

Shape optimisation for acoustic liners design

Gilles TISSOT, Gwénaël GABARD

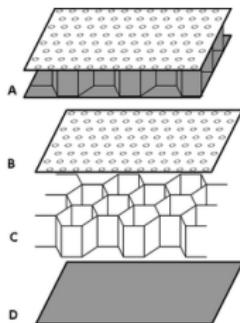
Laboratoire d'Acoustique de l'Université du Mans

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Le Havre, Wednesday, 25 April 2018



Context

Acoustic liners:



Schematic representation.



Integrated in nacelles of aircrafts.

Constraints:

- Low frequency absorption requires deep cavities.
- Challenge to push away this limitation.

MACIA ANR project: LAUM, SAFRAN.

Context

Objective:

- Optimise cavity shape of liners.
- Make it resonate at lower frequencies.

Strategy:

- Shape optimisation.
- Based on frequency response.

Outline

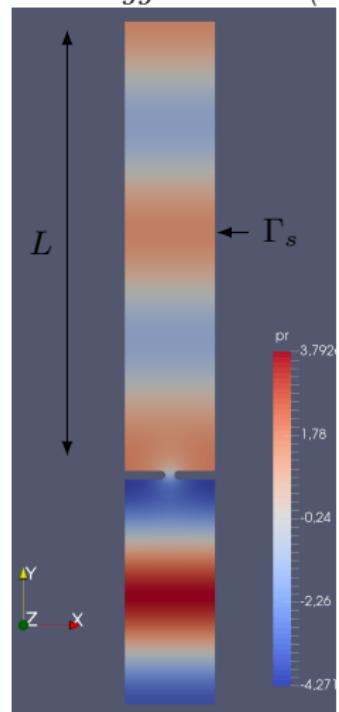
- 1 Governing equations
- 2 Shape optimisation
- 3 Numerical implementation
- 4 Results

Governing equations

Helmholtz equation:

$$\left\{ \begin{array}{l} \Delta p + k^2 p = 0 \quad \text{in } \Omega \\ \nabla p \cdot \mathbf{n} = 0 \quad \text{on } \Gamma_s \text{ (free slip).} \end{array} \right.$$

Bossart et al. (2003)
Berggren et al. (2018)



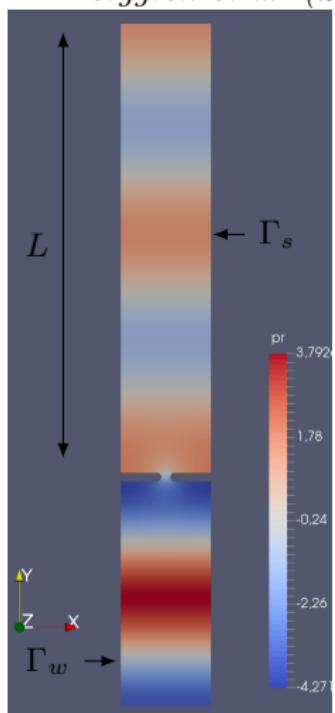
Governing equations

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$$\left\{ \begin{array}{l} \Delta p + k^2 p = 0 \quad \text{in } \Omega \\ \nabla p \cdot \mathbf{n} = 0 \quad \text{on } \Gamma_s \text{ (free slip)} \\ \nabla p \cdot \mathbf{n} = \delta_v \frac{i-1}{2} \Delta_T p - \delta_T k^2 \frac{(i-1)(\gamma-1)}{2} p \\ \qquad \qquad \qquad \text{on } \Gamma_w \text{ (walls).} \end{array} \right.$$

- $\delta_v = \sqrt{\frac{2}{\omega Re_a}}$; $\delta_T = \sqrt{\frac{2}{\omega Re_a Pr}}$.
- $\nabla f = \nabla_T f + \frac{\partial f}{\partial n} \mathbf{n}$;
 $\Delta f = \Delta_T f + \frac{\partial^2 f}{\partial n^2} + \nabla_T \cdot \mathbf{n} \frac{\partial f}{\partial n}$

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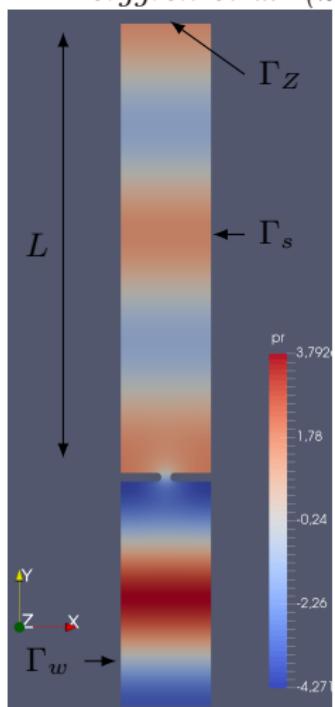


Governing equations

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Bossart et al. (2003)
Berggren et al. (2018)



- Unitary amplitude of incoming plane wave.
- Reflexion coefficient:

$$R = \left(\frac{1}{L_Z} \int_{\Gamma_Z} p(x, L) dx - e^{ikL} \right) e^{ikL}.$$

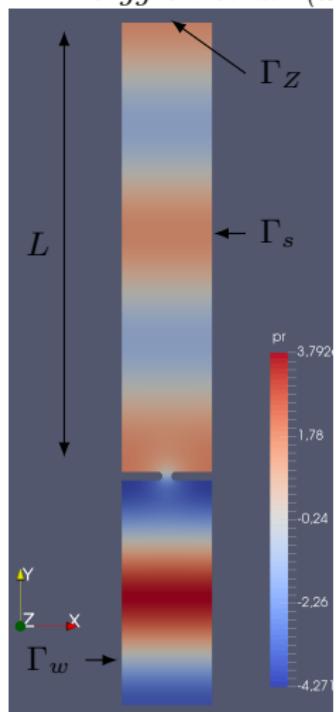
- Impedance: $Z = \frac{1+R}{1-R}$.

Governing equations

Helmholtz equation:

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- Modify cavity shape for improving behaviour: *Target impedance.*

$$\mathcal{J}(\Omega, p) = \frac{1}{2} \int_{k_1}^{k_2} |Z(p, k) - Z_T(k)|^2 dk$$

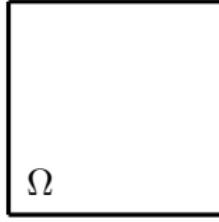


Shape optimisation

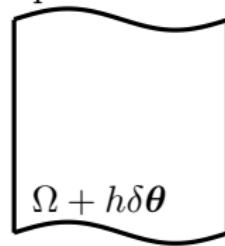
Generalities

Infinitesimal domain variations:

Reference domain:



Shape variation:



h small.

Shape optimisation

Generalities

Allaire (2003,2004)

Cost functional: **unconstrained** minimisation problem

$$\mathcal{J}(\Omega) = \int_{\Omega} f(x) \, dx$$

Shape derivative: $\forall \delta\boldsymbol{\theta}$ we have

$$(\nabla_{\delta\boldsymbol{\theta}} \mathcal{J}, \delta\boldsymbol{\theta}) = \lim_{h \rightarrow 0} \frac{\mathcal{J}(\Omega + h\delta\boldsymbol{\theta}) - \mathcal{J}(\Omega)}{h} = \int_{\Gamma_{\boldsymbol{\theta}}} \delta\boldsymbol{\theta}(x) \cdot \mathbf{n}(x) f(x) \, dx.$$

Shape optimisation

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Descent direction:

$$\mathcal{J}(\Omega + h\delta\boldsymbol{\theta}) = \mathcal{J}(\Omega) + h \int_{\Gamma_{\theta}} \delta\boldsymbol{\theta}(x) \cdot \mathbf{n}(x) f(x) \, dx + \mathcal{O}(h^2)$$

Shape optimisation

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$\Rightarrow \delta\boldsymbol{\theta}(x) = -f(x)\mathbf{n}(x)$ ensures a descent direction!

Indeed: $\mathcal{J}(\Omega + h\delta\boldsymbol{\theta}) = \mathcal{J}(\Omega) - h \int_{\Gamma_{\theta}} f^2(x) \, dx + \mathcal{O}(h^2)$

Shape optimisation

Impedance matching

Bängtsson et al. (2003)

Cost functional:

$$\mathcal{J}(\Omega, p) = \frac{1}{2} \int_{k_1}^{k_2} |Z(p, k) - Z_T(k)|^2 dk$$

subject to

$$\Delta p + k^2 p = 0 \quad + \quad \text{B.C.}$$

Impedance: $Z(p, k) = \frac{e^{-ikL} - e^{ikL} + \bar{p}}{e^{-ikL} + e^{ikL} - \bar{p}}$ with $\bar{p} = \frac{1}{L_Z} \int_{\Gamma_Z} p dl.$

Shape optimisation

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subject to

$$\Delta p + k^2 p = 0 \quad + \quad \text{B.C.}$$

We define the Lagrangian (*constrained* \mapsto *unconstrained*)

$$\mathcal{L}(\Omega, p, \lambda) = \mathcal{J}(\Omega, p) - \text{real} \left(\int_{k_1}^{k_2} (\lambda, \Delta p + k^2 p)_\Omega dk \right)$$

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Derivative with respect to each variable:

$$(\nabla_{\delta\lambda} \mathcal{L}, \delta\lambda) = (\nabla_{\delta p} \mathcal{L}, \delta p) = 0$$

$$(\nabla_{\delta\theta} \mathcal{L}, \delta\theta) = 0 \Rightarrow (\nabla_{\delta\theta} \mathcal{J}, \delta\theta)$$

Impedance matching

Derivative with respect to λ : $\Delta p + k^2 p = 0$.

Impedance matching

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Derivative with respect to p : Adjoint equation

$$\begin{cases} \Delta\lambda + (k^*)^2\lambda = 0 & x \in \Omega \\ \nabla\lambda \cdot \mathbf{n} = 0 & x \in \Gamma_s \\ \nabla\lambda \cdot \mathbf{n} + \delta_v \frac{i+1}{2} \Delta_T \lambda - \delta_T (k^*)^2 \frac{(i+1)(\gamma-1)}{2} \lambda = 0 & x \in \Gamma_w \\ \nabla\lambda \cdot \mathbf{n} - ik^*\lambda = \frac{2}{L_Z} \frac{e^{ikL}(Z(p, k) - Z_T(k))}{\left((e^{-ikL} + e^{ikL} - \bar{p})^2\right)^*} & x \in \Gamma_Z. \end{cases}$$

Impedance matching

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Derivative with respect to Ω : For any displacement direction $\delta\boldsymbol{\theta}$,

$$(\nabla_{\delta\boldsymbol{\theta}} \mathcal{J}, \delta\boldsymbol{\theta}) = \int_{\Gamma_\theta} (\delta\boldsymbol{\theta}(x) \cdot \mathbf{n}(x)) \int_{k_1}^{k_2} \text{Real} \left[-\nabla\lambda^* \cdot \nabla p + k^2 \lambda^* p \right] dk dx.$$

Impedance matching

Algorithm:

Solve iteratively:

- Solve Helmholtz direct $\Rightarrow p(k)$ (*frequency response*).
- Solve Helmholtz adjoint $\Rightarrow \lambda(k)$ (*frequency response*).
- Optimality condition $\Rightarrow \nabla_{\delta\theta}\mathcal{J}$.
- Move shape with linesearch algorithm (*Armijo backtracking*).
- Iterate until convergence.

Numerical implementation

Moës et al. (1999)

Osher et al. (2001)

Protas et al. (2004)

Finite elements with immersed boundary:

- XFEM for integration methods on cut elements (GETFEM++).

- Level set:

$$\begin{cases} \psi = 0 & x \in \Gamma_\theta \\ \psi > 0 & x \in \Omega \\ \psi < 0 & x \notin \Omega. \end{cases}$$

- Transport equation for moving the shape:

$$\frac{\partial \psi}{\partial t} + v \mathbf{n} \cdot \nabla \psi = 0.$$

with

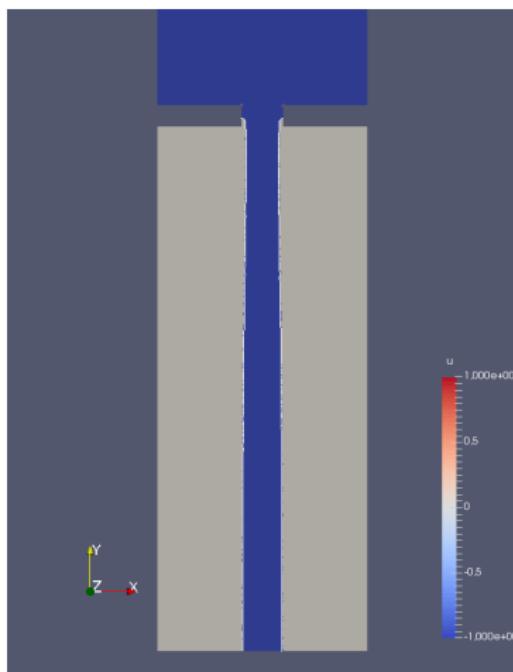
$$v = -\nabla_{\delta\theta} \mathcal{J} \cdot \mathbf{n}. \quad (\text{Regularised by Sobolev Gradient})$$

\mathbf{n} is the outward wall normal direction. SUPG discretisation.

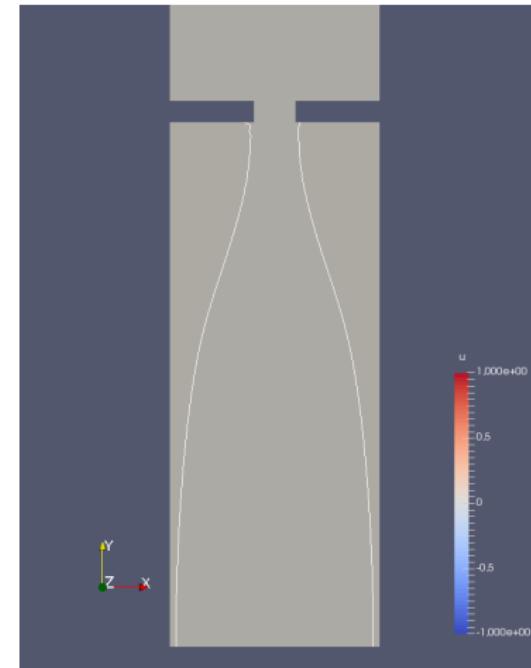


Results

No viscosity: $Z_T = 0$, $k = [0.2 : 0.6]$



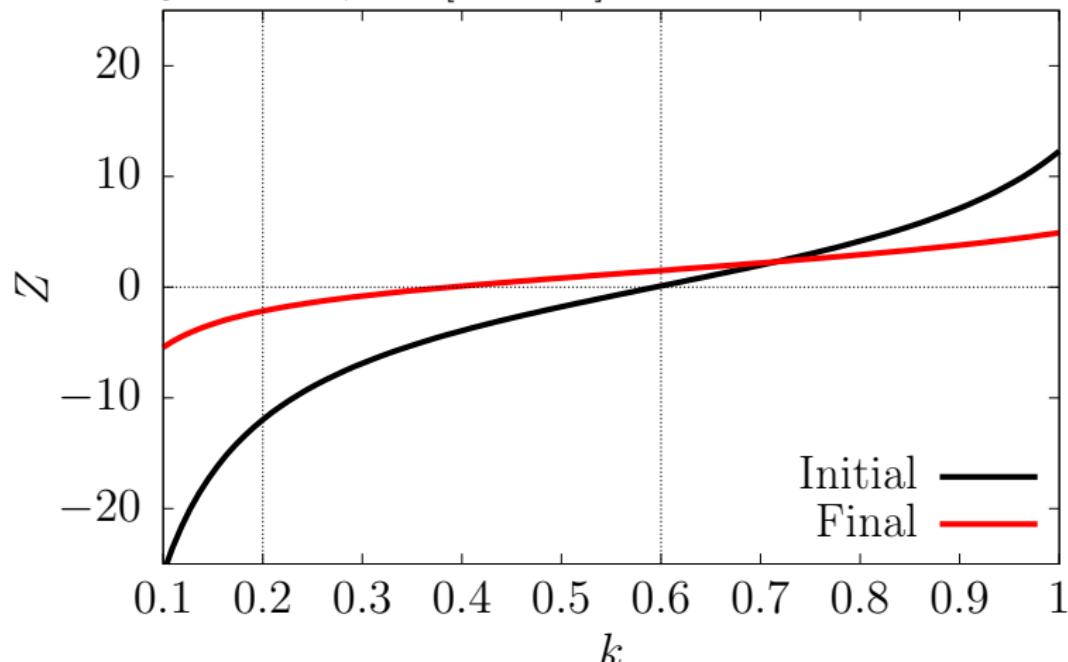
Initial condition.



Converged.

Results

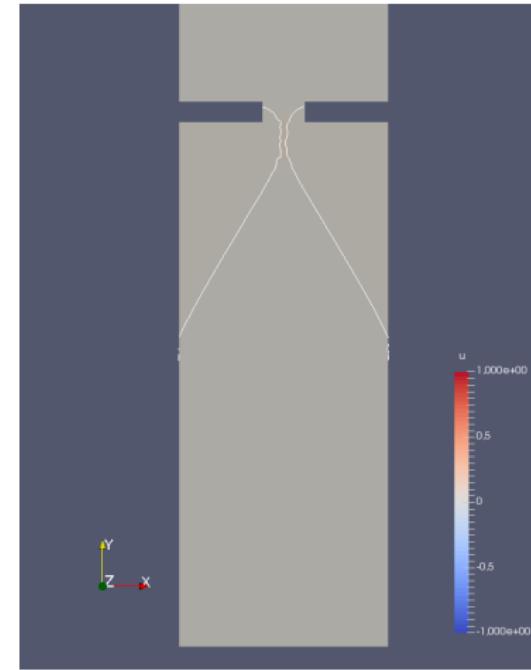
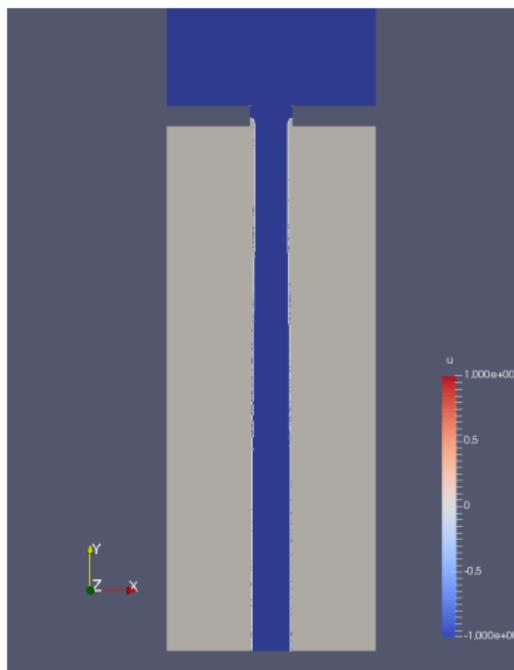
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Reactance $Im(Z)$.

Results

No viscosity: $Z_T = 0$, $k = [0.15 : 0.25]$

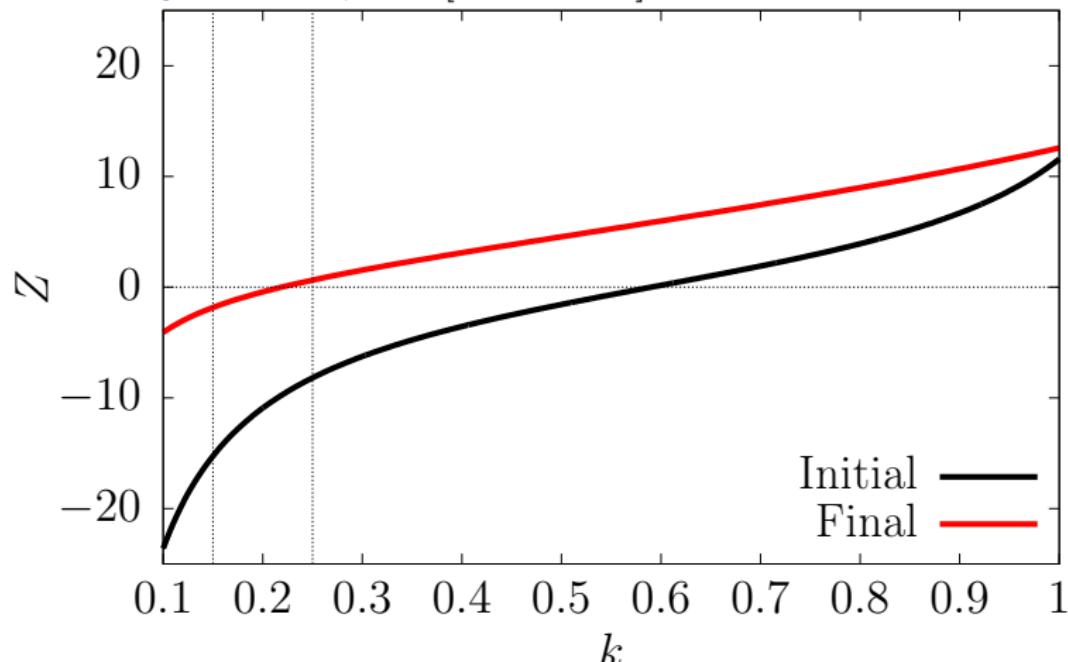


Initial condition.

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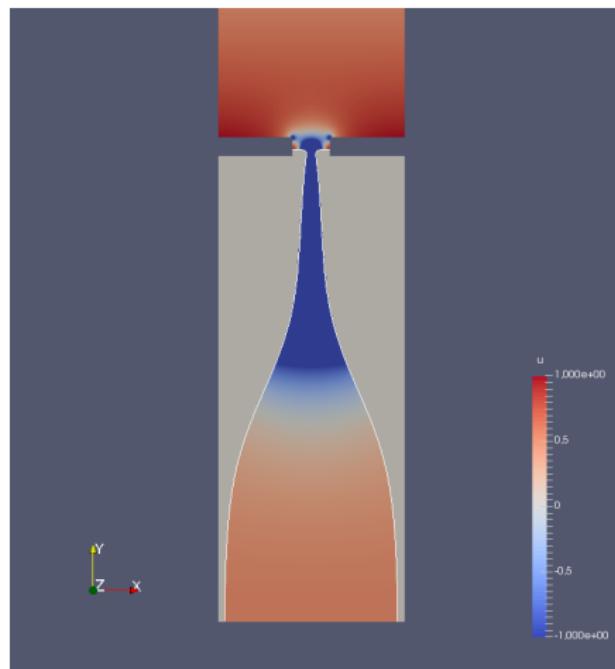
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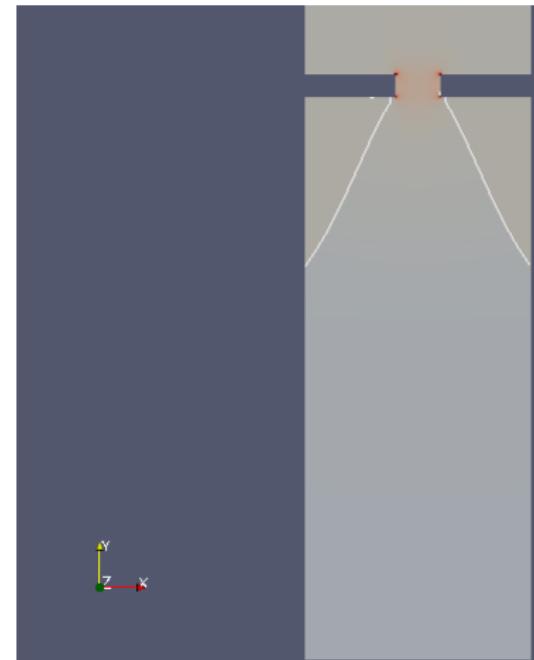
Reactance $Im(Z)$.

Results

Viscosity: $Z_T = 1.0$, $k = [0.2 : 0.6]$



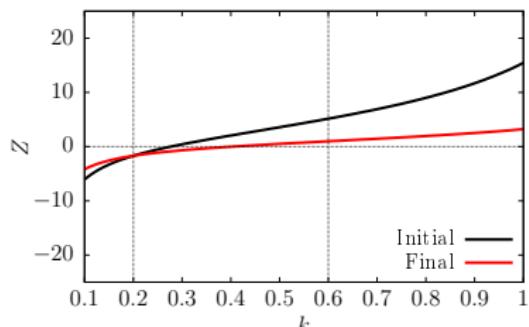
Initial condition.
Coloured by $\nabla_{\delta\theta}\mathcal{J}$.



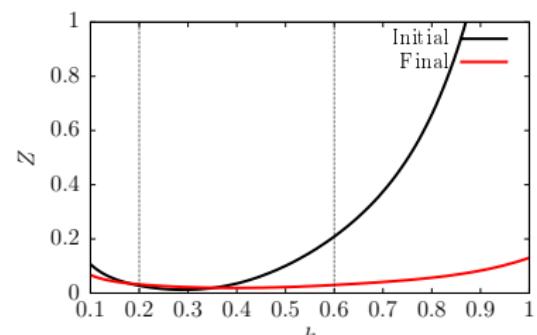
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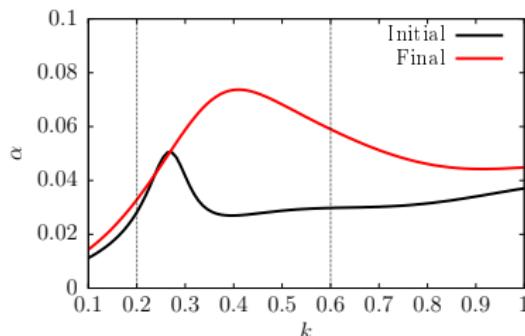
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Reactance $Im(Z)$.



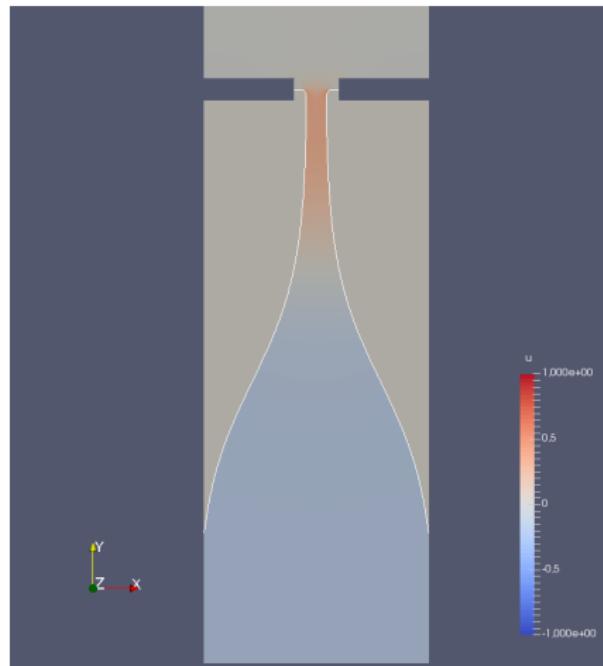
Resistance $Re(Z)$.



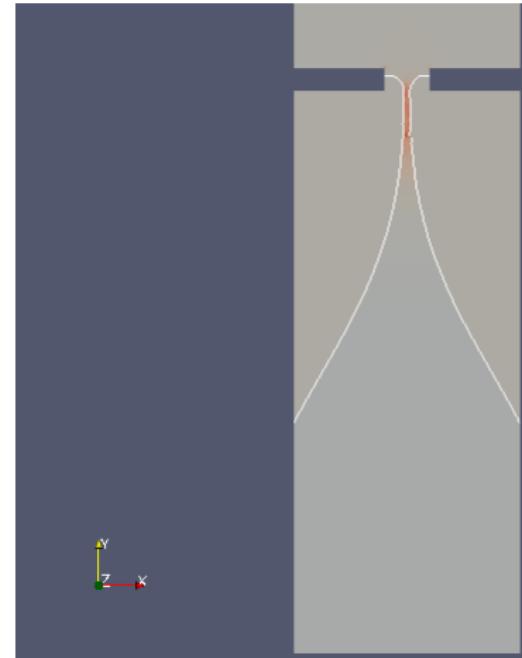
Absorption $\alpha = 1 - |R|^2$.

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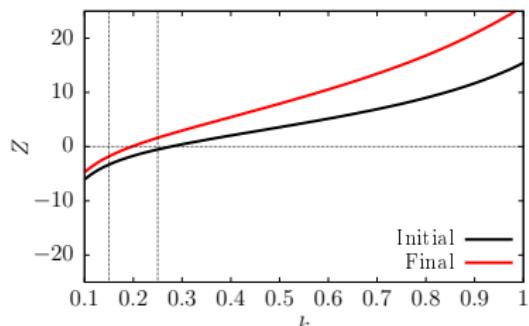
Initial condition.
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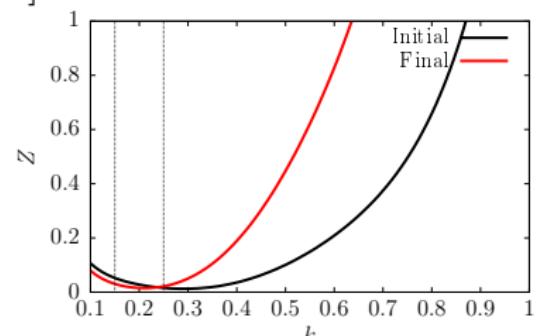
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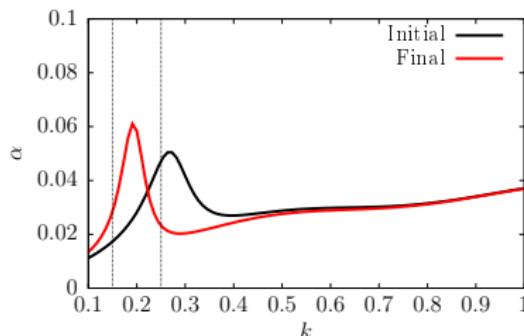
Viscosity: $Z_T = 1.0$, $k = [0.15 : 0.25]$



Reactance $Im(Z)$.



Resistance $Re(Z)$.



Absorption $\alpha = 1 - |R|^2$.

Conclusion

Summary:

- Shape optimisation for impedance matching.
- Viscous model: compromise between Helmholtz and full linearised Navier-Stokes.

Perspectives:

- Optimise from efficient/realistic configuration.
- Different cost functional (Penalty to initial guess, absorption, . . .).

Thank you for your attention.