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# QUANTIFYING THE UNCERTAINTIES INTRODUCED BY DIMENSION REDUCTION IN FLUID DYNAMICS

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# CONTENT

1. Context : observer for wind turbine application
2. Physics, data & reduced order model (ROM)
3. Simulation, measurements & data assimilation
4. Reduced order model under location uncertainty
5. Results



## PART I

CONTEXT :  
OBSERVER FOR WIND  
TURBINE APPLICATIONS



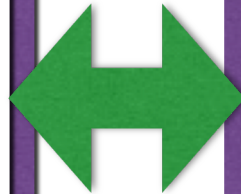
# SCALIAN

## Lab (~ 15 peoples)

Research, R&T, R&D

### Expertise:

- Geophysical fluid dyn.
- Signal, data assimilation
- Machine Learning
- Multi-agents systems
- Drones

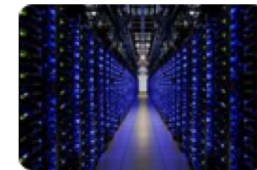
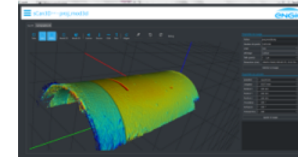


## CEN « Simulation » (~ 70 people)

R&D and engineering

### Expertise:

- Radar, optronics, sonar
- Geophysical fluid dyn.
- Mechanical and thermal



### Business:

- Scientific softwares
- Simulations, HPC
- VR & AR

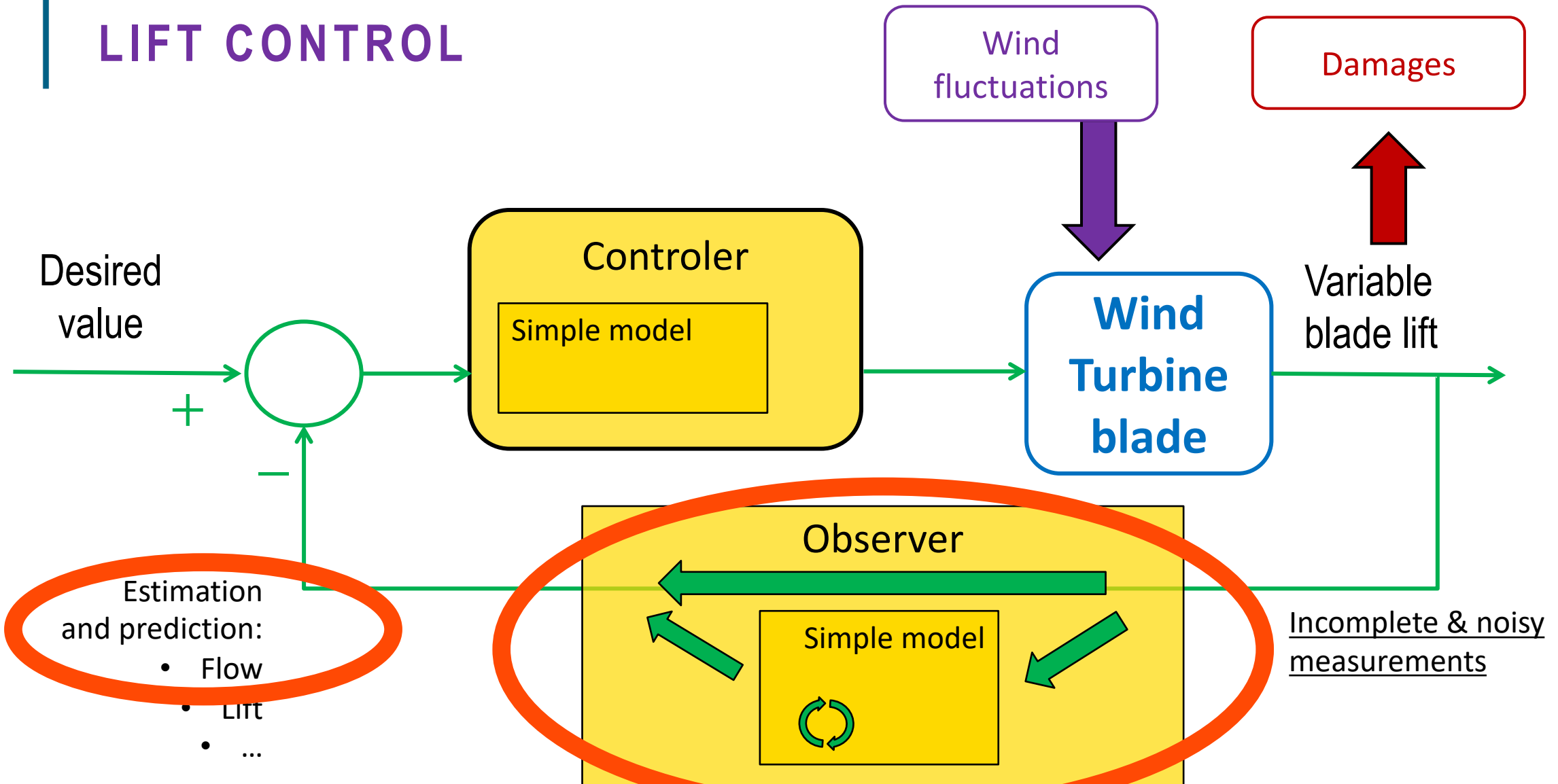


## Other Business Units

~ 2400 people



# WIND TURBINE BLADE LIFT CONTROL



- Estimation and prediction:
- Flow
  - LIFT
  - ...



## PART II

### PHYSICS, DATA & REDUCED ORDER MODEL

# REDUCED ORDER MODEL (ROM)

Solution of an PDE with the form:

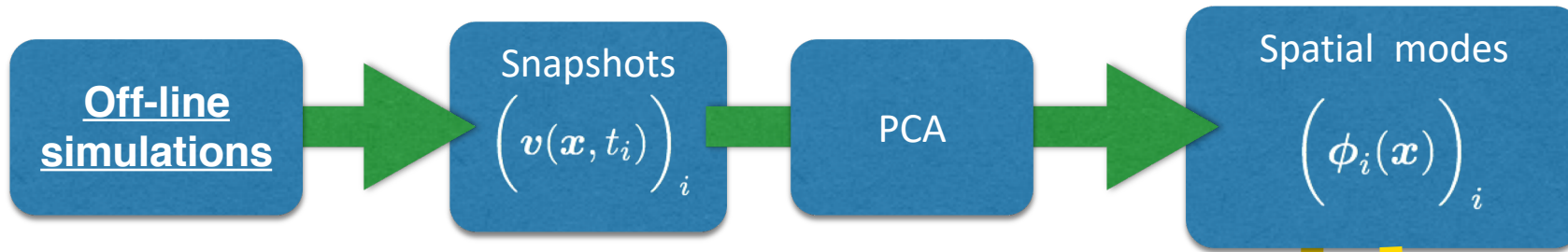
$$v(x, t, \alpha) \approx \sum_{i=0}^n \boxed{b_i(t)} \boxed{\phi_i(x)} \boxed{\gamma_i(\alpha)}$$

Order of magnitude examples in CFD

	Full space	Reduced space
Solution coordinates	$v_q(x_i, t)_{qi}$	$(b_i(t))_i$
Dimension	$M \times d \sim 10^7$	$n \sim 10 - 100$

# POD-GALERKIN

- Principal Component Analysis (PCA) on a *dataset* to reduce the dimensionality:



- Approximation:

$$v(x, t) \approx \sum_{i=0}^n b_i(t) \phi_i(x)$$

Resolved modes

- Projection of the “physics” onto the spatial modes :

$$\int_{\Omega} dx \phi_i(x) \cdot (\text{Physical equation (e.g. Navier-Stokes)})$$

→ ROM for very fast simulation of temporal modes

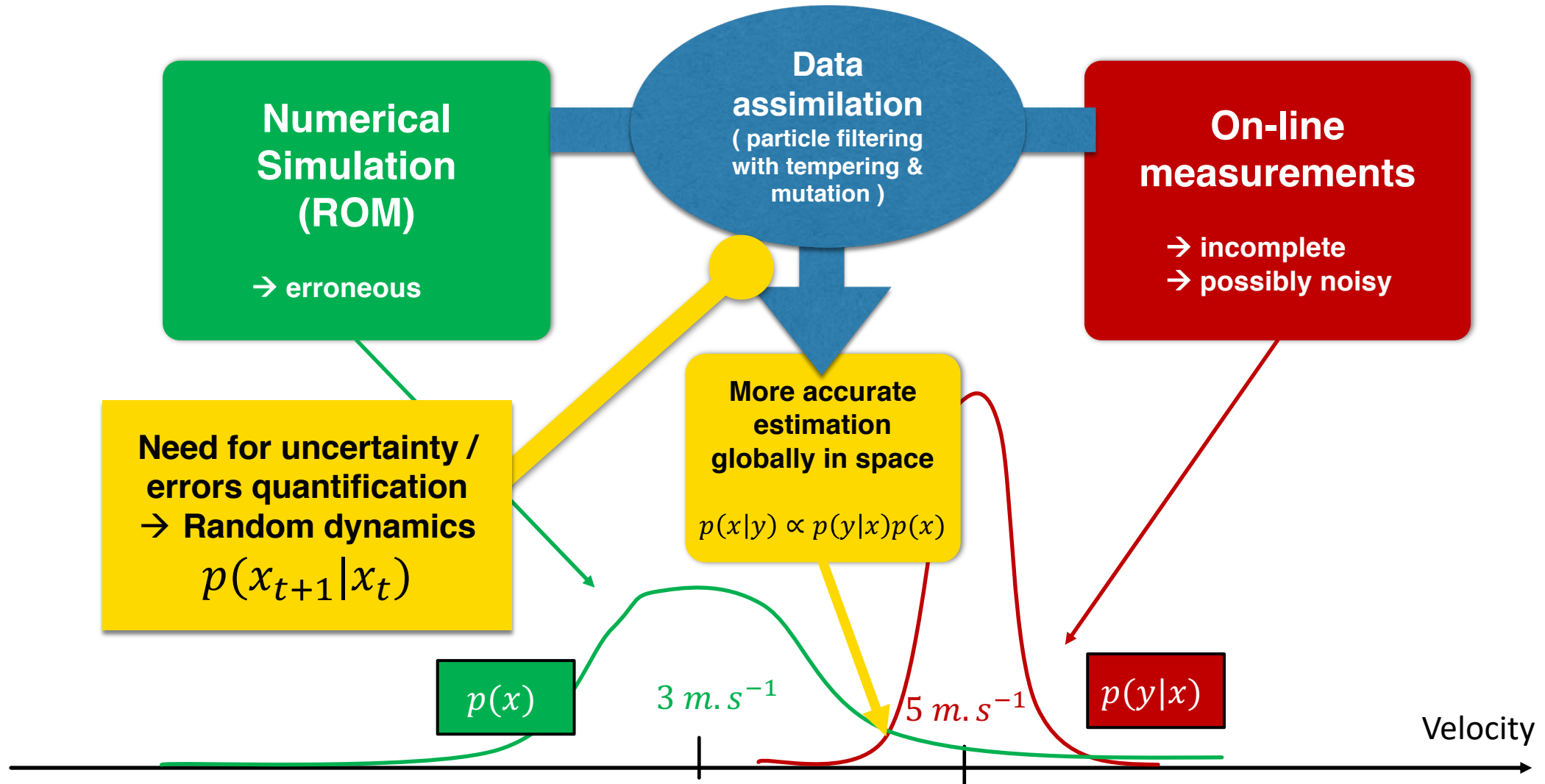




## PART III

### SIMULATION, MEASUREMENTS & DATA ASSIMILATION

# COMBINING SIMULATIONS AND MEASUREMENTS

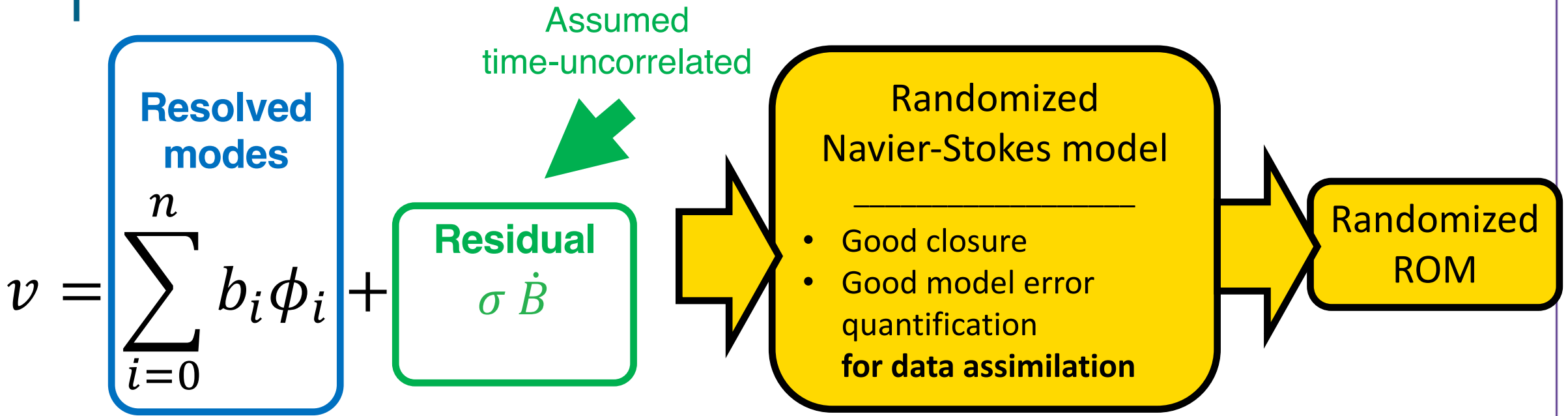




## PART IV

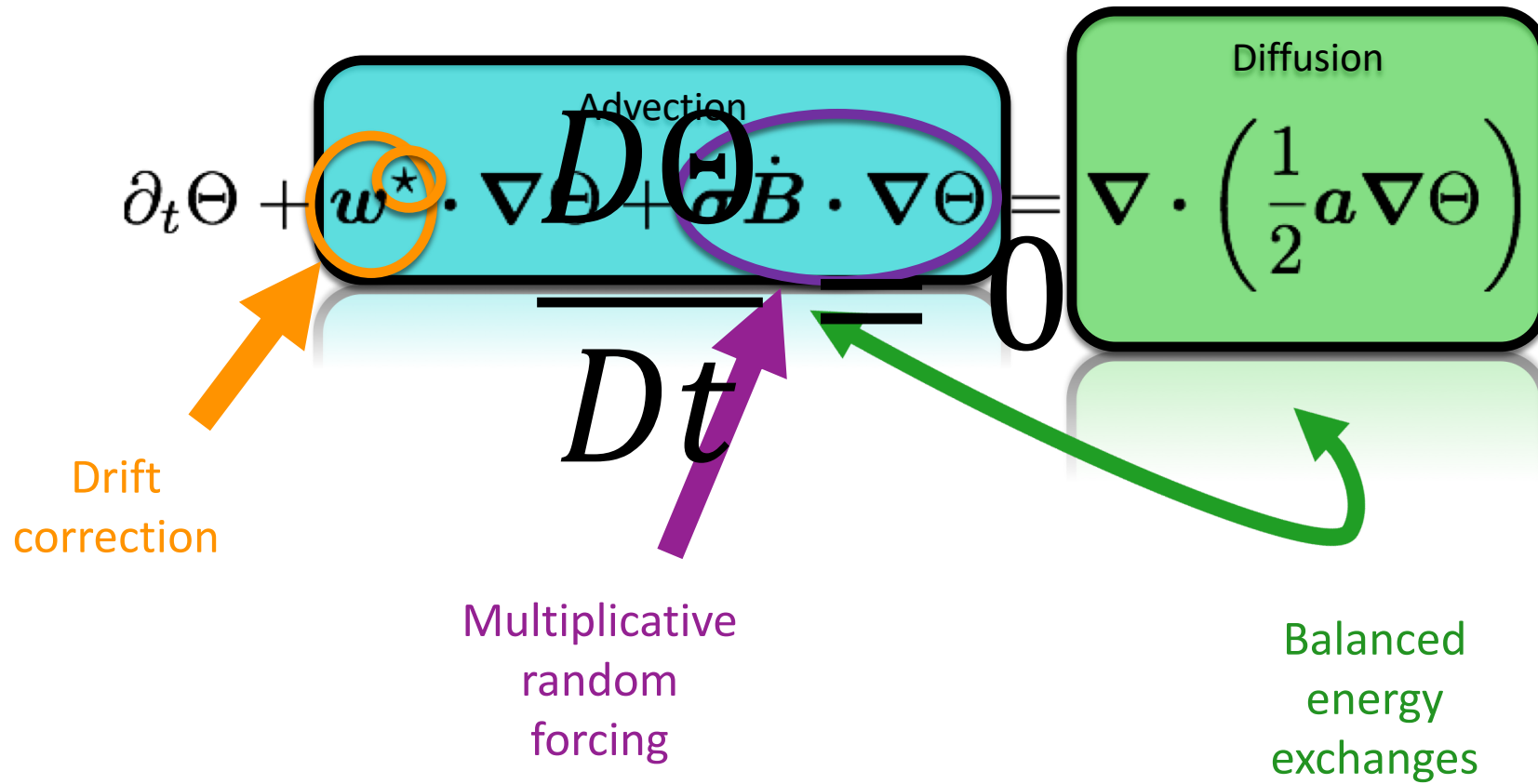
# REDUCED ORDER MODELS UNDER LOCATION UNCERTAINTY

# LOCATION UNCERTAINTY MODELS (LUM)



	LUM	SALT
References :	Memin, 2014 Resseguier et al. 2017 a, b, c, d Cai et al. 2017 Chapron et al. 2018 Yang & Memin 2019	Crisan et al., 2017 Gay-Balmaz & Holm 2017 Cotter and al. 2018 a, b Cotter and al. 2019
	Cotter and al. 2017 Resseguier et al. 2019 a, b	

# MODEL UNDER LOCATION UNCERTAINTY, THE TRACER ADVECTION EXAMPLE



Large scales:  
 $w$

Small scales:  
 $\sigma \dot{B}$

Variance tensor:  
 $a = a(x, x) = \frac{\mathbb{E}\{\sigma dB (\sigma dB)^T\}}{dt}$



# REDUCED MODELS UNDER LOCATION UNCERTAINTY: GALERKIN PROJECTION GIVES SDES FOR RESOLVED MODES

$$\int_{\Omega} \phi_i \cdot (\text{stochastic Navier-Stokes})$$

$$\frac{db_i}{dt} = \boxed{F_i(b)} + \overset{n \times M \quad M \times 1}{\underbrace{(\alpha_{\cdot i} \dot{B}_t)^T}_{\text{multiplicative noise}}} \underbrace{b}_{n \times 1} + \overset{1 \times M \quad M \times 1}{\underbrace{(\theta_i \dot{B}_t)}_{\text{additive noise}}}$$

Large scales:  
 $\frac{d}{dt} \int_{\Omega} \phi_i \cdot u$

Small scales:  
 $\sigma \dot{B}$

Variance tensor:  
 $a = a(x, x) = \frac{\mathbb{E}\{\sigma dB (\sigma dB)^T\}}{dt}$

2<sup>nd</sup> order polynomial:  
coefficients given by physics,

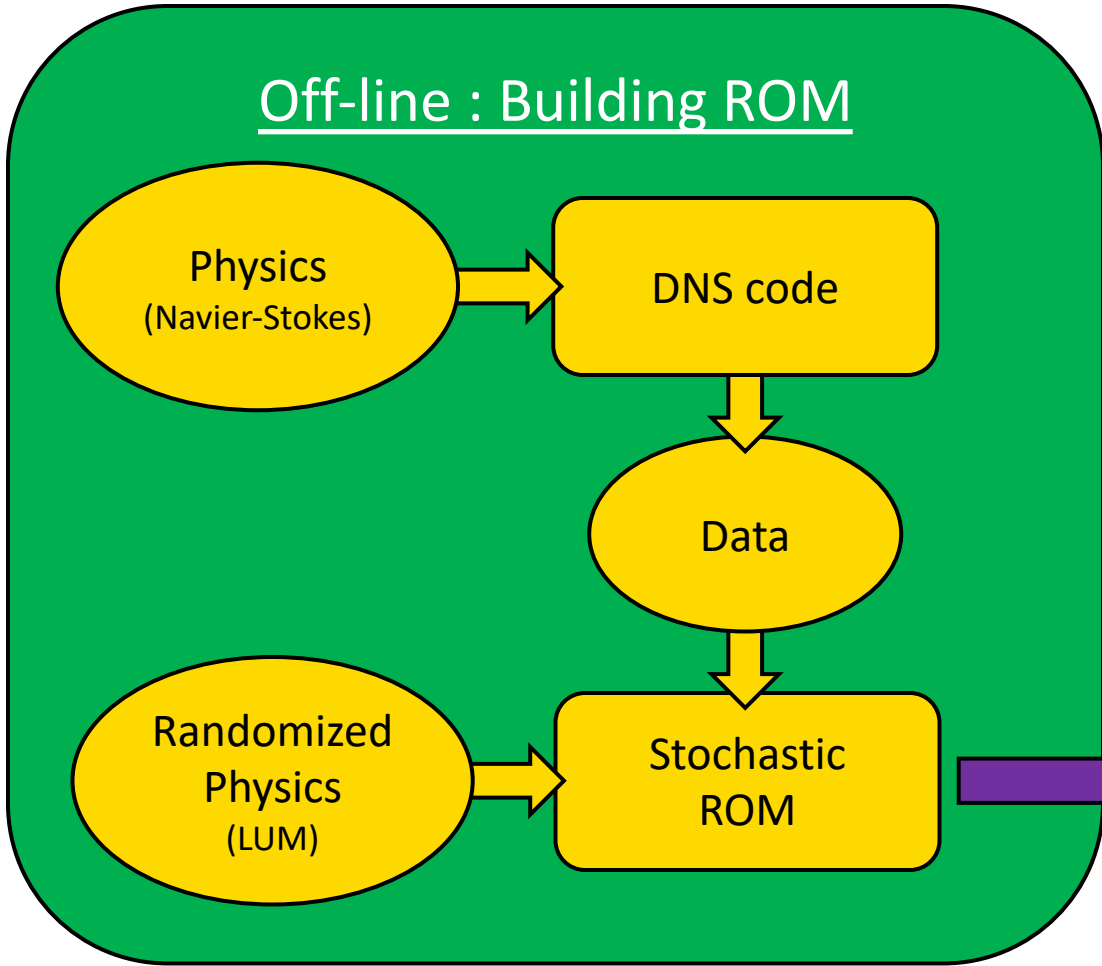
$$(\phi_j)_j \text{ and } a = \overline{\sigma \dot{B} (\sigma \dot{B})^T} \tau$$

**Correlations to estimate**

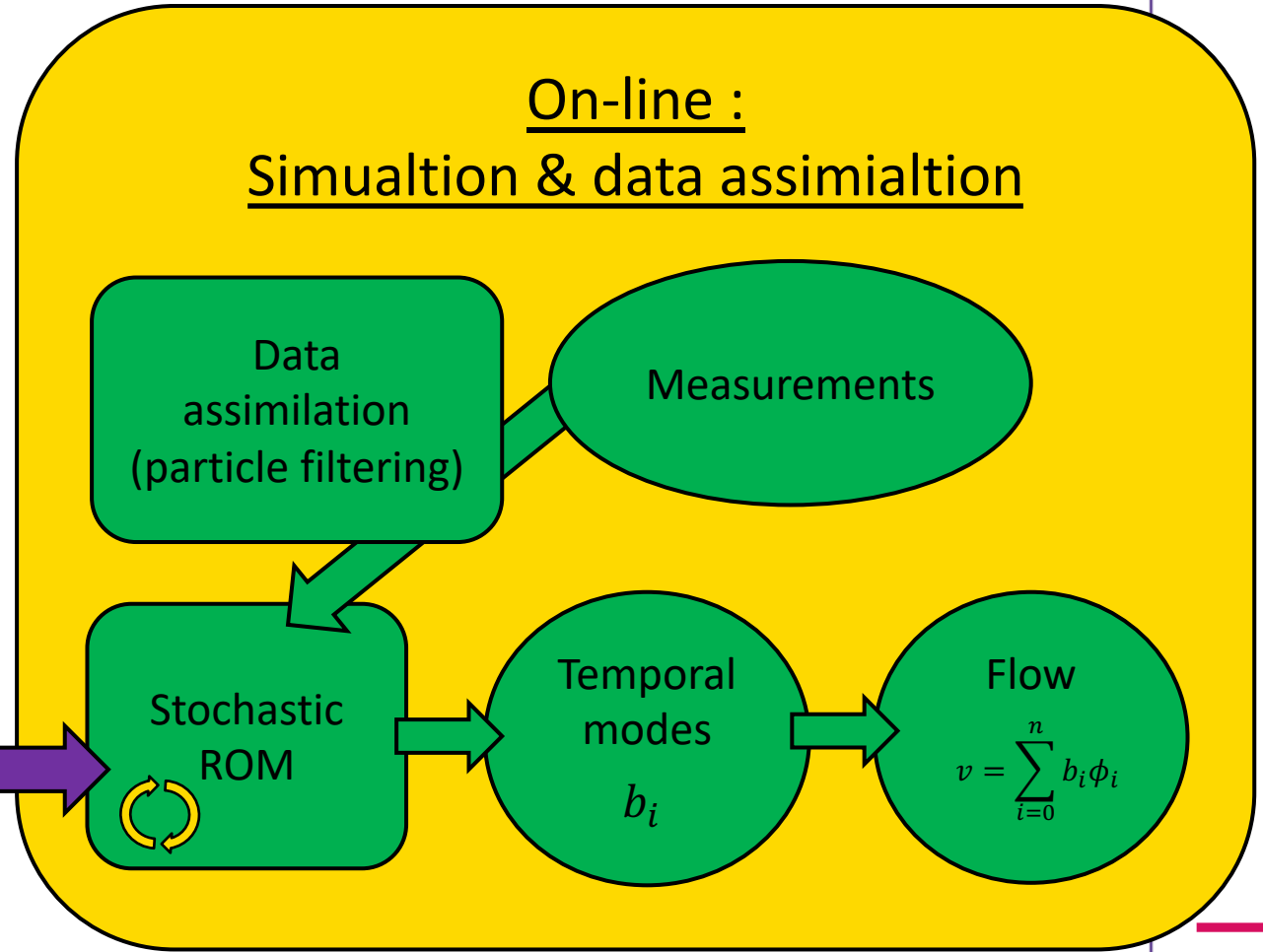
$$\alpha_{pi} \cdot \alpha_{qj}, \alpha_{pi} \cdot \theta_j, \theta_i \cdot \theta_j.$$

# SUMMARY

## Off-line : Building ROM



## On-line : Simulation & data assimilation





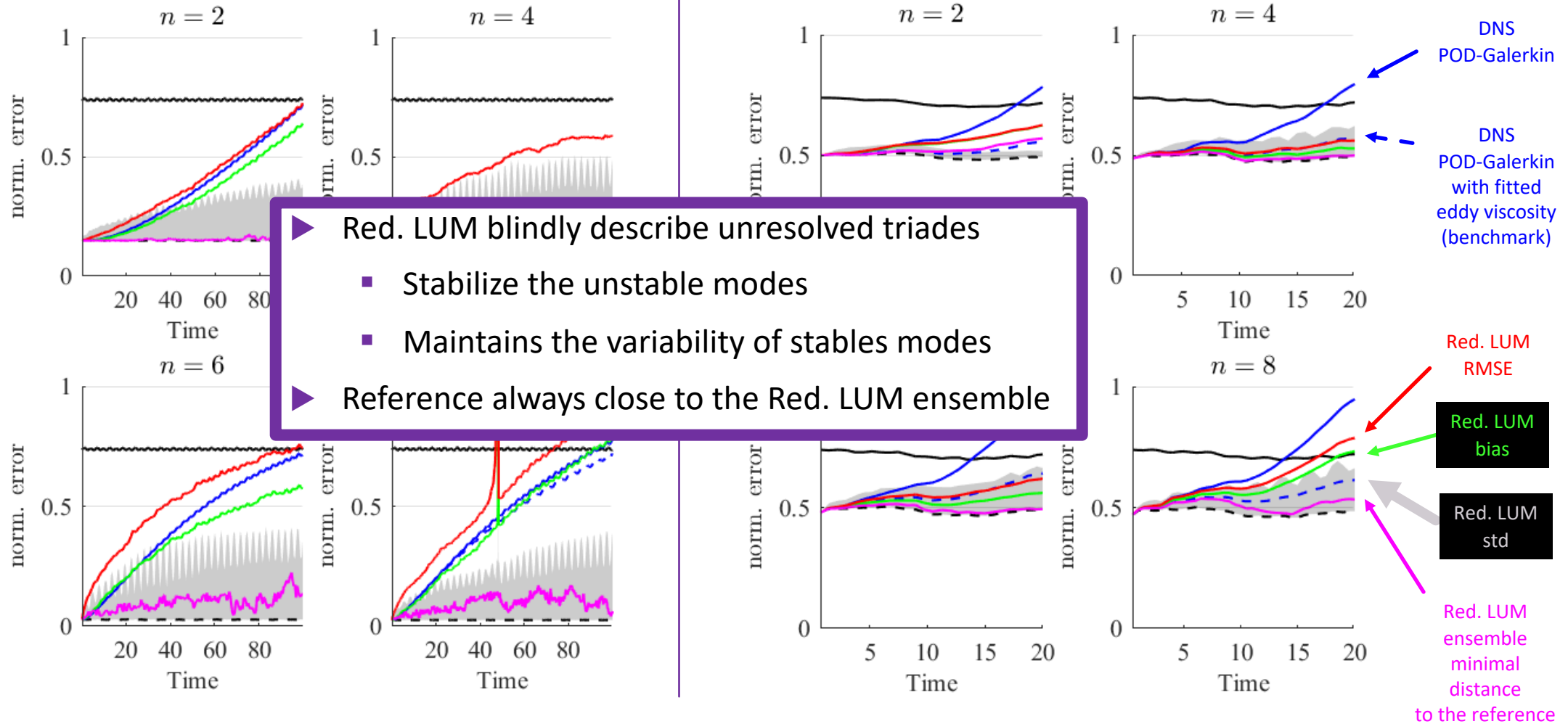
## PART V

RESULTS :  
UQ &  
FAST OBSERVER  
OF THE FLOW

# UNCERTAINTY QUANTIFICATION (no data assimilation)

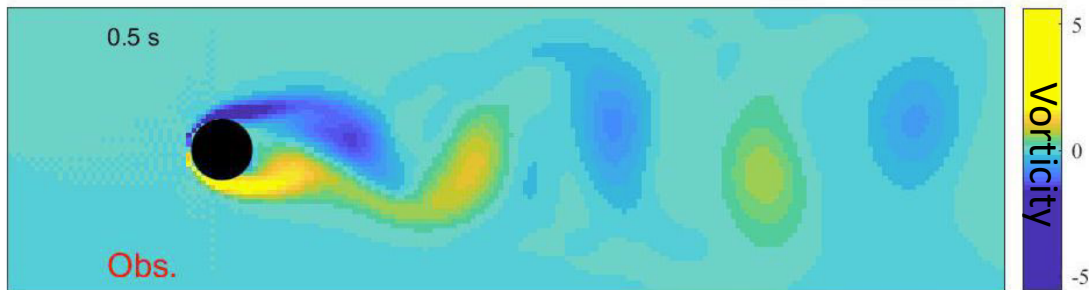
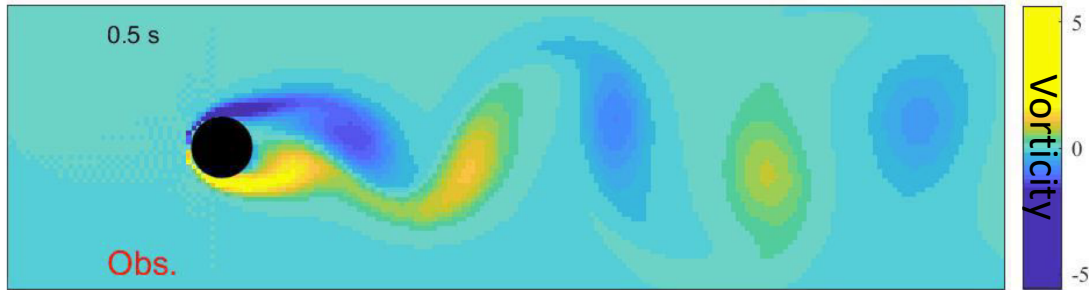
## ► 2D Wake at Re 100

## ► 3D Wake at Re 300



# DATA ASSIMILATION: WAKE AT RE 100

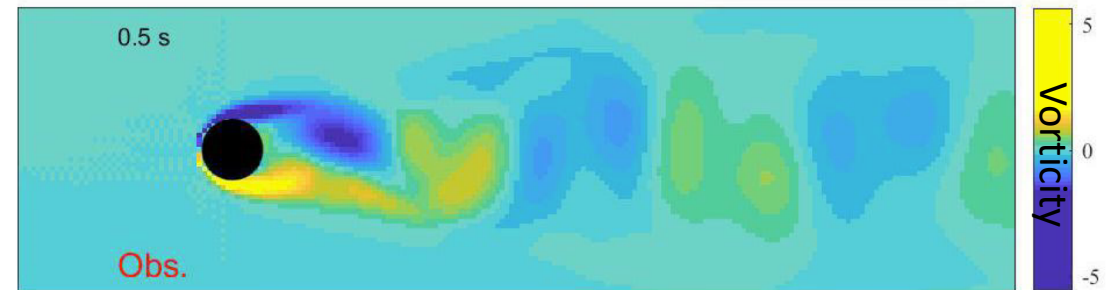
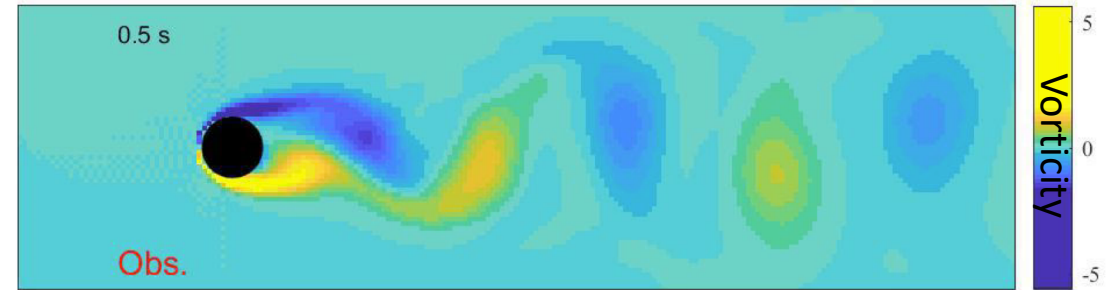
Reference  
(DNS)  
 $10^4$  degrees of freedom



**Our method**  
(Red-LUM-based data-assimilation)  
6 degrees of freedom

Reduced order models with  $n = 6$   
and 2dB-SNR obs. assimilated every 5 sec

Theoretical bound  
(Optimal from 6-d.o.f. linear decomposition)  
6 degrees of freedom



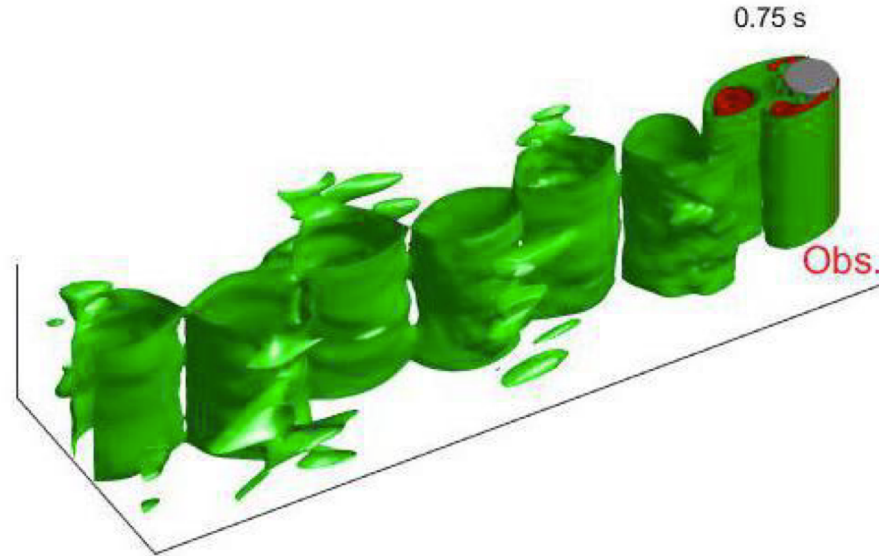
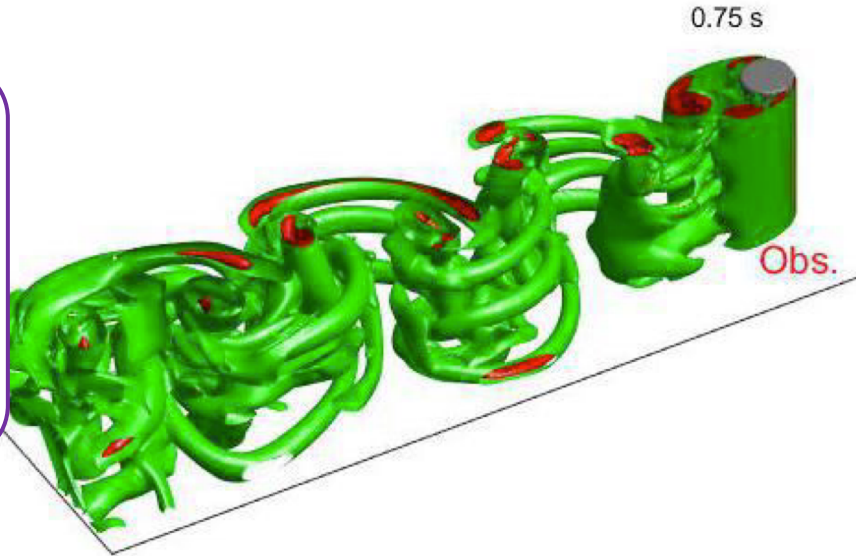
Benchmark  
(POD-ROM (with eddy viscosity) + init. by obs.)  
6 degrees of freedom



# DATA ASSIMILATION: WAKE AT RE 300

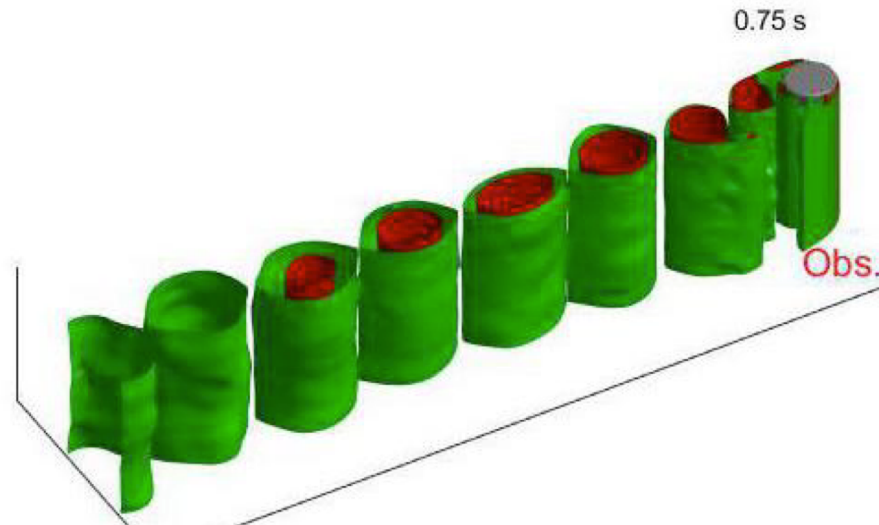
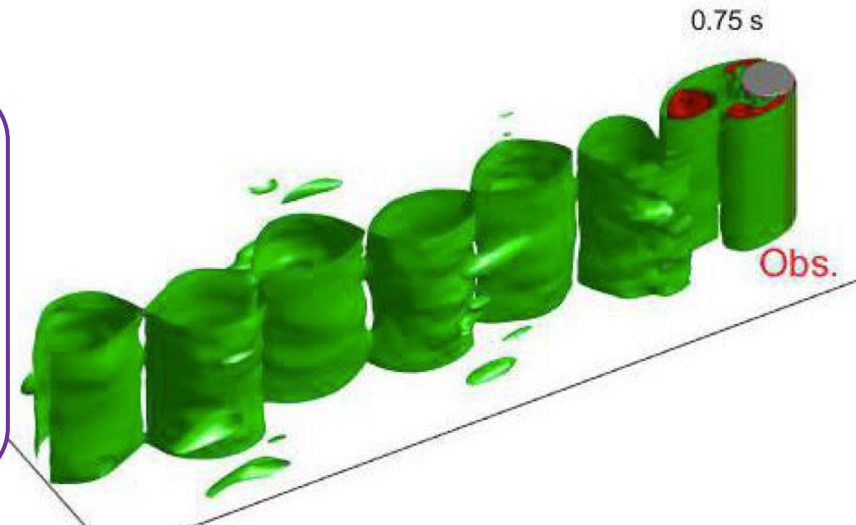
Reduced order models with  $n = 6$   
and 2dB-SNR obs. assimilated every 5 sec

Reference  
(DNS)  
 $10^7$  degrees of freedom



Theoretical bound  
(optimal from  
6-d.o.f. linear  
decomposition)  
6 degrees of freedom

Our method  
(Red-LUM-  
based data-  
assimilation)  
6 degrees of freedom



Benchmark  
(POD-ROM  
(with eddy  
viscosity) +  
init. by obs.)  
6 degrees of freedom



# CONCLUSION

## CONCLUSION

- ▶ Reduced order model (ROM) : for very fast and robust CFD ( $10^7 \rightarrow 6$  degrees of freedom.)
  - Combine data & physics (built off-line)
  - Closure problem handled by LUM
- ▶ Data assimilation : to correct the fast simulation on-line by incomplete/noisy measurements
  - Model error quantification handled by LUM
- ▶ First results
  - Optimal unsteady flow estimation/prediction in the whole spatial domain (large-scale structures)
  - Robust far outside the learning period

## NEXT STEPS

- ▶ Real measurements (PIV, TrimControl, ...)
- ▶ Increasing the degrees of freedom ( $n$ )
- ▶ Parametric ROM
- ▶ Increasing Reynolds (reduced DNS  $\rightarrow$  reduced LES)