Sparse direct methods using low-rank compression
Nested dissection (ND) [George, 1973]
Sparse direct methods using low-rank compression
Nested dissection (ND) [George, 1973]
Sparse direct methods using low-rank compression
Nested dissection (ND) [George, 1973]
Sparse direct methods using low-rank compression
Nested dissection (ND) [George, 1973]
Sparse direct methods using low-rank compression
Symbolic Factorization

Quotient Graph

Matrix
Sparse direct methods using low-rank compression

- **Multifrontal methods**
  - Dense frontal matrices
    [Ghysels et al., 2016, Xia, 2013, Xia et al., 2009, Amestoy et al., 2015, Aminfar et al., 2016, Mary, 2017]

- **Supernodal methods**
  - Dense (supernodal) sub-matrices
    [Pichon et al., 2017a]

Sparse methods towards \( \mathcal{H} \)

- **Advantages**
  - Orderings
  - Symbolic factorization

- Hierarchical compression used locally on sub-matrices
Sparse direct methods using low-rank compression

- **Multifrontal methods**
  - Dense frontal matrices
    [Ghysels et al., 2016, Xia, 2013, Xia et al., 2009, Amestoy et al., 2015, Aminfar et al., 2016, Mary, 2017]

- **Supernodal methods**
  - Dense (supernodal) sub-matrices
    [Pichon et al., 2017a]

**Sparse methods towards $\mathcal{H}$**

- **Advantages**
  - Orderings
  - Symbolic factorization

- Hierarchical compression used locally on sub-matrices
Hierarchical methods for dense linear systems

- Many Formats:
  - $\mathcal{H}$-Matrix, $\mathcal{H}^2$-matrix, HSS, HODLR, BLR

- Clustering method
  - Bisection
    [Hackbusch, 1999, Hackbusch, 2015]
Hierarchical methods for dense linear systems

- Many Formats:
  - $\mathcal{H}$-Matrix, $\mathcal{H}^2$-matrix, HSS, HODLR, BLR

- Clustering method
  - Bisection
    [Hackbusch, 1999, Hackbusch, 2015]
Hierarchical methods for dense linear systems

- Many Formats:
  - $\mathcal{H}$-Matrix, $\mathcal{H}^2$-matrix, HSS, HODLR, BLR

- Clustering method
  - Bisection
    [Hackbusch, 1999, Hackbusch, 2015]
Hierarchical methods for dense linear systems

- Many Formats:
  - $H$-Matrix, $H^2$-matrix, HSS, HODLR, BLR

- Clustering method
  - Bisection
    [Hackbusch, 1999, Hackbusch, 2015]
Hierarchical methods exploiting sparsity

- Many Formats:
  - $\mathcal{H}$-Matrix, $\mathcal{H}^2$-matrix, HSS, HODLR, BLR

- Clustering method
  - Bisection
    - [Hackbusch, 1999, Hackbusch, 2015]
Hierarchical methods exploiting sparsity

- Many Formats:
  - $\mathcal{H}$-Matrix, $\mathcal{H}^2$-matrix, HSS, HODLR, BLR

- Clustering method
  - Nested dissection
    [Le Borne et al., 2007, Grasedyck et al., 2008, Grasedyck et al., 2009, Kriemann, 2013]

$\mathcal{H}$-Matrices towards sparse

- Advantages
  - Global hierarchical structure
  - Compression

- How to manage and locate fill-in?
Problem formulation

Sparse methods towards $\mathcal{H}$

- **Advantages**
  - Orderings lead to less fill-in
  - Symbolic factorization
- **Current limits?**
  - Hierarchical compression used locally on sub-matrices

$\mathcal{H}$-Matrices towards sparse

- **Advantages**
  - Global hierarchical structure
  - Compression
- **Current limits?**
  - How to manage and locate fill-in?
Objectives

Combine hierarchical and sparse techniques

- Number of non-zeros as low as sparse direct methods
- A global hierarchical structure

→ Hierarchical Symbolic Factorization
Nested dissection clustering [George, 1973]

[Le Borne et al., 2007, Grasedyck et al., 2008, Grasedyck et al., 2009, Kriemann, 2013]
Nested dissection clustering [George, 1973]

[Le Borne et al., 2007, Grasedyck et al., 2008, Grasedyck et al., 2009, Kriemann, 2013]
Nested dissection clustering [George, 1973]

[Le Borne et al., 2007, Grasedyck et al., 2008, Grasedyck et al., 2009, Kriemann, 2013]
Nested dissection clustering [George, 1973]

[Le Borne et al., 2007, Grasedyck et al., 2008, Grasedyck et al., 2009, Kriemann, 2013]
Hierarchical Block Symbolic Factorization (TD-HBSF)

Quotient Graph
Hierarchical Block Symbolic Factorization (TD-HBSF)

Quotient Graph
Hierarchical Block Symbolic Factorization (TD-HBSF)

Quotient Graph
Hierarchical Block Symbolic Factorization (TD-HBSF)
Hierarchical Block Symbolic Factorization (BU-HBSF)
Hierarchical Block Symbolic Factorization (BU-HBSF)

Quotient Graph
Hierarchical Block Symbolic Factorization (BU-HBSF)

Quotient Graph
Hierarchical Block Symbolic Factorization (BU-HBSF)

Quotient Graph
Separator Division
Separator Division

Airbus / Inria – Hierarchical Symbolic Factorization for Sparse Matrices
Minimizing extra-diagonal blocks is NP-hard
[Pichon et al., 2017b]
Interactions-Oblivious Separator Bisection

[Le Borne et al., 2007, Grasedyck et al., 2008, Grasedyck et al., 2009, Kriemann, 2013]
Interactions-Aware Flat Separator Division (IA-FSD)
Related to T4 supernodes [Demmel et al., 1999] but too rare and therefore usually rejected
Interactions-Aware Hierarchical Separator Division (IA-HSD)
Interactions-Aware Hierarchical Separator Division (IA-HSD)
Interactions-Aware Leaf Separator Division (IA-LSD)
Using Block Column Symbolic Factorization (BCSF) Information
TD-HBSF Memory Profile

N = \(1 \times 10^6\)

[Graph showing memory usage over time for TD-HBSF, Scotch ND + AMF with markers for Graph, Clustering, and SF.]
BU-HBSF Memory Profile

$N = 1 \times 10^6$

Graph  ●  SF  ●  Clustering

Airbus / Inria – Hierarchical Symbolic Factorization for Sparse Matrices
Conclusions

- Performing a (bottom-up) hierarchical symbolic factorization is nearly costless
- Possible divisions of separators
  - Bisection
  - Interactions-Aware Flat Separator Division (IA-FSD)
  - Interactions-Aware Hierarchical Separator Division (IA-HSD)
  - Interactions-Aware Leaf Separator Division (IA-LSD)

Perspectives & Work in Progress

- Extensive comparison between separator division methods
- Study the impact of each algorithm on numerical factorization
- Parallel implementation
- Extensive comparison with reference sparse solvers
Thank you for your attention!


