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## Multi-Player Bandits Revisited

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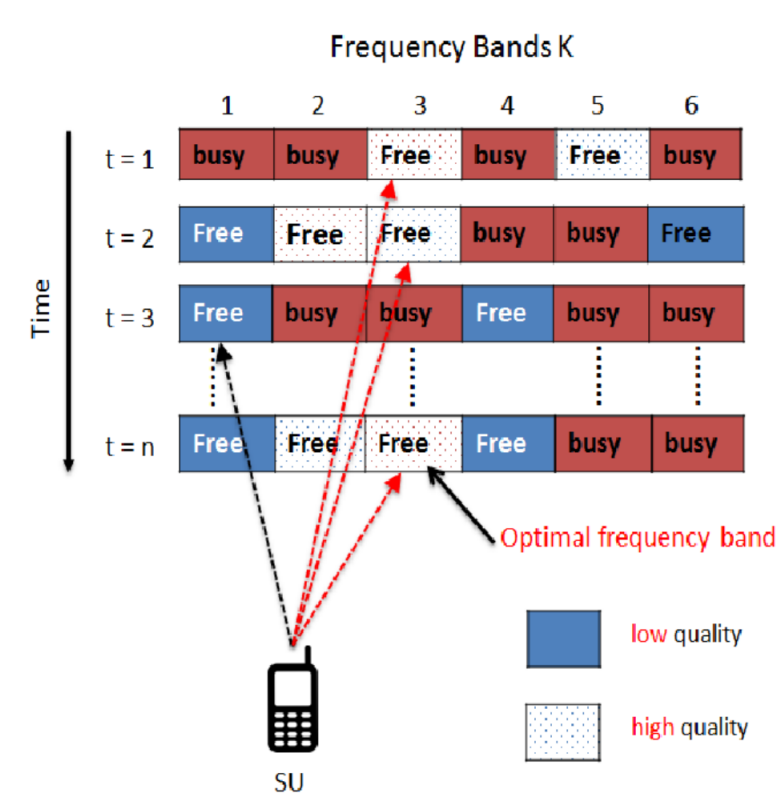
## 1. INTRODUCTION & GOAL

Goal: insert objects in a wireless network, keep a good *Quality of Service*.

- *Hypothesis*: object  $j$  choose channel  $A_j(t) \in \{1, \dots, K\}$ , to use to communicate at time  $t$ .
- *Idea*: use on-line **Machine Learning algorithms** ?
- *Not so easy*: each device takes its own decisions, without central control or communication, has light CPU/memory etc.
- $\implies$  **Solution: Decentralized MAB algorithms!**

## 2. MODEL: TIME/FREQUENCY PROTOCOL

## M PLAYERS IN THE NETWORK



Fix-duration communication. Channels can be free or busy.

- $K$  RF channels (of same bandwidth), e.g.,  $K = 9$ .
- Primary users  $\text{PU}$  create a background traffic.
- Channel  $k$  is busy with mean  $\mu_1, \dots, \mu_K \in [0, 1]$ .
- Sensing gives binary feedback  $Y_{k,t} \sim \mathcal{B}(\mu_k)$ .
- $1 \leq M \leq K$  secondary users  $\text{SU}$ ,  $j \in \{1, \dots, M\}$ .
- Base station  $\text{BS}$  replies if  $\text{SU}$  packet is received.
- Collision indic.  $C^j(t) = \mathbb{1}(\exists j' \neq j, A^j(t) = A^{j'}(t))$ .
- Non-stochastic 0/1 reward  $r^j(t) := Y_{A^j(t),t} \times C^j(t)$ .

## 3. PREVIOUS WORKS (SINCE 2009)

Ideas: 1) use empirical means to learn the  $M$  best arms ( $\mu_1^*, \dots, \mu_M^*$ ), 2) and each user orthogonalize on this set.

- RhoRand: use an index policy (UCB) and random *ranks*, [Anandkumar et al, 2011]
- MEGA: use a simple  $\varepsilon$ -greedy algorithm (hard to tune, and not so efficient), [Avner & Mannor, 2015]
- Musical Chair: pure exploration then random hopping until convergence on  $M$  best arms, [Shamir et al, 2016]
- Some others: TDFS uses a time sharing, RhoLearn uses BayesUCB to learn the ranks, etc.

## 4. OUR PROPOSAL

- 1) kl-UCB  $>$  UCB<sub>1</sub> for best arms identification.
- 2) New algorithm MCTopM for orthogonalization:
  - Estimate the set of  $M$  best arm  $\widehat{M}^j(t)$ ,
  - Randomly play in  $\widehat{M}^j(t)$  until fixed on a good one (no collision: fixed on a “chair”).

## 5. UCB<sub>1</sub> AND kl-UCB INDEXES

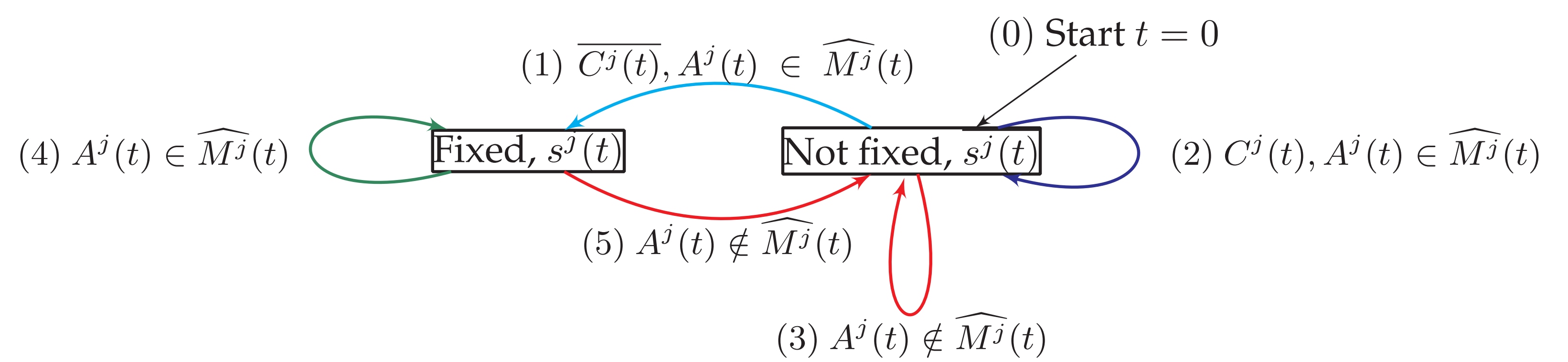
Let  $T_k^j(t)$  the selections of channel  $k$  for player  $j$ ,  $X_k^j(t)$  mean of free sensing.

$$\text{UCB}_1 : g_k^j(t) := \frac{X_k^j(t)}{T_k^j(t)} + \sqrt{\frac{\log(t)}{2T_k^j(t)}}$$

$$\text{kl-UCB} : g_k^j(t) := \sup_{q \in [0,1]} \left\{ q : \text{kl} \left( \frac{X_k^j(t)}{T_k^j(t)}, q \right) \leq \frac{\log(t)}{T_k^j(t)} \right\}$$

[Garivier & Cappé, 2011], [Cappé et al, 2013]

## 7. ILLUSTRATION OF MCTOPM



## 6. MCTOPM ALGORITHM

- 1  $A^j(1) \sim \mathcal{U}(\{1, \dots, K\})$ ,  $C^j(1) = s^j(1) = \text{False}$
- 2 **for**  $t = 0, \dots, T - 1$  **do**
- 3   **if**  $A^j(t) \notin \widehat{M}^j(t)$  **then**   // (3) or (5)
- 4      $A^j(t+1) \sim \mathcal{U}(\widehat{M}^j(t) \cap \{k : g_k^j(t-1) \leq g_{A^j(t)}^j(t-1)\})$
- 5      $s^j(t+1) = \text{False}$    // arm with smaller UCB at  $t-1$
- 6   **else if**  $C^j(t)$  and  $\overline{s^j(t)}$  **then**   // (2)
- 7      $A^j(t+1) \sim \mathcal{U}(\widehat{M}^j(t))$
- 8      $s^j(t+1) = \text{False}$
- 9   **else**   // transition (1) or (4)
- 10     $A^j(t+1) = A^j(t)$    // same arm
- 11     $s^j(t+1) = \text{True}$    // on “chair”
- 12 **end**
- 13 Play arm  $A^j(t+1)$ , get new observations,
- 14 Compute the indexes  $g_k^j(t+1)$ , and set  $\widehat{M}^j(t+1)$  for next step.
- 15 **end**

## 8. THEOREMS

Let  $T_k(T) := \sum_{j=1}^M T_k^j(T)$  total selections of arm  $k$ , and  $\mathcal{C}_k(T)$  counts collisions on arm  $k$ .

$$1. \text{ Regret: } R_T(\mu, M) := \mathbb{E}_\mu \left[ \sum_{t=1}^T \sum_{j=1}^M \mu_j^* - r^j(t) \right] = \left( \sum_{k=1}^M \mu_k^* \right) T - \mathbb{E}_\mu \left[ \sum_{t=1}^T \sum_{j=1}^M r^j(t) \right]$$

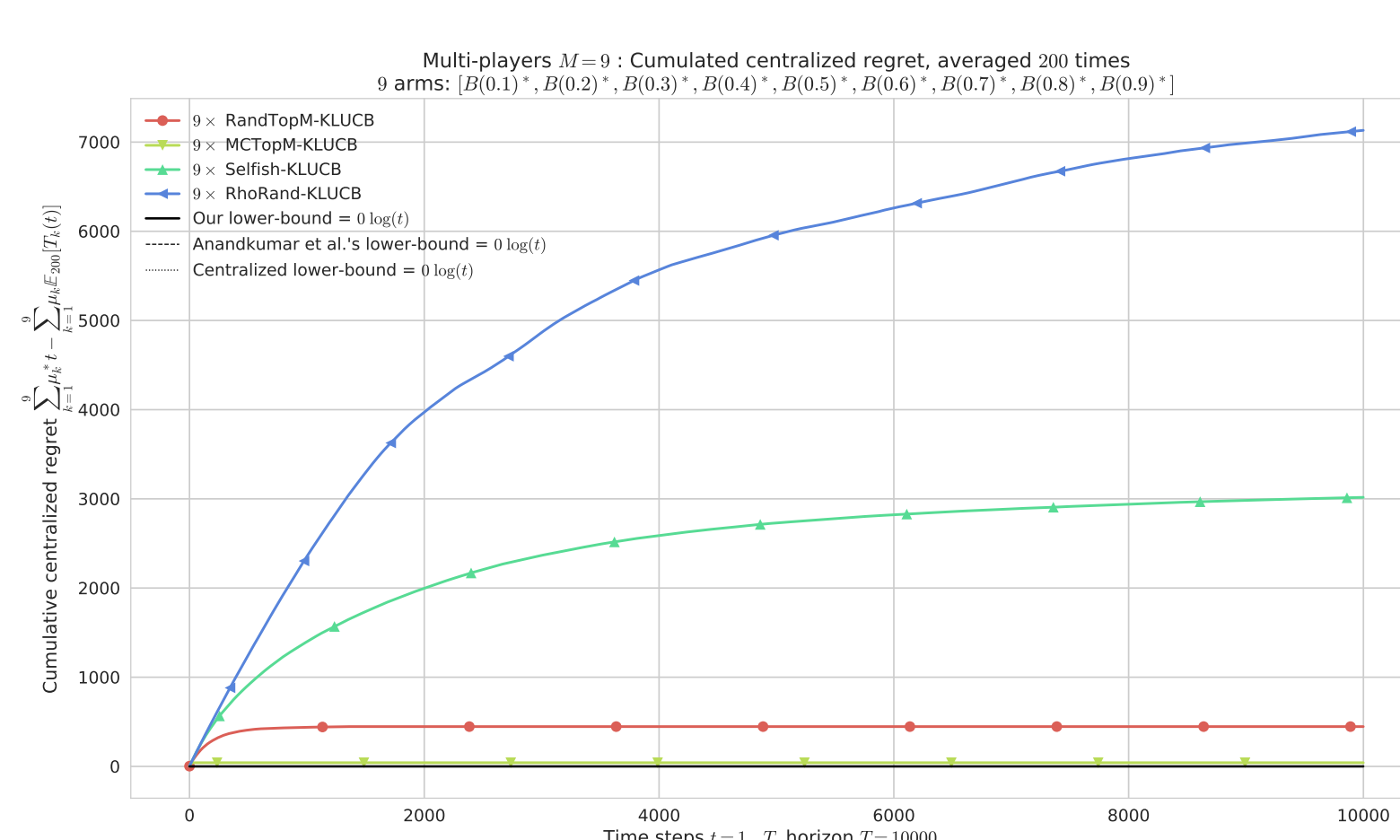
$$R_T = \sum_{k \in M\text{-worst}} (\mu_M^* - \mu_k) \mathbb{E}_\mu [T_k(T)] + \sum_{k \in M\text{-best}} (\mu_k - \mu_M^*) (T - \mathbb{E}_\mu [T_k(T)]) + \sum_{k=1}^K \mu_k \mathbb{E}_\mu [\mathcal{C}_k(T)].$$

2. Lemma for upper-bound: only two terms to focus on (bad selections and collisions).

$$\sum_{k \in M\text{-best}} (\mu_k - \mu_M^*) (T - \mathbb{E}_\mu [T_k(T)]) \leq (\mu_1^* - \mu_M^*) \left( \sum_{k \in M\text{-worst}} \mathbb{E}_\mu [T_k(T)] + \sum_{k \in M\text{-best}} \mathbb{E}_\mu [\mathcal{C}_k(T)] \right).$$

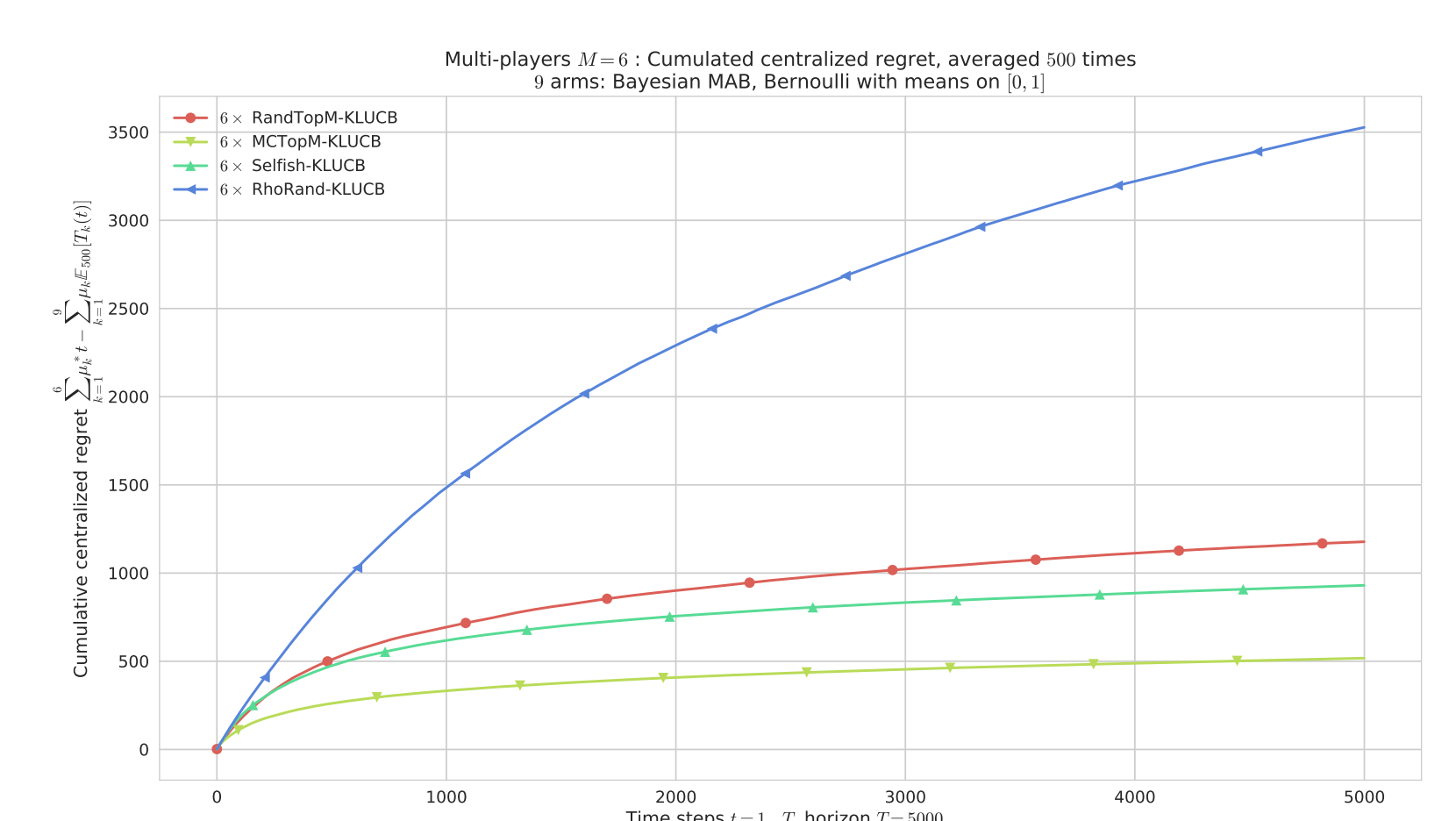
3. If all  $M$  players use MCTopM with kl-UCB,  $\forall \mu, \exists G_{M,\mu}, R_T(\mu, M) \leq G_{M,\mu} \times \log(T) + o(\log T)$ .

### 9.1. ILLUSTRATION FOR $M = K$



Only **RandTopM** and **MCTopM** achieve constant regret in this saturated case (proven).

### 9.2. ILLUSTRATION FOR $M < K$



**RhoRand** < **RandTopM** < **Selfish** < **MCTopM**.

## 10. CONCLUSIONS

- Our algorithm MCTopM is uniformly better than all previous proposals.
- *Good bounds*:  $\log(T)$  collisions, arm switches, bad selections and regret.
- Real-world implementation?  $\leftrightarrow$  Yes, presented at ICT 2018!
- *Future?* learn  $M$ , arrival/departures of users, dynamic problems, jammers etc.

## 11. MAIN REFERENCES

MORE ON-LINE  $\rightarrow$  <http://lbo.k.vu/JdD2018>

- [BBM<sup>+</sup>17] R. Bonnefoi, L. Besson, C. Moy, E. Kaufmann, and J. Palicot (2017). *Multi-Armed Bandit Learning in IoT Networks: Learning helps even in non-stationary settings*. In *12th EAI Conference on Cognitive Radio Oriented Wireless Network and Communication*.
- [BK18] L. Besson and E. Kaufmann (April 2018). *Multi-Player Bandits Revisited*. In *Algorithmic Learning Theory*. Lanzarote, Spain. URL <https://hal.inria.fr/hal-01629733>.
- [B18] Simulation code on [GitHub.com/SMPyBandits/SMPyBandits](https://github.com/SMPyBandits/SMPyBandits), open source (MIT license)!

## 12. THANKS TO ...

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