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## SUBSPACE-BASED DAMAGE DETECTION WITH REJECTION OF THE TEMPERATURE EFFECT AND UNCERTAINTY IN THE REFERENCE

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### ABSTRACT

Temperature variation can be a nuisance that perturbs vibration based structural health monitoring (SHM) approaches for civil engineering structures. In this paper, temperature affected vibration data is evaluated within a stochastic damage detection framework, which relies on a null space based residual. Besides two existing temperature rejection approaches – building a reference state from an averaging method or a piecewise method – a new approach is proposed, using model interpolation. In this approach, a general reference model is obtained from data in the reference state at several known reference temperatures. Then, for a particular tested temperature, a local reference model is derived from the general reference model. Thus, a well fitting reference null space for the formulation of a residual is available when new data is tested for damage detection at an arbitrary temperature. Particular attention is paid to the computation of the residual covariance, taking into account the uncertainty related to the null space matrix estimate. This improves the test performance, contrary to prior methods, for local and global damages, resulting in a higher probability of detection (PoD) for the new interpolation approach compared to previous approaches.

*Keywords: statistical method, subspace-based method, temperature rejection, model interpolation, uncertainty*

### 1. INTRODUCTION

In structural health monitoring of civil engineering structures, the effects of environmental variability often limit the possible application of existing damage diagnosis approaches. Methods that are sensitive to changes due to damages mostly are prone to false alarms, when temperature affects the dynamical behavior of the structure in a more significant way than small, local damages would. Thus, the environmental variability is an crucial issue for the practicability of vibration based SHM. In [1] examples of

SHM projects are listed, which gave evidence of the importance of considering temperature effects.

Several approaches are proposed within the literature for temperature rejection methods in vibration based damage diagnosis tools. Some of them consider data features, which are blind to environmental variations. In [2], a regression analysis of the natural frequencies was performed, requiring the estimation of those quantities. Other approaches [3, 4] adopt a novelty detection point of view, with the definition of the system under normal reference conditions.

In this paper the stochastic subspace-based methods will be used for damage diagnosis robust to temperature variations. These statistical approaches have been shown to be able to handle noises and uncertainties in the reference null space. The stochastic subspace-based method has already been basis for some work on temperature rejection too. In [8], a damage diagnosis test is proposed, where the system parameter matrix estimates are averaged with respect to temperature. This temperature depending average is used for the determination of a reference and therefore for the statistical tests. In [6] the generic stochastic subspace-based method was modified for temperature compensation, by considering the reference measurements piecewise. The temperature parameter is then used to find the best fitting reference null space.

The now proposed temperature rejection approach relies on the model interpolation approach from [7]. Assume that the dynamic behavior of a system depends on a parameter  $p$ , for example, the temperature of a monitored structure. Given some data sets, each being collected on the system when  $p$  stays around a specific value, then each of these data sets can be used to build a model, which is valid locally in the range of  $p$ . Naturally, when  $p$  varies within the same range, new models can be obtained by interpolating the available local models. However, with the stochastic subspace method, the available local models are in state-space form, and each of them are represented in an arbitrary state basis, independent from each other. This lack of coherence between the state bases of the local models makes the interpolation of local models non trivial. The method proposed in [7] has the advantage of being insensitive to this lack of coherence. In the context of the current paper and the temperature rejection, the considered varying physical parameter is assumed to be the temperature.

In the following, the basics of subspace-based damage diagnosis are introduced shortly. In Section 3. two existing temperature rejection approaches for the stochastic subspace-base damage diagnosis are described, and the new approach using model interpolation is derived. After considering the issue of uncertainty considerations, the three temperature rejection approaches are compared regarding their probability of detection in Section 4..

## 2. SUBSPACE-BASED DAMAGE DIAGNOSIS

Assume the structure of interest to be modeled by a linear mechanical model, which is influenced by the temperature  $T$ . At some temperature  $T = T_j$ , the corresponding stochastic discrete time state-space model writes as

$$\begin{aligned} x_{k+1,j} &= A_j x_{k,j} + w_{k,j} \\ y_{k,j} &= C_j x_{k,j} + v_{k,j}, \end{aligned} \tag{1}$$

where  $x \in \mathbb{R}^n$  denotes the state,  $k$  the index of the sample within each experiment,  $N$  the total number of samples for each experiment,  $y \in \mathbb{R}^r$  is the measured output. The system matrices  $A_j = A(T_j)$  and  $C_j = C(T_j)$  are the state transition matrix with dimension  $n \times n$  and the observation matrix with dimension  $r \times n$ , respectively. White noise is assumed for the state noise  $w$  and output noise  $v$ . Denote the cross-covariances between the signal  $y_{k,j}$  and its time-shifted version  $y_{k-i,j}$  as the *output covariances*

$R_{i,j} = \mathbf{E}(y_{k,j}y_{k-i,j}^T) = C_j A_j^{i-1} G_j$ , where  $G_j = \mathbf{E}(x_{k+1,j}y_{k,j}^T)$ . Then a block Hankel matrix is built:

$$\mathcal{H}_j \stackrel{\text{def}}{=} \begin{bmatrix} R_{1,j} & R_{2,j} & \dots & R_{q,j} \\ R_{2,j} & R_{3,j} & \dots & R_{q+1,j} \\ \vdots & \vdots & \ddots & \vdots \\ R_{p+1,j} & R_{p+2,j} & \dots & R_{q+p,j} \end{bmatrix}. \quad (2)$$

By definition  $\mathcal{H}_j$  depends on the system matrices and therefore will change if the dynamical system changes due to damages. For damage diagnosis, the left null space matrix of  $\mathcal{H}_j$  in the reference state is considered. For the so called *reference null space*  $S$  it holds  $S^T \mathcal{H}_j = 0$  for some  $j$ . A data-based residual vector  $\zeta$  can be defined as

$$\zeta = \sqrt{N} \text{vec}(S^T \hat{\mathcal{H}}), \quad (3)$$

with  $\text{vec}(\cdot)$  as the column stacking vectorization operator and  $\hat{\mathcal{H}}$  as the estimate of the Hankel matrix from data in the monitoring state at an arbitrary temperature  $T$ . Notice that for every  $\mathcal{H}_j$ , there exists a different null space  $S_j$ . This makes the choice of a unique null space impossible as temperature varies.

If the system deviates from the reference state,  $S$  is no longer the null space of the monitored data Hankel matrix and the mean value of  $\zeta$  deviates from 0. The evaluation of the residual  $\zeta$  is done by means of some hypothesis tests, as proposed e.g. in [8, 9]. The test writes as

$$t = \zeta^T \Sigma_\zeta^{-1} \zeta, \quad (4)$$

where  $\Sigma_\zeta$  is the asymptotic residual covariance. The test statistic  $t$  is asymptotically  $\chi^2$  distributed and compared to a threshold, which has to be computed in a training phase in the reference state for a given type I error. Note that besides changes due to damage, changes due to temperature will also lead to a deviation of the residual mean from 0 when the temperature for the computation of  $S$  does not correspond to the temperature of  $\hat{\mathcal{H}}$ .

Conventionally, the asymptotic covariance  $\Sigma_{\mathcal{H}} = \lim \text{cov}(\sqrt{N} \text{vec}(\hat{\mathcal{H}}))$  corresponding to the computed matrix  $\hat{\mathcal{H}}$  is considered in the computation of  $\Sigma_\zeta$  as

$$\Sigma_\zeta = \mathcal{J}_{\zeta, \mathcal{H}} \Sigma_{\mathcal{H}} \mathcal{J}_{\zeta, \mathcal{H}}^T \quad (5)$$

where  $\mathcal{J}_{\zeta, \mathcal{H}} = I \otimes S^T$ . An estimate  $\hat{\Sigma}_{\mathcal{H}}$  of the covariance  $\Sigma_{\mathcal{H}}$  is obtained from a sample covariance [9]: A data sample  $y_k$ ,  $k = 1, \dots, N$ , is separated into  $n_b$  blocks, each of the length  $N_b$ , with  $n_b \cdot N_b = N$ . With the output covariance estimates computed for each data block  $h$ , the Hankel matrix  $\hat{\mathcal{H}}^{(h)}$  can be computed and the sample covariance yields

$$\hat{\Sigma}_{\mathcal{H}} = \frac{N_b}{n_b - 1} \sum_{h=1}^{n_b} \text{vec} \left( \hat{\mathcal{H}}^{(h)} - \hat{\mathcal{H}} \right) \text{vec} \left( \hat{\mathcal{H}}^{(h)} - \hat{\mathcal{H}} \right)^T. \quad (6)$$

### 3. TEMPERATURE REJECTION WITHIN SUBSPACE-BASED DAMAGE DETECTION

The stochastic subspace-based method is promising when it comes to damage detection of structures, which are exposed to temperature variations. By choosing an adequate reference null space for the residual and a proper residual covariance for the hypothesis testing, the damage detection tests can be made robust to temperature variation.

In the following two existing temperature rejection approaches – an averaging and a piecewise approach – will be recalled shortly. Finally a new method is introduced, where an interpolated reference null space is considered.

### 3.1. Averaging and piecewise approaches for temperature rejection

A simple temperature rejection approach [8] uses the reference null space of *averaged* Hankel matrices in the reference state and an averaged residual covariance. Measurements  $\mathcal{Y}_j = [y_{1,j} \ y_{2,j} \ \dots \ y_{N,j}]$  at  $j = 1, \dots, m$  different reference temperatures  $T_j$  are merged to one big data set  $\bar{\mathcal{Y}} = [\mathcal{Y}_1 \ \mathcal{Y}_2 \ \dots \ \mathcal{Y}_m]$ . The null space  $\bar{S}$  of the *averaged* system and the corresponding data covariance  $\bar{\Sigma}_{\mathcal{H}}$  follow from Equations (2) and (6), computed on the merged data set  $\bar{\mathcal{Y}}$ . Note that for this method no information about the reference temperatures is used.

Taking into account the temperature measurement information into the temperature rejection method can improve the test performance, as the reference null space can be determined more accurately for a specific temperature range. In [6] an approach was proposed, where reference null space matrices  $S_j$  and corresponding covariances  $\Sigma_{\mathcal{H}_j}$  are estimated “piecewise” on reference data  $\mathcal{Y}_j$  at each available reference temperature  $T_j$ ,  $j = 1, \dots, m$ . In the tested state, the current temperature  $T$  is recorded and the *nearest* reference null space with respect to this temperature is used to compute the residual and the hypothesis test.

### 3.2. Model interpolation for temperature rejection

The basic idea of the new temperature rejection method is to provide an adequate reference null space for each tested temperature  $T$ . This approach is based on a linear parameter varying (LPV) system model interpolation with model reduction method proposed in [7]. In this model interpolation method, the local state-space models (1), obtained at  $j = 1, \dots, m$  reference temperatures  $T_j$ , are combined to build a large global state-space model. Because each local state-space model (1) has been estimated with an arbitrary state basis, it is not possible to directly interpolate the matrices involved in (1) before making the local models coherent. The method proposed in [7] has the advantage of not requiring the formulation of coherent local models. From this LPV model a local model for the required tested temperature  $T$  can be interpolated in the testing step. Then, this local model corresponds well to a reference system at temperature  $T$ , and can be used to obtain the null space and covariance matrices for damage detection. Following [7], the local models (1) are combined into a large state-space model as

$$\begin{aligned} \underline{x}_{k+1} &= \underline{A} \underline{x}_k + \underline{w}_k \\ y_k &= \underline{C}(T) \underline{x}_k + \underline{v}_k, \end{aligned} \quad (7)$$

where

$$\begin{aligned} \underline{x}_k &= [x_{k,1}^T \ \dots \ x_{k,m}^T]^T, \quad \underline{w}_k = [w_{k,1}^T \ \dots \ w_{k,m}^T]^T \\ \underline{C}(T) &= [\rho_1(T)C_1 \ \dots \ \rho_m(T)C_m], \quad \underline{v}_k = \sum_{j=1}^m \rho_j(T) v_{k,j}(t) \\ \underline{A} &= \begin{bmatrix} A_1 & & \\ & \ddots & \\ & & A_m \end{bmatrix}, \quad \underline{G} = \begin{bmatrix} \rho_1(T)G_1 \\ \vdots \\ \rho_m(T)G_m \end{bmatrix}. \end{aligned}$$

The weighting functions  $\rho(T)$  are typically bell-shaped functions, centered at  $T$ , such that

$$\sum_{j=1}^m \rho_j(T) = 1 \quad \text{for all } T \in \mathbb{T} \quad \text{and} \quad \rho_j(T) : \mathbb{T} \Rightarrow [0, 1]. \quad (8)$$

This large global model of order  $mn$  is then reduced. Using the well-known balanced reduction method, due to its reliable numerical behavior, the state vector  $\underline{x}_k$  is transformed linearly, removing the smallest Hankel singular values. Then only the  $n$  largest Hankel singular values are kept, and the resulting state space model is of order  $n$ , like any of the local models. This method was developed based on the availability of system matrix estimates at different temperatures.

However, the damage detection test (4) is not directly formulated with the matrices involved in state-space models like those of (1) or those of the interpolated local model. Instead, it is based on the Hankel matrix of the output covariances and its null space in the reference state of the system. Direct interpolation of the temperature-dependent null space is a non trivial task, because each corresponding local model is related to an arbitrary state basis. In this paper, this interpolation will be made through the interpolation of the Hankel matrices corresponding to different reference temperatures  $T_j$  and the production of a global, interpolated Hankel matrix  $\underline{\mathcal{H}}(T)$  corresponding to the reference state at temperature  $T$ . Considering the definition of the output covariances  $R_i$ , the output covariances of system (7) can be written as

$$\underline{R}_i(T) = \underline{C}(T)\underline{A}^{i-1}\underline{G} = \sum_{j=1}^m \rho_j^2(T)C_j A_j^{i-1} G_j = \sum_{j=1}^m \rho_j^2(T)R_{i,j}. \quad (9)$$

This leads to the interpolated Hankel matrix

$$\underline{\mathcal{H}}(T) = \sum_{j=1}^m \rho_j^2(T)\mathcal{H}_j \quad (10)$$

and thus to the null space matrix  $\underline{S}(T)$  of the interpolated Hankel matrix, i.e. the left null space of (10). The covariance  $\underline{\Sigma}_{\mathcal{H}}(T)$  of the interpolated Hankel matrix follows as

$$\underline{\Sigma}_{\mathcal{H}}(T) = \sum_{j=1}^m \rho_j^4(T)\Sigma_{\mathcal{H}_j}. \quad (11)$$

### 3.3. Behavior of the temperature rejection approaches

For the evaluation of the temperature rejection approaches, hypothesis tests on an 8-mass-spring damper are performed at different tested temperatures between 0°C and 20°C. The temperature effect is modeled as a decrease or an increase of stiffness at all elements of maximum 8%. In the reference state, data is simulated at  $m = 5$  reference temperatures, namely at 0, 5, 10, 15, and 20 °C. Two different data lengths are used (40,000 and 80,000 data samples, respectively) at each considered reference temperature when setting up the reference state, i.e. for the computation of the null space and the covariance matrix.

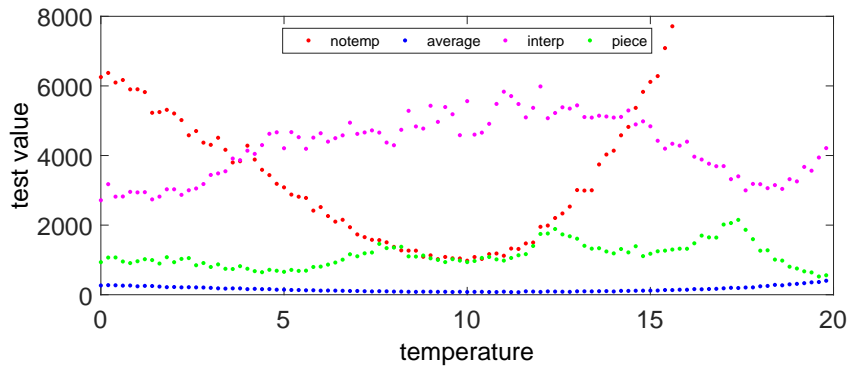
For the damage detection tests, datasets of length  $N=200,000$  are generated in the reference and damaged states in the whole temperature range between 0°C and 20°C for the computation of  $\hat{\mathcal{H}}$ .

The test values in the reference state are plotted in Figure 1 with the different temperature rejection methods. The red lines show the results when 10°C is taken as the reference temperature and no further temperature rejection is considered. It can be observed that for other tested temperatures than the chosen reference temperature, the test values differ significantly from those computed at the reference temperature. This is most likely due to the fact that the considered reference null space does not fit properly to the data, if the temperature in the tested state is different from the one at the reference state, to the point that a change in the system is incorrectly detected. Then, it will not be possible to define a threshold for damage detection. A well defined threshold is mandatory for a good test performance with few false alarms. Notice that the observed effect is independent from the length of the dataset used for the computation of the reference null space.

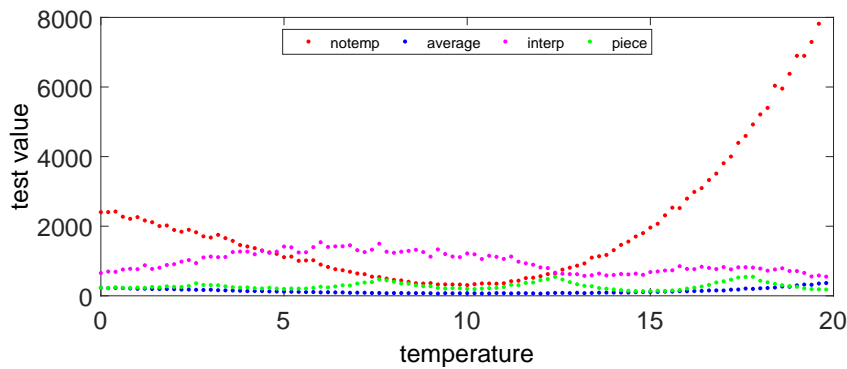
The test values of the averaging method (blue lines) follow a flat curve, and the impact of data length is not obvious. Stability of the test values for different tested temperatures is a first requirement to ensure, so that a reliable threshold avoiding too many false alarms can be found.

For the piecewise approach, the  $m = 5$  reference temperatures 0, 5, 10, 15, and 20 °C can be clearly identified as the minima of the green curves. It can be seen that using more data at each temperature state gives more stable test values. However, if the tested temperature differs from the reference temperature used for the null space computation, the test values are rising, as a change is detected.

The new temperature rejection method aims at providing an adequate reference for each tested temperature (pink lines). With the reference temperatures 0, 5, 10, 15, and 20 °C, the test values in the reference state vary in a certain range. They do not show a specific increase or decrease according to the reference temperatures. The values become more stable when the data length increases.



(a) 40,000 data points at each reference temperature



(b) 80,000 data points at each reference temperature

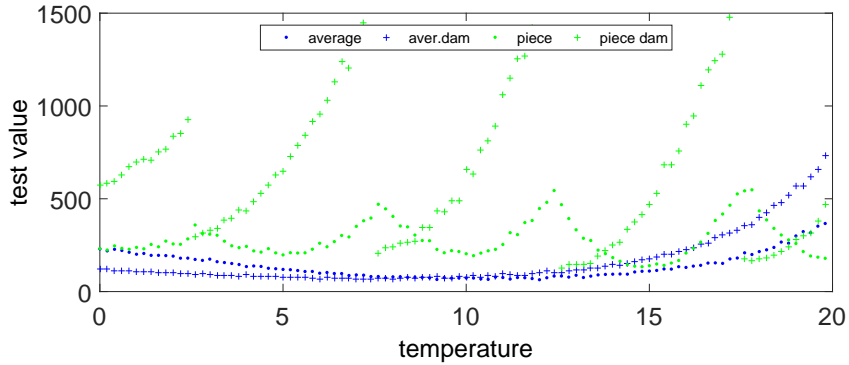
**Figure 1:** Test values in reference case for short and long data sets in the reference state

To consider the performance of the hypothesis test when damage occurs, the shift of the test values in the damaged state from those in the reference state is the relevant criterion. In Figure 2 test values in the reference and in the damaged state are presented, where the long data set in the reference state was considered. The damage is defined as a global damage with 2% loss of stiffness at all elements. To detect global damages, such as the loss of external pre-stressing or extensive cracking, is challenging compared to detecting local ones, because the effect of the damage on the dynamical behavior is similar to the temperature effect.

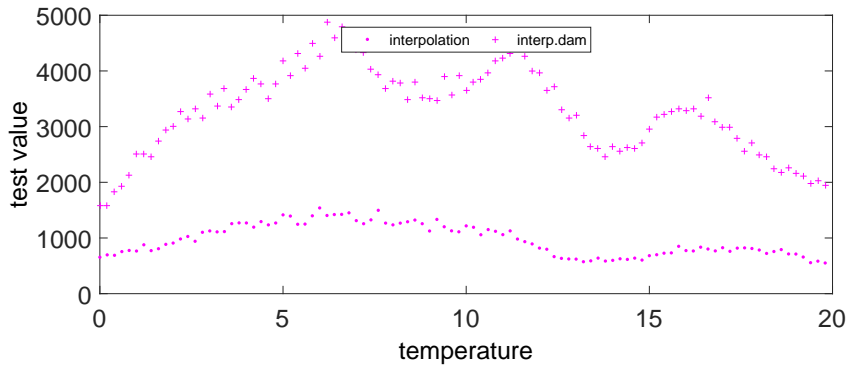
When using the averaging method, the shift of the test values in the damaged case is sometimes positive and sometimes negative, depending on the temperature in the tested state. This can be explained as follows: The reference null space is computed from merged data with mean temperature at 10°C. For tested temperatures under 10°C, temperature and damage affect the stiffness in a similar way but in different directions. The effects of temperature and damage cancel out each other. When they act in the same direction, as it is the case for temperatures above the reference temperature, the shift between reference and damaged test values is very high. However, reducing the number of false alarms in the reference state will require a relatively high threshold for the damage test value, and lots of damages that come along with higher test temperatures will not be detected. Basically, this phenomenon of crossing test value curves is most likely a problem of a badly chosen reference for the computation of the residual.

While the results of the piecewise computation show the same problem of badly fitting reference at each temperature state, the interpolated reference null space overcomes this problem. The test values in the

damaged case are clearly shifted positively from those of the reference state for every tested temperature. Even if the effects of damage and temperature do not have the same direction, they do not cancel out each other. This allows for the definition of an adequate global threshold with few false alarms.



(a) Averaging and piecewise method



(b) Interpolation method

**Figure 2:** Test values in reference and damaged case for global damage

## 4. UNCERTAINTY CONSIDERATIONS FOR SUBSPACE-BASED DAMAGE DETECTION WITH TEMPERATURE EFFECTS

### 4.1. Covariance computation with uncertainties in the reference null space

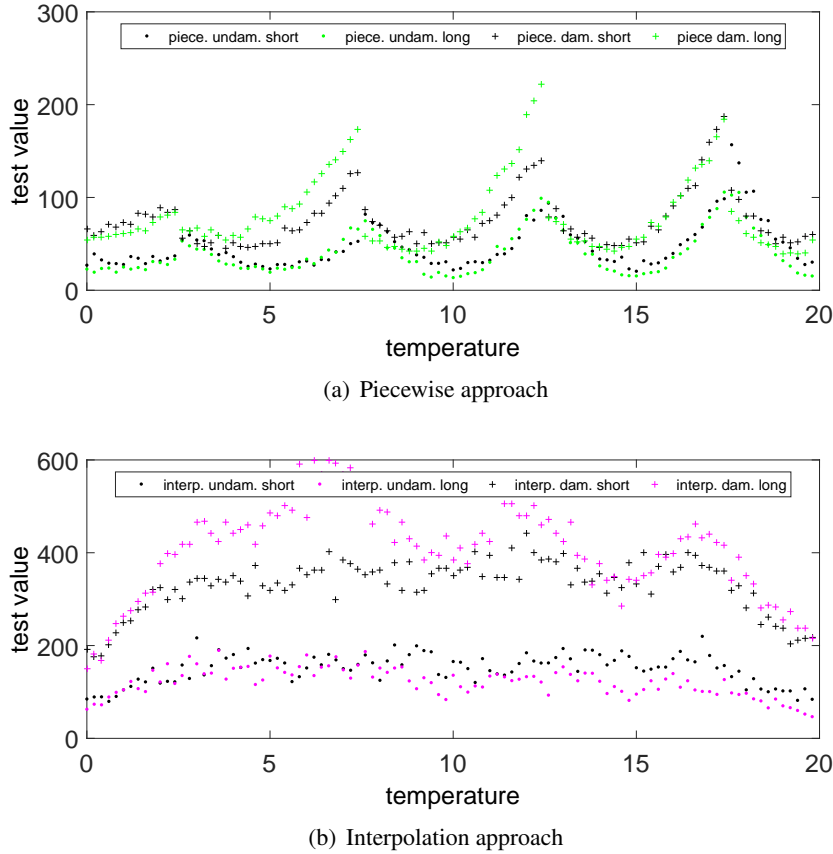
In the subspace-based damage detection the accuracy of the residual covariance estimate is very important for the performance of the hypothesis testing. The conventional computation of the estimate of the residual covariance described in (5) only considers the uncertainties of the data itself. However, the null space matrix  $S$  taken as reference is usually estimated, computed from measurements and thus afflicted with uncertainties. In [10] it is described how those uncertainties can be taken into account. The residual covariance estimate can be formulated as

$$\hat{\Sigma}_{\zeta} = \mathcal{A} \begin{bmatrix} N/M \hat{\Sigma}_{\mathcal{H}} & 0 \\ 0 & \hat{\Sigma}_{\mathcal{H}} \end{bmatrix} \mathcal{A}^T, \text{ with } \mathcal{A} = [\mathcal{J}_{\zeta, S} \quad \mathcal{J}_{S, \mathcal{H}} \quad \mathcal{J}_{\zeta, \mathcal{H}}], \quad (12)$$

where the estimate of the Hankel matrix covariance  $\Sigma_{\mathcal{H}}$  is used.  $N$  and  $M$  are the length of the data sets used in the computation of  $\hat{\mathcal{H}}$  in the testing state and  $S$ , respectively. A detailed definition of the sensitivities  $\mathcal{J}$  can be found in [10]. This extended covariance computation can increase the performance of the damage detection test significantly. Studies in [11] demonstrated, that if the estimation of the reference null space  $S$  is significantly afflicted with uncertainties, because no long data sets in the reference state are available, these uncertainties should not be neglected.



The test values in the undamaged and damaged state considering the uncertainty in the reference are presented in Figure 3 for the piecewise (a) and the interpolation (b) method. Both approaches show high dependency on the data length when the uncertainty of the reference is neglected (see Figure 1). By taking this uncertainty into account, stable results in the undamaged case can be obtained, even for short data lengths. However, the shift of the test values in the damaged case increases when more data is used. This will result in better test performances.



**Figure 3:** Test values (undamaged and damaged) with consideration of uncertainty in the reference for short (40,000) and long (80,000) data sets

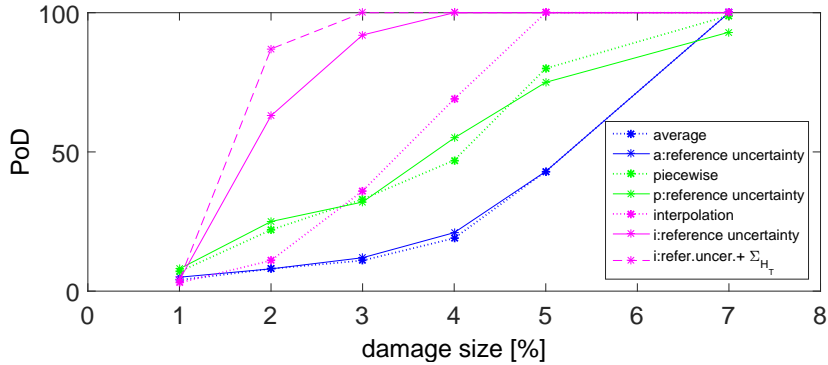
In Figure 4 the probability of detection (PoD) of the 8-mass-spring damper with a local damage at element 3 is plotted with respect to the damage size considering the impact of the uncertainty of  $S$  in the covariance computation. 5 temperatures are taken into account in the reference state for computation of the temperature rejection methods. The considered temperatures in the reference training state and in the testing state are between  $0^{\circ}\text{C}$  and  $20^{\circ}\text{C}$ . The threshold to detect damages is defined such that the false alarm rate is beneath 1% in the reference training state. Note that in the piecewise approach, a threshold is computed individually for each reference temperature state.

It can be seen that for the averaging and the piecewise approach the effect of different uncertainty computations to the PoD of the damage detection test is negligible. In these approaches the main reason for bad test performance is the bad fitting reference null space in the considered tested temperature state.

For the interpolation approach (Figure 2) it has already been shown that a better fitting reference null space is provided. The only issue for the test performance is the uncertainty of the estimated reference null space based on the interpolation. Figure 4 shows that the PoD is improved significantly when taking into account the uncertainty of the estimation of  $S$  in the residual covariance computation. This effect will fade when very long data sets in the reference state are available.

Instead of interpolating the covariance of the Hankel matrix as in (11), it can also be calculated again as

$\Sigma_{\mathcal{H}_T}$  on the data for each tested temperature state using the test data (dashed line) and then used in the residual covariance. This yields a similar performance for the PoD, meaning that the computation of a correct  $S$  is the crucial part of the method.



**Figure 4:** Probability of detection wrt. damage size and the uncertainty on the null space computation

#### 4.2. Test performance with temperature rejection approaches

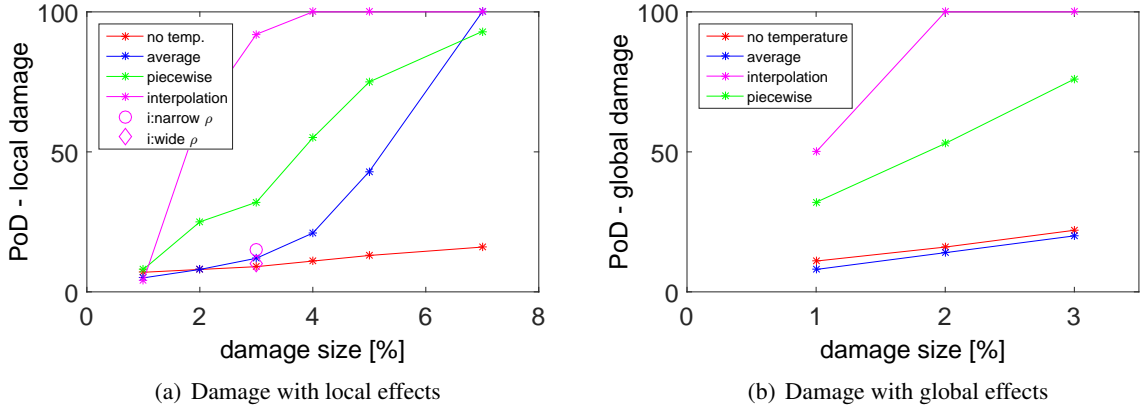
In this section the test performance of the three temperature rejection approaches is evaluated. The performance for different damage types and with respect to the amount of data in the reference state is examined. The covariance in the tests is computed taking into account the uncertainty on the null space as in equation (12). The temperature setup and threshold computation are defined as described in Section 4.1..

In Figure 5a the PoD of a local damage for different damage sizes is shown. Damage is simulated as loss of stiffness in [%] at element 3. Among all presented approaches, temperature rejection leads to better test performance. While the interpolation approach already detects damages of very small damage size, the averaging approach only works properly if the damage is big enough.

It has to be noted that the test performance of the interpolation method relies on the chosen weighting function. The circle and the diamond in Figure 5a denote the PoD of 3% damage, when a very narrow or a very wide bell shaped weighting function is used. It can be seen that the test performance decreases significantly in this case. If the weighting function is very wide and flat, the resulting reference null space will take into account every measured reference temperature point with nearly the same weight – similar to the averaging method. If the weighting function is very narrow, this may lead to a more or less piecewise method. Thus, finding an accurate weighting function is an important issue that should be addressed during the computation of the reference null space. The method here provides the possibility to introduce knowledge of the temperature dependent behavior of the structure into the reference state without requiring explicit knowledge of the temperature modeling. If the temperature effect is mostly linear, good results may be achieved with linear interpolation of the two nearest local models. If sudden temperature effects at specific temperatures are expected, this may also be taken into account by choosing discrete weighting functions.

Global damage (Figure 5b) may be considered as a loss of stiffness at all elements. The resulting effect on the dynamical behavior is very similar to that induced by temperature variations. The averaging method is not able to distinguish between the causes, and thus does not perform better as if no temperature changes would have taken into account. However, the piecewise and the interpolation approach both refer to a reference temperature point for the computation of the reference null space. They test if the system behavior has changed with respect to a particular reference null space and show good test performance. The interpolation method is more sensitive to damages than the piecewise method, due to the well fitting reference null space.

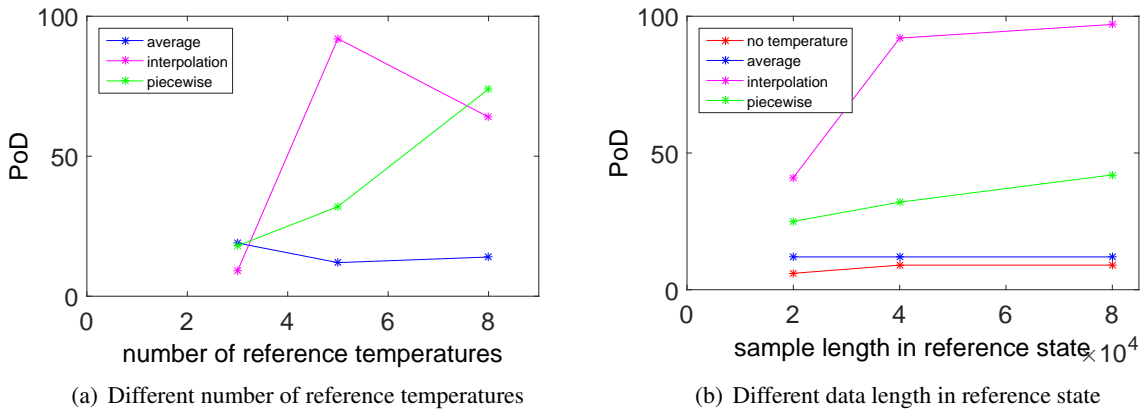
For SHM of civil structures, the availability of data from the reference state always is an issue – for



**Figure 5:** Probability of detection wrt. the damage size for different damage types.

example the temperature in the measured reference states cannot be influenced and data have to be shortened to ensure temperature stability within one given dataset. In Figure 6a the PoD for different numbers of temperatures in the reference state is presented. The PoD of the piecewise approach increases with more reference temperatures. If there are measurements for a dense grid of temperature points, one can choose a quite correct reference null space from measurements and thus good results can be obtained. However, it can be seen, that the performance of the interpolation method depends on an optimization of the weighting function that should be chosen in accordance with the reference temperatures. Here the same weighting function is used for each setup. Since this weighting function was chosen for 5 reference temperatures, it leads to bad test performance for a different number of reference temperatures.

Figure 6b shows that the sample length in the reference state affects the interpolation and the piecewise approach only slightly. Long datasets usually lead to better estimates of the reference null spaces. However, by dealing with the uncertainty of  $S$  in the covariance residual computation as in (12), those uncertainties are already handled.



**Figure 6:** Probability of detection wrt. reference null space computation

## 5. CONCLUSIONS

In this paper a new temperature rejection method for subspace-based damage detection has been proposed and experimentally compared with two existing methods. Based on the interpolation of models, it could be shown that the new interpolation approach gives good results for both global and local damages. The local reference null space computed from an interpolated reference Hankel matrix at each tested

temperature fits better to the test data, than the averaging or the piecewise approaches. Moreover, taking into account the uncertainty in the computation of the reference null space improves significantly the performance of damage detection.

The choice of the weighting function in the interpolation method provides the possibility to introduce specific monitoring rules into the data driven method, without requiring explicit knowledge of the temperature model.

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