



**HAL**  
open science

# Optimisation de forme pour la conception des cavités de liner

Gilles Tissot, Gwénaél Gabard

► **To cite this version:**

Gilles Tissot, Gwénaél Gabard. Optimisation de forme pour la conception des cavités de liner. CFA'18 - 14ème Congrès Français d'acoustique, Apr 2018, Le Havre, France. pp.1-30. hal-01959233

**HAL Id: hal-01959233**

**<https://inria.hal.science/hal-01959233>**

Submitted on 18 Dec 2018

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

# Shape optimisation for acoustic liners design

Gilles TISSOT, Gwénaél GABARD

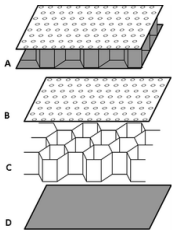
Laboratoire d'Acoustique de l'Université du Mans

Congrès Français d'Acoustique  
Le Havre, Wednesday, 25 April 2018



# Context

## Acoustic liners:



Schematic  
representation.



Integrated in  
nacelles of aircrafts.

## Constraints:

- Low frequency absorption requires deep cavities.
- Challenge to push away this limitation.

*MACIA* ANR project: LAUM, SAFRAN.

# Context

---

## Objective:

- Optimise cavity shape of liners.
- Make it resonate at lower frequencies.

## Strategy:

- Shape optimisation.
- Based on frequency response.

# Outline

---

- 1 Governing equations
- 2 Shape optimisation
- 3 Numerical implementation
- 4 Results

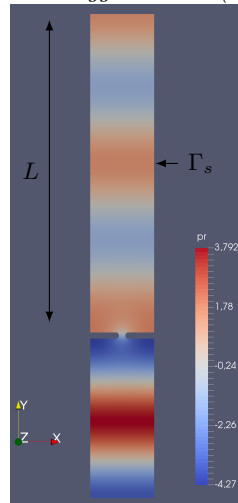
# Governing equations

Helmholtz equation:

$$\left\{ \begin{array}{l} \Delta p + k^2 p = 0 \quad \text{in } \Omega \\ \nabla p \cdot \mathbf{n} = 0 \quad \text{on } \Gamma_s \text{ (free slip)}. \end{array} \right.$$

*Bossart et al. (2003)*

*Berggren et al. (2018)*



# Governing equations

Helmholtz equation:

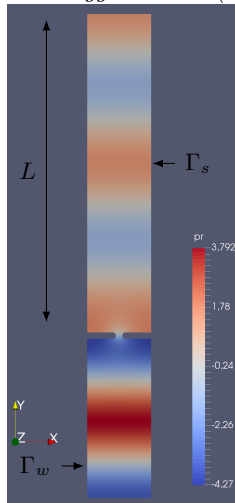
$$\left\{ \begin{array}{l} \Delta p + k^2 p = 0 \quad \text{in } \Omega \\ \nabla p \cdot \mathbf{n} = 0 \quad \text{on } \Gamma_s \text{ (free slip)} \\ \nabla p \cdot \mathbf{n} = \delta_v \frac{i-1}{2} \Delta_T p - \delta_T k^2 \frac{(i-1)(\gamma-1)}{2} p \\ \hspace{15em} \text{on } \Gamma_w \text{ (walls)}. \end{array} \right.$$

- $\delta_v = \sqrt{\frac{2}{\omega Re_a}} \quad ; \quad \delta_T = \sqrt{\frac{2}{\omega Re_a Pr}}$ .

- $\nabla f = \nabla_T f + \frac{\partial f}{\partial \mathbf{n}} \mathbf{n} ;$   
 $\Delta f = \Delta_T f + \frac{\partial^2 f}{\partial n^2} + \nabla_T \cdot \mathbf{n} \frac{\partial f}{\partial n}$

*Bossart et al. (2003)*

*Berggren et al. (2018)*







## Governing equations

Helmholtz equation:

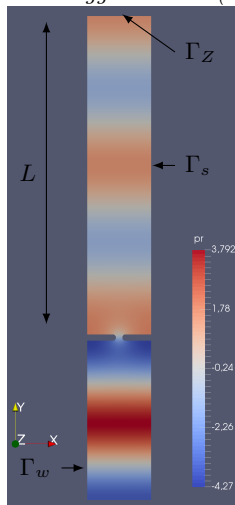
$$\begin{cases} \Delta p + k^2 p = 0 & \text{in } \Omega \\ \nabla p \cdot \mathbf{n} = 0 & \text{on } \Gamma_s \text{ (free slip)} \\ \nabla p \cdot \mathbf{n} = \delta_v \frac{i-1}{2} \Delta_T p - \delta_T k^2 \frac{(i-1)(\gamma-1)}{2} p & \text{on } \Gamma_w \text{ (walls)} \\ \nabla p \cdot \mathbf{n} + ikp = 2ike^{ikL} & \text{on } \Gamma_Z \text{ (Impedance B.C.).} \end{cases}$$

- Modify cavity shape for improving behaviour: *Target impedance*.

$$\mathcal{J}(\Omega, p) = \frac{1}{2} \int_{k_1}^{k_2} |Z(p, k) - Z_T(k)|^2 dk$$

*Bossart et al. (2003)*

*Berggren et al. (2018)*

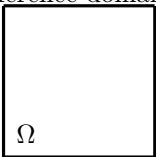


# Shape optimisation

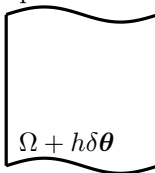
## Generalities

Infinitesimal domain variations:

Reference domain:



Shape variation:



$h$  small.

# Shape optimisation

## Generalities

Cost functional: **unconstrained** minimisation problem *Allaire (2003,2004)*

$$\mathcal{J}(\Omega) = \int_{\Omega} f(x) \, dx$$

Shape derivative:  $\forall \delta\boldsymbol{\theta}$  we have

$$(\nabla_{\delta\boldsymbol{\theta}} \mathcal{J}, \delta\boldsymbol{\theta}) = \lim_{h \rightarrow 0} \frac{\mathcal{J}(\Omega + h\delta\boldsymbol{\theta}) - \mathcal{J}(\Omega)}{h} = \int_{\Gamma_{\theta}} \delta\boldsymbol{\theta}(x) \cdot \mathbf{n}(x) f(x) \, dx.$$

# Shape optimisation

## Generalities

Cost functional: **unconstrained** minimisation problem *Allaire (2003,2004)*

$$\mathcal{J}(\Omega) = \int_{\Omega} f(x) \, dx$$

Shape derivative:  $\forall \delta \boldsymbol{\theta}$  we have

$$(\nabla_{\delta \boldsymbol{\theta}} \mathcal{J}, \delta \boldsymbol{\theta}) = \lim_{h \rightarrow 0} \frac{\mathcal{J}(\Omega + h \delta \boldsymbol{\theta}) - \mathcal{J}(\Omega)}{h} = \int_{\Gamma_{\theta}} \delta \boldsymbol{\theta}(x) \cdot \mathbf{n}(x) f(x) \, dx.$$

Descent direction:

$$\mathcal{J}(\Omega + h \delta \boldsymbol{\theta}) = \mathcal{J}(\Omega) + h \int_{\Gamma_{\theta}} \delta \boldsymbol{\theta}(x) \cdot \mathbf{n}(x) f(x) \, dx + \mathcal{O}(h^2)$$

# Shape optimisation

## Generalities

Cost functional: **unconstrained** minimisation problem *Allaire (2003,2004)*

$$\mathcal{J}(\Omega) = \int_{\Omega} f(x) \, dx$$

Shape derivative:  $\forall \delta\boldsymbol{\theta}$  we have

$$(\nabla_{\delta\boldsymbol{\theta}} \mathcal{J}, \delta\boldsymbol{\theta}) = \lim_{h \rightarrow 0} \frac{\mathcal{J}(\Omega + h\delta\boldsymbol{\theta}) - \mathcal{J}(\Omega)}{h} = \int_{\Gamma_{\theta}} \delta\boldsymbol{\theta}(x) \cdot \mathbf{n}(x) f(x) \, dx.$$

Descent direction:

$$\mathcal{J}(\Omega + h\delta\boldsymbol{\theta}) = \mathcal{J}(\Omega) + h \int_{\Gamma_{\theta}} \delta\boldsymbol{\theta}(x) \cdot \mathbf{n}(x) f(x) \, dx + \mathcal{O}(h^2)$$

$\Rightarrow \delta\boldsymbol{\theta}(x) = -f(x)\mathbf{n}(x)$  ensures a descent direction!

Indeed: 
$$\mathcal{J}(\Omega + h\delta\boldsymbol{\theta}) = \mathcal{J}(\Omega) - h \int_{\Gamma_{\theta}} f^2(x) \, dx + \mathcal{O}(h^2)$$

# Shape optimisation

## Impedance matching

*Bängtsson et al. (2003)*

Cost functional:

$$\mathcal{J}(\Omega, p) = \frac{1}{2} \int_{k_1}^{k_2} |Z(p, k) - Z_T(k)|^2 dk$$

subject to

$$\Delta p + k^2 p = 0 \quad + \quad \text{B.C.}$$

Impedance:  $Z(p, k) = \frac{e^{-ikL} - e^{ikL} + \bar{p}}{e^{-ikL} + e^{ikL} - \bar{p}}$  with  $\bar{p} = \frac{1}{L_Z} \int_{\Gamma_Z} p dl$ .

# Shape optimisation

## Impedance matching

Bängtsson et al. (2003)

Cost functional:

$$\mathcal{J}(\Omega, p) = \frac{1}{2} \int_{k_1}^{k_2} |Z(p, k) - Z_T(k)|^2 dk$$

subject to

$$\Delta p + k^2 p = 0 \quad + \quad \text{B.C.}$$

We define the Lagrangian (*constrained*  $\mapsto$  *unconstrained*)

$$\mathcal{L}(\Omega, p, \lambda) = \mathcal{J}(\Omega, p) - \text{real} \left( \int_{k_1}^{k_2} (\lambda, \Delta p + k^2 p)_\Omega dk \right)$$

# Shape optimisation

## Impedance matching

Bärgtsson et al. (2003)

Cost functional:

$$\mathcal{J}(\Omega, p) = \frac{1}{2} \int_{k_1}^{k_2} |Z(p, k) - Z_T(k)|^2 dk$$

subject to

$$\Delta p + k^2 p = 0 \quad + \quad \text{B.C.}$$

We define the Lagrangian (*constrained*  $\mapsto$  *unconstrained*)

$$\mathcal{L}(\Omega, p, \lambda) = \mathcal{J}(\Omega, p) - \text{real} \left( \int_{k_1}^{k_2} (\lambda, \Delta p + k^2 p)_\Omega dk \right)$$

Derivative with respect to each variable:

$$(\nabla_{\delta\lambda} \mathcal{L}, \delta\lambda) = (\nabla_{\delta p} \mathcal{L}, \delta p) = 0$$

$$(\nabla_{\delta\theta} \mathcal{L}, \delta\theta) = 0 \Rightarrow (\nabla_{\delta\theta} \mathcal{J}, \delta\theta)$$



# Impedance matching

---

Derivative with respect to  $\lambda$ :  $\Delta p + k^2 p = 0$ .

# Impedance matching

Derivative with respect to  $\lambda$ :  $\Delta p + k^2 p = 0$ .

Derivative with respect to  $p$ : Adjoint equation

$$\left\{ \begin{array}{ll} \Delta \lambda + (k^*)^2 \lambda = 0 & x \in \Omega \\ \nabla \lambda \cdot \mathbf{n} = 0 & x \in \Gamma_s \\ \nabla \lambda \cdot \mathbf{n} + \delta_v \frac{i+1}{2} \Delta_T \lambda - \delta_T (k^*)^2 \frac{(i+1)(\gamma-1)}{2} \lambda = 0 & x \in \Gamma_w \\ \nabla \lambda \cdot \mathbf{n} - ik^* \lambda = \frac{2}{LZ} \frac{e^{ikL}(Z(p, k) - Z_T(k))}{\left( (e^{-ikL} + e^{ikL} - \bar{p})^2 \right)^*} & x \in \Gamma_Z. \end{array} \right.$$

# Impedance matching

Derivative with respect to  $\lambda$ :  $\Delta p + k^2 p = 0$ .

Derivative with respect to  $p$ : Adjoint equation

$$\begin{cases} \Delta \lambda + (k^*)^2 \lambda = 0 & x \in \Omega \\ \nabla \lambda \cdot \mathbf{n} = 0 & x \in \Gamma_s \\ \nabla \lambda \cdot \mathbf{n} + \delta_v \frac{i+1}{2} \Delta_T \lambda - \delta_T (k^*)^2 \frac{(i+1)(\gamma-1)}{2} \lambda = 0 & x \in \Gamma_w \\ \nabla \lambda \cdot \mathbf{n} - ik^* \lambda = \frac{2}{LZ} \frac{e^{ikL}(Z(p, k) - Z_T(k))}{\left( (e^{-ikL} + e^{ikL} - \bar{p})^2 \right)^*} & x \in \Gamma_Z. \end{cases}$$

Derivative with respect to  $\Omega$ : For any displacement direction  $\delta \boldsymbol{\theta}$ ,

$$(\nabla_{\delta \boldsymbol{\theta}} \mathcal{J}, \delta \boldsymbol{\theta}) = \int_{\Gamma_\theta} (\delta \boldsymbol{\theta}(x) \cdot \mathbf{n}(x)) \int_{k_1}^{k_2} \text{Real} [-\nabla \lambda^* \cdot \nabla p + k^2 \lambda^* p] dk dx.$$

# Impedance matching

---

## Algorithm:

Solve iteratively:

- Solve Helmholtz direct  $\Rightarrow p(k)$  (*frequency response*).
- Solve Helmholtz adjoint  $\Rightarrow \lambda(k)$  (*frequency response*).
- Optimality condition  $\Rightarrow \nabla_{\delta\theta}\mathcal{J}$ .
- Move shape with linesearch algorithm (*Armijo backtracking*).
- Iterate until convergence.

## Numerical implementation

*Moës et al. (1999)*

Finite elements with immersed boundary:

*Osher et al. (2001)*

- **XFEM** for integration methods on cut elements (GETFEM++).

*Protas et al. (2004)*

- **Level set:**

$$\begin{cases} \psi = 0 & x \in \Gamma_\theta \\ \psi > 0 & x \in \Omega \\ \psi < 0 & x \notin \Omega. \end{cases}$$

- Transport equation for moving the shape:

$$\frac{\partial \psi}{\partial t} + v \mathbf{n} \cdot \nabla \psi = 0.$$

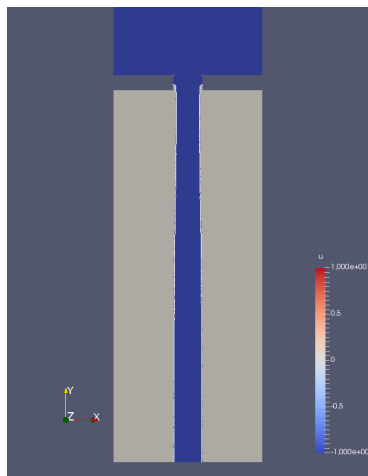
with

$$v = -\nabla_{\delta\theta} \mathcal{J} \cdot \mathbf{n}. \quad (\text{Regularised by Sobolev Gradient})$$

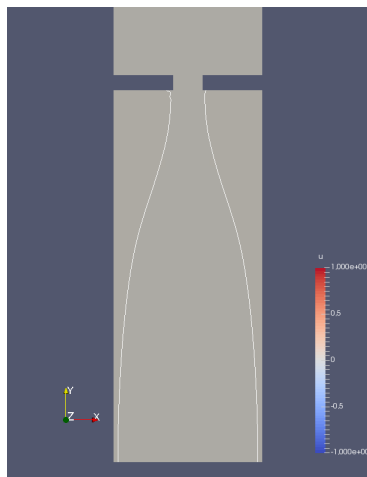
$\mathbf{n}$  is the outward wall normal direction. SUPG discretisation.

# Results

No viscosity:  $Z_T = 0$ ,  $k = [0.2 : 0.6]$



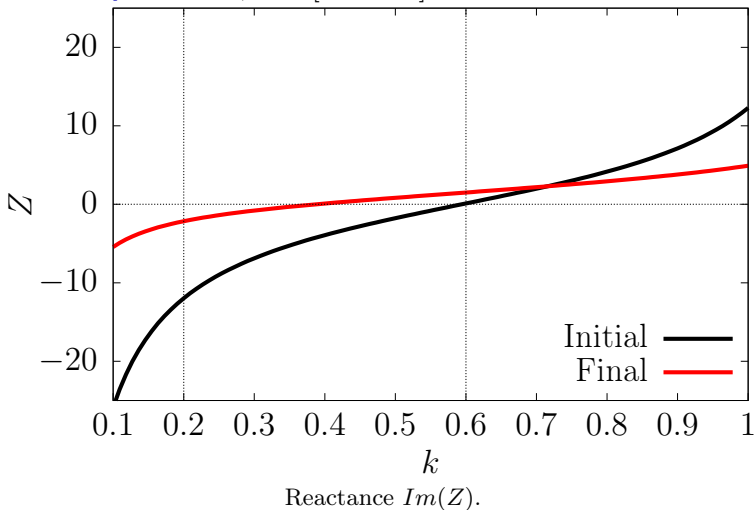
Initial condition.



Converged.

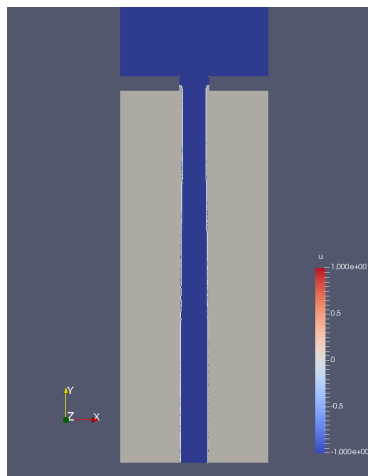
# Results

No viscosity:  $Z_T = 0$ ,  $k = [0.2 : 0.6]$

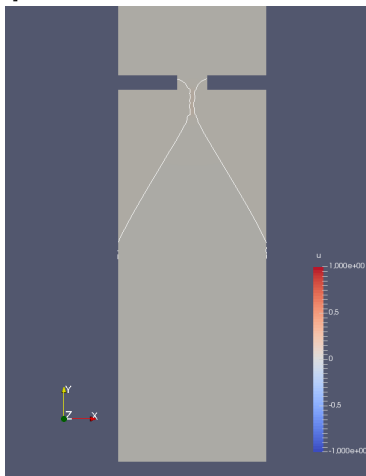


# Results

No viscosity:  $Z_T = 0$ ,  $k = [0.15 : 0.25]$



Initial condition.

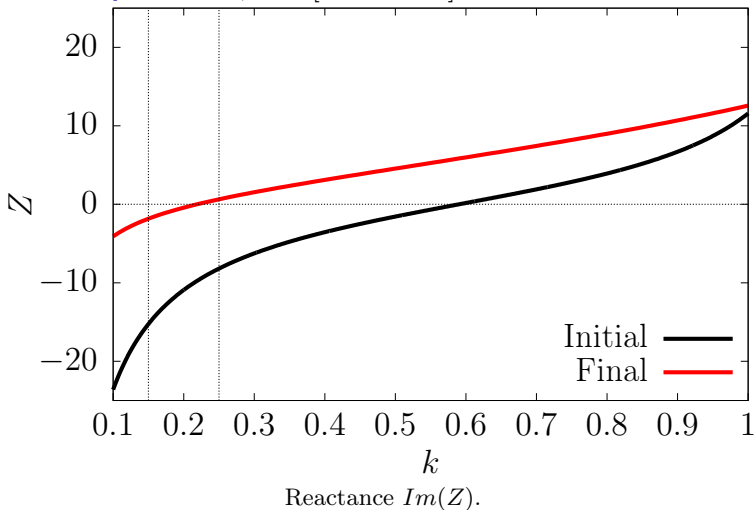


Converged.



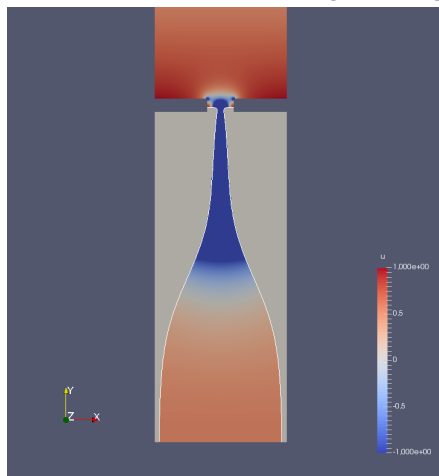
# Results

No viscosity:  $Z_T = 0$ ,  $k = [0.15 : 0.25]$

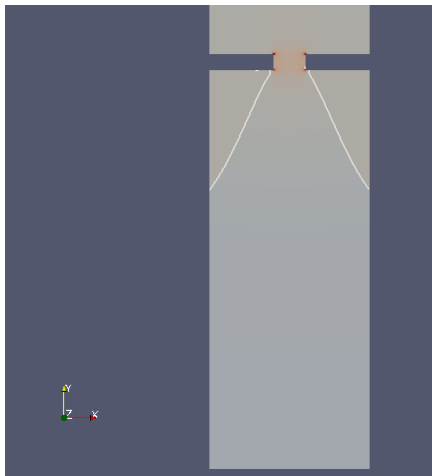


# Results

Viscosity:  $Z_T = 1.0$ ,  $k = [0.2 : 0.6]$



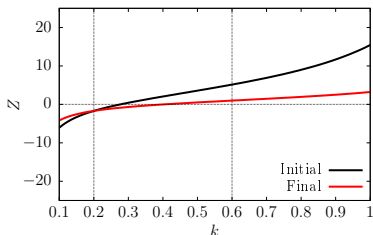
Initial condition.  
Coloured by  $\nabla_{\delta\theta} \mathcal{J}$ .



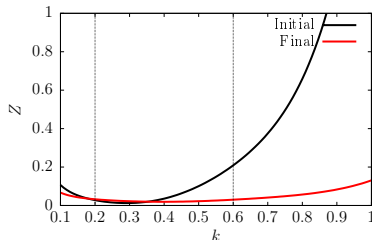
Converged.

# Results

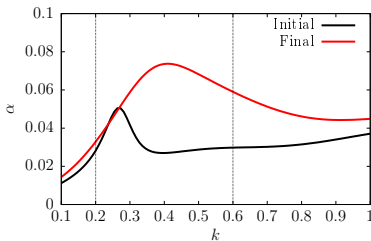
Viscosity:  $Z_T = 1.0$ ,  $k = [0.2 : 0.6]$



Reactance  $Im(Z)$ .



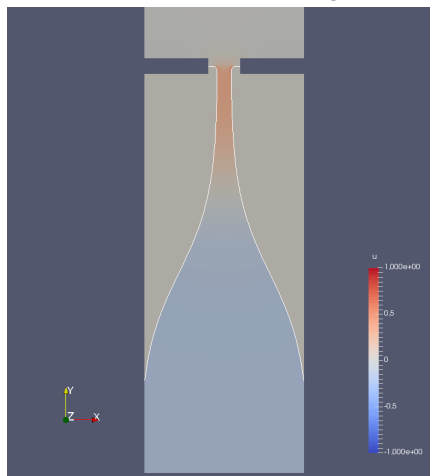
Resistance  $Re(Z)$ .



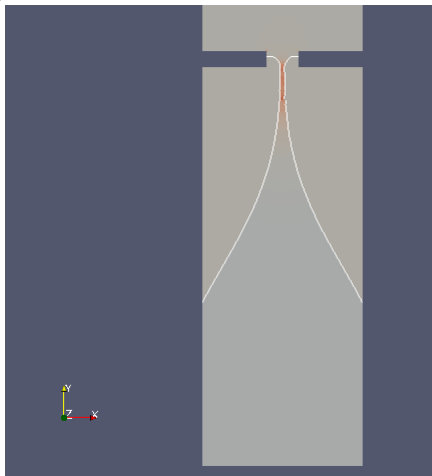
Absorption  $\alpha = 1 - |R|^2$ .

# Results

Viscosity:  $Z_T = 1.0$ ,  $k = [0.15 : 0.25]$



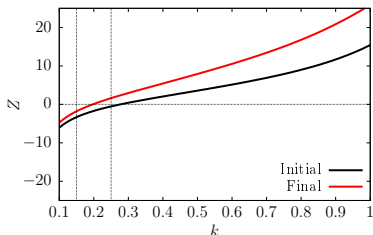
Initial condition.  
Coloured by  $\nabla_{\delta\theta}\mathcal{J}$ .



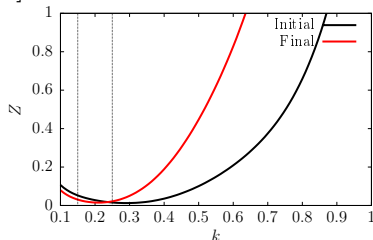
Converged.

# Results

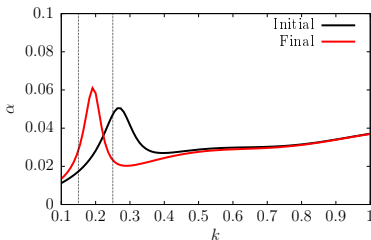
Viscosity:  $Z_T = 1.0$ ,  $k = [0.15 : 0.25]$



Reactance  $Im(Z)$ .



Resistance  $Re(Z)$ .



Absorption  $\alpha = 1 - |R|^2$ .

# Conclusion

---

## Summary:

- Shape optimisation for impedance matching.
- Viscous model: compromise between Helmholtz and full linearised Navier-Stokes.

## Perspectives:

- Optimise from efficient/realistic configuration.
- Different cost functional (Penalty to initial guess, absorption, ...).

Thank you for your attention.