

A Bilevel Model for Large-scale Time-and-Level-of-Use Pricing

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Introduction, context, design goals



Problem context

Changing context for power grids:

- ▶ Integration of Distributed Renewable Sources
- ▶ Easier, automated data exchange
- ▶ Contribution of energy users to better grid operations

Objectives for the supplier:

- ▶ Generation planning requires information on demand
- ▶ More than forecast, **guarantee** on consumption
- ▶ Demand Response still not targeting residential users

Design goals

- ▶ Price-based Demand Response (vs. incentive-based)
- ▶ Offer users cost reductions for this guarantee on demand
- ▶ Maintain privacy and user-side flexibility by design
- ▶ Minimal information exchange and user actions

Time-and-Level-of-Use tariff



Time-and-Level-of-Use tariff

Introduced by Gomez-Herrera & Anjos [2], extension of Time-of-Use pricing.
Policy described by:

- ▶ K , booking fee
- ▶ π^L , lower tariff decreasing with capacity
- ▶ π^H , higher tariff increasing with capacity

User cost = Booking cost + Expected energy cost

User cost(cap) = $K \cdot cap + \text{Expected energy consumption} \times Price(cap)$

TLOU operational steps

1. Supplier sends pricing information
2. Users book a capacity
3. After consumption, total cost of each user is computed and billed

⇒ Done for each time frame, day-ahead or other settings.

⇒ One leader, multi-follower Stackelberg game.

First study: supplier - one consumer \Leftrightarrow one set of prices for each consumer [1].

Current work: multi-consumers for one pricing.

User cost for one price setting

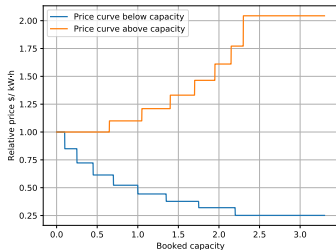


Figure: Higher and lower tariff curves

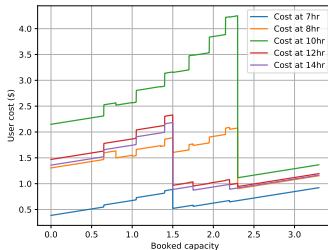


Figure: User cost vs booked capacity

User reaction

What if the user has a storage capacity, can change its consumption?

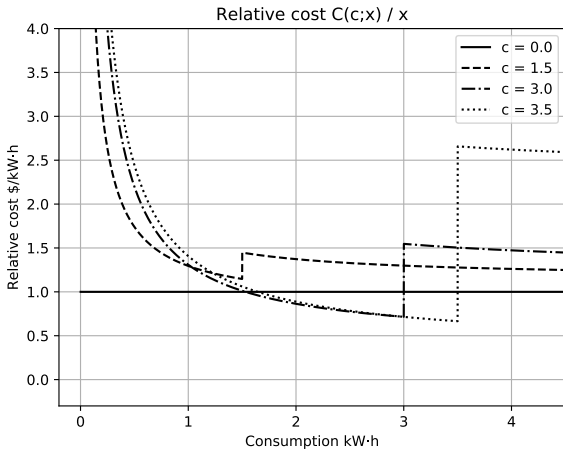


Figure: Relative cost of consumption for a given capacity

Bilevel formulation



Consumption levels

- ▶ User decision only depends on the supplier pricing & their distribution.
- ▶ No aggregation or law of large number effect at the consumer level.
- ▶ Devices may have **boolean**, **discrete** or **continuous** activation.

Consumption distributions using mixed probabilities

Continuous probability distribution with mass points.

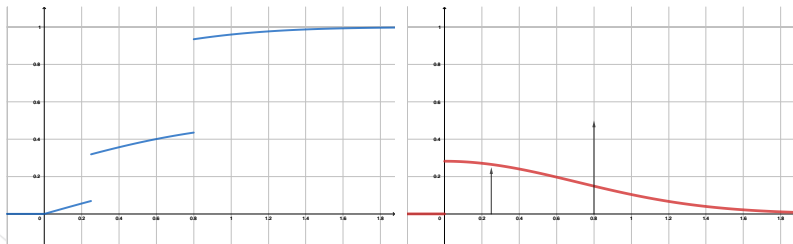


Figure: Mixed continuous discrete CDF Figure: Continuous PDF and mass points

NB: PDF undefined, requires generalized functions or "distributions".

Decision variables

Supplier chooses a price setting:

- ▶ $K \geq 0$
- ▶ $\pi^L \in \mathbb{R}^{|\pi^L|} \geq 0$
- ▶ $\pi^H \in \mathbb{R}^{|\pi^H|} \geq 0$

$\pi^L(0) = \pi^H(0)$: the baseline Time-of-Use price.

Users $u \in U$:

- ▶ Booked capacity for the time frame $c_u \geq 0 \forall u \in U$

Generation-side constraints

Ideally, the supplier would like to bound the total consumption:

$$\underline{X} \leq \sum_{u \in U} X_u \leq \bar{X} \quad (1)$$

They do not however control consumption, they can create an incentive for users to commit to capacities respecting those bounds:

$$\underline{c} \leq \sum_{u \in U} c_u \leq \bar{c} \quad (2)$$

A naive formulation

$$\max_{c, K, \pi^L, \pi^H} \sum_{u \in U} C_u(c_u, K, \pi^L(c_u), \pi^H(c_u)) \quad (3)$$

s. t.

$$c_u \in \arg \min_c C_u(c, K, \pi^L(c), \pi^H(c)) \quad (4)$$

$$\underline{c} \leq \sum_{u \in U} c_u \leq \bar{c} \quad (5)$$

$$(K, \pi^L, \pi^H) \in \Phi \quad (6)$$

Hard?

- ▶ Functions C_u appearing both in objective and constraints
⇒ non-linear, non-continuous (mass points and price breakpoints)
- ▶ Constraint (4): bilevel structure.

Solution method



Leveraging the cost structure (1)

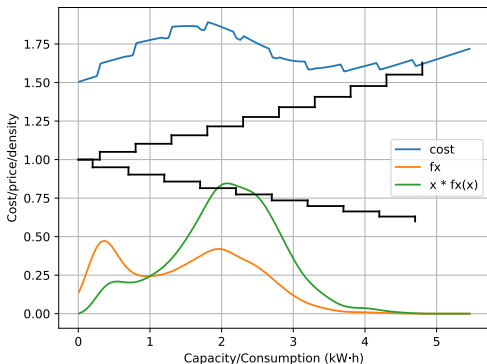


Figure: Example of user expected cost vs booked capacity

Low inter-discontinuity variability.

Leveraging the cost structure (2)

Optimal booked capacity only on discontinuities:

- ▶ Steps of the higher price curve (left limit)
- ▶ Steps of the lower price curve (right limit)
- ▶ Mass points of the consumption (right limit)

Reformulating the user continuous variable, $\forall u \in U$:

$$c_u = \sum_{k \in S_u} z_{uk} \cdot C_{uk} \quad (7)$$

$$\sum_{k \in S_u} z_{uk} = 1 \quad (8)$$

$$z_{uk} \in \mathbb{B} \quad \forall k \in S_u \quad (9)$$

$S_u = C_L \cup C_H \cup X_u^m$: optimality candidates for user u

Solution method

1. Network utilization bounds $[\underline{c}, \bar{c}]$

- ▶ Involve only binary variables z_{uk}
- ▶ Infeasible combinations \mathcal{I} can be found iteratively and added to the constraints:

$$\sum_{i \in \mathcal{I}} z_i \leq |\mathcal{I}| - 1 \Leftrightarrow \text{SOS1}(z_i, i \in \mathcal{I}) \quad (10)$$

2. Lower-level optimality: for any user u , if a solution k is picked, we have:

$$\hat{z}_{uk} = 1 \Rightarrow C_{uk} < C_{uj} \quad \forall j \in S_u \setminus \{k\} \quad (11)$$

All other solutions must be costlier.

Conclusion and perspectives

Bilevel MINLP \Rightarrow MILP.

Cuts leverage problem structure, compare with an initially complete model.

How reasonable is the regularity assumption?

How conservative should supplier be on user behavior? Models for risk aversion?

Modeling mixed continuous discrete probability distributions:

<https://github.com/matbesancon/MixedDistributions.jl>

Time-and-Level-of-Use pricing modeling tool:

<https://github.com/matbesancon/TLOU.jl>

References



References I



Mathieu Besançon, Miguel F. Anjos, Luce Brotcorne, and Juan A. Gomez-Herrera.

A Bilevel Framework for Optimal Price-Setting of Time-and-Level-of-Use Tariffs.

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Juan A. Gomez-Herrera and Miguel F. Anjos.

Optimization-based estimation of power capacity profiles for activity-based residential loads.

International Journal of Electrical Power & Energy Systems, 104:664–672, jan 2019.

Model user conservativeness & risk aversion

If two solutions are equivalent to the user, which one would they choose?
Bilevel optimization commonly uses **optimistic**, **pessimistic** formulations.

When a decision incurs more action than others (booking versus not booking), it may be penalized by a small difference. For a solution k to be picked:

$$\mathcal{C}_u(C_{uk}) \leq \mathcal{C}_u(C_{uj}) + \delta_u \quad \forall j \neq k \quad (12)$$

Adding a perceived risk factor [1], β_u :

$$\mathcal{C}_u(C_{uk}) \leq \mathcal{C}_u(C_{uj}) + \delta_u + \beta_u \cdot \left[(\pi^H(C_{uk}) - \pi^L(C_{uk})) - (\pi^H(C_{uj}) - \pi^L(C_{uj})) \right] \quad \forall j \neq k \quad (13)$$

Expression of \mathcal{C}_u

$$E_u^-(c) = P[X \leq c] \cdot \mathbb{E}[X|X \leq c] \quad (14)$$

$$E_u^+(c) = P[X > c] \cdot \mathbb{E}[X|X > c] \quad (15)$$

$$\mathbb{E}[X] = E_u^-(c) + E_u^+(c) \quad (16)$$

The user cost $\mathcal{C}_u(c)$ can be computed at any c :

$$\mathcal{C}_u(c) = K \cdot c + \pi^L(c) \cdot E_u^-(c) + \pi^H(c) \cdot E_u^+(c) \quad (17)$$

Evaluation of \mathcal{C}_u at a capacity c

Separation of the mass points:

$$X_u^-(c) = \{x^m \in X_u^m, x^m \leq c\} = \{x_1, x_2 \dots x_b\} \quad (18a)$$

$$X_u^+(c) = \{x^m \in X_u^m, x^m > c\} = \{x_{b+1}, x_{b+2} \dots x_n\} \quad (18b)$$

$$\begin{aligned} \mathcal{C}_u(c) = & K \cdot c + \sum_{i=0}^{b-1} \pi^L(c) \cdot x_i \cdot p_i + \int_{x_i}^{x_{i+1}} \pi^L(c) \cdot f_X(x) \cdot x \cdot dx + \\ & \pi^L(c) \cdot p_b \cdot x_b \int_{x_b}^c \pi^L(c) \cdot f_X(x) \cdot x \cdot dx + \\ & \int_c^{x_{b+1}} \pi^H(c) \cdot f_X(x) \cdot x \cdot dx + \\ & \sum_{i=b+1}^{n-1} \pi^H(c) \cdot x_i \cdot p_i + \int_{x_i}^{x_{i+1}} \pi^H(c) \cdot f_X(x) \cdot x \cdot dx + \\ & \pi^H(c) \cdot x_n \cdot p_n + \int_{x_n}^{\infty} \pi^H(c) \cdot f_X(x) \cdot x \cdot dx \quad (19) \end{aligned}$$

MILP formulation

$$\max_{z, \mu, K, \pi^L, \pi^H} \sum_{u \in U} \mu_u \quad (20)$$

s.t.

$$\mu_u \leq \gamma_{uj} + (1 - z_{uj}) \cdot \pi_0 \cdot \mathbb{E}[X] \quad \forall u \in U \forall j \in S_u \quad (21)$$

$$\mu_u \geq \gamma_{uj} - (1 - z_{uj}) \cdot \pi_0 \cdot \mathbb{E}[X] \quad \forall u \in U \forall j \in S_u \quad (22)$$

$$\mu_u \leq \gamma_{ui} - \alpha_{uji} + (1 - z_{uj} + z_{ui}) \cdot \pi_0 \cdot \mathbb{E}[X] \quad \forall u \in U \forall i \in S_u \forall j \in S_u \setminus \{i\} \quad (23)$$

$$\sum_{j \in S_u} z_{uj} = 1 \quad \forall u \in U \quad (24)$$

$$\underline{c} \leq \sum_{u \in U} \sum_{j \in S_u} C_{uj} \cdot z_{uj} \leq \bar{c} \quad (25)$$

$$\gamma_{uj} = K \cdot C_{uj} + \pi^L(C_{uj}) \cdot E^-(C_{uj}) + \pi^H(C_{uj}) \cdot E^+(C_{uj}) \quad (26)$$

$$\alpha_{uji} = \delta_u + \beta_u \cdot (\pi^H(C_{uj}) - \pi^L(C_{uj})) - \beta_u \cdot (\pi^H(C_{ui}) - \pi^L(C_{ui})) \quad (27)$$

$$(K, \pi^L, \pi^H) \in \Phi \quad (28)$$

$$z_{uj} \in \mathbb{B}, \mu_u \geq 0 \quad (29)$$