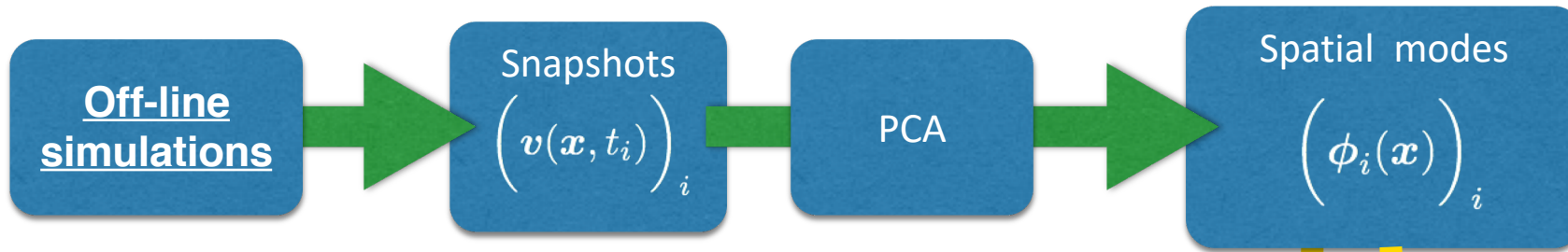
$$v(x, t, \alpha) \approx \sum_{i=0}^n \boxed{b_i(t)} \boxed{\phi_i(x)} \boxed{\gamma_i(\alpha)}$$

Order of magnitude examples in CFD

	Full space	Reduced space
Solution coordinates	$v_q(x_i, t)_{qi}$	$(b_i(t))_i$
Dimension	$M \times d \sim 10^7$	$n \sim 10 - 100$

POD-GALERKIN

- Principal Component Analysis (PCA) on a *dataset* to reduce the dimensionality:



- Approximation:

$$v(x, t) \approx \sum_{i=0}^n b_i(t) \phi_i(x)$$

Resolved modes

- Projection of the “physics” onto the spatial modes :

$$\int_{\Omega} dx \phi_i(x) \cdot (\text{Physical equation (e.g. Navier-Stokes)})$$

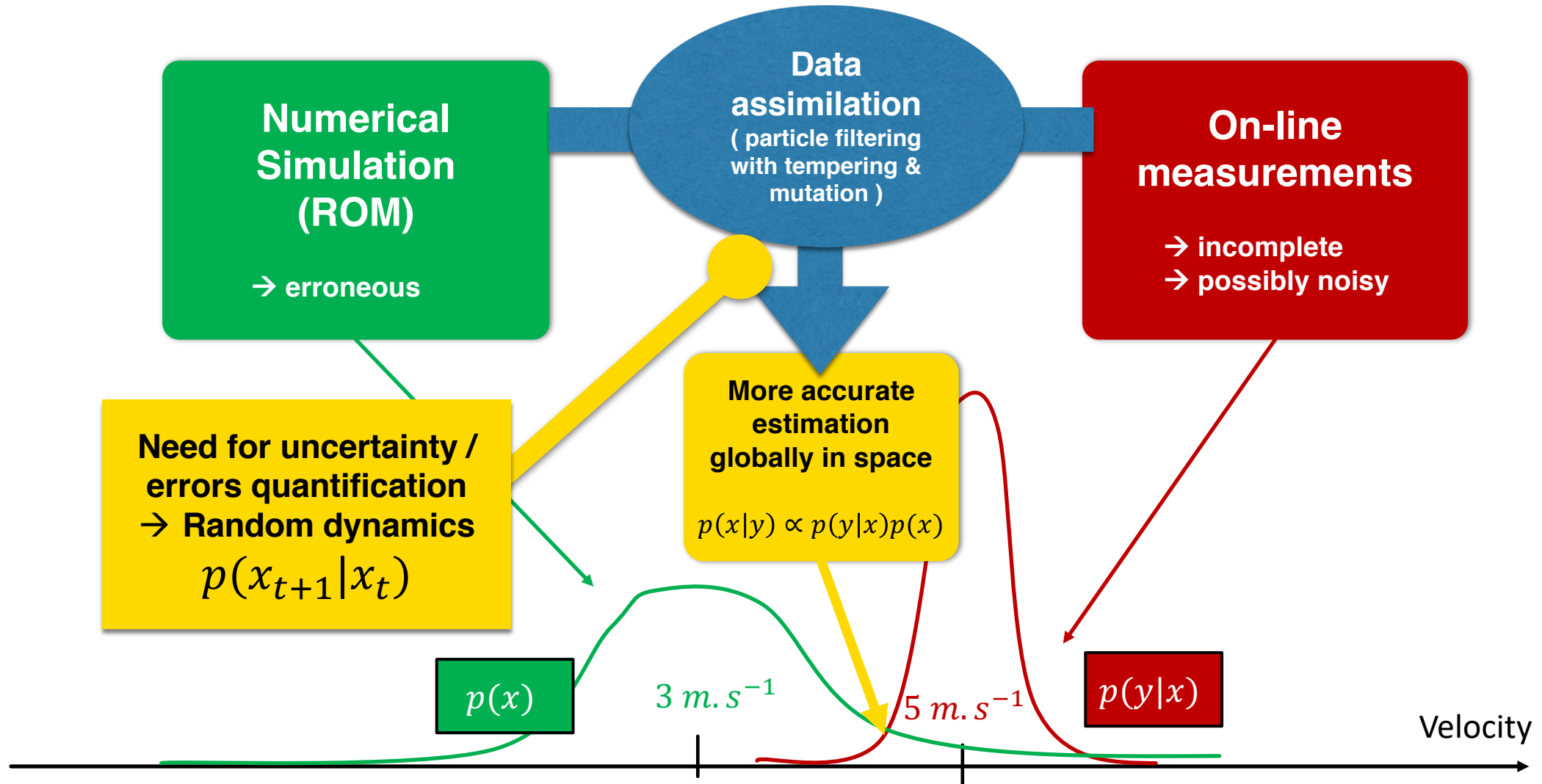
→ ROM for very fast simulation of temporal modes



PART III

SIMULATION, MEASUREMENTS & DATA ASSIMILATION

COMBINING SIMULATIONS AND MEASUREMENTS

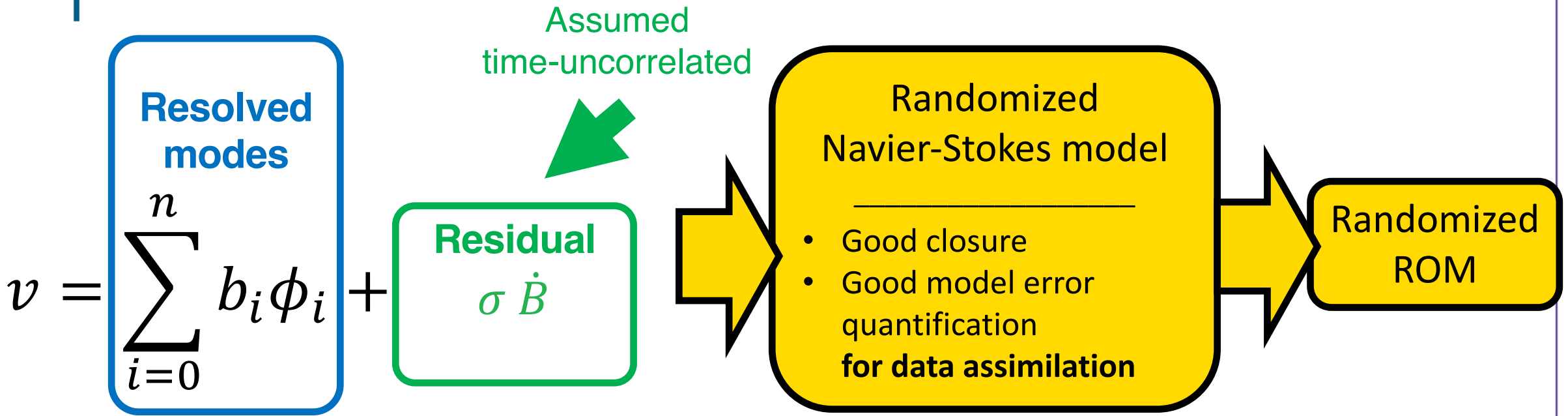




PART IV

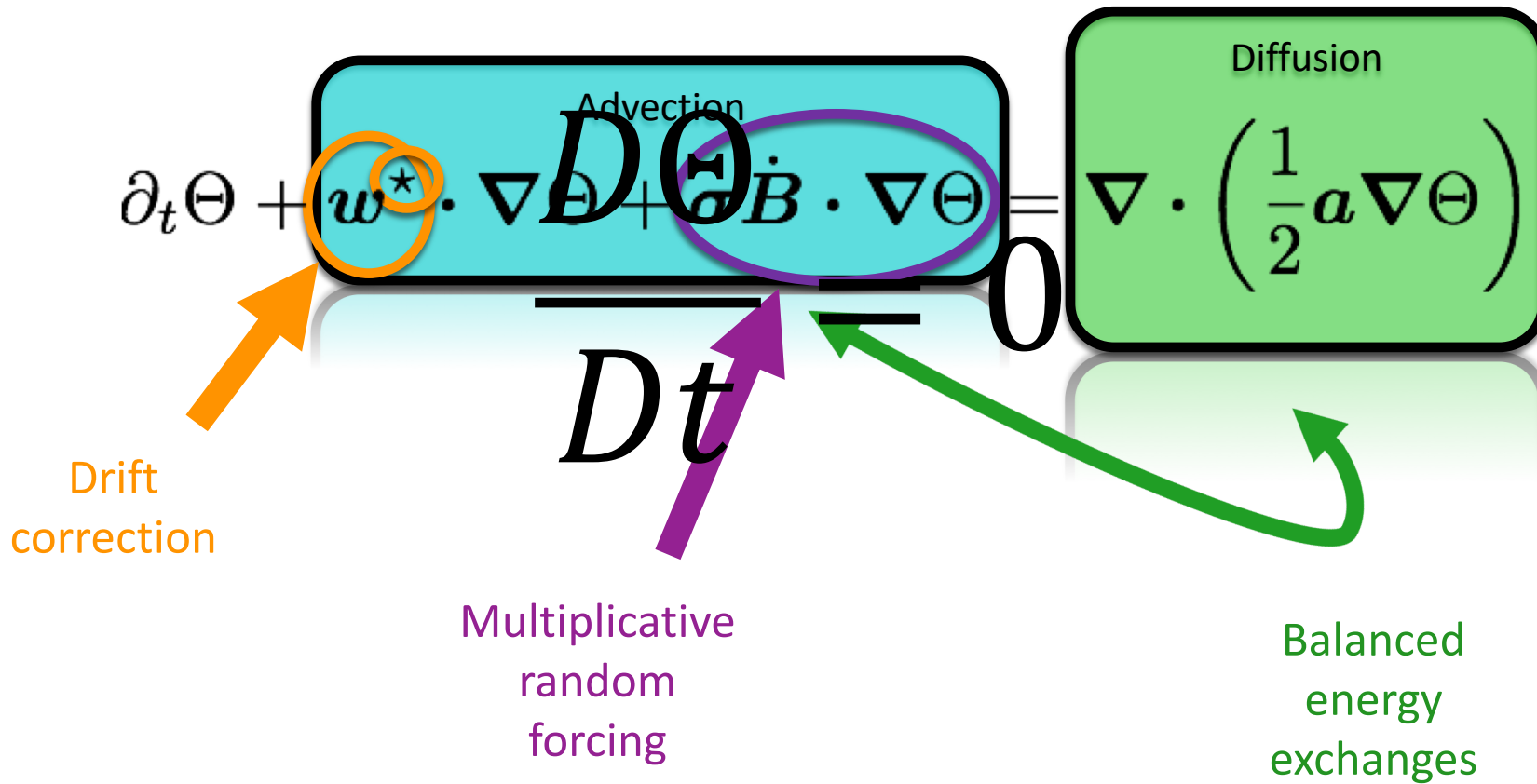
REDUCED ORDER MODELS UNDER LOCATION UNCERTAINTY

LOCATION UNCERTAINTY MODELS (LUM)



	LUM	SALT
References :	Memin, 2014 Resseguier et al. 2017 a, b, c, d Cai et al. 2017 Chapron et al. 2018 Yang & Memin 2019	Holm, 2015 Holm and Tyranowski, 2016 Arnaudon et al. 2017 Crisan et al., 2017 Gay-Balmaz & Holm 2017 Cotter and al. 2018 a, b Cotter and al. 2019
	Cotter and al. 2017 Resseguier et al. 2019 a, b	

MODEL UNDER LOCATION UNCERTAINTY, THE TRACER ADVECTION EXAMPLE



Large scales:
 w

Small scales:
 $\sigma \dot{B}$

Variance tensor:
 $a = a(x, x) = \frac{\mathbb{E}\{\sigma d\mathbf{B} (\sigma d\mathbf{B})^T\}}{dt}$

REDUCED MODELS UNDER LOCATION UNCERTAINTY: GALERKIN PROJECTION GIVES SDES FOR RESOLVED MODES

$$\int_{\Omega} \phi_i \cdot (\text{stochastic Navier-Stokes})$$

$$\frac{db_i}{dt} = \boxed{F_i(b)} + \overset{n \times M \quad M \times 1}{\boxed{(\alpha_{\cdot i} \dot{B}_t)^T}} \overset{n \times 1}{b} + \overset{1 \times M \quad M \times 1}{\boxed{(\theta_i \dot{B}_t)}}$$

multiplicative noise additive noise

Large scales:
 $\frac{d}{dt} \langle \phi_i \phi_j \rangle$

Small scales:
 $\sigma \dot{B}$

Variance tensor:
 $a = a(x, x) = \frac{\mathbb{E}\{\sigma dB (\sigma dB)^T\}}{dt}$

2nd order polynomial:
coefficients given by physics,

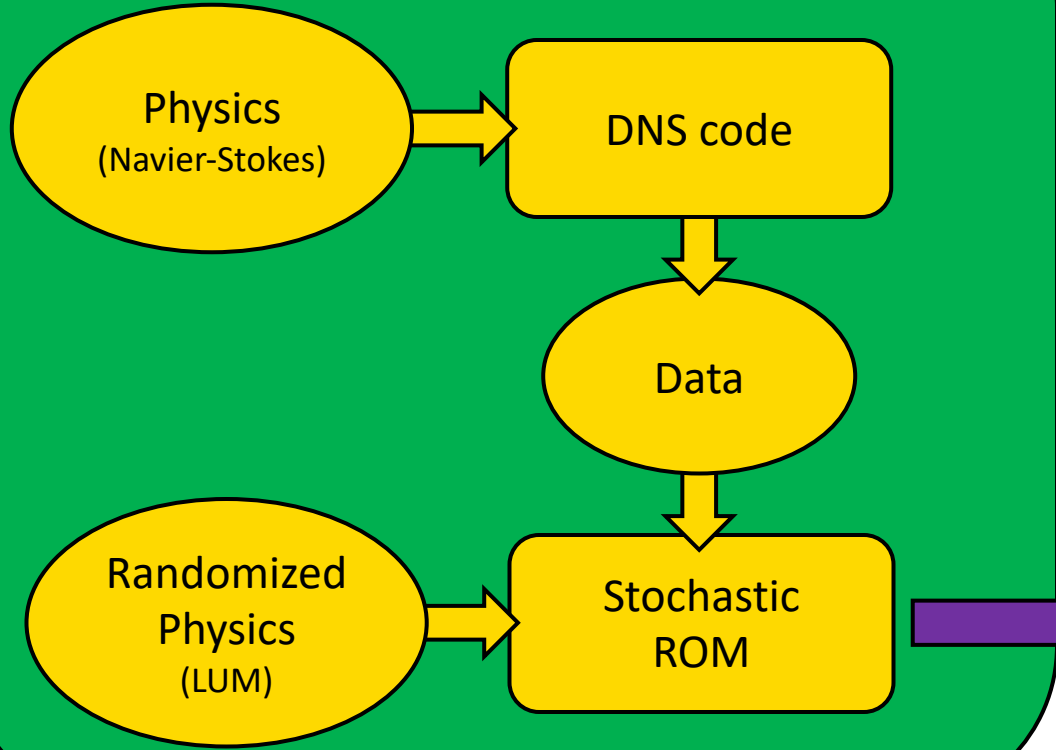
$$(\phi_j)_j \quad \text{and} \quad a = \overline{\sigma \dot{B} (\sigma \dot{B})^T} \tau$$

Correlations to estimate

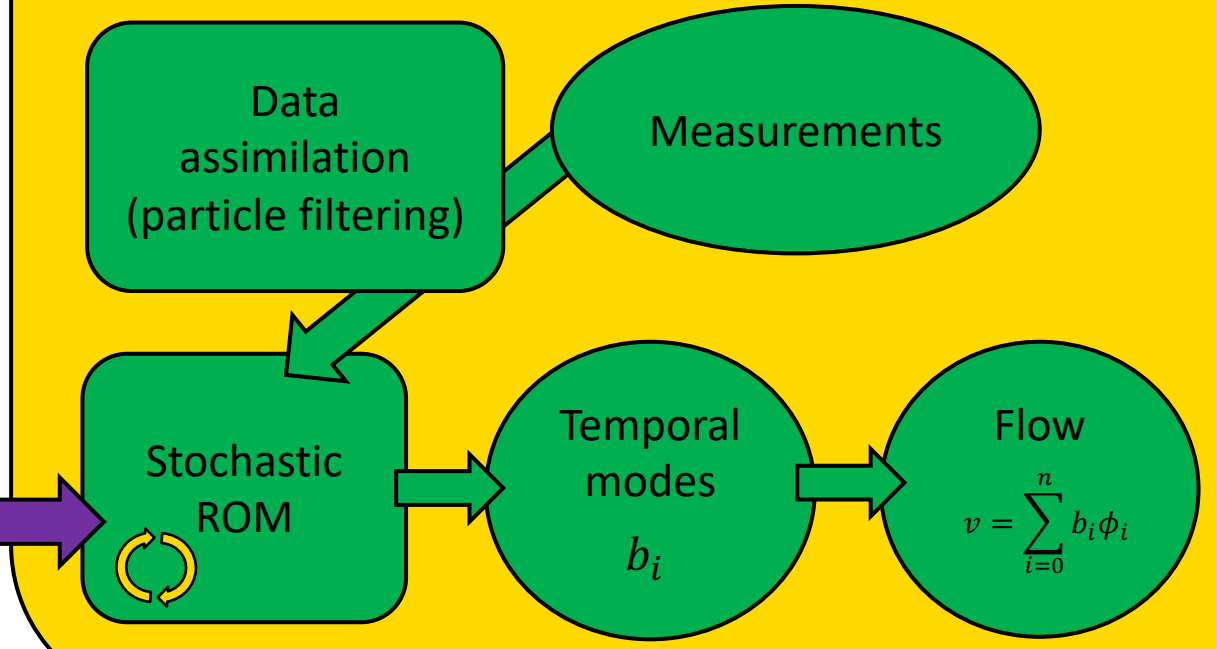
$$\alpha_{pi} \cdot \alpha_{qj}, \quad \alpha_{pi} \cdot \theta_j, \quad \theta_i \cdot \theta_j.$$

SUMMARY

Off-line : Building ROM



On-line : Simulation & data assimilation



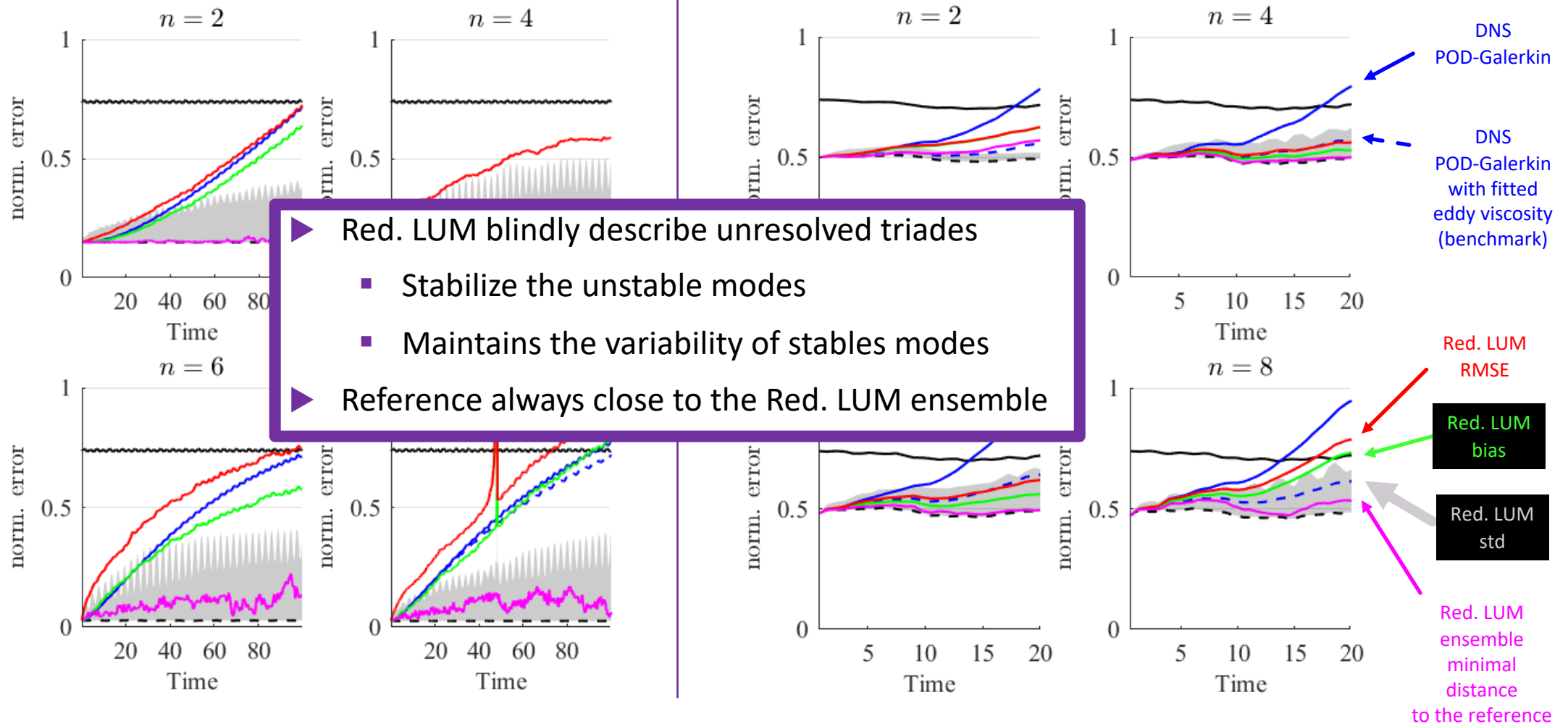
PART V

RESULTS :
UQ &
FAST OBSERVER
OF THE FLOW

UNCERTAINTY QUANTIFICATION (no data assimilation)

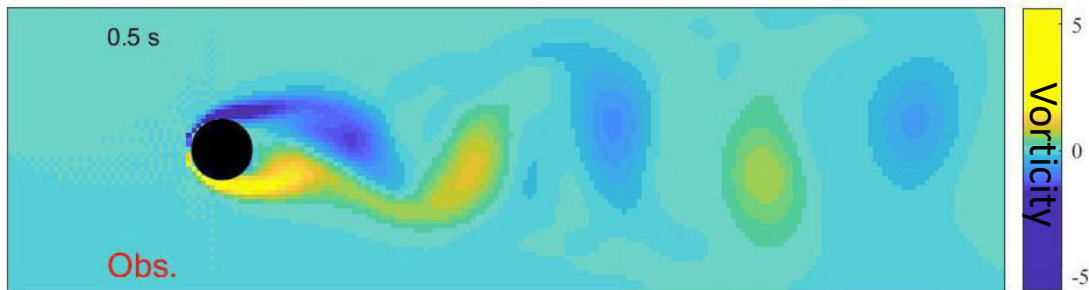
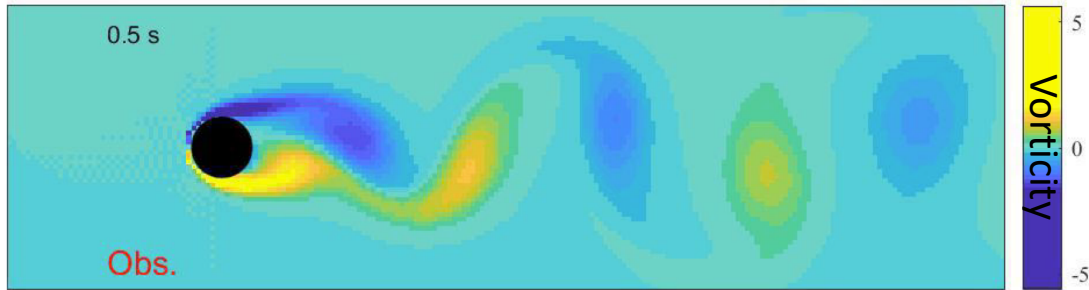
► 2D Wake at Re 100

► 3D Wake at Re 300



DATA ASSIMILATION: WAKE AT RE 100

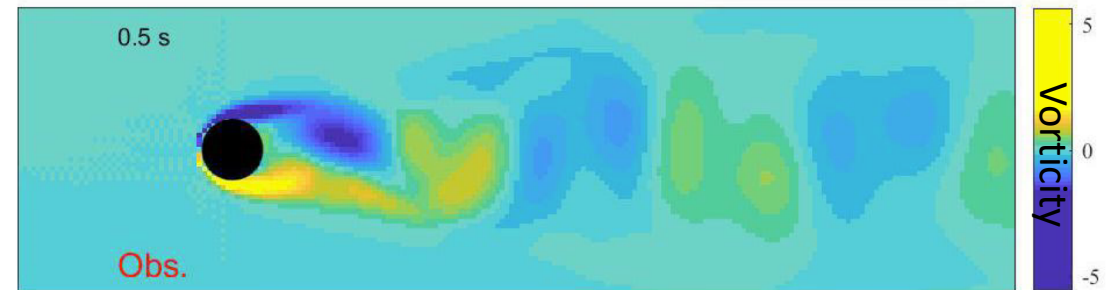
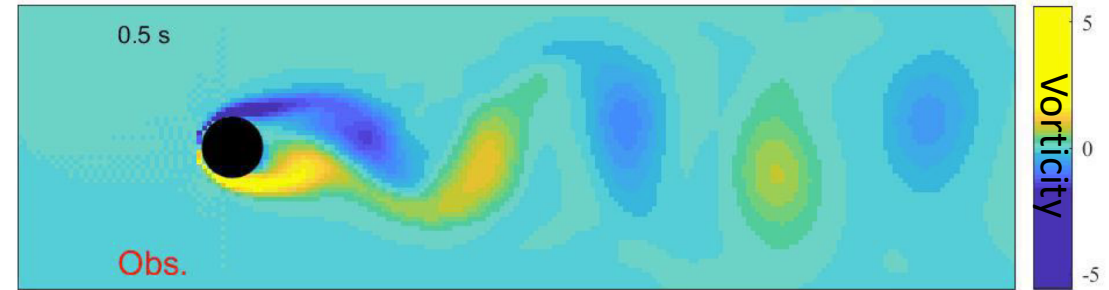
Reference
(DNS)
 10^4 degrees of freedom



Our method
(Red-LUM-based data-assimilation)
6 degrees of freedom

Reduced order models with $n = 6$
and 2dB-SNR obs. assimilated every 5 sec

Theoretical bound
(Optimal from 6-d.o.f. linear decomposition)
6 degrees of freedom

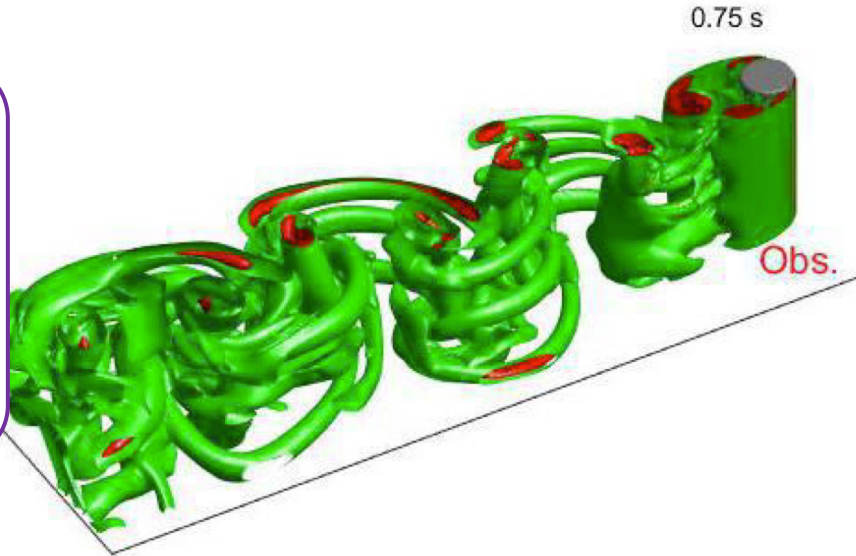


Benchmark
(POD-ROM (with eddy viscosity) + init. by obs.)
6 degrees of freedom

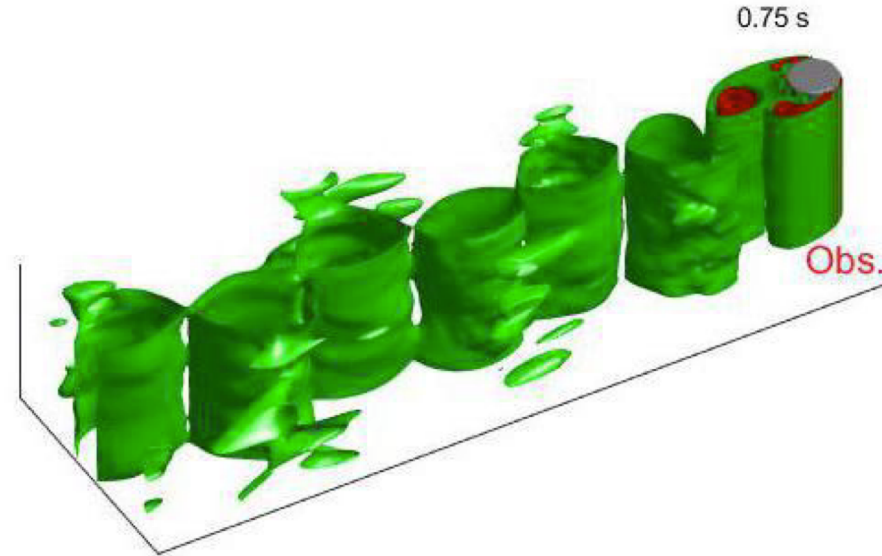
DATA ASSIMILATION: WAKE AT RE 300

Reduced order models with $n = 6$
and 2dB-SNR obs. assimilated every 5 sec

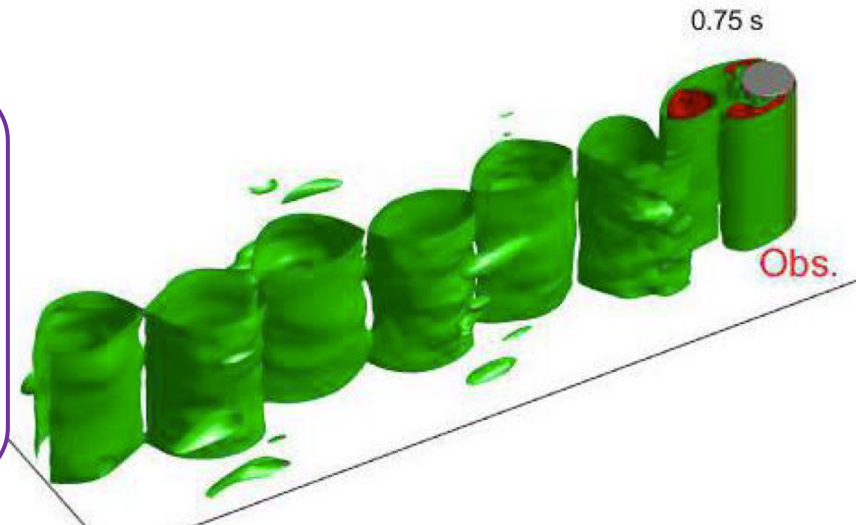
Reference
(DNS)
 10^7 degrees of freedom



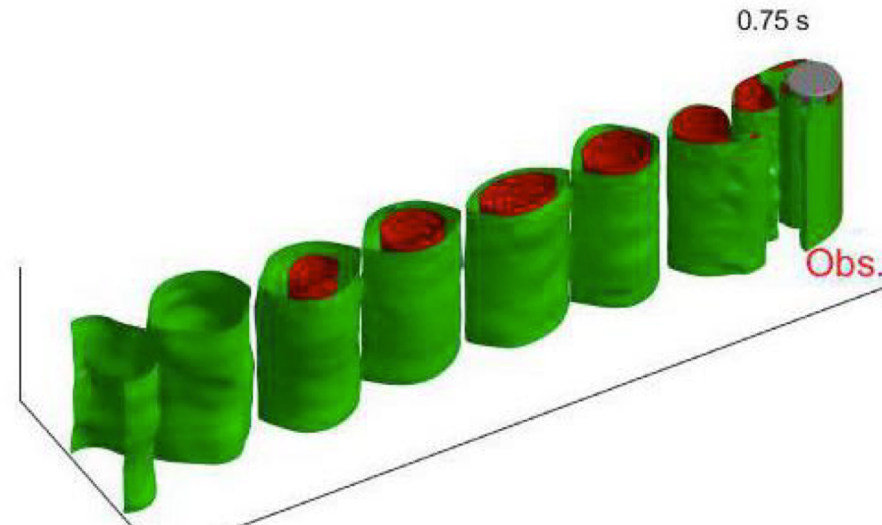
Theoretical bound
(optimal from
6-d.o.f. linear
decomposition)
6 degrees of freedom



Our method
(Red-LUM-
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6 degrees of freedom



Benchmark
(POD-ROM
(with eddy
viscosity) +
init. by obs.)
6 degrees of freedom





CONCLUSION

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- ▶ Reduced order model (ROM) : for very fast and robust CFD ($10^7 \rightarrow 6$ degrees of freedom.)
 - Combine data & physics (built off-line)
 - Closure problem handled by LUM
- ▶ Data assimilation : to correct the fast simulation on-line by incomplete/noisy measurements
 - Model error quantification handled by LUM
- ▶ First results
 - Optimal unsteady flow estimation/prediction in the whole spatial domain (large-scale structures)
 - Robust far outside the learning period

NEXT STEPS

- ▶ Real measurements (PIV, TrimControl, ...)
- ▶ Increasing the degrees of freedom (n)
- ▶ Parametric ROM
- ▶ Increasing Reynolds (reduced DNS \rightarrow reduced LES)