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On CCZ-Equivalence, Extended-Affine Equivalence and Function Twisting

Anne Canteaut, Léo Perrin

October 9, 2018

Journées C2



Cryptographic Properties

$F : \mathbb{F}_2^n \rightarrow \mathbb{F}_2^m$ and $G : \mathbb{F}_2^n \rightarrow \mathbb{F}_2^m$ are functions (e.g. S-Boxes).

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Definition (DDT/LAT)

The D(ifference) D(istribution) T(able) of $F : \mathbb{F}_2^n \rightarrow \mathbb{F}_2^m$ is

$$\mathcal{D}_F(\alpha, \beta) = \# \{x, F(x \oplus \alpha) \oplus F(x) = \beta\}$$

The L(inear) A(pproximation) T(able) of $F : \mathbb{F}_2^n \rightarrow \mathbb{F}_2^m$ is

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Big APN Problem

Is there an APN permutation on $2t$ bits such that $\max(\text{DDT}) = 2$?

Equivalence Relations (1/2)

Definition (Affine-Equivalence)

F and G are *affine equivalent* if $G(x) = (B \circ F \circ A)(x)$, where A, B are affine permutations.

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Definition (EA-Equivalence; EA-mapping)

F and G are *Extended Affine equivalent* if, up to translations, $G(x) = (B \circ F \circ A)(x) + C(x)$, where A, B, C are linear and A, B are permutations; so that

$$\{(x, G(x)), \forall x \in \mathbb{F}_2^n\} = \begin{bmatrix} A^{-1} & 0 \\ CA^{-1} & B \end{bmatrix} (\{(x, F(x)), \forall x \in \mathbb{F}_2^n\}) .$$

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Affine permutations with such linear part are **EA-mappings**.

Equivalence Relations (2/2)

Definition (CCZ-Equivalence)

$F : \mathbb{F}_2^n \rightarrow \mathbb{F}_2^m$ and $G : \mathbb{F}_2^n \rightarrow \mathbb{F}_2^m$ are *C(arlet)-C(harpin)-Z(inoviev)* equivalent if, up to translations,

$$\Gamma_G = \{(x, G(x)), \forall x \in \mathbb{F}_2^n\} = L(\{(x, F(x)), \forall x \in \mathbb{F}_2^n\}) = L(\Gamma_F),$$

where $L : \mathbb{F}_2^{n+m} \rightarrow \mathbb{F}_2^{n+m}$ is a linear permutation.

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- CCZ-equivalence preserves the distribution of the coefficients in the DDT and the LAT.
- It does **not** preserve bijectivity.
- It does **not** preserve the algebraic degree.

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- CCZ-equivalence preserves the distribution of the coefficients in the DDT and the LAT.
- It does **not** preserve bijectivity.
- It does **not** preserve the algebraic degree.
- It plays a crucial role in the investigation of the big APN problem.

The Problem with CCZ-Equivalence

Admissible Mapping

For $F : \mathbb{F}_2^n \rightarrow \mathbb{F}_2^m$, the affine permutation L is **admissible for F** if

$$L(\{(x, F(x)), \forall x \in \mathbb{F}_2^n\}) = \{(x, G(x)), \forall x \in \mathbb{F}_2^n\}$$

for a well defined function $G : \mathbb{F}_2^n \rightarrow \mathbb{F}_2^m$.

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- 1 How do we find admissible mapping?
- 2 Is there a simpler way of seeing CCZ-equivalence?
- 3 How do we know if a function is CCZ-equivalent to a permutation?

Structure of this talk

- 1 CCZ-Equivalence and Vector Spaces of 0
- 2 Function Twisting
- 3 Necessary and Efficient Conditions for CCZ-Equivalence to a Permutation
- 4 Conclusion

Outline

- 1 **CCZ-Equivalence and Vector Spaces of 0**
- 2 Function Twisting
- 3 Necessary and Efficient Conditions for CCZ-Equivalence to a Permutation
- 4 Conclusion

Plan of this Section

- 1 **CCZ-Equivalence and Vector Spaces of 0**
 - Vector Spaces of Zeroes
 - Partitioning a CCZ-Class into EA-Classes
- 2 Function Twisting
- 3 Necessary and Efficient Conditions for CCZ-Equivalence to a Permutation
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Walsh Zeroes

Definition (Walsh Zeroes)

The *Walsh zeroes* of $F : \mathbb{F}_2^n \rightarrow \mathbb{F}_2^m$ is the set

$$\mathcal{Z}_F = \{u \in \mathbb{F}_2^n \times \mathbb{F}_2^m, \mathcal{W}_F(u) = 0\} \cup \{0\}.$$

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For all $F : \mathbb{F}_2^n \rightarrow \mathbb{F}_2^m$, we have

$$\mathcal{W}_F(\alpha, 0) = \sum_{x \in \mathbb{F}_2^n} (-1)^{\alpha \cdot x + 0 \cdot F(x)} = 0$$

so that

$$\mathcal{V} = \{(x, 0^m), \forall x \in \mathbb{F}_2^n\} \subset \mathcal{Z}_F \subset \mathbb{F}_2^{n+m}$$

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Note that if $\Gamma_G = L(\Gamma_F)$, then $\mathcal{Z}_G = (L^T)^{-1}(\mathcal{Z}_F)$.

Admissibility for F

Lemma

Let $L : \mathbb{F}_2^{n+m} \rightarrow \mathbb{F}_2^{n+m}$ be a linear permutation. It is admissible for $F : \mathbb{F}_2^n \rightarrow \mathbb{F}_2^m$ if and only if

$$L^T(\mathcal{V}) \subseteq \mathcal{Z}_F$$

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Example

EA-mappings are admissible for all $F : \mathbb{F}_2^n \rightarrow \mathbb{F}_2^m$:

$$\begin{bmatrix} A & 0 \\ C & B \end{bmatrix}^T (\mathcal{V}) = \begin{bmatrix} A^T & C^T \\ 0 & B^T \end{bmatrix} \left(\left\{ \begin{bmatrix} x \\ 0 \end{bmatrix}, \forall x \in \mathbb{F}_2^n \right\} \right) = \mathcal{V}.$$

Permutations

We define

$$\mathcal{V}^\perp = \{(\mathbf{0}^n, y), \forall y \in \mathbb{F}_2^m\} \subset \mathbb{F}_2^{n+m}.$$

Lemma

$F : \mathbb{F}_2^n \rightarrow \mathbb{F}_2^m$ is a permutation if and only if

$$\mathcal{V}^\perp \subset \mathcal{Z}_F.$$

EA-classes imply vector spaces

Lemma

let F , G and G' be such that $\Gamma_G = L(\Gamma_F)$ and $\Gamma_{G'} = L'(\Gamma_F)$.
If $L(\mathcal{V}) = L'(\mathcal{V})$, then G and G' are EA-equivalent.

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Can we use this knowledge to partition a CCZ-class into its EA-classes?

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The Lemma gives us hope!

1 EA-class \implies 1 vector space of zeroes of dimension n in \mathcal{Z}_n

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Reality takes it back...

The converse of the lemma is wrong.

Outline

- 1 CCZ-Equivalence and Vector Spaces of 0
- 2 Function Twisting**
- 3 Necessary and Efficient Conditions for CCZ-Equivalence to a Permutation
- 4 Conclusion

Plan of this Section

- 1 CCZ-Equivalence and Vector Spaces of 0
- 2 **Function Twisting**
 - The Twist
 - $\text{CCZ} = \text{EA} + \text{Twist}$
- 3 Necessary and Efficient Conditions for CCZ-Equivalence to a Permutation
- 4 Conclusion

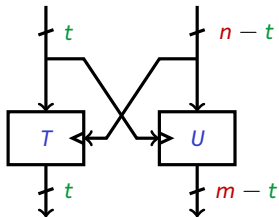
EA-equivalence is a simple sub-case of CCZ-Equivalence...

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What must we add to EA-equivalence to fully describe CCZ-Equivalence?

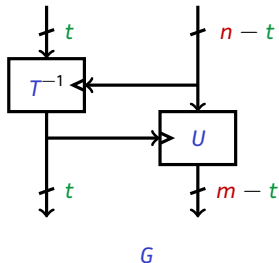
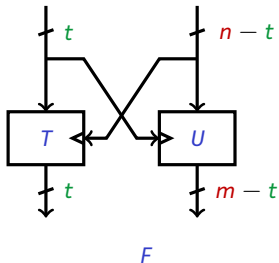
Definition of the Twist

Any function $F : \mathbb{F}_2^n \rightarrow \mathbb{F}_2^m$ can be projected on $\mathbb{F}_2^t \times \mathbb{F}_2^{m-t}$.



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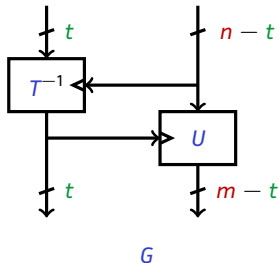
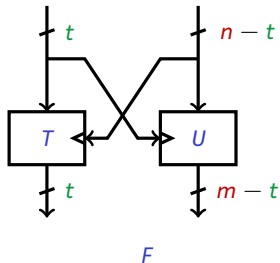
If T is a permutation for all secondary inputs, then we define the t -twist equivalent of F as G , where

$$G(x, y) = (T_y^{-1}(x), U_{T_y^{-1}(x)}(y))$$

for all $(x, y) \in \mathbb{F}_2^t \times \mathbb{F}_2^{n-t}$.

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The identity is a 0-twist, functional inversion is an n -twist.

Swap Matrices

The **swap matrix** permuting \mathbb{F}_2^{n+m} is defined for $t \leq \min(n, m)$ as

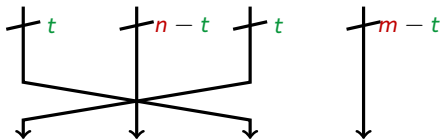
$$M_t = \begin{bmatrix} 0 & 0 & I_t & 0 \\ 0 & I_{n-t} & 0 & 0 \\ I_t & 0 & 0 & 0 \\ 0 & 0 & 0 & I_{m-t} \end{bmatrix}.$$

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It has a simple interpretation:

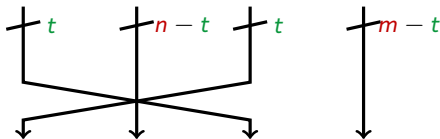


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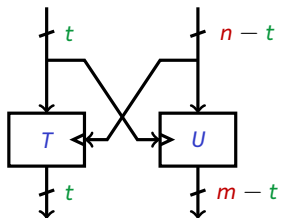
It has a simple interpretation:



For all $t \leq \min(n, m)$, M_t is an **orthogonal** and **symmetric involution**.

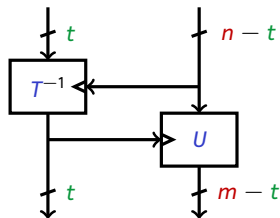
Swap Matrices and Twisting

$$F : \mathbb{F}_2^n \rightarrow \mathbb{F}_2^m$$



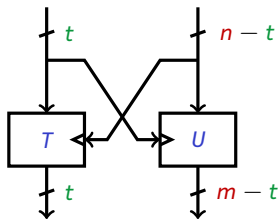
t -twist

$$G : \mathbb{F}_2^n \rightarrow \mathbb{F}_2^m$$



Swap Matrices and Twisting

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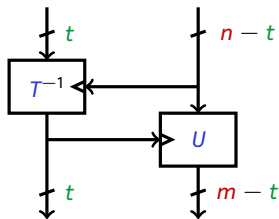


$$\Gamma_F = \{ (x, F(x)), \forall x \in \mathbb{F}_2^n \}$$

t -twist

M_t

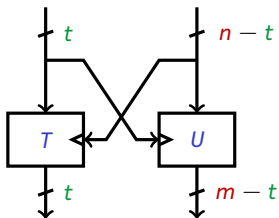
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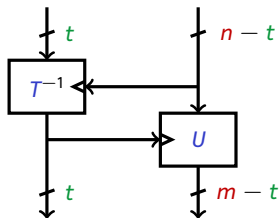
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$$\Gamma_G = \{ (x, G(x)), \forall x \in \mathbb{F}_2^n \}$$

\longleftrightarrow t -twist \longleftrightarrow

\longleftrightarrow M_t \longleftrightarrow

$$\mathcal{W}_F(u) = \mathcal{W}_G(M_t(u))$$

Twisting and CCZ-Class

Lemma

Twisting preserves the CCZ-equivalence class.

Main Result

Theorem

If $F : \mathbb{F}_2^n \rightarrow \mathbb{F}_2^m$ and $G : \mathbb{F}_2^n \rightarrow \mathbb{F}_2^m$ are CCZ-equivalent, then

$$\Gamma_G = (B \times M_t \times A)(\Gamma_F),$$

where A and B are EA-mappings and where

$$t = \dim(\text{proj}_{\mathcal{V}^\perp}((A^T \times M_t \times B^T)(\mathcal{V}))) .$$

In other words, EA-equivalence and twists are sufficient to fully describe CCZ-equivalence!

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In other words, EA-equivalence and twists are sufficient to fully describe CCZ-equivalence!

Corollary

If a function is CCZ-equivalent but not EA-equivalent to another function, then they have to be EA-equivalent to functions for which a t -twist is possible.

Outline

- 1 CCZ-Equivalence and Vector Spaces of 0
- 2 Function Twisting
- 3 Necessary and Efficient Conditions for CCZ-Equivalence to a Permutation**
- 4 Conclusion

Plan of this Section

- 1 CCZ-Equivalence and Vector Spaces of 0
- 2 Function Twisting
- 3 **Necessary and Efficient Conditions for CCZ-Equivalence to a Permutation**
 - Efficient Criterion
 - Applications to APN Functions
- 4 Conclusion

Another Problem

How do we know if a function is CCZ-equivalent to a permutation?

Remainder

Recall that F is a permutation if and only if $\mathcal{V} \subset \mathcal{Z}_F$ and $\mathcal{V}^\perp \subset \mathcal{Z}_F$.

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Lemma

G is CCZ-equivalent to a permutation if and only if

$$V = L(\mathcal{V}) \subset \mathcal{Z}_G \text{ and } V' = L(\mathcal{V}^\perp) \subset \mathcal{Z}_G$$

for some linear permutation L . Note that

$$\text{span}(V \cup V') = \mathbb{F}_2^n \times \mathbb{F}_2^m.$$

Projected Spaces Criterion

Key observation

The projections

$$p : (x, y) \mapsto x \text{ and } p' : (x, y) \mapsto y$$

mapping $\mathbb{F}_2^n \times \mathbb{F}_2^m$ to \mathbb{F}_2^n and \mathbb{F}_2^m respectively are **linear**.

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We deduce that $\dim(p(V)) + \dim(p(V')) \leq n$

Projected Spaces Criterion

If $F : \mathbb{F}_2^n \rightarrow \mathbb{F}_2^m$ is CCZ-equivalent to a permutation, then there are at least two subspaces of dimension $n/2$ in $p(\mathcal{Z}_F)$ and in $p'(\mathcal{Z}_F)$.

QAM

Yu et al. (DCC'14) generated **8180** 8-APN quadratic functions from
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QAM

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None of them are CCZ-equivalent to a permutation

Göloğlu's Candidates (1/2)

Göloğlu's introduced APN functions

$$f_k : x \mapsto x^{2^k+1} + (x + x^{2^{n/2}})^{2^k+1}$$

for $n = 4t$. They have the *subspace property* of the Kim mapping.

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*Unfortunately, f_k are not equivalent to permutations on $n = 4, 8$ and does not seem to be equivalent to one on $n = 12$ (we say "it does not seem to be equivalent to a permutation" since checking the existence of CCZ-equivalent permutations **requires huge amount of computing and is infeasible on $n = 12$; our program was still running at the time of writing**).*

Göloğlu's Candidates (2/2)

n	cardinal proj.	time proj. (s)	time BasesExtraction (s)
12	1365	0.066	0.0012
16	21845	16.79	0.084
20	349525	10096.00	37.48

Time needed to show that f_k is **not** CCZ-equivalent to a permutation.

Outline

- 1 CCZ-Equivalence and Vector Spaces of 0
- 2 Function Twisting
- 3 Necessary and Efficient Conditions for CCZ-Equivalence to a Permutation
- 4 Conclusion**

Plan of this Section

- 1 CCZ-Equivalence and Vector Spaces of 0
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- 4 Conclusion
 - Summary
 - Open Problems

Conclusion

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- $CCZ = EA + \text{Twist}$, both of which have a simple interpretation.
- Efficient criteria to know if a function is CCZ-equivalent to a permutation...
- ... implemented using a very efficient vector space extraction algorithm (not presented)

Open problem

How can we efficiently check the EA-equivalence of two functions?

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<https://eprint.iacr.org/2018/713>

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Thank you!