



An upscaled DGTD method for time-domain electromagnetics

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An upscaled DGTD method for time-domain electromagnetics

Multiscale and multiphysics computation and applications (2P3A)

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Universidad Católica de Valparaíso, Chile
Frédéric Valentin
LNCC, Petropolis, Brazil



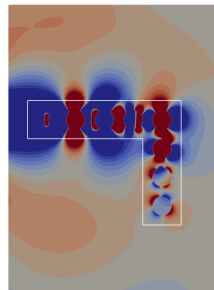
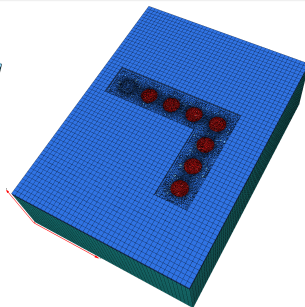
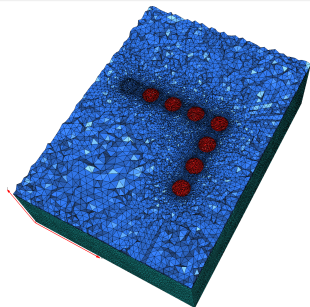
Laboratório
Nacional de
Computação
Científica



Progress In Electromagnetics Research Symposium - PIERS 2018
Toyama, Japan, August 1-4, 2018

Computational nanophotonics: modeling challenges

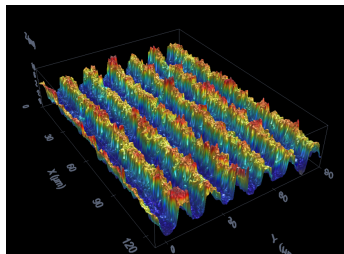
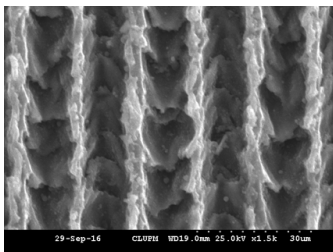
- Material properties:
 - Homogeneous v.s. heterogeneous;
 - Isotropic v.s. anisotropic;
 - Physical dispersion (local v.s. non-local);
 - Linear v.s. non-linear.
- Multiscale problems in space and time



L-shaped waveguide formed of seven 50 nm diameter Au spheres in vacuum, with a 75 nm center-to-center spacing (computational domain: 550 nm $\times$ 750 nm $\times$ 400 nm).

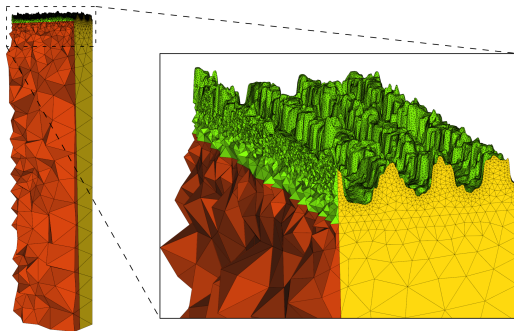
A concrete example in photovoltaics

- Collaboration with Division of Renewable Energies, CIEMAT, Spain
- Study of thin-film silicon-heterojunction solar cells
- Texturization technique based on controlled ablation of the surface by laser scanning



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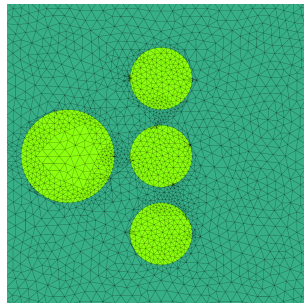
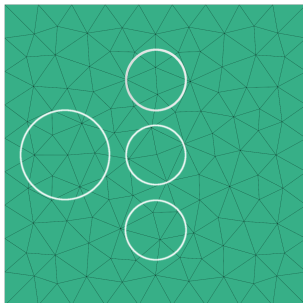


- As-grown TCO: $10\ \mu\text{m}$
- Crystalline silicon: $510\ \mu\text{m}$
- Surface: $100\ \mu\text{m} \times 100\ \mu\text{m}$

Objectives

Designing a family of methods which are:

- Highly flexible in the discretization process;
- Well suited to (highly) heterogeneous problems;
- Well suited to multiscale problems;
- Preserving high order convergence;
- Well adapted to massively parallel computing.



1 Multiscale Hybrid-Mixed (MHM) method

2 MHM-induced local time stepping

3 Numerical results in the 2D case

4 Closure

- 1 Multiscale Hybrid-Mixed (MHM) method
- 2 MHM-induced local time stepping
- 3 Numerical results in the 2D case
- 4 Closure

MHM method in a nutshell

- Finite element type method
- Accurate on general and coarse meshes
- Induce an a posteriori error estimator (to drive mesh adaptivity)
- Locally conservative
- Naturally adapted to HPC

- R. Araya and C. Harder and D. Paredes and F. Valentin
Multiscale hybrid-mixed method
SIAM J. Numer. Anal., Vol. 51, No. 6 (2013)
- C. Harder and D. Paredes and F. Valentin
A family of multiscale hybrid-mixed finite element methods for the Darcy equation with rough coefficients
J. Comput. Phys., Vol. 245 (2013)

- Our goal: adaptation of the MHM framework to wave propagation contexts
- First target: system of time-domain Maxwell equations

Basic principles

- One among several solution strategies recently devised for multiscale problems
- A two level method
- First level
 - Discretization based on a very coarse and general mesh
 - The skeleton of this mesh is the support of a **hybrid variable**
 - Hybrid variable problem discretized by a *classical* finite element formulation based on **upscaled basis functions**
- Second level
 - Each (macro-)element of the first level mesh is triangulated (second level mesh)
 - Local problems defined on each macro-element of the first level mesh
 - Continuous case: analytical solution for the local problems
 - Discrete case: **DGTD^a** method for the local problems

^aDiscontinuous Galerkin Time-Domain

Time-domain Maxwell equations - Initial and boundary value problem

$$\begin{cases} \varepsilon \partial_t \mathbf{E} - \mathbf{curl} \mathbf{H} = -\mathbf{J}, & \text{in } \Omega \times [0, T] \\ \mu \partial_t \mathbf{H} + \mathbf{curl} \mathbf{E} = 0, & \text{in } \Omega \times [0, T] \\ \mathbf{n} \times \mathbf{E} = 0, & \text{on } \Gamma_m \times [0, T] \\ \mathbf{n} \times \mathbf{E} + \mathbf{n} \times (\mathbf{n} \times \mathbf{H}) = \mathbf{n} \times \mathbf{E}^{\text{inc}} + \mathbf{n} \times (\mathbf{n} \times \mathbf{H}^{\text{inc}}), & \text{on } \Gamma_a \times [0, T] \\ \mathbf{E} = \mathbf{E}_0, & \text{in } \Omega \times \{0\} \quad \text{and} \quad \mathbf{H} = \mathbf{H}_0, & \text{in } \Omega \times \{0\} \end{cases}$$

Functional space setting

$\mathcal{T}_H$ a simplicial **macro-mesh** of the computational domain and $\mathcal{T}_{\Delta T} = [0, T] = \bigcup_{n=1}^N I_n$

$$\mathbf{V} := \{\mathbf{v} \in \mathbf{L}^2(\Omega) : \mathbf{v}|_K \in \mathbf{H}^1(K) \text{ for all } K \in \mathcal{T}_H\}$$

$\mathbf{L}$ is the space of the restriction of the tangential component of functions in $\mathbf{H}(\mathbf{curl}; \Omega)$ to ∂K

$$\mathbf{L} := \left\{ \mathbf{v} \times \mathbf{n}^K|_{\partial K} \in \mathbf{H}^{-1/2}(\partial K) \text{ for all } K \in \mathcal{T}_H \text{ with } \mathbf{v} \in \mathbf{H}(\mathbf{curl}; \Omega) \right\}$$

with $\mathbf{n}^K$ being the outward normal vector on ∂K .

Multiscale Hybrid-Mixed method

MHM for the time-domain Maxwell equations: principle of the method

Localization: on each I_n , for $K \in \mathcal{T}_h$

find $(\mathbf{E}, \mathbf{H}) \in \mathbf{V} \times \mathbf{V}$ such that

$$\begin{cases} (\varepsilon \partial_t \mathbf{E}, \mathbf{v})_K - (\mathbf{curl} \mathbf{H}, \mathbf{v})_K = (\mathbf{J}, \mathbf{v})_K, \quad \forall \mathbf{v} \in \mathbf{V}, \\ (\mu \partial_t \mathbf{H}, \mathbf{w})_K + (\mathbf{curl} \mathbf{E}, \mathbf{w})_K = 0, \quad \forall \mathbf{w} \in \mathbf{V}. \end{cases}$$

+ Continuity conditions at the end points of I_n

Localization+hybridization: on each I_n , for $K \in \mathcal{T}_h$

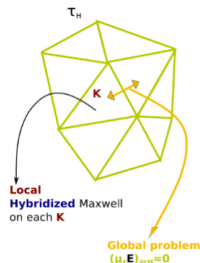
find $(\mathbf{E}, \mathbf{H}, \boldsymbol{\lambda}) \in \mathbf{V} \times \mathbf{V} \times \boldsymbol{\Lambda}$ such that

$$\begin{cases} (\varepsilon \partial_t \mathbf{E}, \mathbf{v})_K - (\mathbf{H}, \mathbf{curl} \mathbf{v})_K = (\mathbf{J}, \mathbf{v})_K - (\boldsymbol{\lambda}, \mathbf{v})_{\partial K}, \quad \forall \mathbf{v} \in \mathbf{V}, \\ (\mu \partial_t \mathbf{H}, \mathbf{w})_K + (\mathbf{curl} \mathbf{E}, \mathbf{w})_K = 0, \quad \forall \mathbf{w} \in \mathbf{V}. \end{cases}$$

+ Continuity conditions at the end points of I_n

Global problem: $(\boldsymbol{\nu}, \mathbf{E})_{\partial \mathcal{T}_H} = 0, \quad \forall \boldsymbol{\nu} \in \boldsymbol{\Lambda}$

It can be shown that $\boldsymbol{\lambda} = -\mathbf{n} \times \mathbf{H}$ on $\partial K \times [0, T]$



Multiscale Hybrid-Mixed method

MHM for the time-domain Maxwell equations: principle of the method

Splitting of unknowns: $\mathbf{E} = \mathbf{E}^J + \mathbf{E}^\lambda$ and $\mathbf{H} = \mathbf{H}^J + \mathbf{H}^\lambda$

Contribution of the current: on each I_n , for $K \in \mathcal{T}_h$

find $(\mathbf{E}^J, \mathbf{H}^J) \in \mathbf{V} \times \mathbf{V}$ such that

$$\begin{cases} (\varepsilon \partial_t \mathbf{E}^J, \mathbf{v})_K - (\mathbf{H}^J, \operatorname{curl} \mathbf{v})_K = (\mathbf{J}, \mathbf{v})_K, \quad \forall \mathbf{v} \in \mathbf{V}, \\ (\mu \partial_t \mathbf{H}^J, \mathbf{w})_K + (\operatorname{curl} \mathbf{E}^J, \mathbf{w})_K = 0, \quad \forall \mathbf{w} \in \mathbf{V}. \end{cases}$$

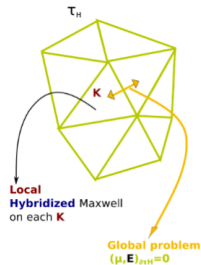
Contribution of the hybrid variable: on each I_n , for $K \in \mathcal{T}_h$

find $(\mathbf{E}^\lambda, \mathbf{H}^\lambda) \in \mathbf{V} \times \mathbf{V}$ such that

$$\begin{cases} (\varepsilon \partial_t \mathbf{E}^\lambda, \mathbf{v})_K - (\mathbf{H}^\lambda, \operatorname{curl} \mathbf{v})_K = -(\boldsymbol{\lambda}, \mathbf{v})_{\partial K}, \quad \forall \mathbf{v} \in \mathbf{V}, \\ (\mu \partial_t \mathbf{H}^\lambda, \mathbf{w})_K + (\operatorname{curl} \mathbf{E}^\lambda, \mathbf{w})_K = 0, \quad \forall \mathbf{w} \in \mathbf{V}. \end{cases}$$

Global problem: : on each I_n , for $K \in \mathcal{T}_h$

find $\boldsymbol{\lambda} \in \boldsymbol{\Lambda}$ such that $(\boldsymbol{\nu}, \mathbf{E}^\lambda)_{\partial \mathcal{T}_H} = -(\boldsymbol{\nu}, \mathbf{E}^J)_{\partial \mathcal{T}_H}, \quad \forall \boldsymbol{\nu} \in \boldsymbol{\Lambda}$



Principle of the method: one-level MHM

- λ_H piecewise constant in time on each I_n
- Choose Λ_H with $\dim(\Lambda_H) < +\infty$ and seek $\lambda_H \in \Lambda_H$
- Decompose λ_H on a basis (Ψ_i) of Λ_H

$$\lambda_H^n = \sum_{i=1}^{\dim(\Lambda_H)} \beta_i^n \Psi_i$$

Define the set (η_i^E, η_i^H) , $i \in [1, \dim(\Lambda_H)]$ solution of

$$\begin{cases} \left(\varepsilon \partial_t \eta_i^E, \mathbf{v} \right)_K - \left(\eta_i^H, \operatorname{curl} \mathbf{v} \right)_K = - \left(\Psi_i, \mathbf{n} \cdot \mathbf{K}, \mathbf{v} \right)_{\partial K}, \quad \forall \mathbf{v} \in \mathbf{V}, \\ \left(\mu \partial_t \eta_i^H, \mathbf{w} \right)_K + \left(\operatorname{curl} \eta_i^E, \mathbf{w} \right)_K = 0, \quad \forall \mathbf{w} \in \mathbf{V}. \end{cases}$$

(η_i^E, η_i^H) are the multiscale basis functions

- On each I_n it can be shown that $\mathbf{E}^{\lambda_H}$ and $\mathbf{H}^{\lambda_H}$ can be defined as

$$\mathbf{E}^{\lambda_H} = \sum_{i=1}^{\dim(\Lambda_H)} \beta_i^n \eta_i^E, \quad \mathbf{H}^{\lambda_H} = \sum_{i=1}^{\dim(\Lambda_H)} \beta_i^n \eta_i^H$$

Multiscale Hybrid-Mixed method

MHM for the time-domain Maxwell equations

Finally, in each K (and if $\mathbf{J}$ does not depend on time)

$$\mathbf{E}^n = \sum_{i=1}^{\dim(\Lambda_H)} \beta_i^n \boldsymbol{\eta}_i^{\mathbf{E}} + \mathbf{E}^{\mathbf{J}}, \quad \mathbf{H}^n = \sum_{i=1}^{\dim(\Lambda_H)} \beta_i^n \boldsymbol{\eta}_i^{\mathbf{H}} + \mathbf{H}^{\mathbf{J}}$$

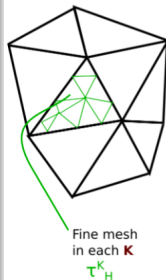
Principle of the method: two-level method

Discretization in the DGTD framework

- For each K , define $\mathbf{V}_h(K)$ as the discretization of $\mathbf{V}$
- In our case, $\mathbf{V}_h(K)$ is the DG discontinuous finite element space
- Formulate a family of local discrete **independent** problems stated on **macro-elements** K

$$\begin{cases} \left(\varepsilon \partial_t \boldsymbol{\eta}_i^{\mathbf{E}}, \mathbf{v} \right)_K - \left(\boldsymbol{\eta}_i^{\mathbf{H}}, \mathbf{curl} \, \mathbf{v} \right)_K = - \left(\boldsymbol{\Psi}_i \mathbf{n} \cdot \mathbf{n}^K, \mathbf{v} \right)_{\partial K}, \quad \forall \mathbf{v} \in \mathbf{V}_h(K), \\ \left(\mu \partial_t \boldsymbol{\eta}_i^{\mathbf{H}}, \mathbf{w} \right)_K + \left(\mathbf{curl} \, \boldsymbol{\eta}_i^{\mathbf{E}}, \mathbf{w} \right)_K = 0, \quad \forall \mathbf{w} \in \mathbf{V}_h(K). \end{cases}$$

$\boldsymbol{\eta}_i^{\mathbf{E}}$ and $\boldsymbol{\eta}_i^{\mathbf{H}}$ are sought in $\mathbf{V}_h(K)$ for each $i \in [1, \dim(\Lambda_H)]$



Multiscale Hybrid-Mixed method

MHM for the time-domain Maxwell equations

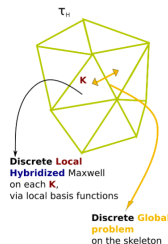
Principle of the method: two-level method

Discretization in the DGTD framework

- For each K , define $\mathbf{V}_h(K)$ as the discretization of $\mathbf{V}$
- In our case, $\mathbf{V}_h(K)$ is the DG discontinuous finite element space
- Formulate a family of local discrete **independent** problems stated on **macro-elements** K
- Formulate a global discrete **coarse** problem stated on the skeleton of $\mathcal{T}_h$

$$\left(\nu_H, \mathbf{E}^{\lambda_H} \right)_{\partial \mathcal{T}_H} = - \left(\nu_H, \mathbf{E}^J \right)_{\partial \mathcal{T}_H}, \quad \forall \nu_H \in \Lambda_H$$

$\Rightarrow$ leads the β_i^n coefficients



MHM for the time-domain Maxwell equations

Summary

1 Local problem associated to $\mathbf{J}$

Find $(\mathbf{E}^{\mathbf{J}}, \mathbf{H}^{\mathbf{J}}) \in \mathbf{V}_h(K) \times \mathbf{V}_h(K)$ such that

$$\begin{cases} (\varepsilon \partial_t \mathbf{E}^{\mathbf{J}}, \mathbf{v})_K - (\mathbf{H}^{\mathbf{J}}, \operatorname{curl} \mathbf{v})_K = (\mathbf{J}, \mathbf{v})_K, \quad \forall \mathbf{v} \in \mathbf{V}_h(K), \\ (\mu \partial_t \mathbf{H}^{\mathbf{J}}, \mathbf{w})_K + (\operatorname{curl} \mathbf{E}^{\mathbf{J}}, \mathbf{w})_K = 0, \quad \forall \mathbf{w} \in \mathbf{V}_h(K). \end{cases}$$

2 Multiscale basis functions

Find $(\boldsymbol{\eta}_i^{\mathbf{E}}, \boldsymbol{\eta}_i^{\mathbf{H}}) \in \mathbf{V}_h(K) \times \mathbf{V}_h(K)$ such that

$$\begin{cases} (\varepsilon \partial_t \boldsymbol{\eta}_i^{\mathbf{E}}, \mathbf{v})_K - (\boldsymbol{\eta}_i^{\mathbf{H}}, \operatorname{curl} \mathbf{v})_K = -(\boldsymbol{\Psi}_i \mathbf{n} \cdot \mathbf{n}^K, \mathbf{v})_{\partial K}, \quad \forall \mathbf{v} \in \mathbf{V}_h(K), \\ (\mu \partial_t \boldsymbol{\eta}_i^{\mathbf{H}}, \mathbf{w})_K + (\operatorname{curl} \boldsymbol{\eta}_i^{\mathbf{E}}, \mathbf{w})_K = 0, \quad \forall \mathbf{w} \in \mathbf{V}_h(K). \end{cases}$$

3 Global problem

Find $(\beta_1^n, \dots, \beta_{\dim(\boldsymbol{\Lambda}_H)}^n) \in \mathbb{R}$ such that

$$\sum_{i=1}^{\dim(\boldsymbol{\Lambda}_H)} \beta_i^n (\boldsymbol{\nu}_H, \boldsymbol{\eta}_i^{\mathbf{E}})_{\partial \mathcal{T}_H} = -(\boldsymbol{\nu}_H, \mathbf{E}^{\mathbf{J}})_{\partial \mathcal{T}_H}, \quad \forall \boldsymbol{\nu}_H \in \boldsymbol{\Lambda}_H$$

MHM for the time-domain Maxwell equations

Summary

Fully discrete local problem for $(\mathbf{E}^J, \mathbf{H}^J)$

$$\begin{cases} \left(\mu \frac{\mathbf{H}^{n,J} - \mathbf{H}^{n-1,J}}{\Delta T}, \mathbf{w}_h \right)_K + \left(\mathbf{E}^{n-\frac{1}{2},J}, \text{curl } \mathbf{w}_h \right)_K = 0 \\ \left(\varepsilon \frac{\mathbf{E}^{n+\frac{1}{2},J} - \mathbf{E}^{n-\frac{1}{2},J}}{\Delta T}, \mathbf{v}_h \right)_K - \left(\mathbf{H}^{n,J}, \text{curl } \mathbf{v}_h \right)_K = (\mathbf{J}^n, \mathbf{v}_h)_K \end{cases}$$

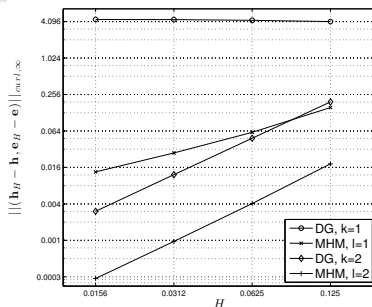
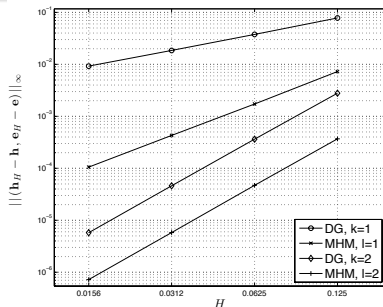
Global problem

$$\sum_{i=1}^{\dim(\Lambda_H)} \beta_i^n (\nu_H, \boldsymbol{\eta}_i^E)_{\partial T_H} = -(\nu_H, \mathbf{E}^{n+\frac{1}{2},J})_{\partial T_H}$$

Multiscale Hybrid-Mixed method

MHM for the time-domain Maxwell equations

- S. Lanteri, D. Paredes, C. Scheid and F. Valentin
SIAM J. Multisc. Model. Simul., 2018
- Second level solver: centered flux DGTD method with explicit LF2 time-stepping
- Formulation and theoretical study in the 3D case
- Numerical study in the 2D case



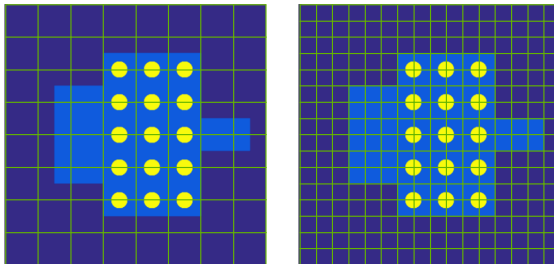
Standing wave in a PEC cavity: comparison between the MHM method and the DG method with respect to the $\|(\mathbf{H} - \mathbf{H}_H, \mathbf{E} - \mathbf{E}_H)\|_\infty$ (left) and $\|(\mathbf{H} - \mathbf{H}_H, \mathbf{E} - \mathbf{E}_H)\|_{\text{curl}, \infty}$ (right) norms.

Multiscale Hybrid-Mixed method

MHM for the time-domain Maxwell equations

Numerical study in 2D

- Nanophotonic waveguide of photonic crystal type
- Holes of silicium in silica encapsulated in air
- Second level discretization: non-dissipative DGTD method with explicit time-stepping and quadratic elements
- Coarse problem: linear elements



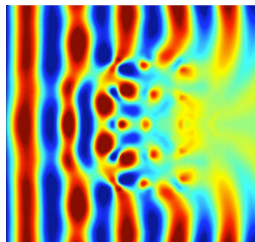
Nanowaveguide problem: coarse quadrangular meshes

Multiscale Hybrid-Mixed method

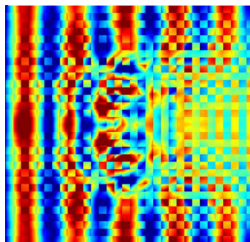
MHM for the time-domain Maxwell equations

Numerical study in 2D

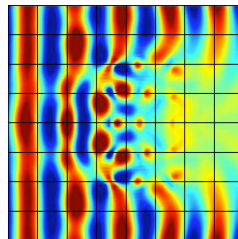
- Nanophotonic waveguide of photonic crystal type
- Holes of silicium in silica encapsulated in air
- Second level discretization: non-dissipative DGTD method with explicit time-stepping and quadratic elements
- Coarse problem: linear elements



DGTD method
with 589,824 DoF



DGTD method
with 4,608 DoF



MHM-DGTD method
with 9,216 DoF

Nanowaveguide problem: contour lines of the amplitude of the electric field at a given time

- 1 Multiscale Hybrid-Mixed (MHM) method
- 2 **MHM-induced local time stepping**
- 3 Numerical results in the 2D case
- 4 Closure

MHM-induced local time stepping

- ΔT : 1st-level (coarse) time step
- Δt^K : 2nd-level (fine) time step verifying the CFL condition for the local problem in K
- Goal: assess the global stability, accuracy and efficiency of MHM-DGTD method

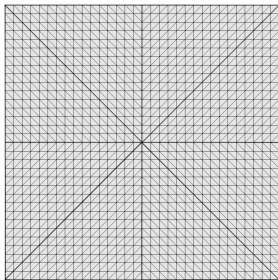
```
Given  $\mathbf{H}^0$  and  $\mathbf{E}^{\frac{1}{2}}$ ;  
for  $n = 1 \rightarrow N$  do  
  for  $k \in \mathcal{T}_H$  do  
    Set  $\mathbf{E}_K^{n,0} = \mathbf{E}_K^{n-1,M_k}$  and  $\mathbf{H}_K^{n,0} = \mathbf{H}_K^{n-1,M_k}$ ;  
    for  $m = 1 \rightarrow M_K$  do  
      if  $n = 1$  then  
        Solve (2) to determine  $\eta_K^{m+\frac{1}{2},E}$  and  $\eta_K^{m,H}$ ;  
      end  
      Solve (1) to determine and  $\mathbf{E}_K^{n,m+\frac{1}{2}}$ , and  $\mathbf{H}_K^{n,m}$ ;  
      Assemble  $\mathbf{E}_K^{n,m+\frac{1}{2}}$ , to the RHS of (3) ;  
    end  
  end  
  Solve (3) to determine  $\beta^n$ ;  
  Update  $\mathbf{E}_K^{n,M_K+\frac{1}{2}} = \beta^n \eta_K^E + \mathbf{E}^{n+\frac{1}{2}}$ ,  
  and  $\mathbf{H}_K^{n,M_K} = \beta^n \eta_K^H + \mathbf{H}^n$ ,  $\forall K \in \mathcal{T}_H$  ;  
end
```

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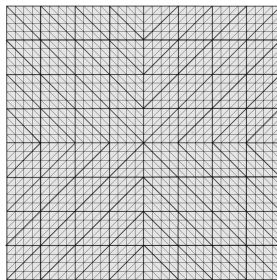
Validation: space refinement

- Standing wave in a PEC cavity
- Macro/micro meshes such that $h = H/2^\beta$
- $\mathbb{P}^k - \mathbb{P}^{k^a}$ polynomial order with k in $\{1, 2, 3, 4\}$

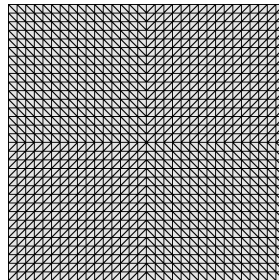
^aFor $\mathbf{V}_h(K)$ and Λ_h respectively.



(a) $H = h \times 2^5$



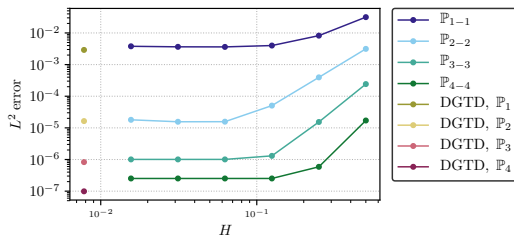
(b) $H = h \times 2^2$



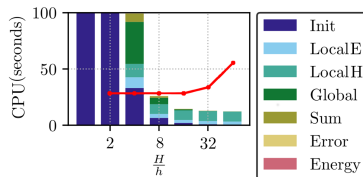
(c) $H = h \times 2^0$

Validation: spatial refinement ($H - h$)

- Standing wave in a PEC cavity
- Macro/micro meshes such that $h = H/2^\beta$
- $\mathbb{P}_k - \mathbb{P}_k$ polynomial order with k in $\{1, 2, 3, 4\}$



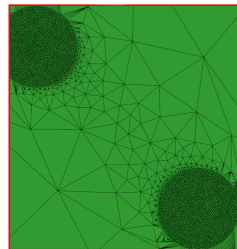
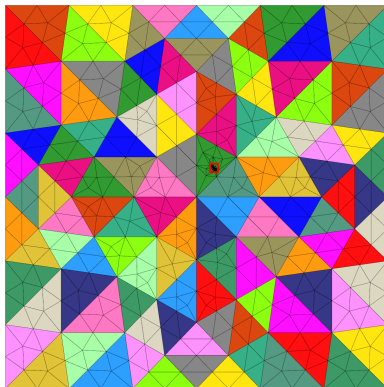
(d) Maximum L_2 error for different H



(e) Breakdown of CPU time

Validation: temporal refinement ($\Delta T - \Delta t$)

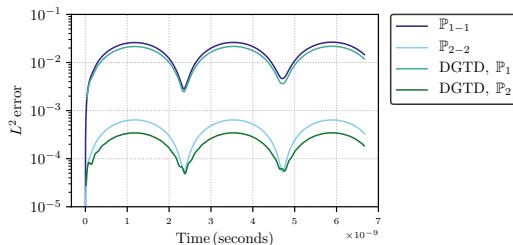
- Standing wave in a PEC cavity
- Locally refined mesh such that $h_{max}/h_{min} = 1500$



Multiscale mesh with zoom in the refined part

Validation: temporal refinement ($\Delta T - \Delta t$)

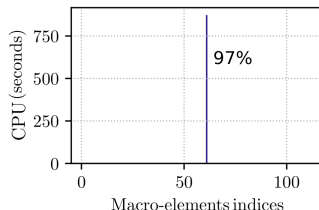
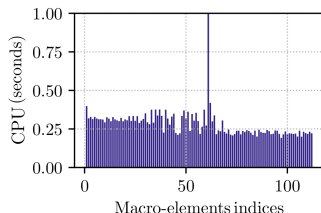
- Standing wave in a PEC cavity
- Locally refined mesh such that $h_{max}/h_{min} = 1500$



L_2 error without local timestepping

Validation: temporal refinement ($\Delta T - \Delta t$)

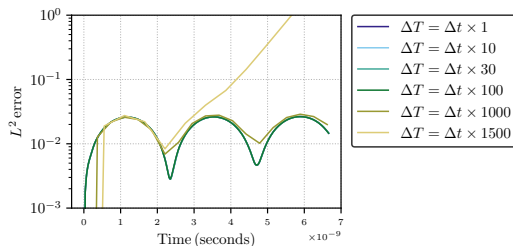
- Standing wave in a PEC cavity
- Locally refined mesh such that $h_{max}/h_{min} = 1500$
- Interpolation orders: $\mathbb{P}_1 - \mathbb{P}_1$



Computation time for multiscale configuration

Validation: temporal refinement ($\Delta T - \Delta t$)

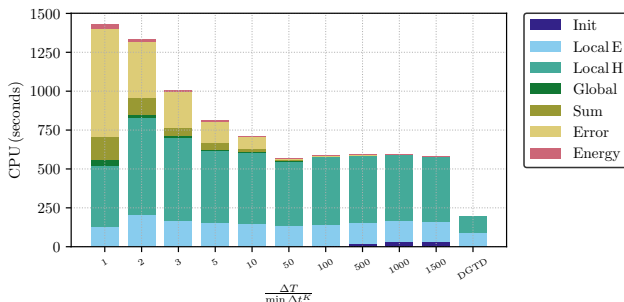
- Standing wave in a PEC cavity
- Locally refined mesh such that $h_{max}/h_{min} = 1500$
- Interpolation orders: $\mathbb{P}_1 - \mathbb{P}_1$



L^2 error for multiscale configuration

Validation: temporal refinement ($\Delta T - \Delta t$)

- Standing wave in a PEC cavity
- Locally refined mesh such that $h_{max}/h_{min} = 1500$
- Interpolation orders: $\mathbb{P}_1 - \mathbb{P}_1$



Computation time for multiscale configuration

- 1 Multiscale Hybrid-Mixed (MHM) method
- 2 MHM-induced local time stepping
- 3 Numerical results in the 2D case
- 4 Closure

Summary of the main features of the MHM method

- First level elements can be meshed independently to yield the second level mesh
- Non-conformity is naturally allowed
- Local problems are independent i.e. can be solved in parallel
- Local time-stepping is possible i.e. local problems can be time advanced with their own time step (or time scheme)
- In the discrete case, first level problem is a linear system with a symmetric positive definite matrix

Open questions and future works

- Stability analysis of the MHM-induced local time stepping method
- A priori convergence analysis (ongoing)
- Implementation in 3D
- High performance computing issues (*data-based* versus *task-based* parallelization)
- Extension to frequency-domain (ongoing)

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Thank you for your attention !



NACHOS project-team

- Numerical methods and high performance algorithms for the numerical modeling of wave interaction with complex geometries/media
- Common project-team with J.A. Dieudonné Mathematics Laboratory UMR CNRS 7351, Côte d'Azur University