

#### H-Matrix Solver Applied to Coupled FEM-BEM Aeroacoustics Simulations

Guillaume Sylvand

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# H-MATRIX SOLVER APPLIED TO COUPLED FEM-BEM AEROACOUSTICS SIMULATIONS

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Guillaume SYLVAND - Airbus Central R&T

JOSO - March 12th, 2019.

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• Airbus Central R&T

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## • Airbus Central R&T

• Departement "Virtual Product Engineering"

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- Departement "Virtual Product Engineering"
- Joint Work with

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    - Aurélien Falco (WIP) H-matrix for FEM/BEM system



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• Sound propagation in a air flow

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- Depends on the phase of the flight
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  - Take-off

Take-Off



#### **Acoustics Sources**

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#### **Acoustics Sources**

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  - Cruise: CROR ?
  - · Approach/Landing

#### Approach





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• Frequency domain, 3D to solve Helmholtz equation

$$-\frac{1}{ik}\oint_{\Gamma\times\Gamma}\frac{\partial^2 G}{\partial\nu_x\partial\nu_y}q(y)q^t(x)dydx = -\frac{1}{ik}\int_{\Gamma}\frac{\partial u_{inc}(x)}{\partial\nu}q^t(x)dx.$$

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- Drawback:
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  - Dense linear system

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Boundary element and finite element coupling for aeroacoustics simulations, by Balin et al., JCP 2015.





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  - much faster, but not fast enough (many iterations, many RHS)



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#### **H-matrix solver**

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- Based on recursive bisection











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## H-Matrix performance

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# H-Matrix performance (2)

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10

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## **BEM limitations**

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- Propagation through jet flow (and interaction with the wing) must be taken into account
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• Volume mesh (tetrahedras) in the jet flow (cylinder, potential flow)



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- Volume mesh (tetrahedras) in the jet flow (cylinder, potential flow)
- Surface mesh = outer surface of the volume domain + the rest of the plane
- Same unknowns, same mesh: same linear system



# **FEM/BEM linear system**



## **FEM/BEM linear system**

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• Build D-C.A<sup>-1</sup>.B  $\rightarrow$  D' and b<sub>s</sub>-C.A<sup>-1</sup>.b<sub>v</sub>  $\rightarrow$  b'<sub>s</sub>



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#### **FEM/BEM Solver**

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  - Solve D'.xs=b's
- FMM-based approach: iterative solver using FMM and on-the-fly Schur complement
  - Solve iteratively (D<sub>FMM</sub>-C.A<sup>-1</sup>.B).x<sub>s</sub>=b'<sub>s</sub>







### H-matrix for FEM/BEM

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- Note: separates volume and surface parts



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## H-matrix for FEM/BEM: performances

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- Example 1: Surface=130kdof, volume=300kdof, RHS=122
   Direct Solver: 9h on 48 cores
   H-mat Solver: 20min on 24 cores
   Speedup: x55
- Example 2: Surface=600kdof, volume=4Mdof, RHS=2896, 4 problems FMM Solver: 65h on 192 cores (≈90 iterations) H-mat Solver: 12h on 24 cores (mat sizes after facto: 16+18+66 Gb) Speedup: x41



• Use ideas from the "sparse solver" community

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  - Nested dissection (minimize fill-in)
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- Use ideas from the "sparse solver" community
  - Nested dissection (minimize fill-in)
  - Symbolic factorization (optimize storage)







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• Mixed Ternary/Binary cluster tree

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- As expected, reduces time and memory consumption
- WIP, to get closer to native sparse solvers on pure FEM problems
- For FEM-BEM, this is the current best approach



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 Mumps-Hmat coupling: multifacto based on H-mat clustering

# **Other Possible Approaches**

- Mumps-Hmat coupling: multifacto based on H-mat clustering
- Unique Hmat for surface AND volume









(b) BEM/FEM subsystem.



#### **FEM/BEM Results**

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- FEM-BEM modelisation is a good compromise between cost and accuracy

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- FEM-BEM modelisation is a good compromise between cost and accuracy
- H-matrix solver shows excellent performance both on BEM only or coupled FEM-BEM systems



## Perspectives

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- PhD position in Inria Bordeaux for HPC / algorithm