



On subspace trails cryptanalysis

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On subspace trails cryptanalysis

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Outline

The AES and the distinguisher of [GRR17]

- The AES

- The distinguisher of Grassi, Rechberger and Rønjom

Proof for the distinguisher

- Case of the AES

- Towards a more general lemma

- Example on another SPN: Midori

Conclusion

The AES and the distinguisher of [GRR17]

The AES

The distinguisher of Grassi, Rechberger and Rønjom

Proof for the distinguisher

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The AES

NIST standard since 2001, SPN on **10 rounds**, **128-bit** blocks [DR02].

$$x = \begin{pmatrix} \textcolor{red}{x_0} & x_4 & x_8 & x_{12} \\ \textcolor{red}{x_1} & x_5 & x_9 & x_{13} \\ \textcolor{red}{x_2} & x_6 & x_{10} & x_{14} \\ \textcolor{red}{x_3} & x_7 & x_{11} & x_{15} \end{pmatrix} \in \mathbb{F}_{2^8}^{16}$$

$$\text{S-box} \begin{cases} \mathbb{F}_{2^8} & \rightarrow \mathbb{F}_{2^8} \\ x_i & \mapsto y_i \end{cases}$$

$$SR(y) = \begin{pmatrix} \textcolor{red}{y_0} & y_4 & y_8 & y_{12} \\ y_5 & y_9 & y_{13} & \textcolor{red}{y_1} \\ y_{10} & y_{14} & \textcolor{red}{y_2} & y_6 \\ y_{15} & \textcolor{red}{y_3} & y_7 & y_{11} \end{pmatrix}$$

ShiftRows *SR*

$$MC(z) = \begin{pmatrix} 2 & 3 & 1 & 1 \\ 1 & 2 & 3 & 1 \\ 1 & 1 & 2 & 3 \\ 3 & 1 & 1 & 2 \end{pmatrix} \times z$$

MixColumns *MC*

The AES and the distinguisher of [GRR17]

The AES

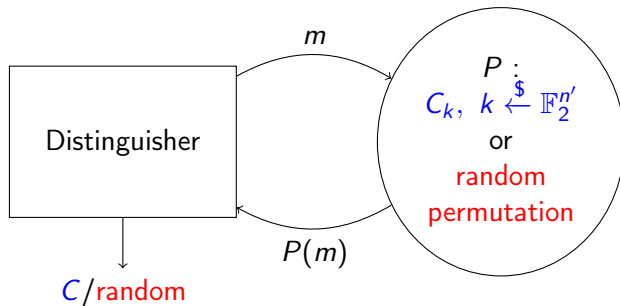
The distinguisher of Grassi, Rechberger and Rønjom

Proof for the distinguisher

Conclusion

What is a distinguisher ?

Let C_k be a cipher with key k ,



Distinguisher \rightarrow **attack** (on more rounds).

Grassi, Rechberger and Rønjom at Eurocrypt 2017 [GRR17]

$\rightarrow C = 5$ AES rounds.

Some definitions...

$$\mathbb{K} = \mathbb{F}_{2^8} \quad \begin{pmatrix} x_0 & x_4 & x_8 & x_{12} \\ x_1 & x_5 & x_9 & x_{13} \\ x_2 & x_6 & x_{10} & x_{14} \\ x_3 & x_7 & x_{11} & x_{15} \end{pmatrix} \in \mathcal{M}_4(\mathbb{K}) \quad x_i \in \mathbb{K}$$

$$\begin{pmatrix} x_0 & 0 & 0 & 0 \\ x_1 & 0 & 0 & 0 \\ x_2 & 0 & 0 & 0 \\ x_3 & 0 & 0 & 0 \end{pmatrix} \in \mathcal{C}_0$$

Columns

$$\mathcal{C}_i = \text{vect}_{\mathbb{K}}(e_{0,i}, e_{1,i}, e_{2,i}, e_{3,i})$$

$$\begin{pmatrix} 0 & x_0 & 0 & y_0 \\ 0 & x_1 & 0 & y_1 \\ 0 & x_2 & 0 & y_2 \\ 0 & x_3 & 0 & y_3 \end{pmatrix} \in \mathcal{C}_{\{1,3\}}$$

$$I \subseteq \llbracket 0, 3 \rrbracket :$$

$$\mathcal{C}_I = \bigoplus_{i \in I} \mathcal{C}_i.$$

$$\begin{pmatrix} x_0 & 0 & 0 & 0 \\ 0 & x_1 & 0 & 0 \\ 0 & 0 & x_2 & 0 \\ 0 & 0 & 0 & x_3 \end{pmatrix} \in \mathcal{D}_0,$$

Diagonals:
 $\mathcal{D}_i = SR^{-1}(\mathcal{C}_i)$

$$\begin{pmatrix} x_0 & 0 & 0 & 0 \\ 0 & 0 & 0 & x_1 \\ 0 & 0 & x_2 & 0 \\ 0 & x_3 & 0 & 0 \end{pmatrix} \in \mathcal{ID}_0,$$

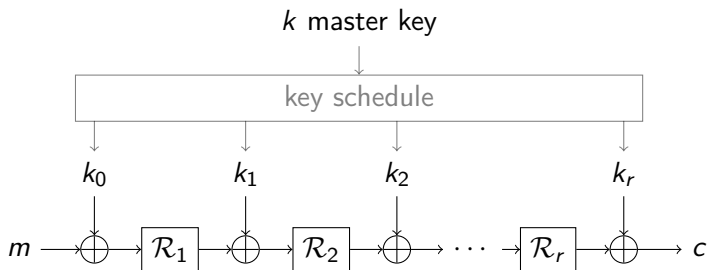
Anti-diagonals:
 $\mathcal{ID}_i = SR(\mathcal{C}_i)$

$$\begin{pmatrix} 2 \cdot x_0 & x_1 & x_2 & 3 \cdot x_3 \\ x_0 & x_1 & 3 \cdot x_2 & 2 \cdot x_3 \\ x_0 & 3 \cdot x_1 & 2 \cdot x_2 & x_3 \\ 3 \cdot x_0 & 2 \cdot x_1 & x_2 & x_3 \end{pmatrix} \in \mathcal{M}_0.$$

Mixed:
 $\mathcal{M}_i = MC(\mathcal{ID}_i)$

$$\mathcal{D}_I \xrightarrow{S} \mathcal{D}_I \xrightarrow{SR} \mathcal{C}_I \xrightarrow{MC} \mathcal{C}_I \xrightarrow{S} \mathcal{C}_I \xrightarrow{SR} \mathcal{ID}_I \xrightarrow{MC} \mathcal{M}_I$$

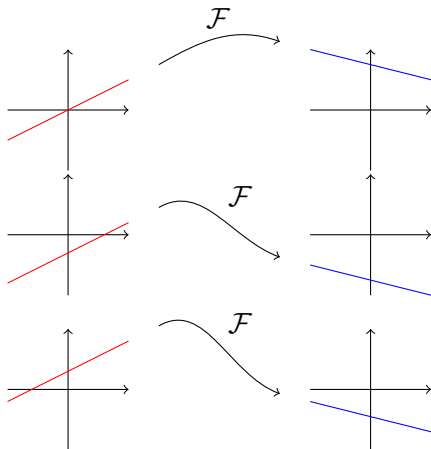
The AES is a key-alternating blockcipher



Subspace trails

Definition ([LTW18])

We have $U \xRightarrow{\mathcal{F}} V$ if $\forall a \in \mathbb{K}^N, \exists b \in \mathbb{K}^N : \mathcal{F}(U + a) \subseteq V + b$.



Examples:

- ▶ $\{0\} \xRightarrow{\mathcal{F}} \{0\}$
- ▶ $U \xRightarrow{\mathcal{F}} \mathbb{K}^N$
- ▶ $\mathcal{D}_I \xRightarrow{\mathcal{R}} \mathcal{C}_I$
- ▶ $\mathcal{C}_I \xRightarrow{\mathcal{R}} \mathcal{M}_I$

$$\mathcal{D}_0 \xRightarrow{\mathcal{R}} \mathcal{C}_0$$

$$\forall a, \forall x,$$

$$\begin{aligned}
 & \begin{pmatrix} x_0 & 0 & 0 & 0 \\ 0 & x_1 & 0 & 0 \\ 0 & 0 & x_2 & 0 \\ 0 & 0 & 0 & x_3 \end{pmatrix} \xrightarrow{+a} \begin{pmatrix} x_0 + a_0 & * & * & * \\ * & x_1 + a_1 & * & * \\ * & * & x_2 + a_2 & * \\ * & * & * & x_3 + a_3 \end{pmatrix} \\
 & \xrightarrow{\mathcal{S}} \begin{pmatrix} y_0 & * & * & * \\ * & y_1 & * & * \\ * & * & y_2 & * \\ * & * & * & y_3 \end{pmatrix} \xrightarrow{SR} \begin{pmatrix} y_0 & * & * & * \\ y_1 & * & * & * \\ y_2 & * & * & * \\ y_3 & * & * & * \end{pmatrix} \\
 & \xrightarrow{MC} \begin{pmatrix} \vdots & * & * & * \\ \vdots & * & * & * \\ MC(y) & * & * & * \\ \vdots & * & * & * \end{pmatrix}
 \end{aligned}$$

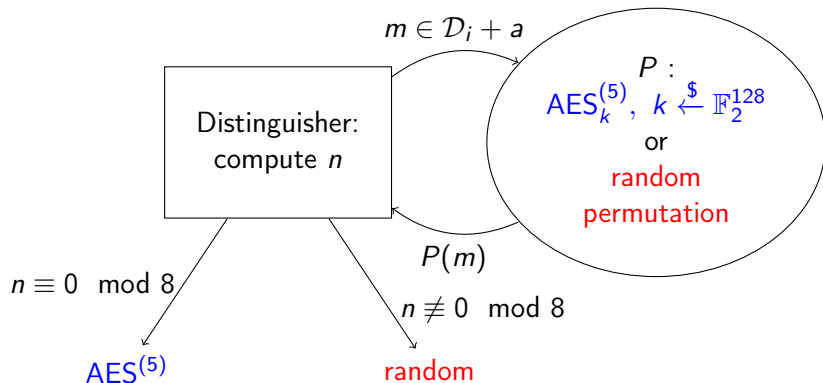
The distinguisher

Theorem ([GRR17])

Let $a \in \mathcal{M}_4(\mathbb{K})$, $i \in \llbracket 0, 3 \rrbracket$, $J \subseteq \llbracket 0, 3 \rrbracket$. We define

$$n = \#\{ \{p^0, p^1\} \in \mathcal{P}^2(\mathcal{D}_i + a) \mid \mathcal{R}^5(p^0) + \mathcal{R}^5(p^1) \in \mathcal{M}_J \}.$$

Then $n \equiv 0 \pmod{8}$.



The AES and the distinguisher of [GRR17]

Proof for the distinguisher

Case of the AES

Towards a more general lemma

Example on another SPN: Midori

Conclusion

A key lemma

Lemma ([GRR17])

Let $a \in \mathcal{M}_4(\mathbb{K})$, $I \subset \llbracket 0, 3 \rrbracket$, $J \subseteq \llbracket 0, 3 \rrbracket$. We define

$$n = \#\{ \{p^0, p^1\} \in \mathcal{P}^2(\mathcal{M}_I + a) \mid \mathcal{R}(p^0) + \mathcal{R}(p^1) \in \mathcal{D}_J \}.$$

Then $n \equiv 0 \pmod{8}$.

$$\overbrace{\mathcal{D}_I \xrightarrow{\mathcal{R}} \mathcal{C}_I \xrightarrow{\mathcal{R}} \mathcal{M}_I}^2 \xrightarrow[\text{Lemma } \mathcal{R}]{1} \overbrace{\mathcal{D}_J \xrightarrow{\mathcal{R}} \mathcal{C}_J \xrightarrow{\mathcal{R}} \mathcal{M}_J}^2$$

Our contribution starts here

- ▶ Search for the underlying property ;
- ▶ write a better proof for it to come out;
- ▶ generalize ?

Step 1: equivalence relation between pairs

In \mathcal{M}_0 ,

$$\left\{ \begin{pmatrix} 2 \cdot x_0 & x_1 & z_2 & 3 \cdot z_3 \\ x_0 & x_1 & 3 \cdot z_2 & 2 \cdot z_3 \\ x_0 & 3 \cdot x_1 & 2 \cdot z_2 & z_3 \\ 3 \cdot x_0 & 2 \cdot x_1 & z_2 & z_3 \end{pmatrix}, \begin{pmatrix} 2 \cdot y_0 & y_1 & z_2 & 3 \cdot z_3 \\ y_0 & y_1 & 3 \cdot z_2 & 2 \cdot z_3 \\ y_0 & 3 \cdot y_1 & 2 \cdot z_2 & z_3 \\ 3 \cdot y_0 & 2 \cdot y_1 & z_2 & z_3 \end{pmatrix} \right\}$$

$$\sim$$

$$\left\{ \begin{pmatrix} 2 \cdot x_0 & y_1 & w_2 & 3 \cdot w_3 \\ x_0 & y_1 & 3 \cdot w_2 & 2 \cdot w_3 \\ x_0 & 3 \cdot y_1 & 2 \cdot w_2 & w_3 \\ 3 \cdot x_0 & 2 \cdot y_1 & w_2 & w_3 \end{pmatrix}, \begin{pmatrix} 2 \cdot y_0 & x_1 & w_2 & 3 \cdot w_3 \\ y_0 & x_1 & 3 \cdot w_2 & 2 \cdot w_3 \\ y_0 & 3 \cdot x_1 & 2 \cdot w_2 & w_3 \\ 3 \cdot y_0 & 2 \cdot x_1 & w_2 & w_3 \end{pmatrix} \right\}$$

$$\left\{ \begin{pmatrix} 2 \cdot x_0 & x_1 & z_2 & 3 \cdot z_3 \\ x_0 & x_1 & 3 \cdot z_2 & 2 \cdot z_3 \\ x_0 & 3 \cdot x_1 & 2 \cdot z_2 & z_3 \\ 3 \cdot x_0 & 2 \cdot x_1 & z_2 & z_3 \end{pmatrix}, \begin{pmatrix} 2 \cdot y_0 & y_1 & z_2 & 3 \cdot z_3 \\ y_0 & y_1 & 3 \cdot z_2 & 2 \cdot z_3 \\ y_0 & 3 \cdot y_1 & 2 \cdot z_2 & z_3 \\ 3 \cdot y_0 & 2 \cdot y_1 & z_2 & z_3 \end{pmatrix} \right\}$$

$$\sim$$

$$\left\{ \begin{pmatrix} 2 \cdot x_0 & y_1 & w_2 & 3 \cdot w_3 \\ x_0 & y_1 & 3 \cdot w_2 & 2 \cdot w_3 \\ x_0 & 3 \cdot y_1 & 2 \cdot w_2 & w_3 \\ 3 \cdot x_0 & 2 \cdot y_1 & w_2 & w_3 \end{pmatrix}, \begin{pmatrix} 2 \cdot y_0 & x_1 & w_2 & 3 \cdot w_3 \\ y_0 & x_1 & 3 \cdot w_2 & 2 \cdot w_3 \\ y_0 & 3 \cdot x_1 & 2 \cdot w_2 & w_3 \\ 3 \cdot y_0 & 2 \cdot x_1 & w_2 & w_3 \end{pmatrix} \right\}$$

Definition

Let $\{p^0, p^1\}$ a pair of states from $\mathcal{M}_I + a$. The **information set** K of the pair $\{p^0, p^1\}$ is $\{k \in \llbracket 0, 3 \rrbracket \mid \exists i \in I : x_{i,k} \neq y_{i,k}\}$.

It is $K = \{0, 1\}$ in the example.

$$\left\{ \begin{pmatrix} 2 \cdot x_0 & x_1 & z_2 & 3 \cdot z_3 \\ x_0 & x_1 & 3 \cdot z_2 & 2 \cdot z_3 \\ x_0 & 3 \cdot x_1 & 2 \cdot z_2 & z_3 \\ 3 \cdot x_0 & 2 \cdot x_1 & z_2 & z_3 \end{pmatrix}, \begin{pmatrix} 2 \cdot y_0 & y_1 & z_2 & 3 \cdot z_3 \\ y_0 & y_1 & 3 \cdot z_2 & 2 \cdot z_3 \\ y_0 & 3 \cdot y_1 & 2 \cdot z_2 & z_3 \\ 3 \cdot y_0 & 2 \cdot y_1 & z_2 & z_3 \end{pmatrix} \right\}$$

$$\sim$$

$$\left\{ \begin{pmatrix} 2 \cdot x_0 & y_1 & w_2 & 3 \cdot w_3 \\ x_0 & y_1 & 3 \cdot w_2 & 2 \cdot w_3 \\ x_0 & 3 \cdot y_1 & 2 \cdot w_2 & w_3 \\ 3 \cdot x_0 & 2 \cdot y_1 & w_2 & w_3 \end{pmatrix}, \begin{pmatrix} 2 \cdot y_0 & x_1 & w_2 & 3 \cdot w_3 \\ y_0 & x_1 & 3 \cdot w_2 & 2 \cdot w_3 \\ y_0 & 3 \cdot x_1 & 2 \cdot w_2 & w_3 \\ 3 \cdot y_0 & 2 \cdot x_1 & w_2 & w_3 \end{pmatrix} \right\}$$

Definition

Let $P = \{p^0, p^1\}$, $Q = \{q^0, q^1\} \in \mathcal{P}^2(\mathcal{M}_I + a)$. We have $P \sim Q$ if:

- ▶ K is the information set of $P \Rightarrow K$ is the information set of Q .
- ▶ $\forall k \in K, \exists b \in \{0, 1\} : \forall i \in I, q_{i,k}^0 = p_{i,k}^b$ et $q_{i,k}^1 = p_{i,k}^{1-b}$.

\sim is an equivalence relation on $\mathcal{P}^2(\mathcal{M}_I + a)$.

Lemma

The function

$$\begin{aligned} f : \mathcal{P}^2(\mathcal{M}_I + a) &\longrightarrow \mathcal{M}_4(\mathbb{K}) \\ \{p^0, p^1\} &\longmapsto \mathcal{R}(p^0) + \mathcal{R}(p^1) \end{aligned}$$

is constant on the equivalence classes of \sim .

Proposition

Let \mathfrak{C} be an equivalence class K . Then

$$\#\mathfrak{C} = 2^{|K|-1+8|I|(4-|K|)} \equiv 0 \pmod{8}.$$

Lemma

If

$$n = \#\{ \{p^0, p^1\} \in \mathcal{P}^2(\mathcal{M}_I + a) \mid \mathcal{R}(p^0) + \mathcal{R}(p^1) \in \mathcal{D}_J \},$$

then $n \equiv 0 \pmod{8}$.

Proof.

$$\begin{aligned}
 n &= \#f^{-1}(\mathcal{D}_J) \\
 &= \sum_{\mathfrak{C} \in \mathcal{P}^2(\mathcal{M}_I + a) / \sim} \#(f^{-1}(\mathcal{D}_J) \cap \mathfrak{C}) \\
 &= \sum_{\mathfrak{C} \in \mathcal{P}^2(\mathcal{M}_I + a) / \sim} 1_{\tilde{f}(\mathfrak{C}) \in \mathcal{D}_J} \# \mathfrak{C} \\
 &\equiv 0 \pmod{8}
 \end{aligned}$$

What about the branch number ?

In [GRR17], the proof needs maximal branch number. But...

Proposition ([GRR16])

Let $I, J \subseteq \llbracket 0, 3 \rrbracket$ and b be the differential branch number of MC. Then

$$|I| + |J| < b \quad \Rightarrow \quad \mathcal{D}_I \cap \mathcal{M}_J = \{0\}$$

If $\{p^0, p^1\} \in \mathcal{P}^2(\mathcal{M}_I + a)$ has information set K ,

$$p^0 + p^1 \in \mathcal{C}_K \text{ and then } \mathcal{R}(p^0) + \mathcal{R}(p^1) \in \mathcal{M}_K.$$

If $|K| < b - |J|$, $\mathcal{M}_K \cap \mathcal{D}_J = \{0\}$ and $\mathcal{R}(p^0) + \mathcal{R}(p^1) \notin \mathcal{D}_J$.

Lemma

If

$$n = \#\{ \{p^0, p^1\} \in \mathcal{P}^2(\mathcal{M}_I + a) \mid \mathcal{R}(p^0) + \mathcal{R}(p^1) \in \mathcal{D}_J \},$$

then $n \equiv 0 \pmod{8}$.

Proof.

$$\begin{aligned}
 n &= \sum_{\mathfrak{C} \in \mathcal{P}^2(\mathcal{M}_I + a) / \sim} 1_{\tilde{f}(\mathfrak{C}) \in \mathcal{D}_J} \# \mathfrak{C} \\
 &= \sum_{h=0}^4 \sum_{\mathfrak{C}: |K(\mathfrak{C})|=h} 1_{\tilde{f}(\mathfrak{C}) \in \mathcal{D}_J} \# \mathfrak{C} \\
 &= \sum_{h=\textcolor{red}{b}-|J|}^4 \sum_{\mathfrak{C}: |K(\mathfrak{C})|=h} 1_{\tilde{f}(\mathfrak{C}) \in \mathcal{D}_J} \# \mathfrak{C}
 \end{aligned}$$

The AES and the distinguisher of [GRR17]

Proof for the distinguisher

- Case of the AES

- Towards a more general lemma

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Conclusion

Definition

Let $V \subseteq \mathbb{K}^N$ be a \mathbb{K} -subspace. We say V is compatible with \mathcal{S} if it has a basis g such that its matrix in basis f is of the form:

$$\begin{pmatrix}
 * & \dots & * & & & & \\
 \vdots & \lambda_{0,\ell,i} & \vdots & 0 & & 0 & \\
 * & \dots & * & & & & \\
 & & & * & \dots & * & \\
 & 0 & & \vdots & \lambda_{k,\ell,i} & \vdots & 0 \\
 & & & * & \dots & * & \\
 & & & & & & * & \dots & * \\
 & 0 & & 0 & & \vdots & \lambda_{h-1,\ell,i} & \vdots & \\
 & & & & & * & \dots & * \\
 & 0 & & 0 & & & 0 & & \\
 & \uparrow & & \uparrow & & & \uparrow & & \\
 & g_{0,i} & & g_{k,i} & & & g_{h-1,i} & &
 \end{pmatrix}$$

$$\begin{pmatrix} 2 & & & & & & & \\ 1 & & & & & & & \\ 1 & & & & & & & \\ 3 & & & & & & & \\ & 1 & & & & & & \\ & 1 & & & & & & \\ & 3 & & & & & & \\ & 2 & & & & & & \\ & & 1 & & & & & \\ & & 3 & & & & & \\ & & 2 & & & & & \\ & & 1 & & & & & \\ & & & 3 & & & & \\ & & & 2 & & & & \\ & & & 1 & & & & \\ & & & 1 & & & & \end{pmatrix}$$

\mathcal{M}_0 is compatible with \mathcal{S}_{AES} .

Likewise, $\mathcal{M}_0 \cap \mathcal{C}_{0,1}$ is compatible with \mathcal{S}_{AES} .

$$\begin{pmatrix} 2 \cdot x_0 & x_1 & 0 & 0 \\ x_0 & x_1 & 0 & 0 \\ x_0 & 3 \cdot x_1 & 0 & 0 \\ 3 \cdot x_0 & 2 \cdot x_1 & 0 & 0 \end{pmatrix} \in \mathcal{M}_0 \cap \mathcal{C}_{0,1}.$$

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Midori

Midori, presented at Asiacrypt 2015 [BBI⁺15].

Goal: low energy consumption.

- ▶ $\mathcal{R} : \mathbb{F}_2^{128} \rightarrow \mathbb{F}_2^{128}$
- ▶ S-box: $\mathbb{F}_{2^8} \rightarrow \mathbb{F}_{2^8}$
- ▶ \mathcal{L} :
 - ▶ ShuffleCell SC (more complex ShiftRows)
 - ▶ MixColumns MC

$$M_{MC} = \begin{pmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix}$$

$$\overbrace{\mathcal{D}'_I \xRightarrow{\mathcal{R}} C_I \xRightarrow{\mathcal{R}} \mathcal{M}'_I}^2 \quad \overbrace{\text{Generalized Lemma}}^1 \quad \overbrace{\mathcal{D}'_J \xRightarrow{\mathcal{R}} C_J \xRightarrow{\mathcal{R}} \mathcal{M}'_J}^2$$

$\mathcal{R} \dashrightarrow$

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What now ?

- ▶ The generalization can be useful (the distinguisher can be easily transposed)
but cannot give better results!
- ▶ Working on subspace trails [LTW18].

$$\overbrace{D'_I \xRightarrow{\mathcal{R}} C_I \xRightarrow{\mathcal{R}} M'_I}^2 \quad \xrightarrow[\mathcal{R}]{\text{Generalized Lemma}} \quad \overbrace{D'_J \xRightarrow{\mathcal{R}} C_J \xRightarrow{\mathcal{R}} M'_J}^2$$



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