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Identification by data assimilation of the sensitivity of wavepackets in jets to non-linear effects

Gilles Tisot^{1,2,3}, Mengqi Zhang⁴, Francisco C. Lajús Jr.^{1,5},
André V.G. Cavalieri¹, Peter Jordan⁴, Tim Colonius⁶

Colloque National d'Assimilation de Données
September 28, 2018

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⁵ Universidade Federal de Santa Catarina, Florianópolis, SC, Brazil.

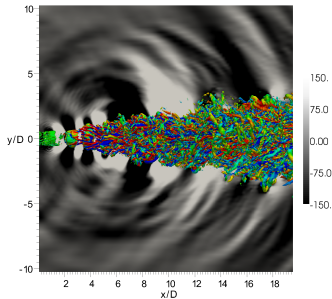
⁶ California Institute of Technology, Pasadena, CA, USA.

Motivations

Jet noise:



São Paulo airport.



G. Daviller (2011).

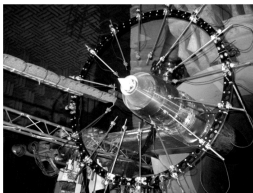
- Jet noise dominant during take off.
- Becomes limiting for specifications.
- Noise comes from the flow.

Motivations

Wavepackets

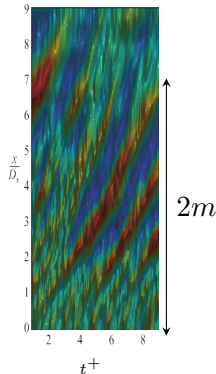
Where does the noise come from?

- Turbulent stochastic eddies?
- Or something more organised?
- Wavepackets in pressure/velocity field.



Tinney & Jordan 2008; Co-axial transonic heated jet $Re = 5 \times 10^6$.

Jordan & Colonius (2013)
Cavalieri et al. (2012,2013)
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Near-field pressure.

- Acoustic directivity as **extended source** (low azimuthal angles).

Motivations

Wavepackets

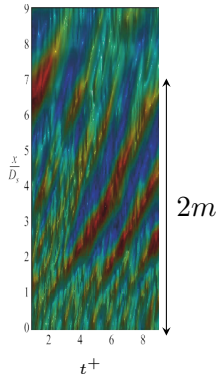
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Near-field pressure.

- Acoustic directivity as **extended source** (low azimuthal angles).

Source: likely a wavepacket shape.

Motivations

Wavepackets

Jordan & Colonius (2013)

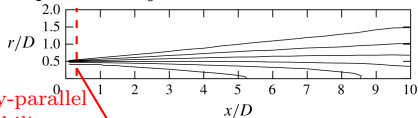
Cavalieri et al. (2013)

Sinha et al. (2014)

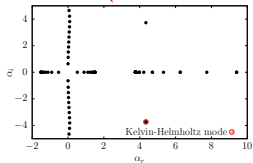
Baqui et al. (2015)

Wavepackets: Propagated linear instability waves

Experimental jet **mean flow**.



Locally-parallel
Stability



Motivations

Wavepackets

Jordan & Colonius (2013)

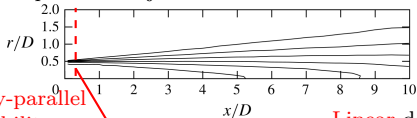
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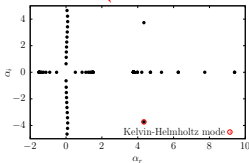
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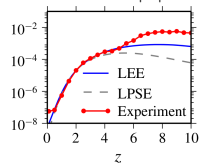
Locally-parallel
Stability



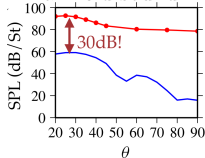
Linear downstream
propagation of the
Kelvin-Helmholtz mode.

PSE
(Parabolised Stability Eq.)

Centerline $|u|^2$:



Far-field sound:



**Non-linearities important for far-field prediction...
...toward low-order non-linear wavepacket model.**

Motivations

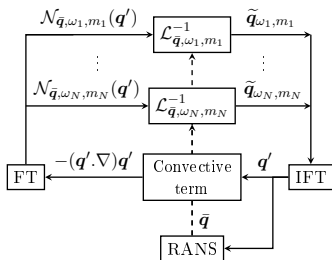
Non-linearity

Non-linearity as an “external forcing”

Landhal (1967)
 McKeon & Sharma (2010,2013)
 Moarref et al. (2013)
 Towne et al. (2015)

Navier-Stokes in the frequency-azimuthal mode domain:

$$\mathcal{L}_{\bar{q},\omega,m}\tilde{q}_{\omega,m} = \mathcal{N}_{\bar{q},\omega,m}(q').$$



Motivations

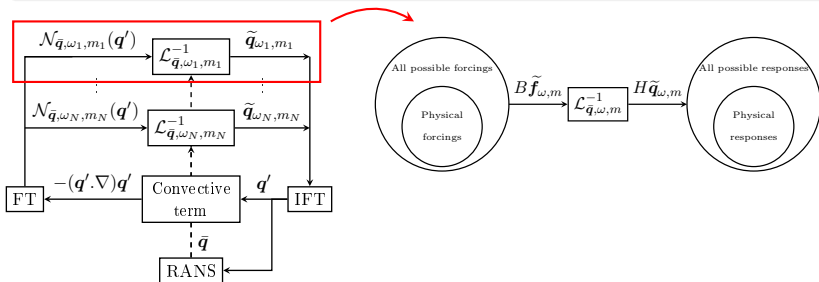
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Identify relevant non-linearities.

Motivations

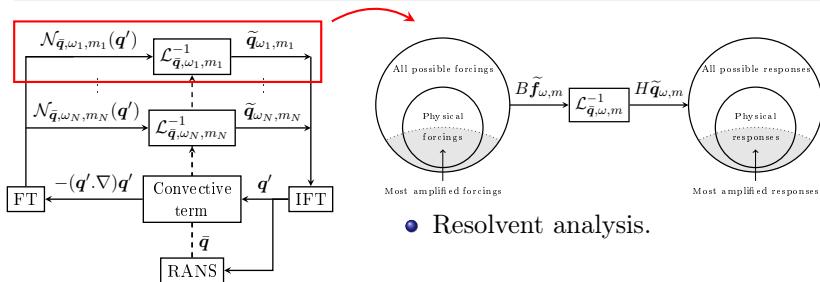
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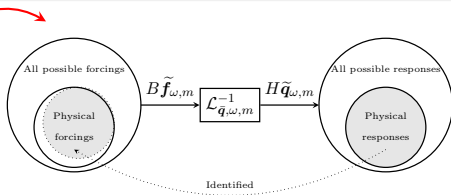
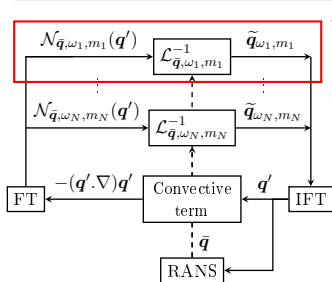
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- Resolvent analysis.
- Inverse problem: PSE-4D-Var.

Identify relevant non-linearities.

Locally parallel resolvent analysis

Model

Compressible Navier-Stokes equations:

$$\begin{cases} \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0 \\ \frac{\partial \rho \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla)(\rho \mathbf{u}) = -\nabla p + \nabla \cdot \tau \\ \rho \frac{\partial \gamma r T}{\partial t} + \rho (\mathbf{u} \cdot \nabla)(\gamma r T) = -p(\nabla \cdot \mathbf{u}) + \tau : \nabla \mathbf{u} + \frac{1}{RePr} \nabla \cdot (\mu \nabla T) \end{cases}$$

with $\tau = \frac{1}{Re_a} (\mu (\nabla \mathbf{u} + (\nabla \mathbf{u})^T) + (\mu_B - \frac{2}{3}\mu) (\nabla \cdot \mathbf{u}) \mathbb{I})$; $p = \rho r T$.

Locally parallel resolvent analysis

Model

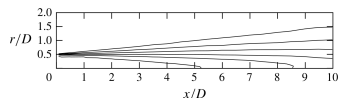
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Linearised Navier-Stokes Equations: *over mean flow $U(y)$*

$$(\rho, \mathbf{u}, T)^T = \mathbf{q}_\phi(\mathbf{x}, t) = \bar{\mathbf{q}}(\mathbf{x}) + \mathbf{q}'(\mathbf{x}, t)$$



Locally parallel resolvent analysis

Model

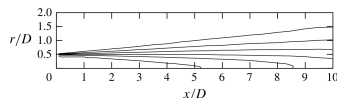
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Locally parallel assumption + Fourier Transform:

$$\mathbf{q}'(x, r, \theta, t) = \frac{1}{(2\pi)^3} \sum_m \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \tilde{\mathbf{q}}_{\omega, m, \alpha}(r) e^{i(m\theta - \omega t + \alpha x)} d\alpha d\omega.$$

Locally parallel resolvent analysis

Linearised problem: *eigenvalue problem* ($D(\alpha, \omega, m) = 0$).

$$\mathcal{L}_{\bar{\mathbf{q}}, \alpha, \omega, m} \tilde{\mathbf{q}}_{\alpha, \omega, m} = 0.$$

Locally parallel resolvent analysis

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Linearised problem with external forcing: ($D(\alpha, \omega, m) \neq 0$).

$$H \tilde{\mathbf{q}}_{\alpha, \omega, m} = H \mathcal{L}_{\bar{q}, \alpha, \omega, m}^{-1} B \tilde{\mathbf{f}}_{\bar{q}, \alpha, \omega, m}. \quad (H \text{ restricts on } \mathbf{u} \text{ comp.})$$

Locally parallel resolvent analysis

Resolvent analysis

Linearised problem with external forcing:

$$H\tilde{\mathbf{q}}_{\alpha,\omega,m} = \underbrace{H\mathcal{L}_{\tilde{\mathbf{q}},\alpha,\omega,m}^{-1}B}_{\text{SVD}}\tilde{\mathbf{f}}_{\tilde{\mathbf{q}},\alpha,\omega,m}.$$

Locally parallel resolvent analysis

Resolvent analysis

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$$H\tilde{\mathbf{q}}_{\alpha,\omega,m} = \underbrace{H\mathcal{L}_{\tilde{\mathbf{q}},\alpha,\omega,m}^{-1}B}_{\text{SVD}} \tilde{\mathbf{f}}_{\tilde{\mathbf{q}},\alpha,\omega,m}.$$

SVD to maximise Rayleigh quotient:

$$\max_{\tilde{\mathbf{f}}_{\alpha,\omega,m}} \frac{\|H\tilde{\mathbf{q}}_{\alpha,\omega,m}\|^2}{\|\tilde{\mathbf{f}}_{\alpha,\omega,m}\|^2} = \frac{\|(H\mathcal{L}_{\tilde{\mathbf{q}},\alpha,\omega,m}^{-1}B)\tilde{\mathbf{f}}_{\alpha,\omega,m}\|^2}{\|\tilde{\mathbf{f}}_{\alpha,\omega,m}\|^2}.$$

Locally parallel resolvent analysis

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Most amplified harmonic forcing/response modes:

$$H\mathcal{L}_{\tilde{\mathbf{q}},\alpha,\omega,m}^{-1}B = U\Sigma V^*$$

$$H\mathcal{L}_{\tilde{\mathbf{q}},\alpha,\omega,m}^{-1}B\mathbf{V}_i = \sigma_i\mathbf{U}_i$$

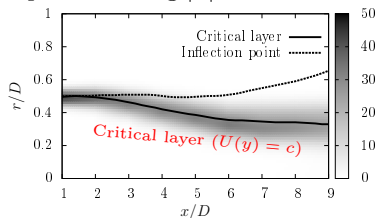
with $U = (\mathbf{U}_1, \dots, \mathbf{U}_N)$, $V = (\mathbf{V}_1, \dots, \mathbf{V}_N)$ and $\Sigma = \text{diag}(\sigma_1, \dots, \sigma_N)$.

Locally parallel resolvent analysis

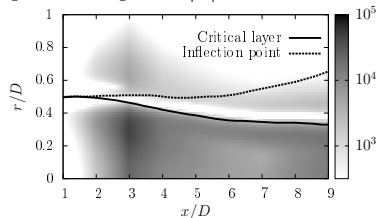
Resolvent analysis

$m = 0$ (most acoustically efficient), $St = 0.6$, $\alpha = \alpha_{PSE}$.

Optimal forcing $|u|^2$:



Optimal response $|u|^2$:

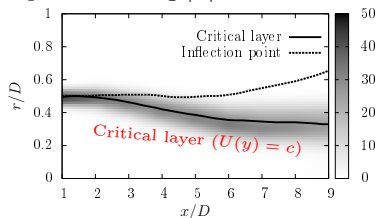


Locally parallel resolvent analysis

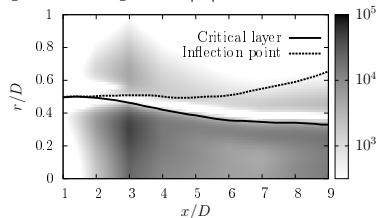
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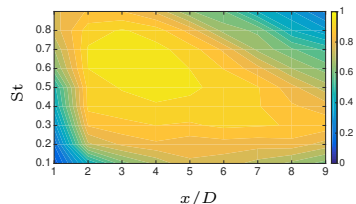


Optimal response $|u|^2$:

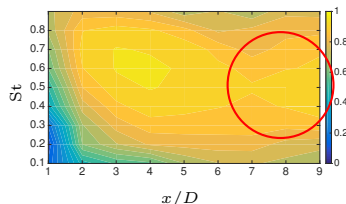


Normalised radial inner product

PSE vs Exp.:



Optimal response vs Exp.:



Forced wavepacket represents well downstream region.

Parabolised stability equations:

Herbert (1997)

- ~~Locally parallel~~ → Slowly divergent.

$$\tilde{\mathbf{q}}_{\omega,m}(x,r) = \mathbf{q}(x,r) e^{i \int_0^x \alpha(\xi) d\xi}.$$

- We neglect viscosity.

Parabolised stability equations:

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Model propagated downstream:

$$\begin{cases} E \frac{\partial \mathbf{q}}{\partial x} + (A + \alpha B) \mathbf{q} = 0 \\ \left(\mathbf{q}, \frac{\partial \mathbf{q}}{\partial x} \right)_r = 0 \\ \mathbf{q}(0) = \mathbf{q}_{\text{K-H}}, \end{cases}$$

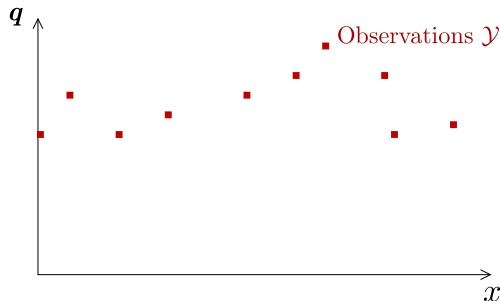
Inflow condition: Kelvin-Helmholtz mode from locally parallel stability analysis.

PSE-4D-Var

4D-Var

Observations: \mathcal{Y} : PSD $(|u|^2, |v|^2)^T$

Papadakis (2007)
Ansaldo (2015)



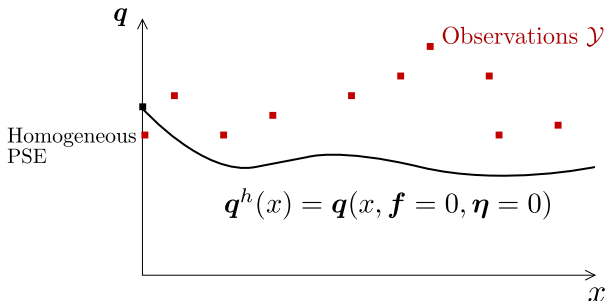
PSE-4D-Var

4D-Var

Model: Parabolised Stability Equations (PSE)

*Papadakis (2007)**Ansaldi (2015)*

$$\begin{cases} E \frac{\partial \mathbf{q}}{\partial x} + (A + \alpha B) \mathbf{q} = \mathbf{f} \\ \left(\mathbf{q}, \frac{\partial \mathbf{q}}{\partial x} \right)_r = 0 \\ \mathbf{q}(0, r) = \mathbf{q}_0 + \boldsymbol{\eta}, \quad \alpha(0) = \alpha_0, \end{cases} \quad \text{with} \quad \tilde{\mathbf{q}}_{\omega, m}(x) = \mathbf{q}(x) e^{i \int_0^x \alpha(\xi) d\xi}$$

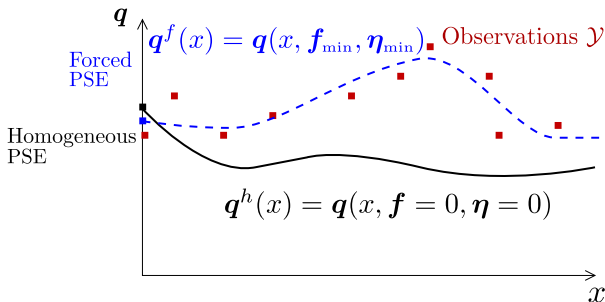


PSE-4D-Var

4D-Var

4D-Var: search for $(\mathbf{f}_{\min}, \boldsymbol{\eta}_{\min}) = \operatorname{argmin}(\mathcal{J}(\mathbf{q}, \alpha, \mathbf{f}, \boldsymbol{\eta}))$ Papadakis (2007)
Ansaldo (2015)

$$\mathcal{J} = \frac{1}{2} \int_0^L \|\mathbb{H}(\mathbf{q}, \alpha) - \mathcal{Y}\|_{W_o}^2 dx + \frac{1}{2} \|\mathbb{H}_L - \mathcal{Y}_L\|_{W_T}^2 \\ + \frac{1}{2} \int_0^L \|\mathbf{f}\|_{W_f}^2 dx + \frac{1}{2} \|\boldsymbol{\eta}\|_{W_\eta}^2.$$



PSE-4D-Var

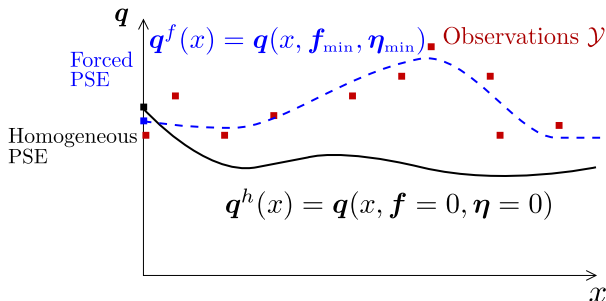
4D-Var

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What are the missing non-linearities \mathbf{f} in the linear model?

Solved using adjoint method (adjoint PSE).



PSE-4D-Var

4D-Var

Adjoint equation:

$$\begin{cases} -E^+ \frac{\partial \boldsymbol{\lambda}}{\partial x} + \left(A + \alpha B - \frac{\partial E}{\partial x} \right)^+ \boldsymbol{\lambda} + \frac{\partial \mathbf{q}}{\partial x} (\zeta - \zeta^*) - \mathbf{q} \frac{\partial \zeta^*}{\partial x} = RHS_1, \\ (B\mathbf{q}, \boldsymbol{\lambda})_r = RHS_2, \\ E^+ \boldsymbol{\lambda}(L, r) = RHS_3, \\ \zeta(L) = 0. \end{cases}$$

Optimality condition:

$$\begin{cases} \frac{\partial J}{\partial \mathbf{f}} = \boldsymbol{\lambda}(x, r) + W_f \mathbf{f} \\ \frac{\partial J}{\partial \boldsymbol{\eta}} = E^+ \boldsymbol{\lambda}(0, r) + \mathbf{q}_0 \zeta^*(0) + W_\eta \boldsymbol{\eta}. \end{cases}$$

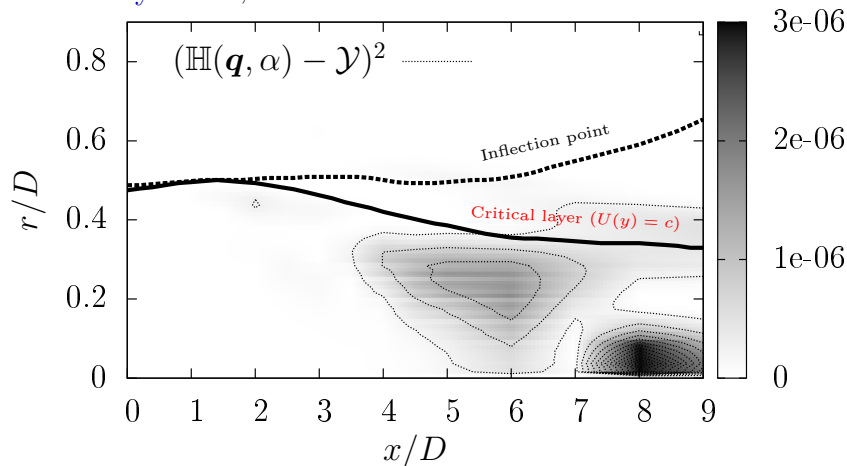
- Solved iteratively (*steepest descent*).
- Weights determined by L-curve method.



PSE-4D-Var

Sensitivity

Sensitivity: $m = 0$, $St = 0.6$.

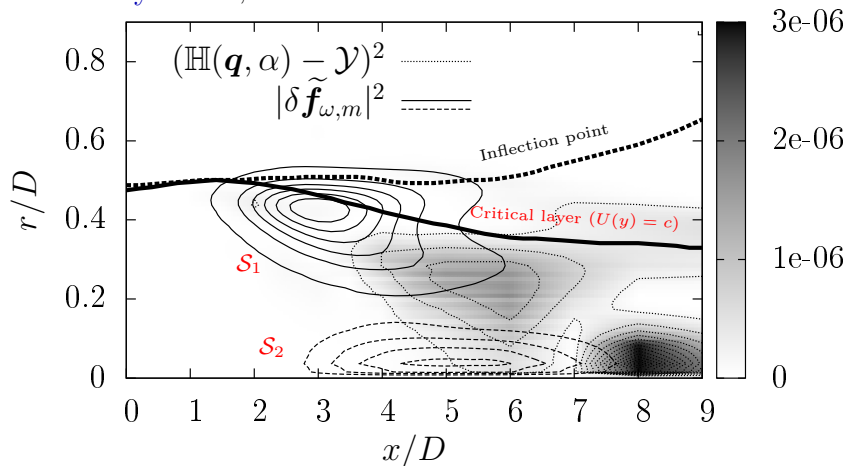


$|u|^2$, grayscale=observation error.

PSE-4D-Var

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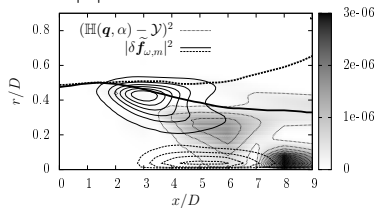
$|u|^2$, grayscale=observation error, contour=sensitivity.

PSE-4D-Var

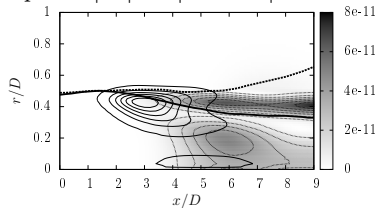
Sensitivity

Sensitivity: $m = 0$, $St = 0.6$.

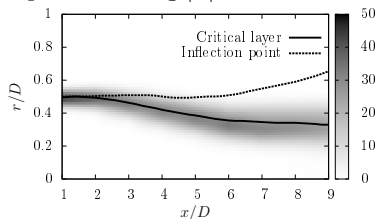
Error $|u|^2$:



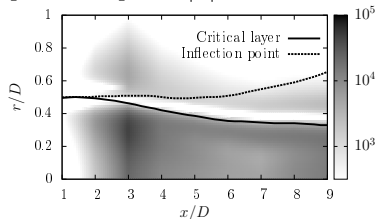
Response $|\delta u|^2 = |u^f - u^h|^2$:



Optimal forcing $|u|^2$:



Optimal response $|u|^2$:



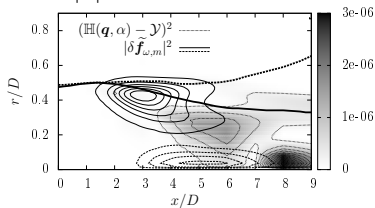
Predicted by locally parallel resolvent analysis.

PSE-4D-Var

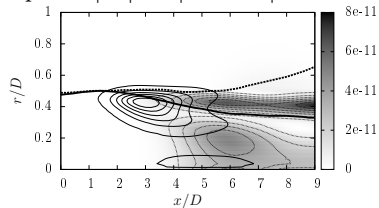
Sensitivity

Sensitivity: $m = 0$, $St = 0.6$.

Error $|u|^2$:



Response $|\delta u|^2 = |u^f - u^h|^2$:



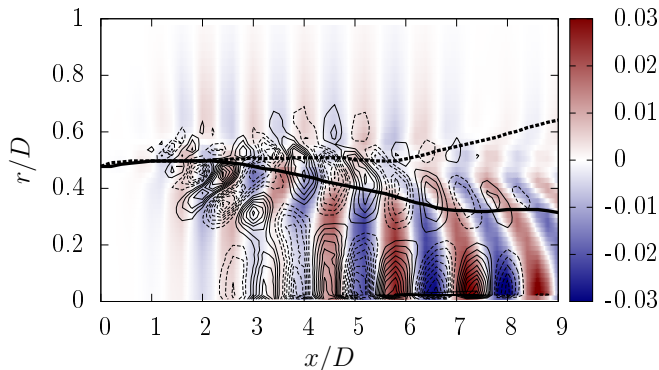
Inflow condition not sensitive:

- Critical layer sensitivity when neutral ($x \approx 3$).
- Kelvin-Helmholtz growth dominates upstream (modal behaviour).
- Homogeneous PSE works upstream.

PSE-4D-Var

Converged

Converged 4D-Var:.

Real(u), $St=0.6$ 

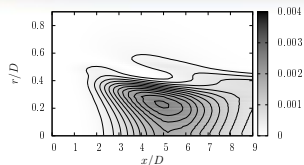
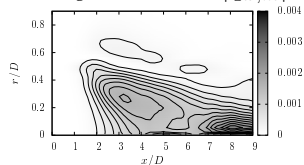
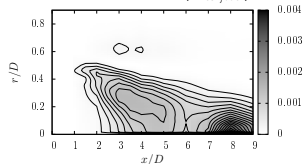
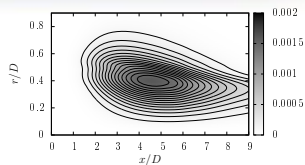
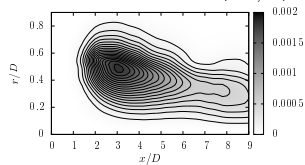
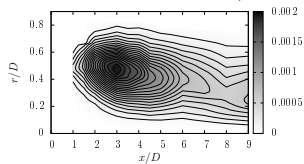
High forcing near:

- Critical layer.
- Centerline.

Converged results conserve the same trend.

PSE-4D-Var

Converged

Homogeneous PSE $|\tilde{q}_{\omega,m}^u|^2$ Forced PSE $|\tilde{q}_{\omega,m}^u|^2$ Observation y^u Homogeneous PSE $|\tilde{q}_{\omega,m}^v|^2$ Forced PSE $|\tilde{q}_{\omega,m}^v|^2$ Observation y^v

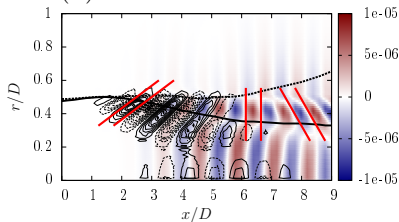
Match experiments.

PSE-4D-Var

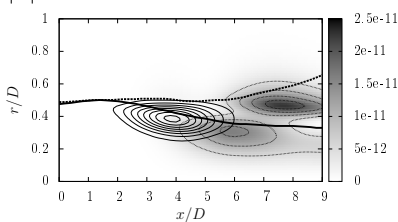
Orr mechanism

Sensitivity: contours=forcing; color=infinitesimal response. *Orr (1907)*
Boyd (1983)
Garnaud (2013)
Jiménez (2013,2015)

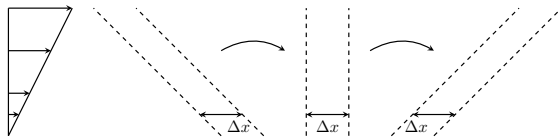
Real(u):



$|v|^2$:



Orr mechanism:



Vorticity conservation
 \Rightarrow Growth of $|v|^2$.

Tilting suggesting Orr mechanism in space.

PSE-4D-Var

Orr mechanism

Orr model: for Couette flow ($U(y) = Sy$).

Orr (1907)
Case (1960)

Temporal Orr model: *vorticity convected by shear*

$$\left(\frac{\partial \cdot}{\partial t} - Sy \frac{\partial \cdot}{\partial x} \right) \nabla^2 \psi(x, y, t) = 0, \quad \psi \text{ stream func.}$$

$$\nabla^2 \psi(x, y, t) = F(x - Syt, y).$$

Spatial Orr model: *Fourier Transform in time*

$$\nabla^2 \tilde{\psi}(x, y, \omega) = \tilde{F}_2(y) e^{i \frac{\omega x}{Sy}}.$$

with

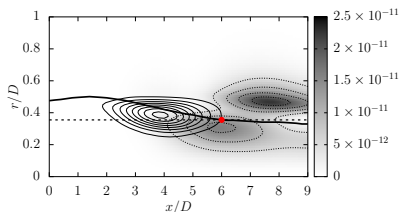
$$\tilde{F}_2(y) = \frac{1}{Sy} \tilde{F} \left(\frac{\omega}{Sy}, y \right) \quad \text{FFT in } x \text{ of } F(x, y).$$

$$\tilde{F}_2(y) = \nabla^2 \tilde{\psi}(0, y, \omega)$$

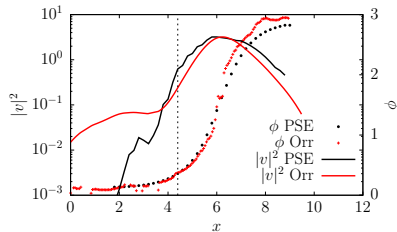
PSE-4D-Var

Orr mechanism

Jet locally approximated by Couette flow!



*Point used for matching flows
(max(v) at critical layer).*



*Comparison PSE sensitivity / Orr
model.*

Orr mechanism quantitatively confirmed.

Conclusion

Summary

- **Forced wavepacket** consistent with experiment.
- **Critical layer** highly sensitive region to non-linearity.
- Along the critical layer, shear convects and tilts the response to non-linearities, leading to an amplification by **Orr mechanism**.
- PSE-4D-Var powerful tool.