H-Matrix Solver Applied to Coupled FEM-BEM Aeroacoustics Simulations

Guillaume Sylvand

H-MATRIX SOLVER APPLIED TO COUPLED FEM-BEM AEROACOUSTICS SIMULATIONS

AIRBUS

Guillaume SYLVAND - Airbus Central R&T

JOSO - March 12th, 2019.

Context

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  - Departement "Virtual Product Engineering"

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    - Aurélien Falco (WIP) H-matrix for FEM/BEM system

Aeroacoustics

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- Sound propagation in a air flow

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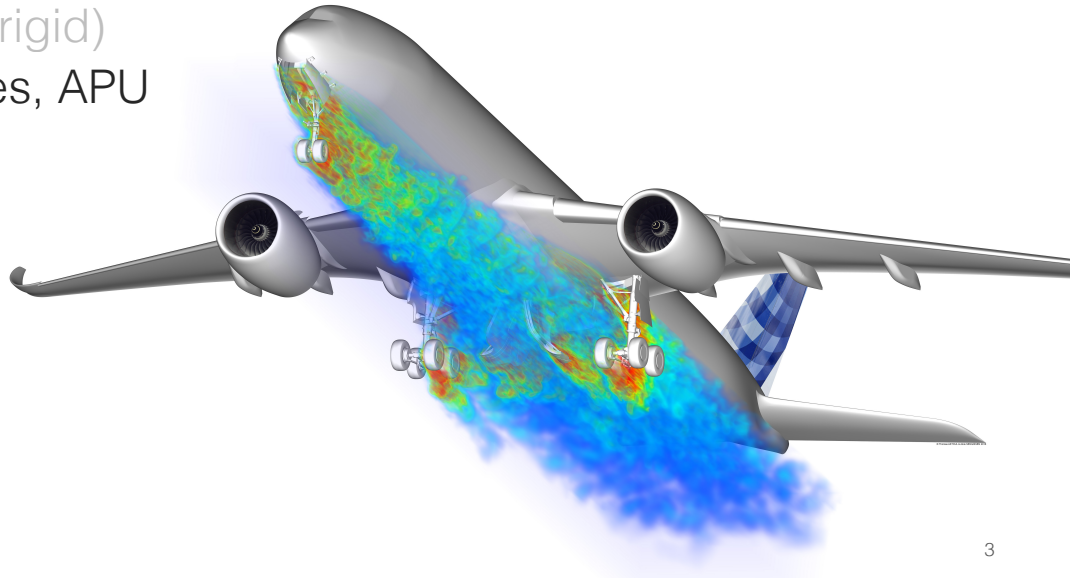

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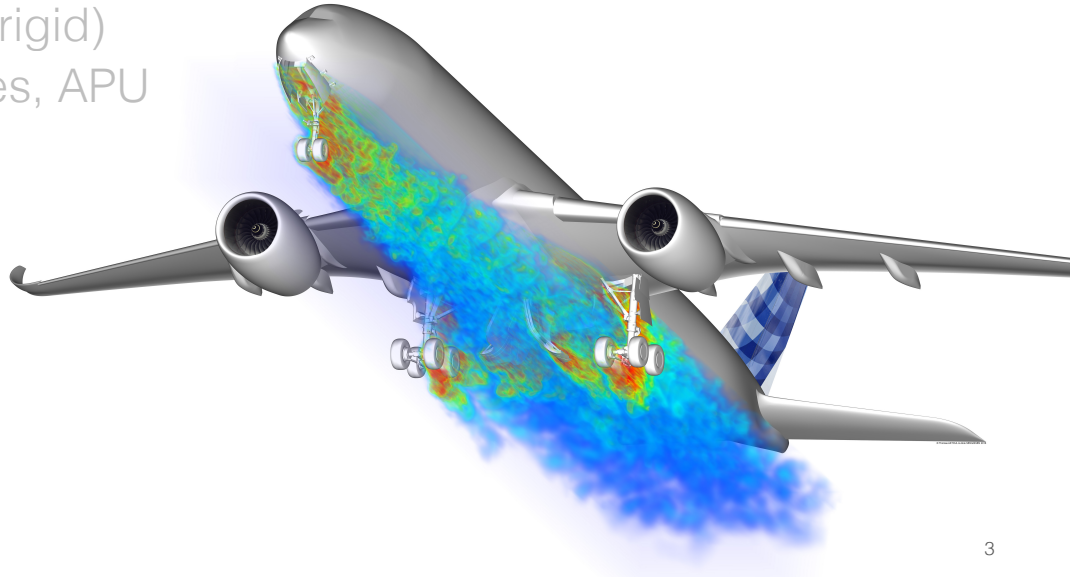

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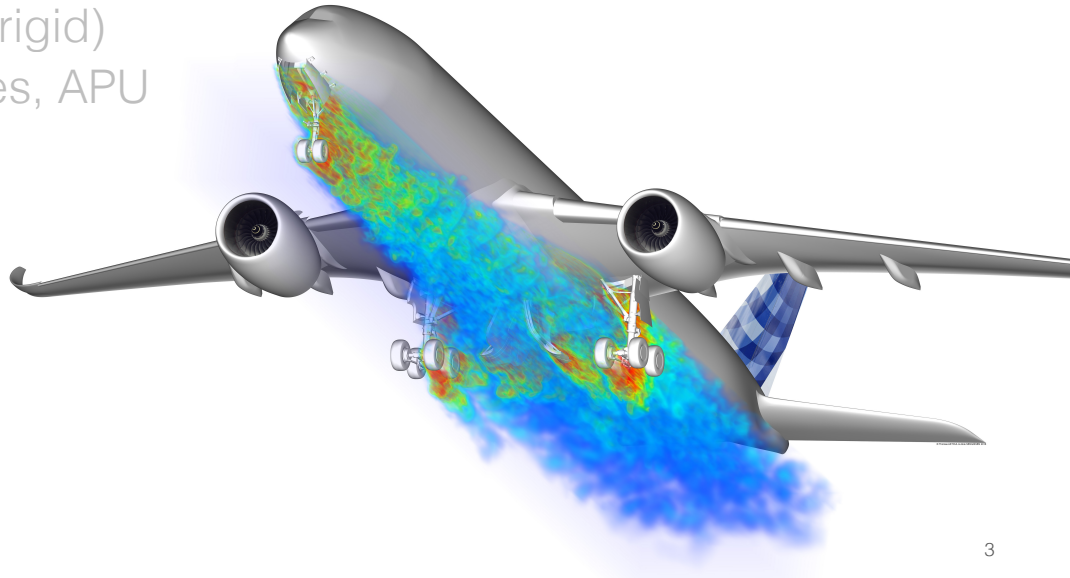

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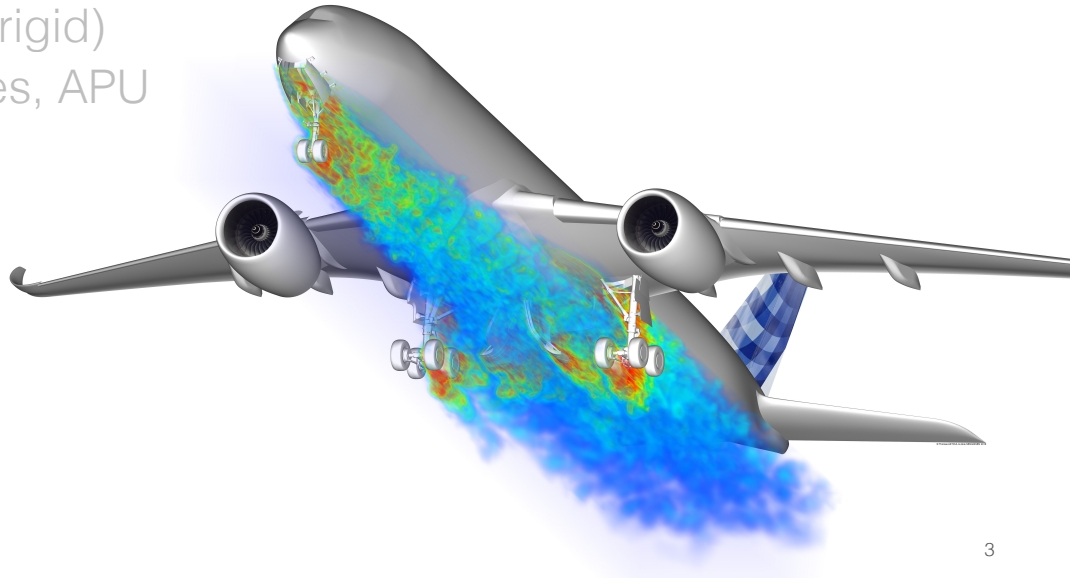

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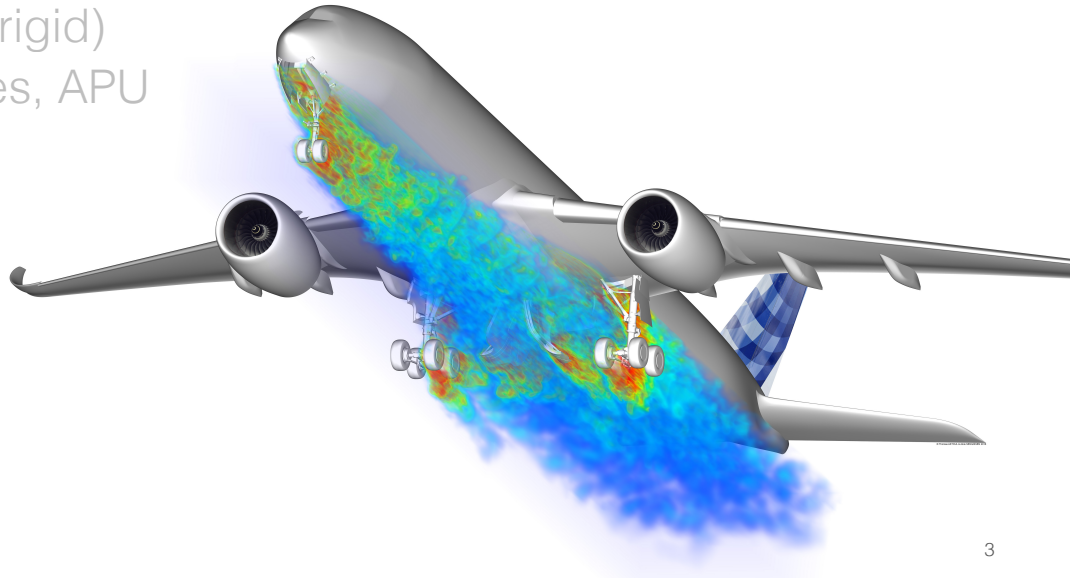

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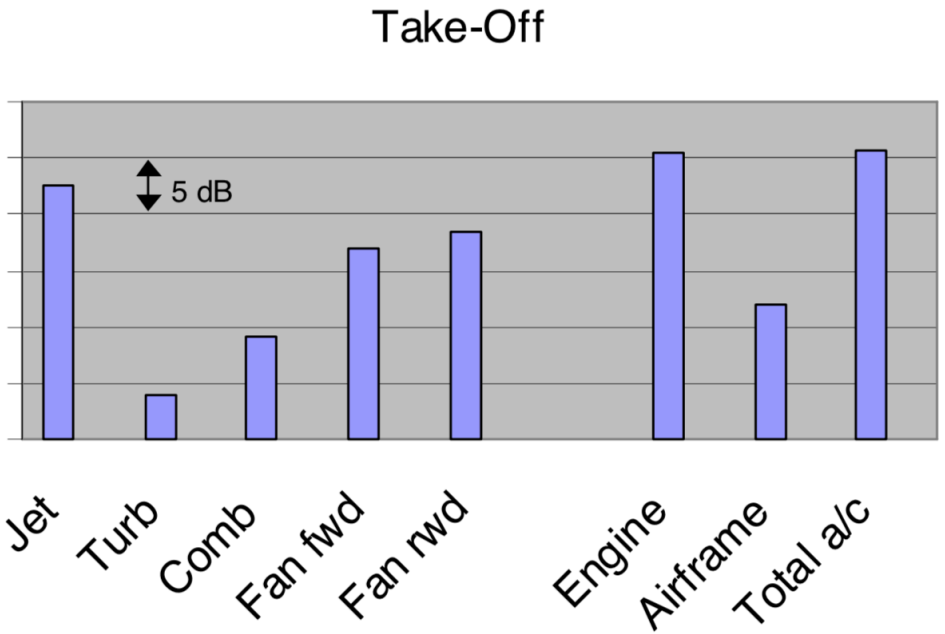

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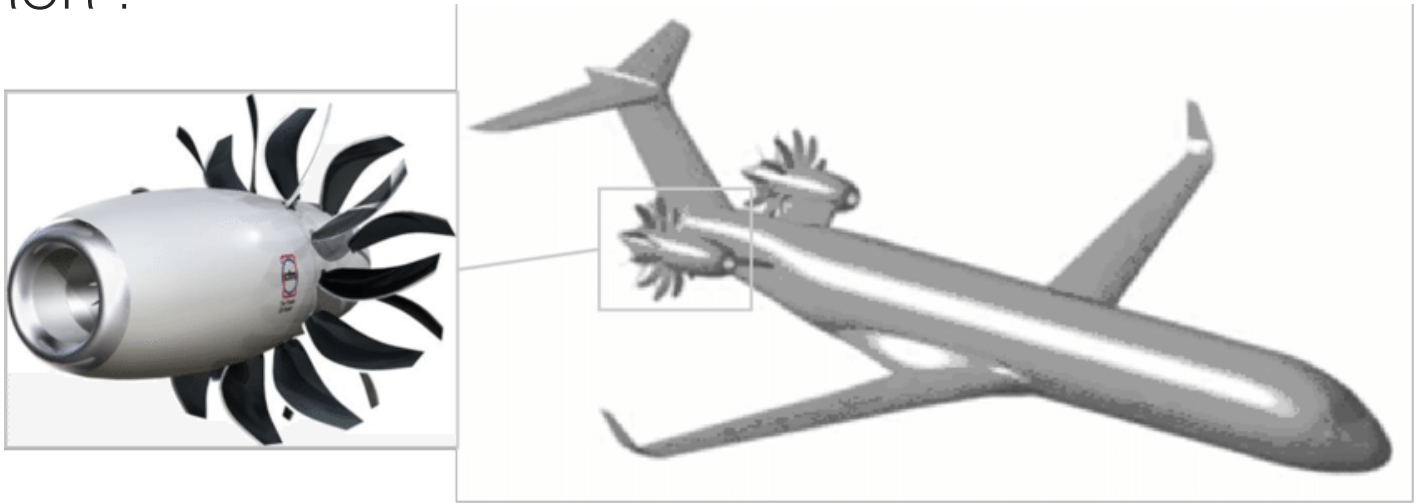

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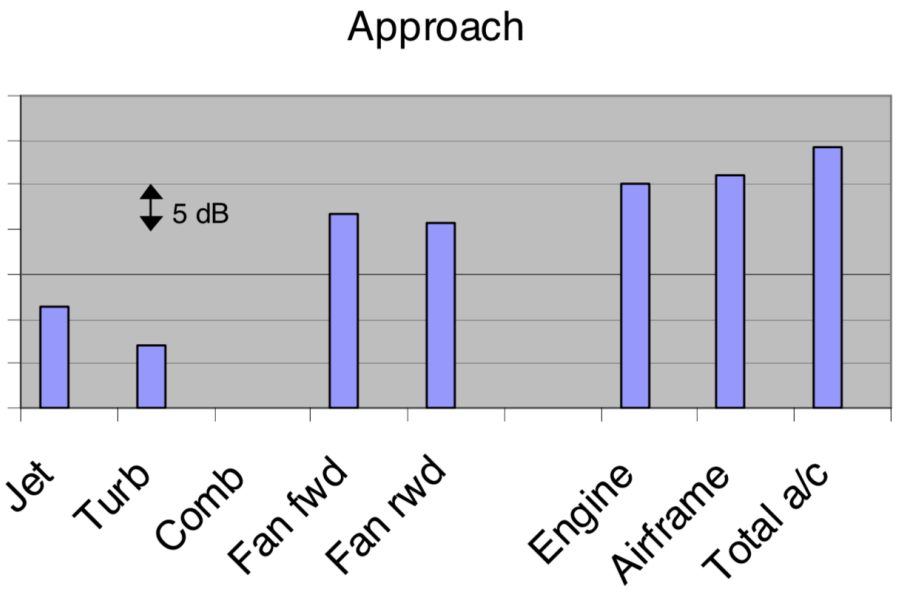

BEM Formulation

AIRBUS

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- Frequency domain, 3D to solve Helmholtz equation

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- Interest:

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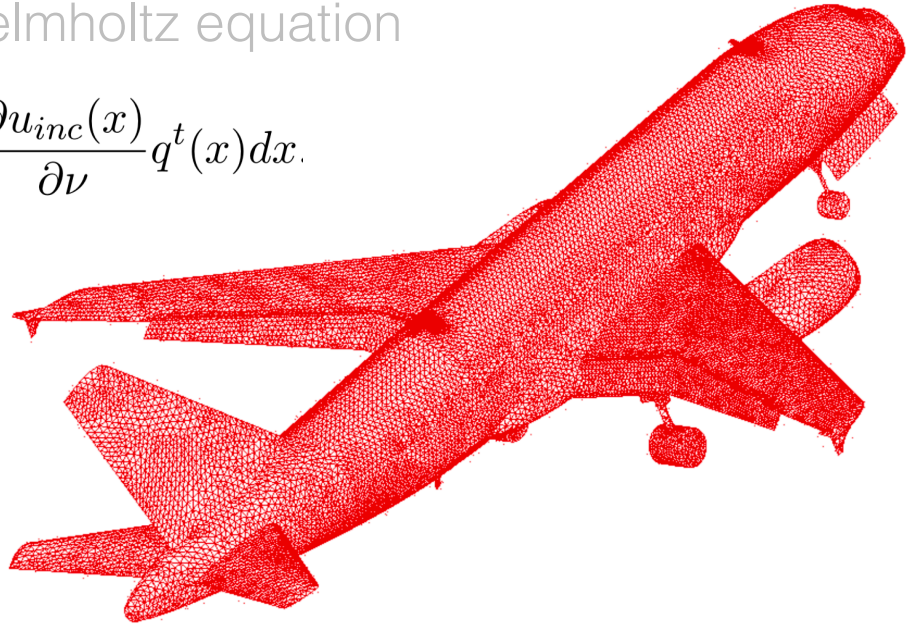

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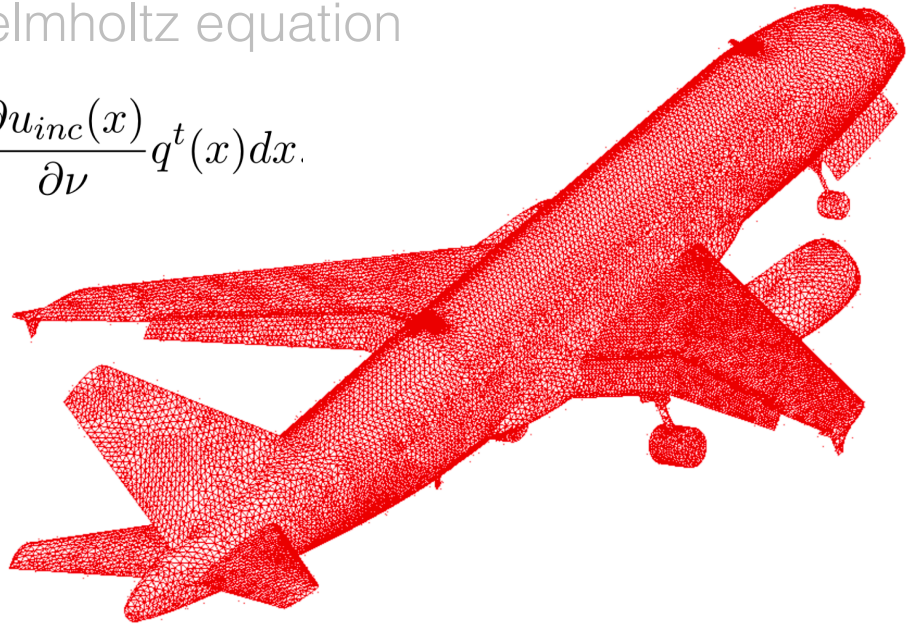

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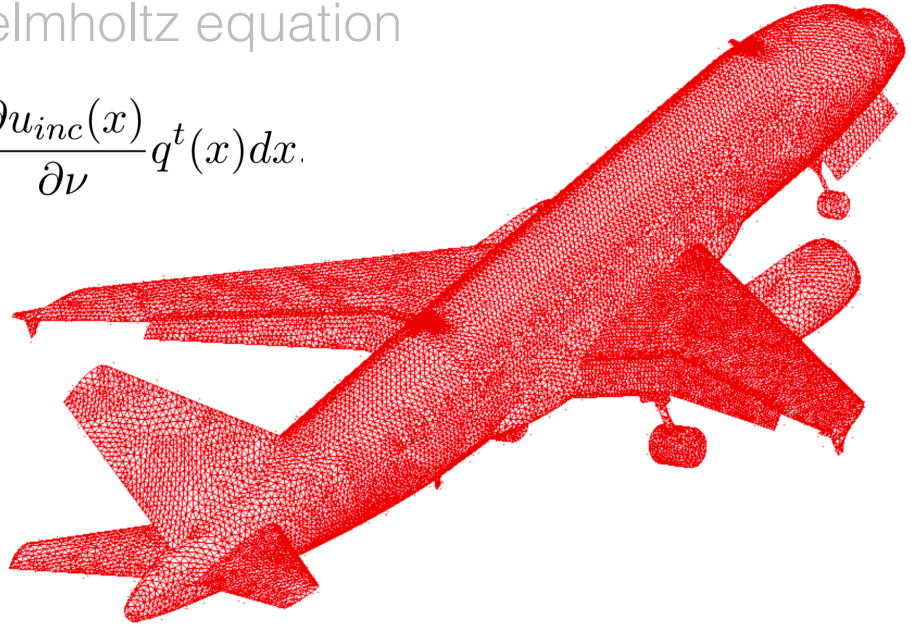

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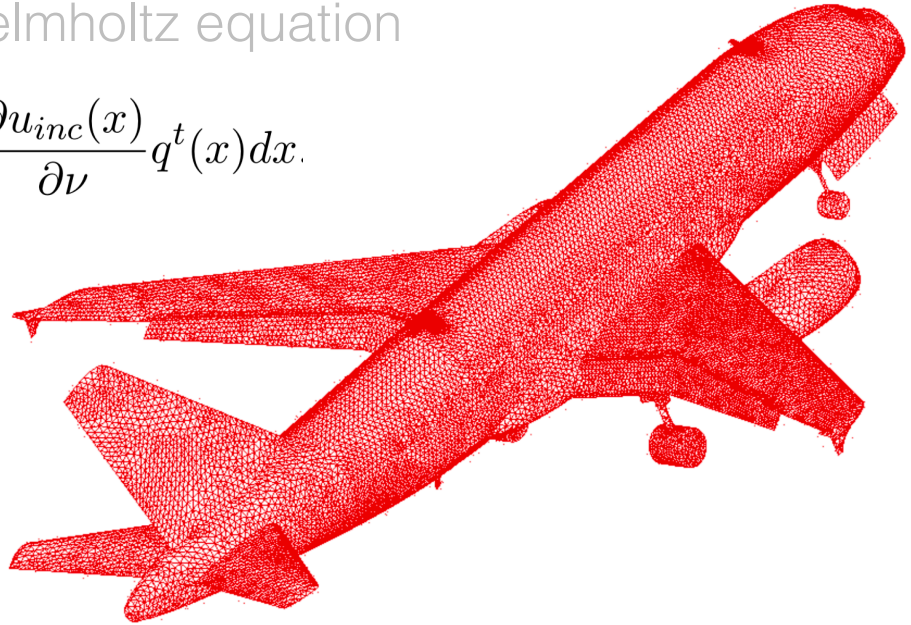

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- Drawback:
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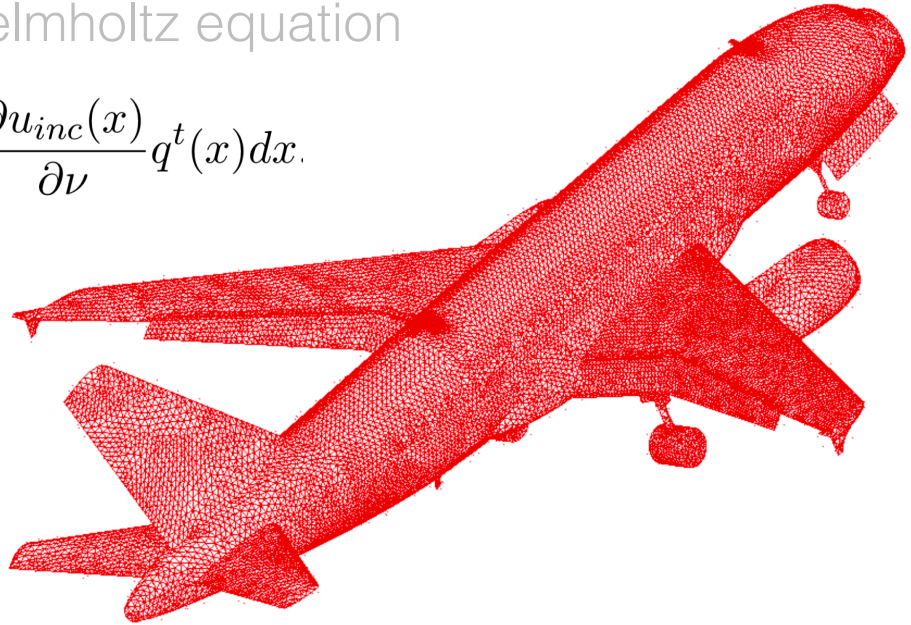
  - Dense linear system



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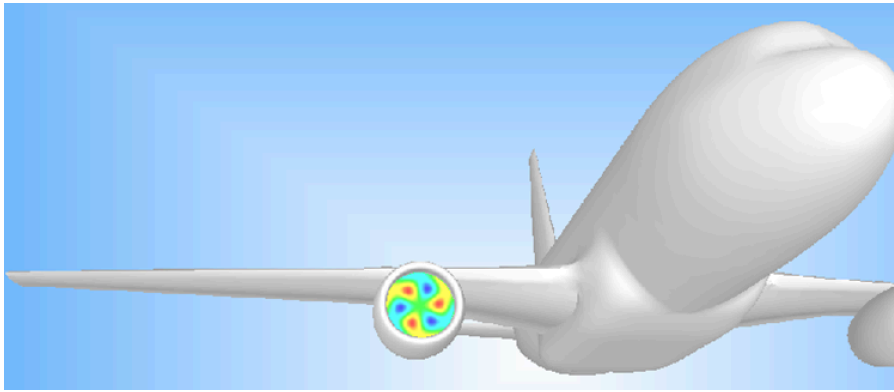

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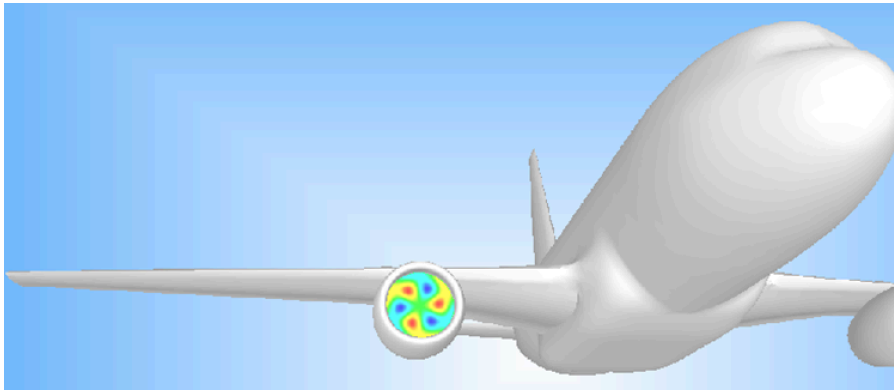

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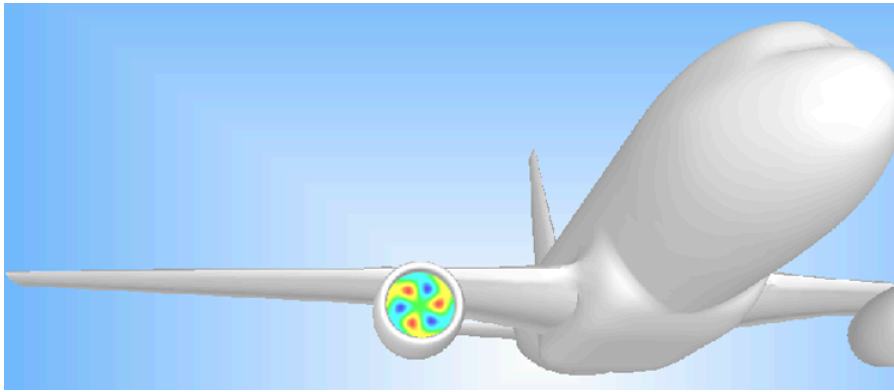

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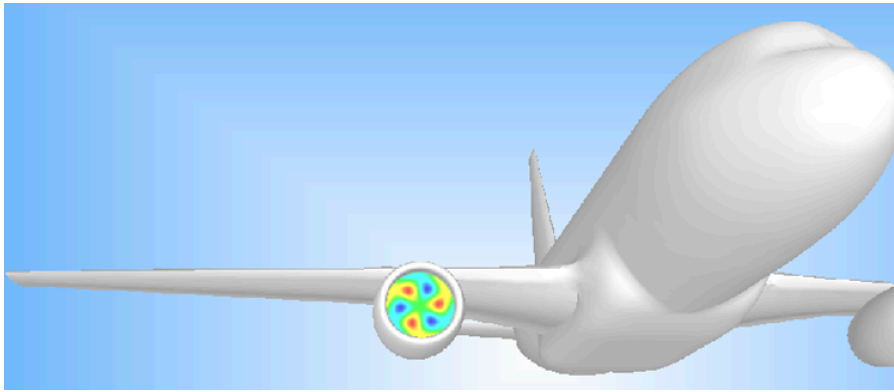

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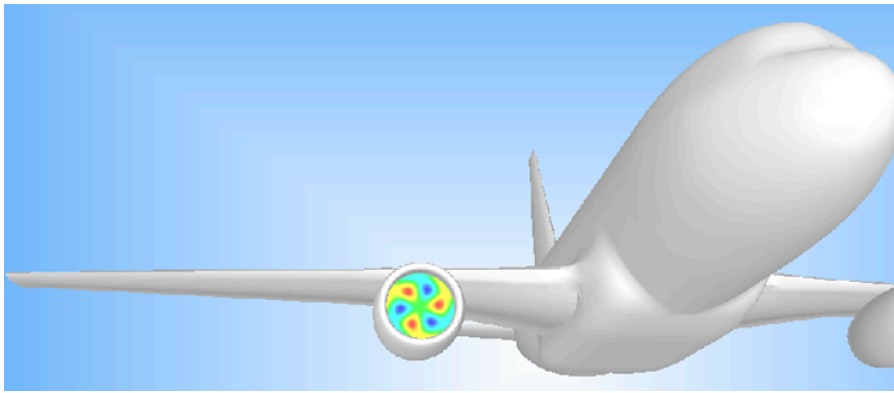
Boundary element and finite element coupling for aeroacoustics simulations, by Balin et al., JCP 2015.



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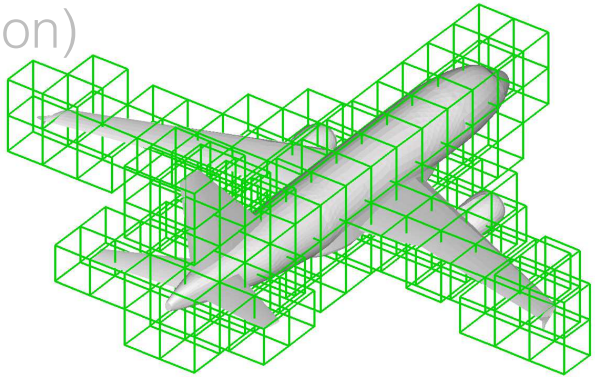

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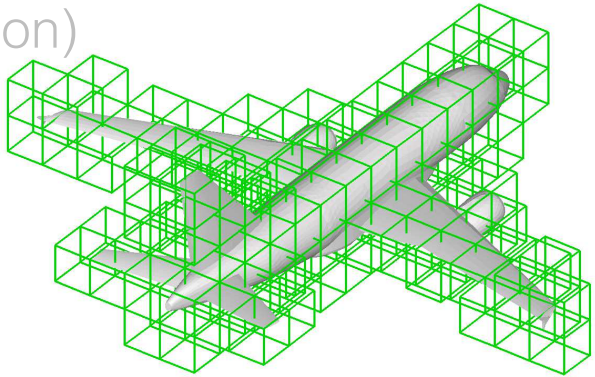

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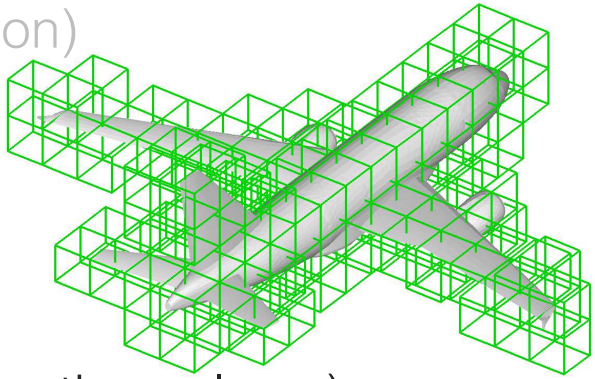

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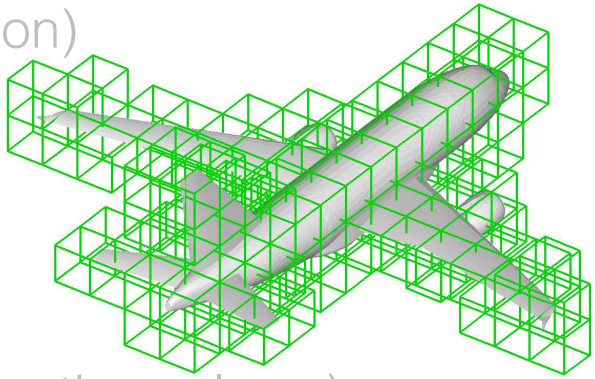

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H-matrix solver

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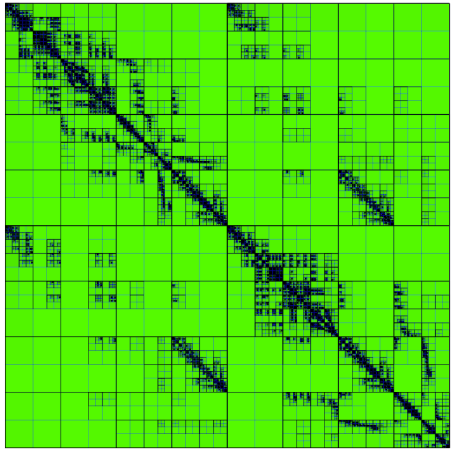

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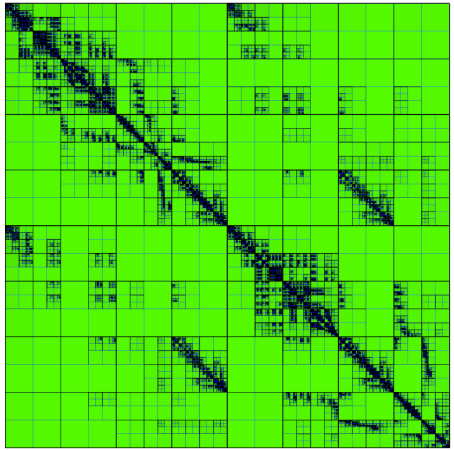

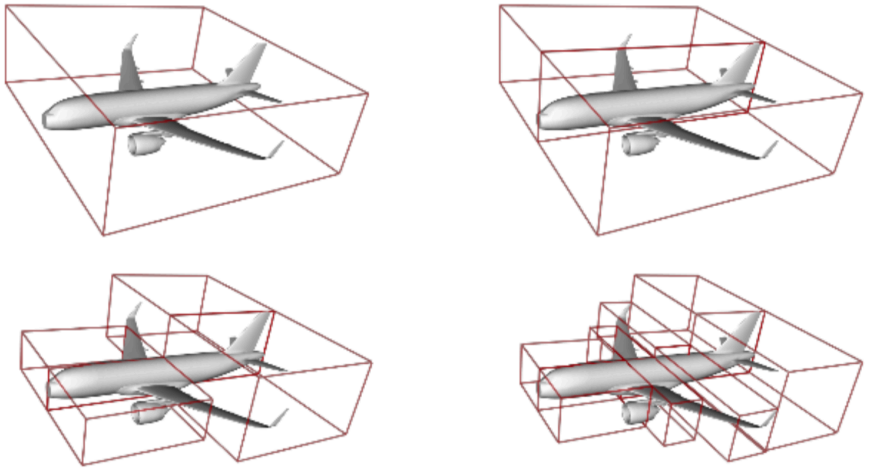

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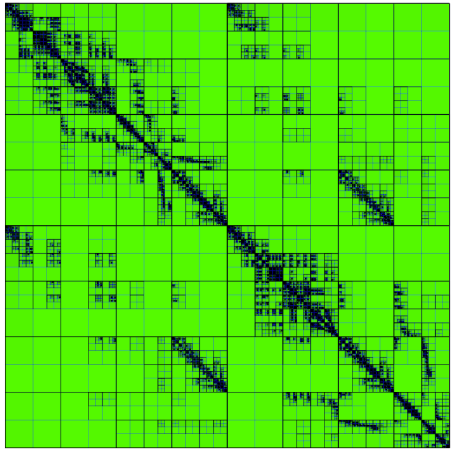

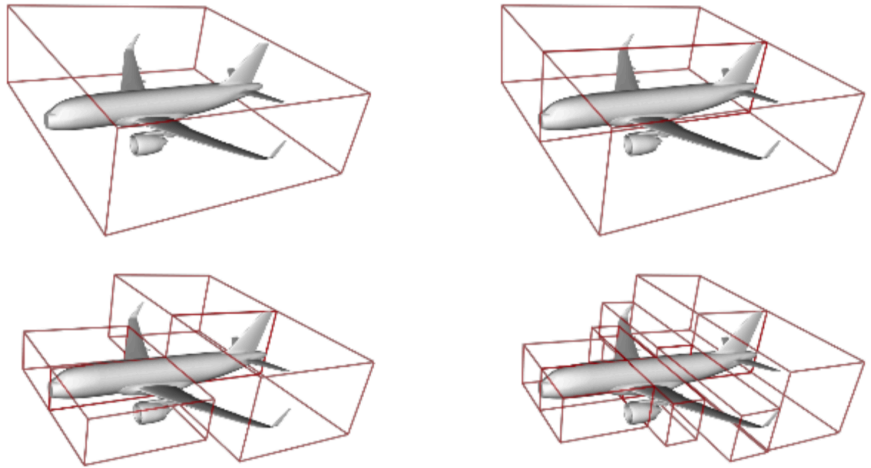

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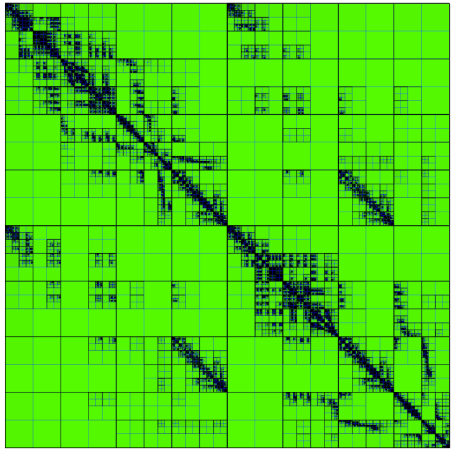

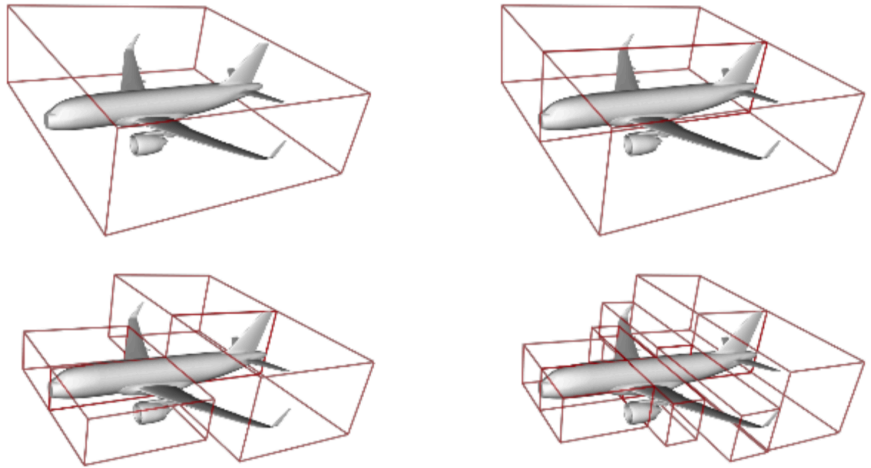

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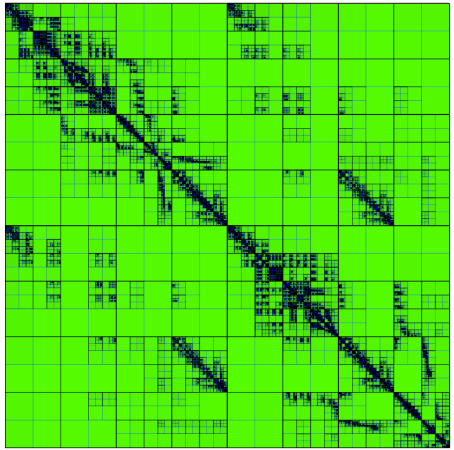

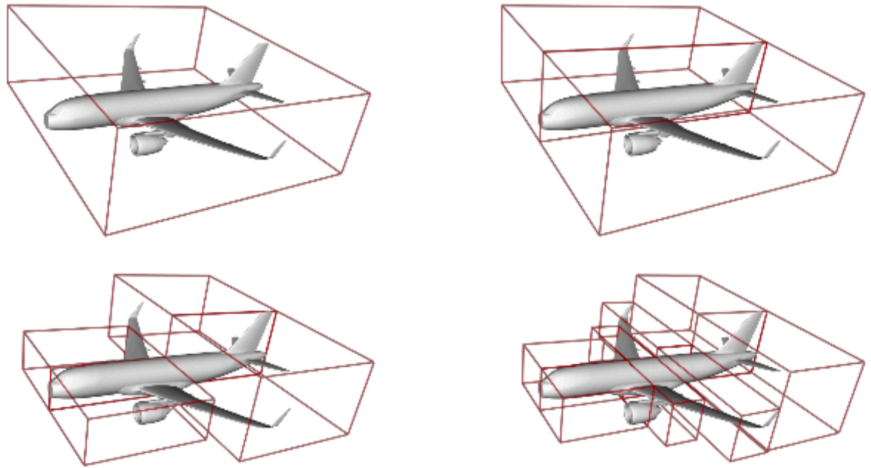

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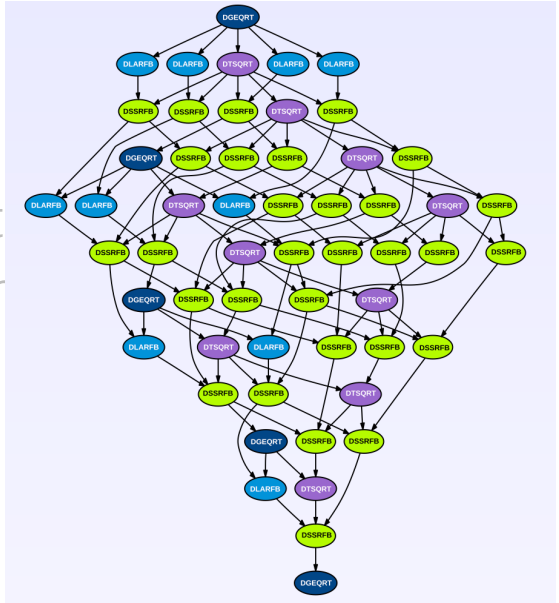
  - Hmat (airbus) : faster, parallel, not free and not free. tasks based.



H-Matrix performance

AIRBUS

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AIRBUS

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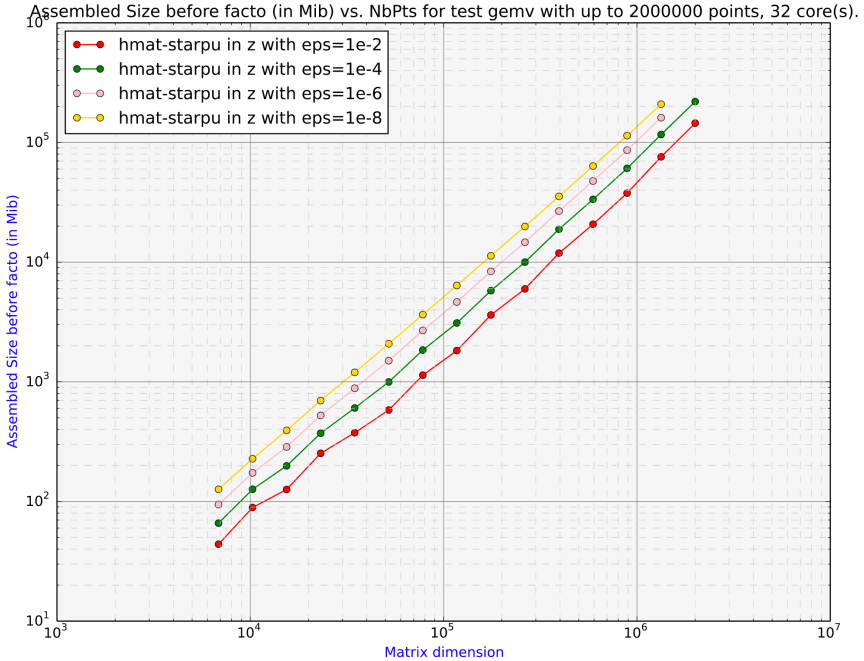

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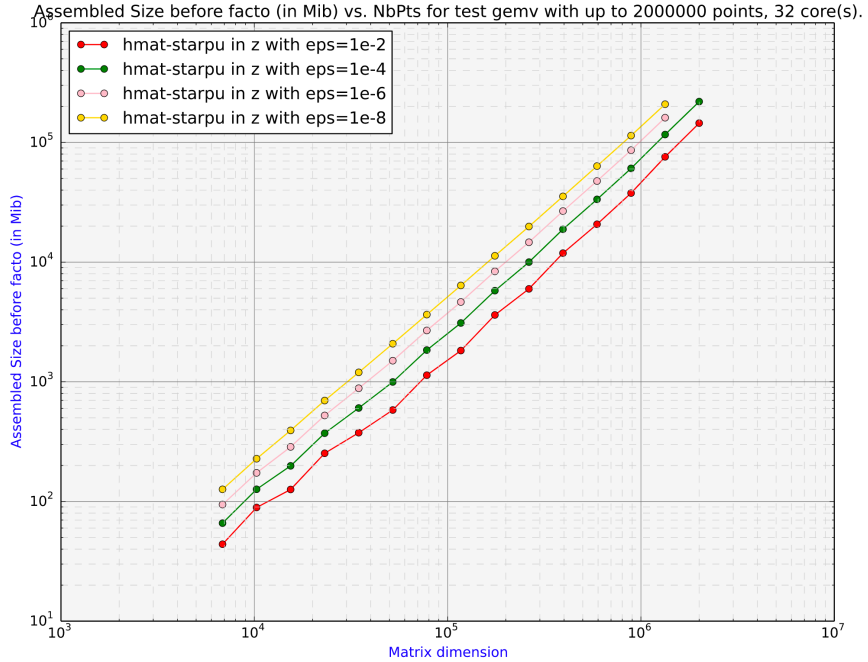

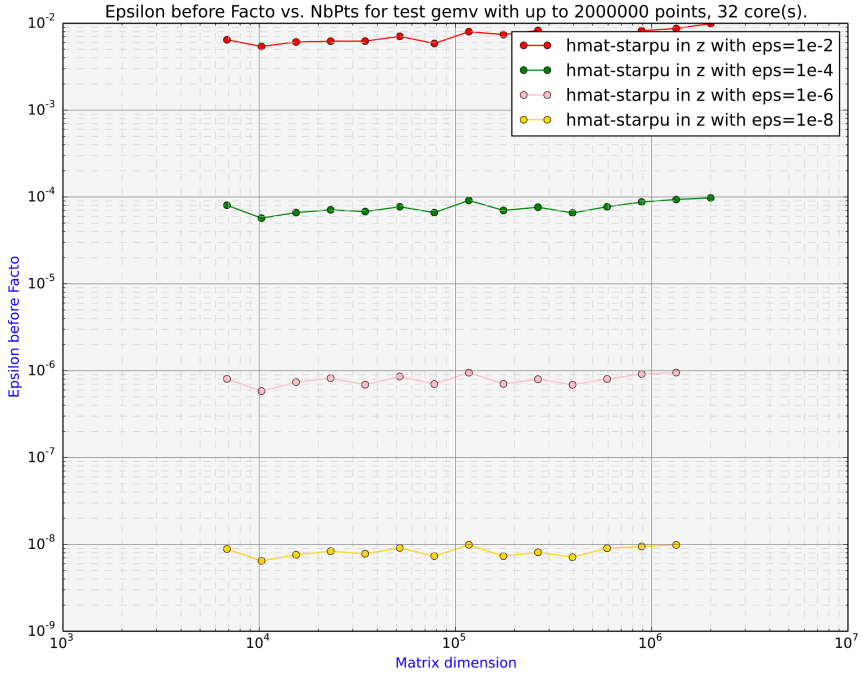

H-Matrix performance (2)

AIRBUS

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AIRBUS

- Test on Factorization

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AIRBUS

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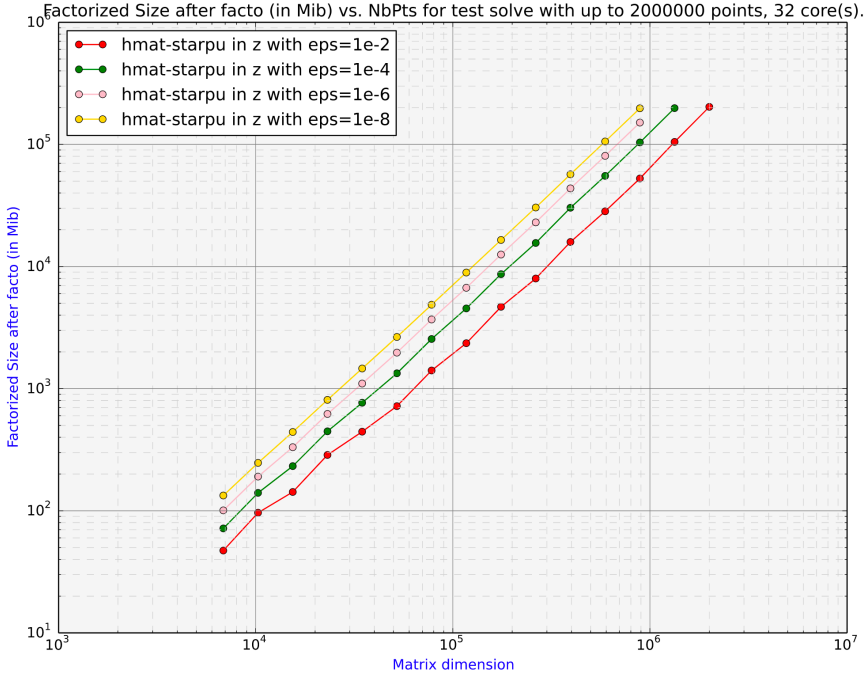
- size = $n^{1,50}$



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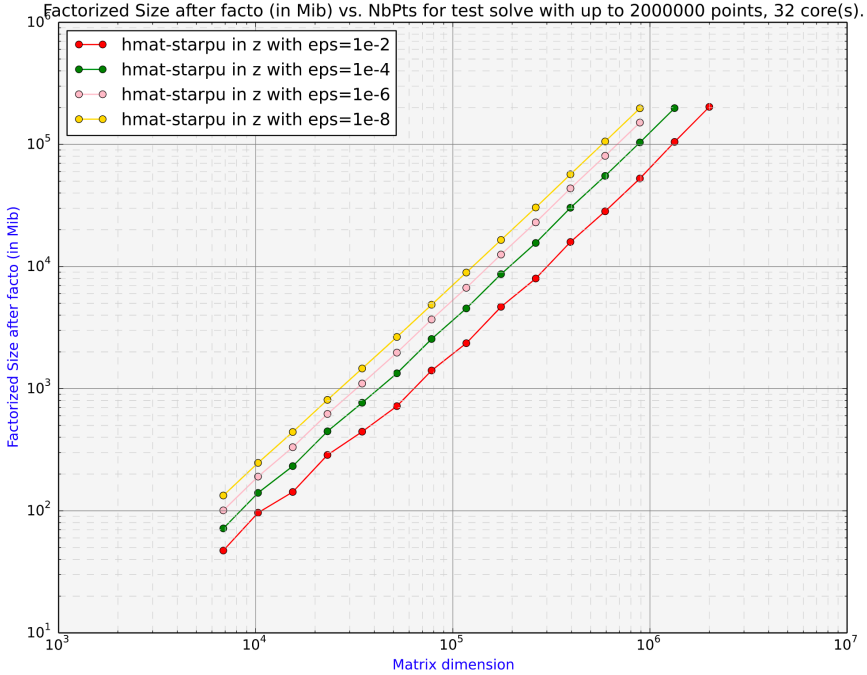

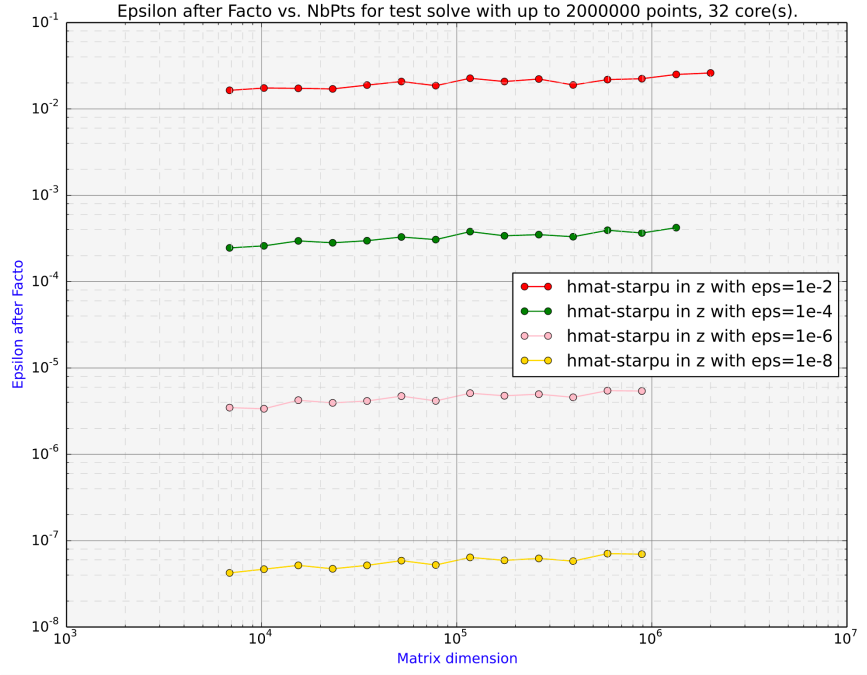

H-Matrix performance (3)

AIRBUS

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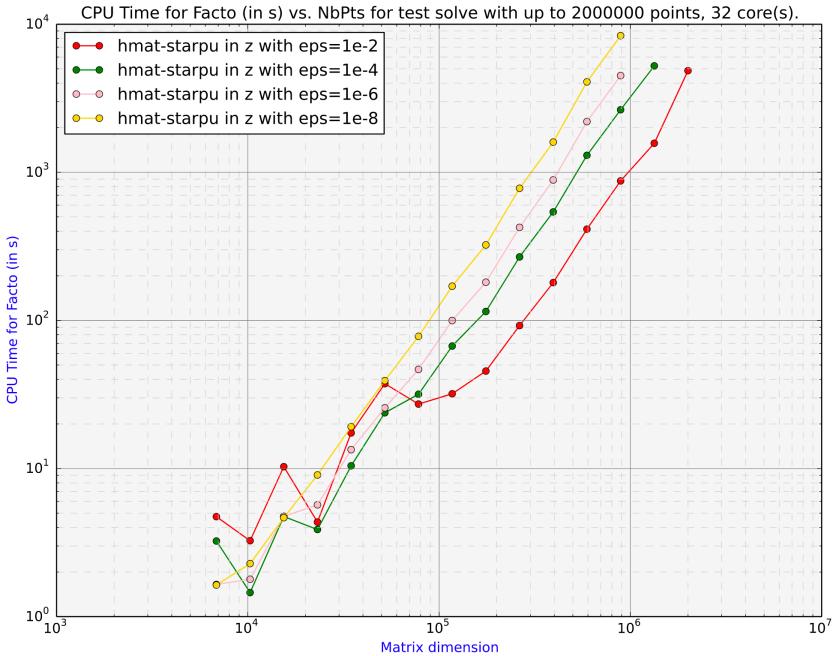
AIRBUS

- Factorization: CPU time = $n^{1,80}$



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AIRBUS

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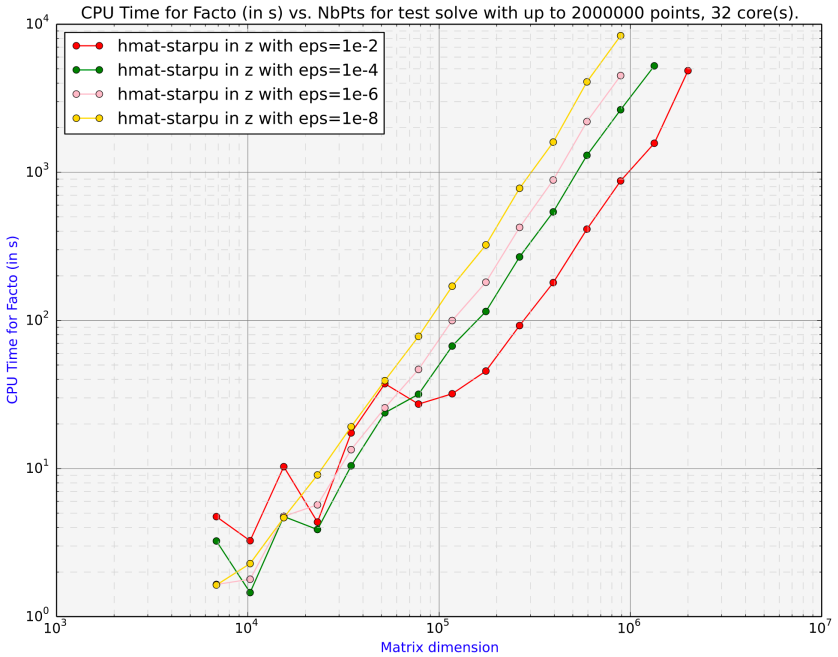

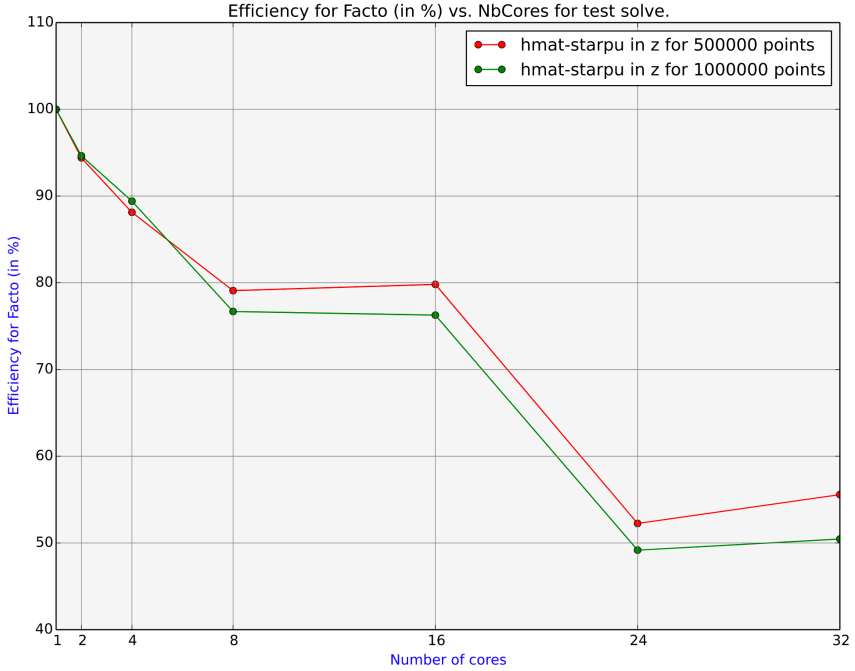

BEM limitations

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AIRBUS

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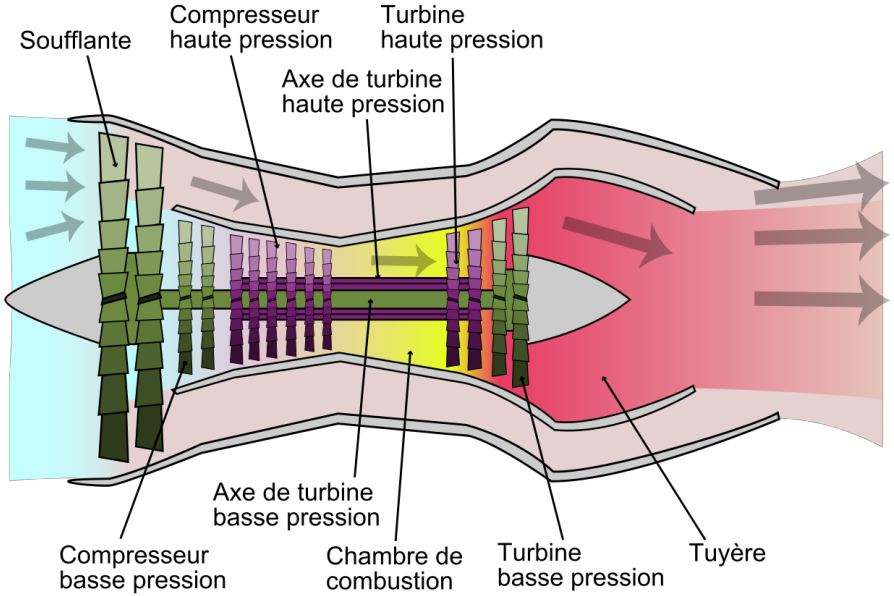

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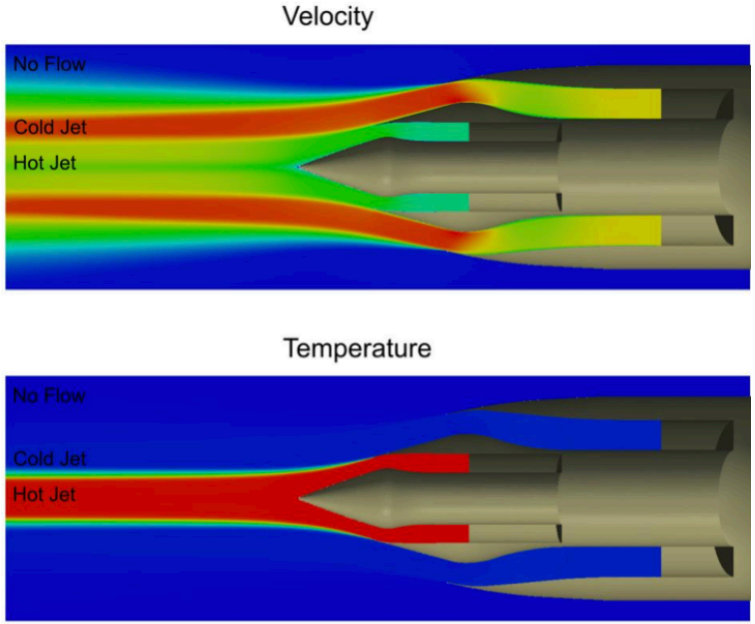

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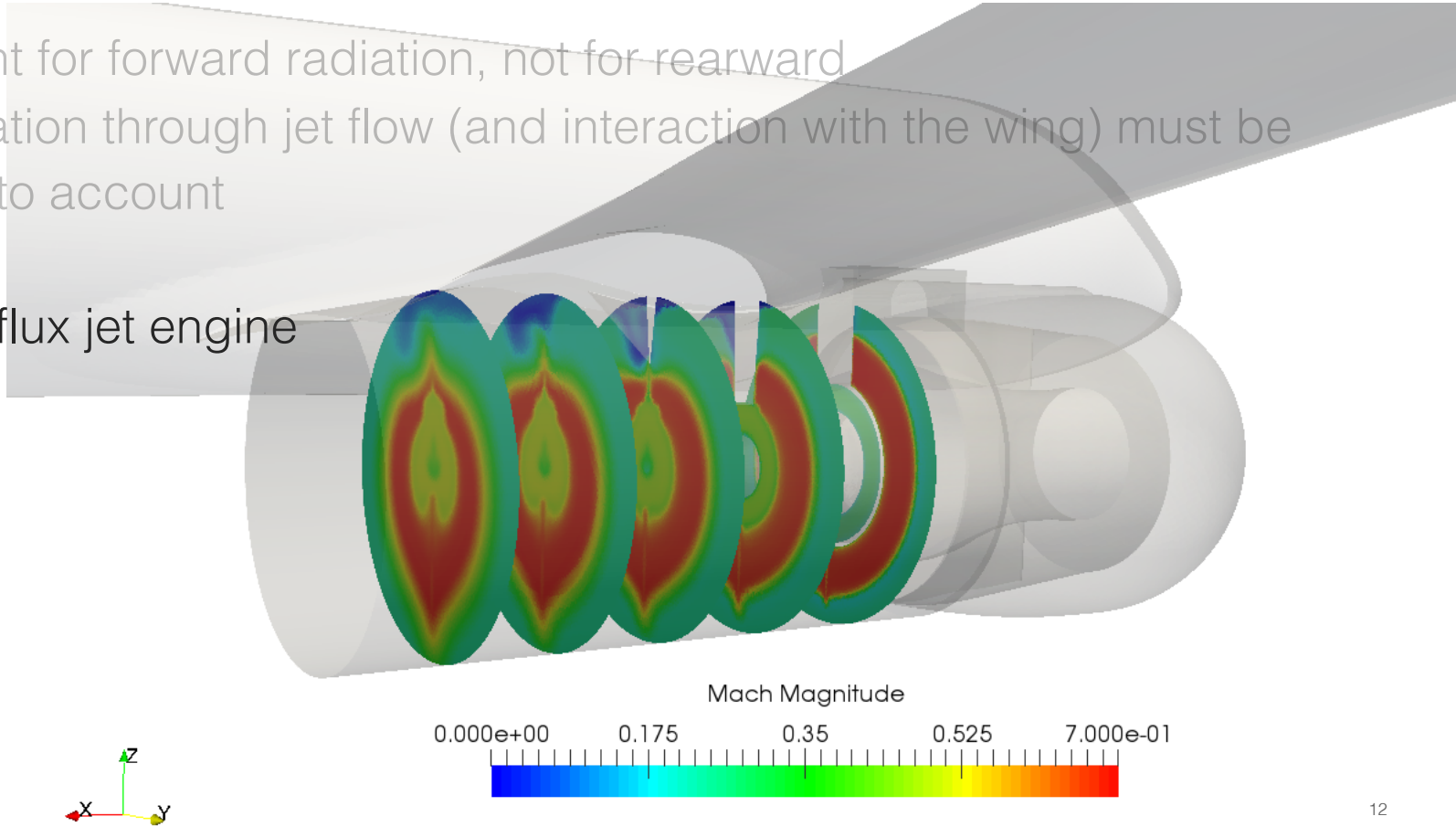

FEM/BEM coupling

AIRBUS

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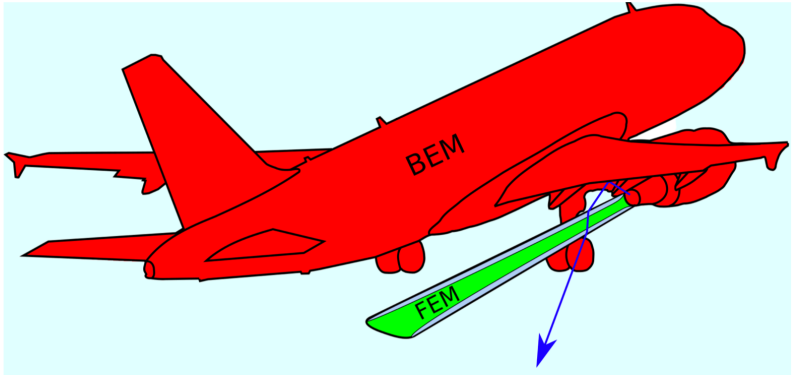
- Volume mesh (tetrahedras) in the jet flow (cylinder, potential flow)



FEM/BEM coupling

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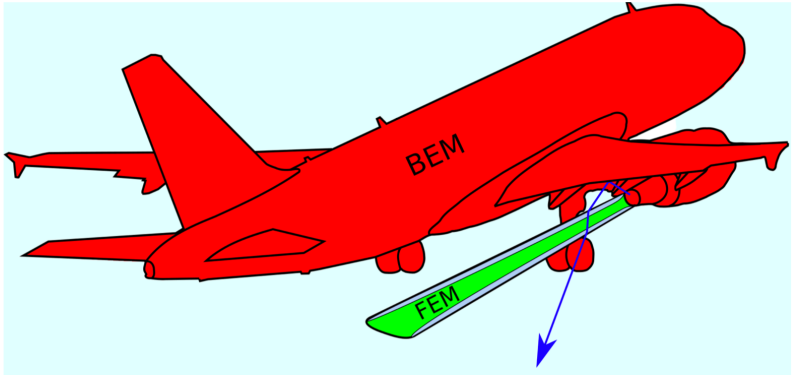

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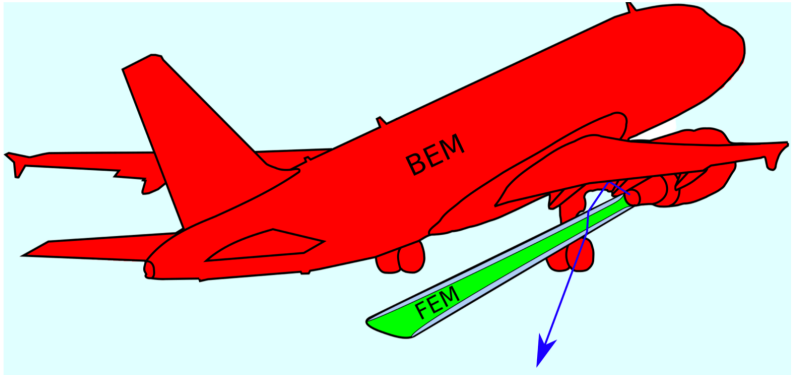
- Same unknowns, same mesh: same linear system



FEM/BEM linear system

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- Surface mesh: from 10k to 1M dofs

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FEM/BEM linear system

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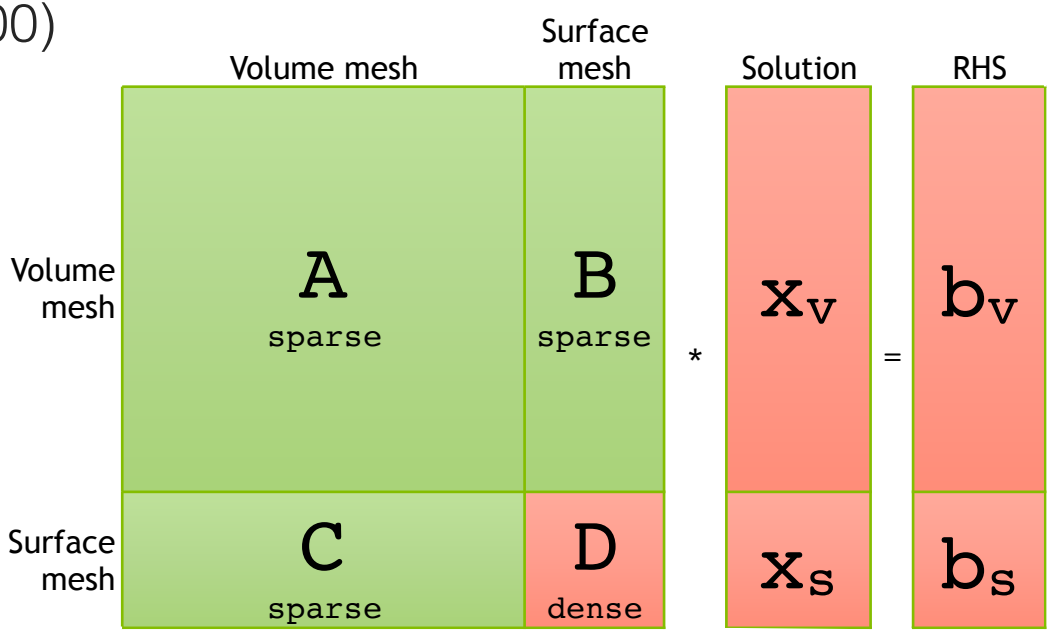

FEM/BEM Solver

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FEM/BEM Solver

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FEM/BEM Solver

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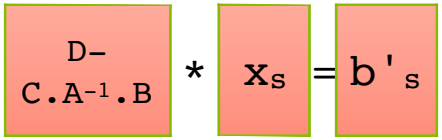
  - Build $D-C.A^{-1}.B \rightarrow D'$ and $b_s-C.A^{-1}.b_v \rightarrow b'_s$



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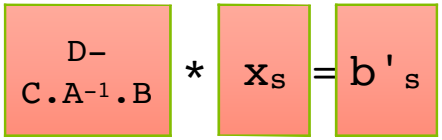

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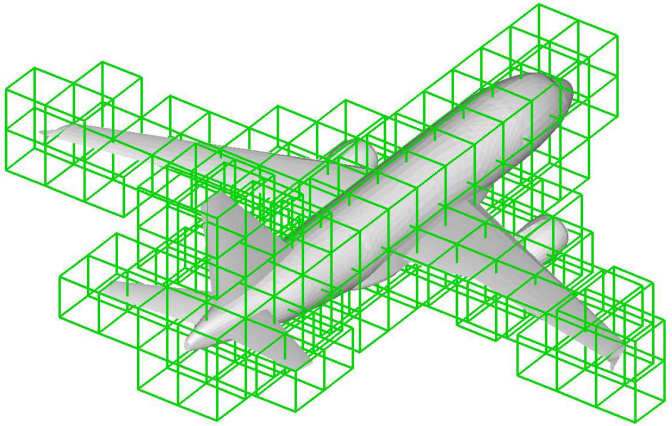

FEM/BEM Solver

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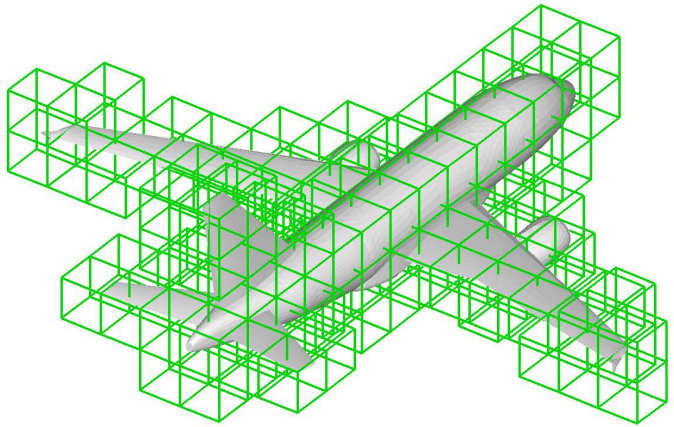

H-matrix for FEM/BEM

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- We can use H-matrix on volume mesh

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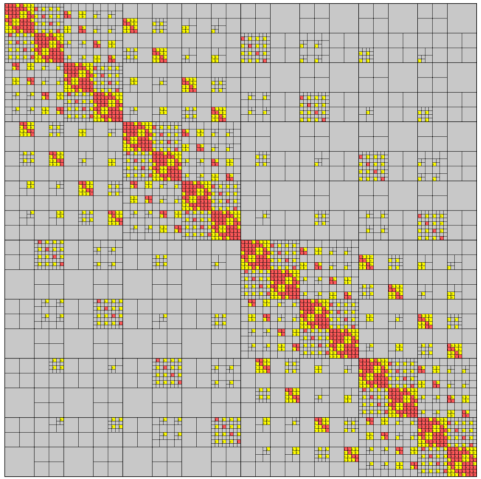
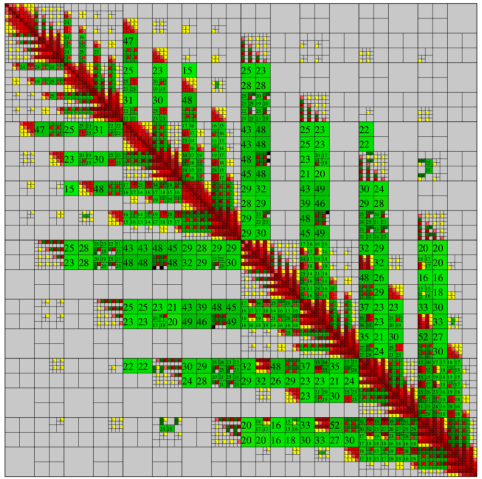


(a) Sparse matrix before factorization, with a bisection clustering.



(b) Matrix after factorization, with a bisection clustering.

H-matrix for FEM/BEM

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- We can use H-matrix on volume mesh
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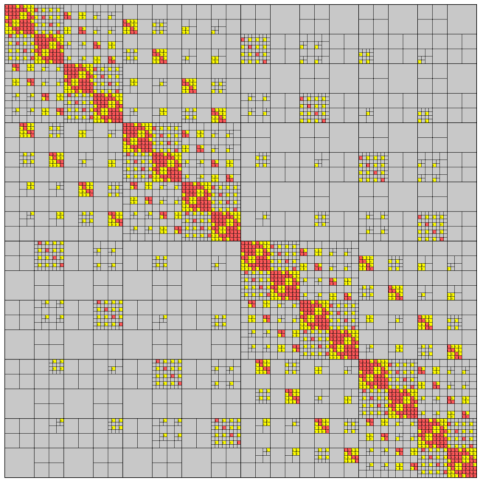
- Note: separates volume and surface parts



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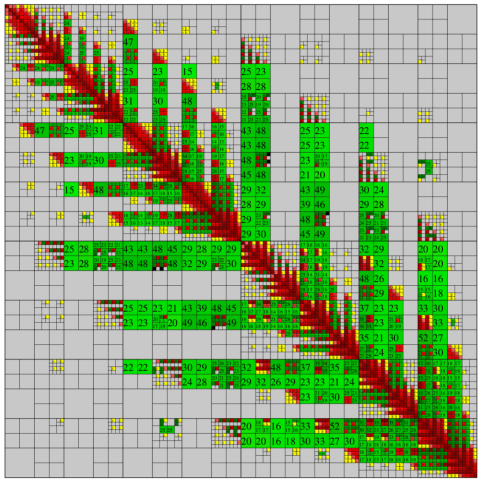

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H-matrix for FEM/BEM: performances

AIRBUS

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- Example 1: Surface=130kdof, volume=300kdof, RHS=122
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  - H-mat Solver: 20min on 24 cores
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- Example 2: Surface=600kdof, volume=4Mdof, RHS=2896, 4 problems
  - FMM Solver: 65h on 192 cores (≈90 iterations)
  - H-mat Solver: 12h on 24 cores (mat sizes after facto: 16+18+66 Gb)
    - Speedup: x41

Improve sparse H-matrix

AIRBUS

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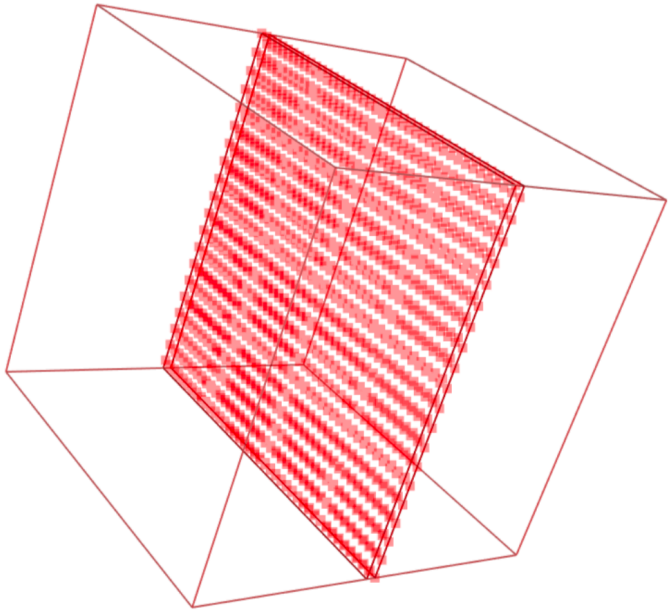

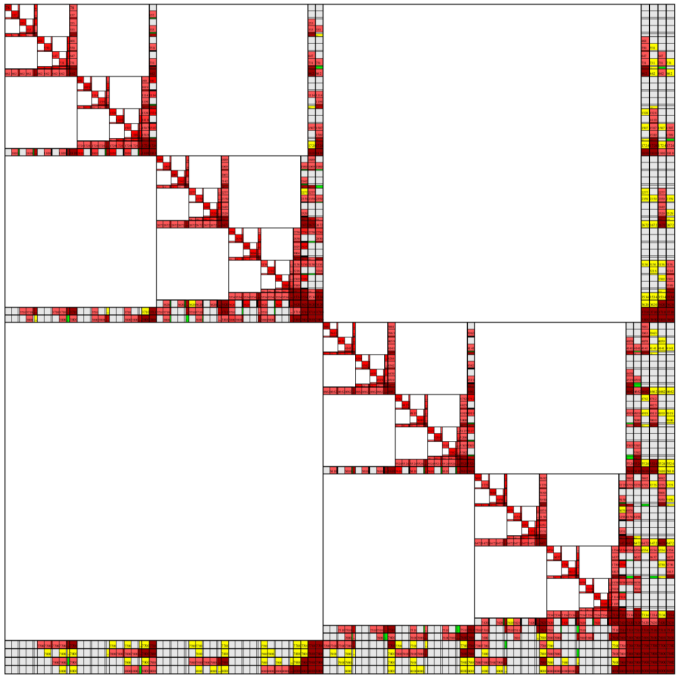

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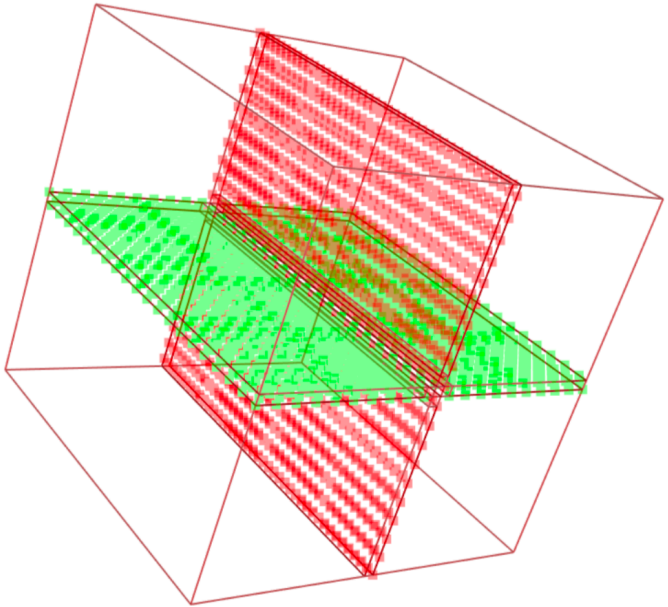

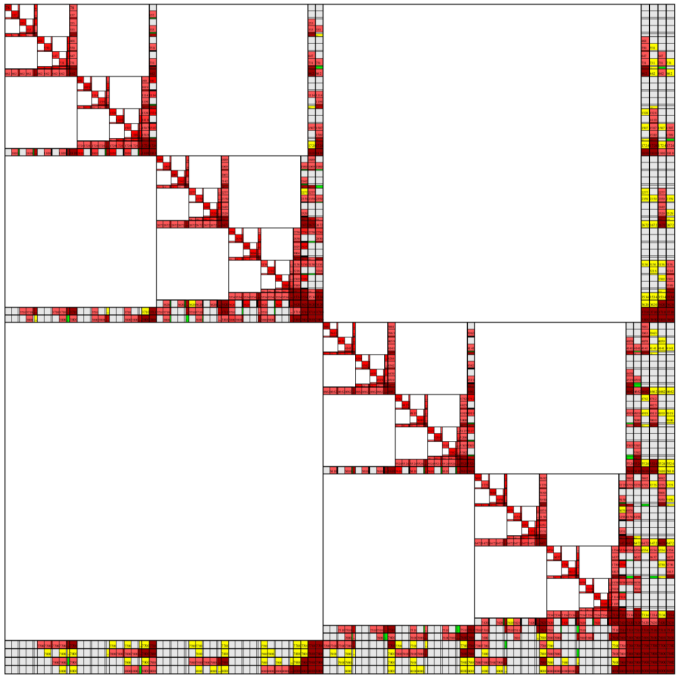

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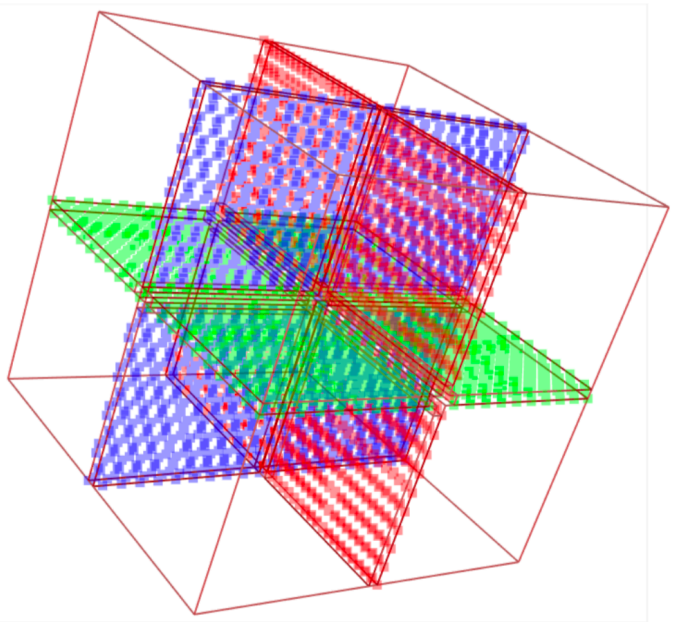

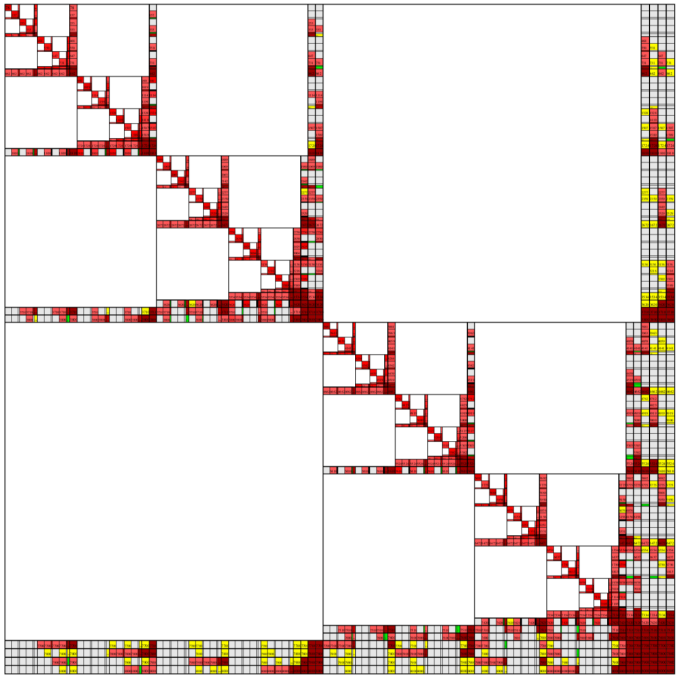

Improve sparse H-matrix

AIRBUS

- Use ideas from the "sparse solver" community
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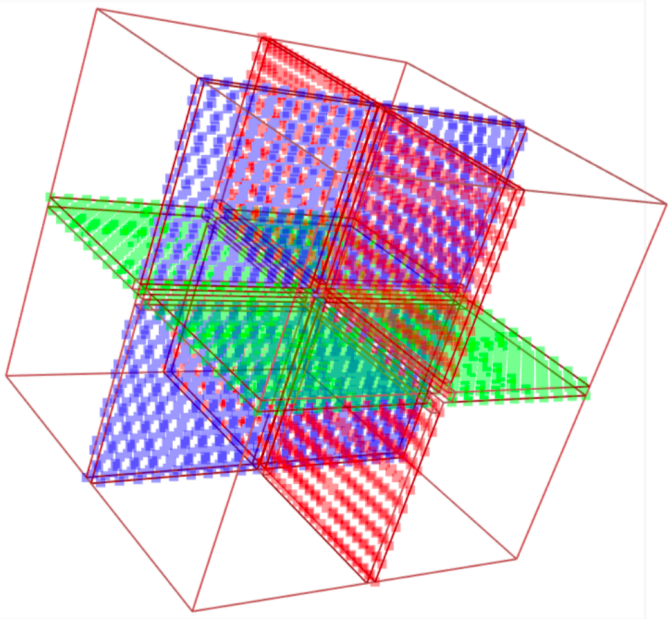
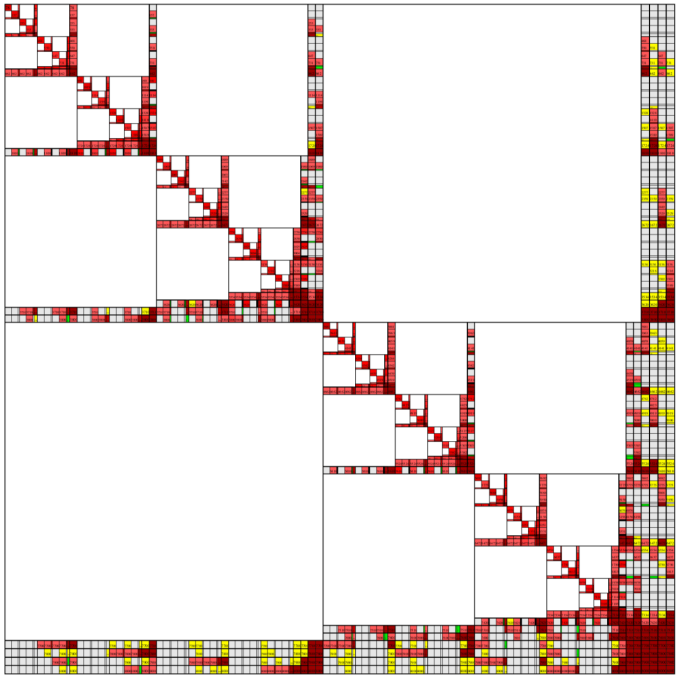
  - Symbolic factorization (optimize storage)





Impact on H-matrix

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Impact on H-matrix

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Performance of improved H-matrix

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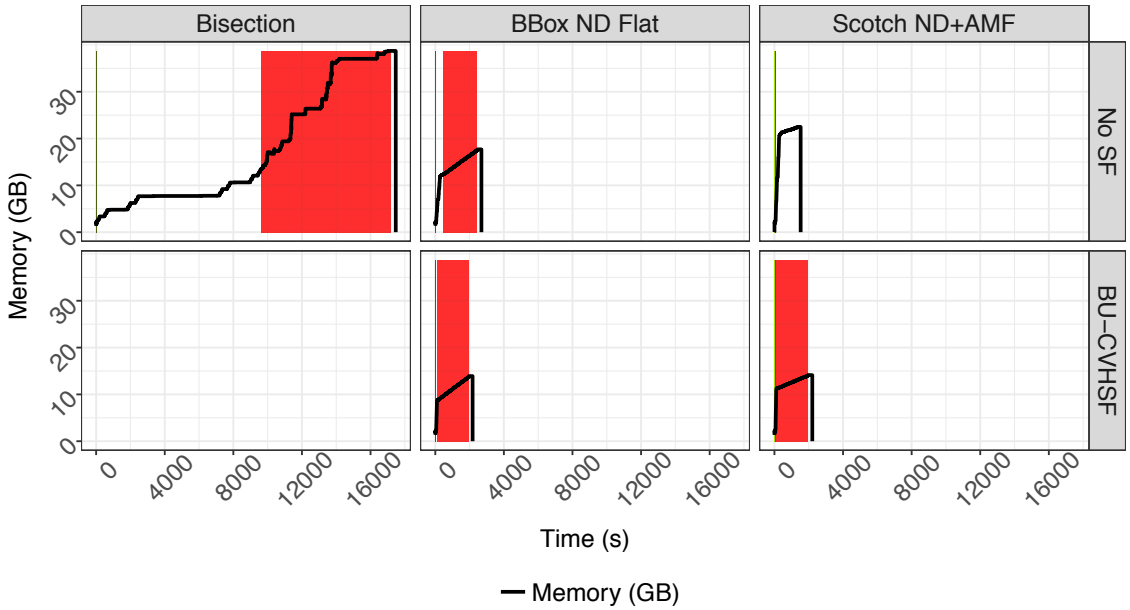
AIRBUS

- As expected, reduces time and memory consumption



Performance of improved H-matrix

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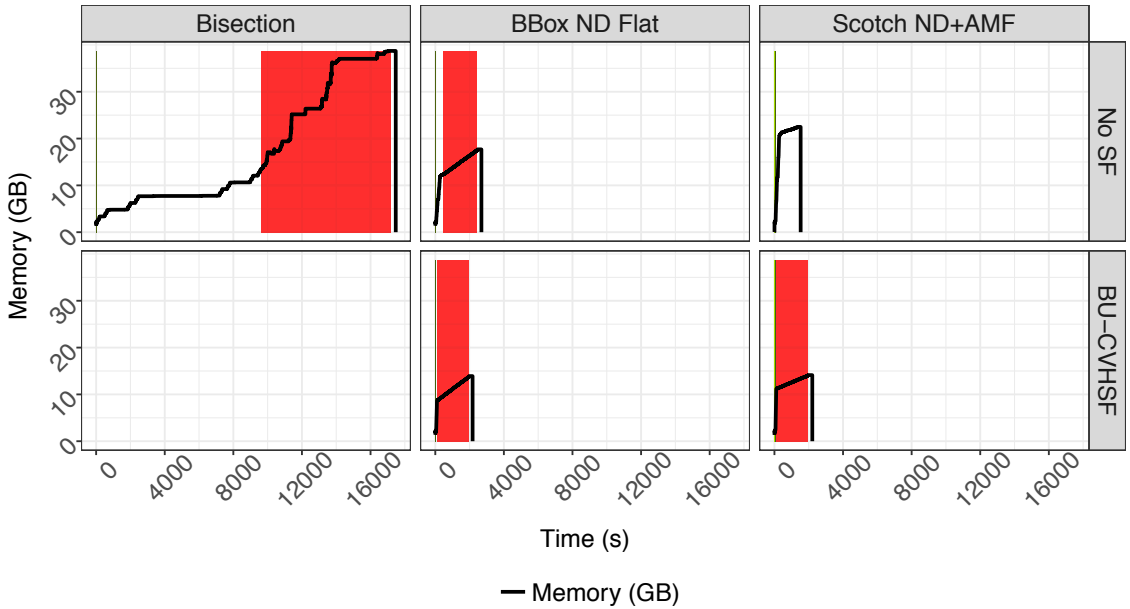

Performance of improved H-matrix

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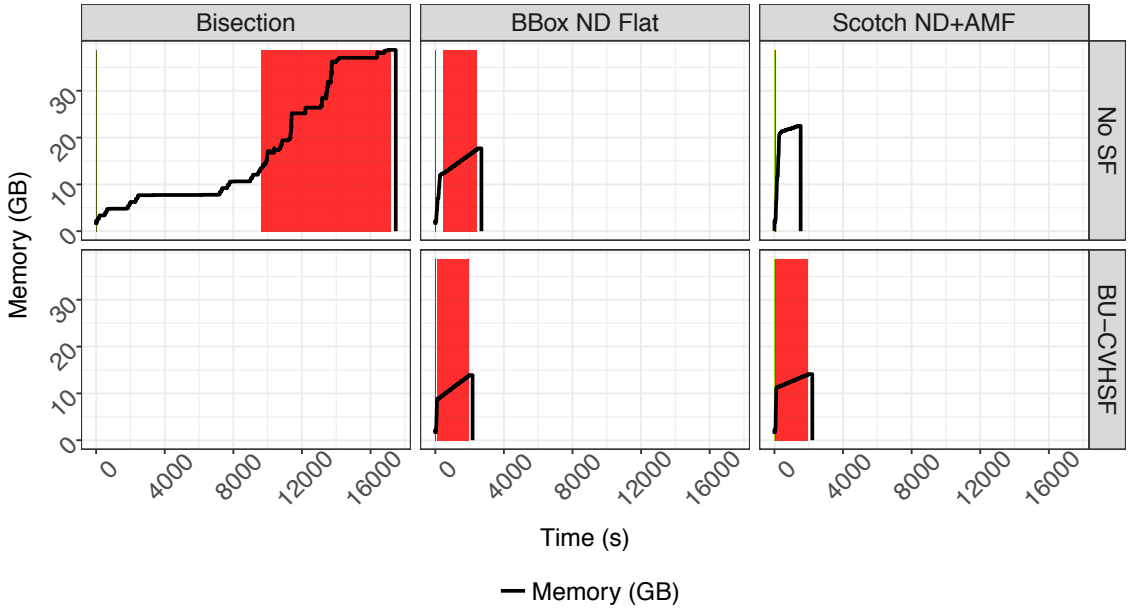
- For FEM-BEM, this is the current best approach



Other Possible Approaches

AIRBUS

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- Mumps-Hmat coupling: multi-facto based on H-mat clustering

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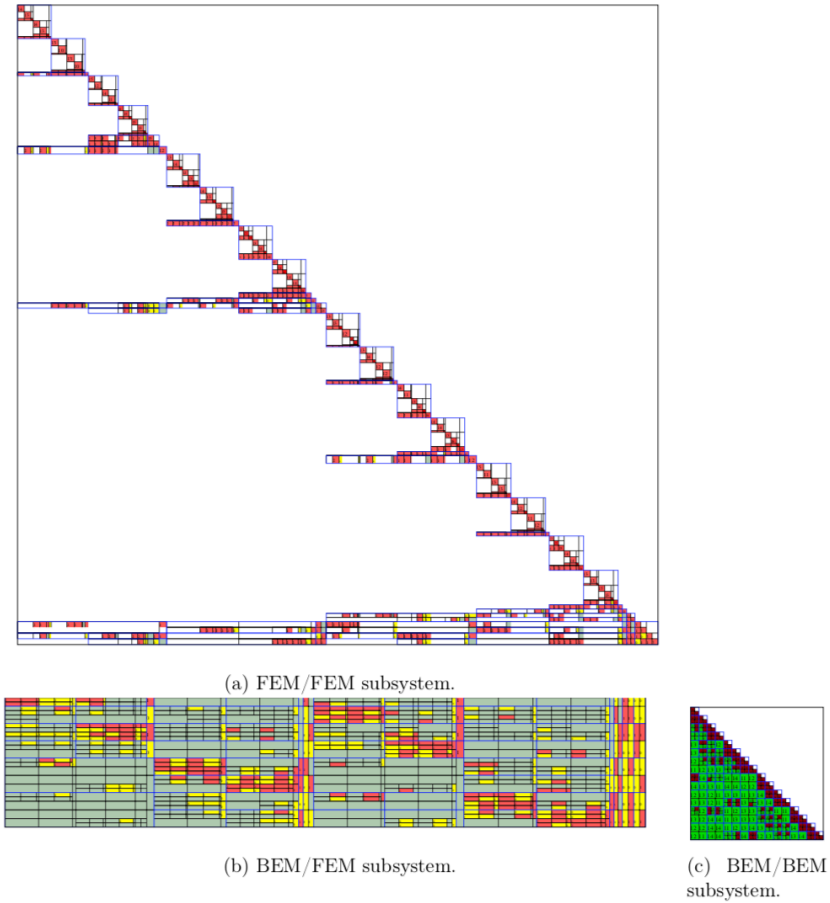
- Unique Hmat for surface AND volume



(a) FEM/FEM subsystem.

(b) BEM/FEM subsystem.

(c) BEM/BEM subsystem.

FEM/BEM Results

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FEM/BEM Results

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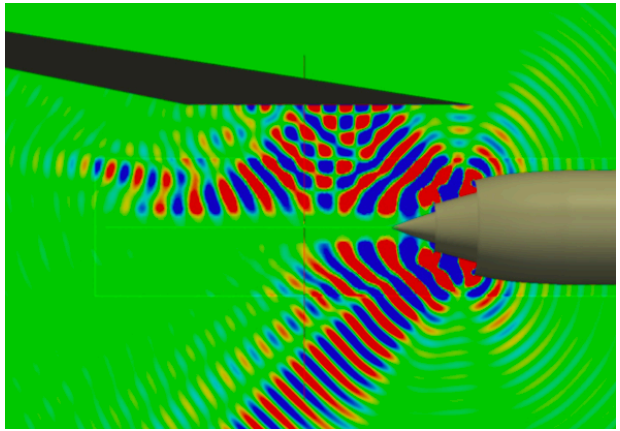

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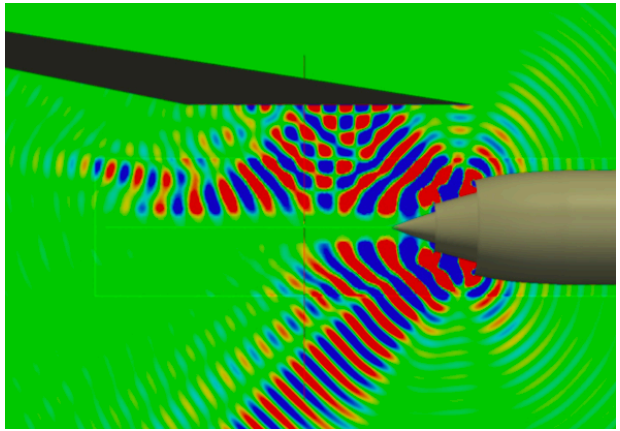
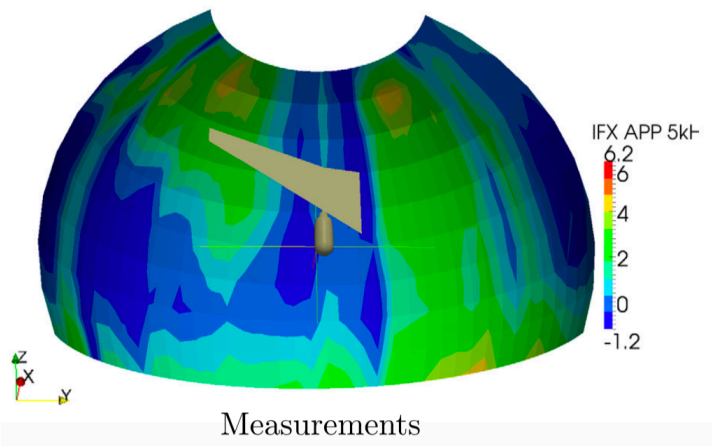
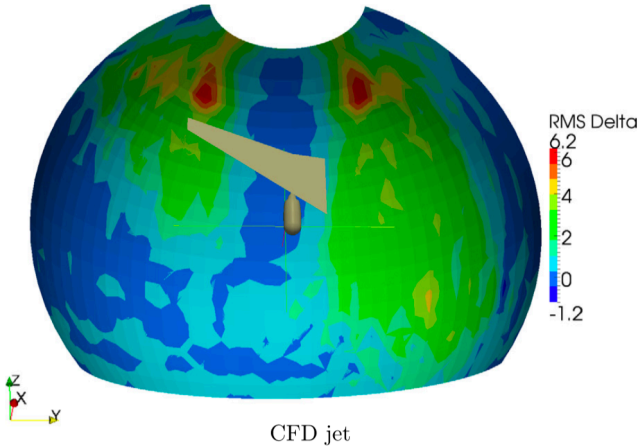




Measurements



CFD jet

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- FEM-BEM modelisation is a good compromise between cost and accuracy
- H-matrix solver shows excellent performance both on BEM only or coupled FEM-BEM systems

Perspectives

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- PhD position in Inria Bordeaux for HPC / algorithm