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New Results on Quantum Symmetric Cryptanalysis

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ERC project QUASYModo

JNIM Avril 2018
Outline

► Introduction
On Quantum-Safe *Symmetric* Cryptography

► Efficient Quantum Collision Search
joint work with A. Chailloux and A. Schrottenloher
[Asiacrypt17]

► On Modular Additions
joint work with X. Bonnetain
Symmetric Cryptography
Classical Cryptography

Enable secure communications even in the presence of malicious adversaries.

Asymmetric (e.g. RSA) (*no key exchange/computationally costly*)
- Security based on well-known hard mathematical problems (e.g. factorization).

Symmetric (e.g. AES) (*key exchange needed/efficient*)
- Ideal security defined by generic attacks ($2^{|K|}$).
- Need of continuous security evaluation (cryptanalysis).

⇒ Hybrid systems! (e.g. in SSH)
Symmetric primitives

Block ciphers, (stream ciphers, hash functions..)

Message decomposed into blocks, each transformed by the same function $E_K$.

$E_K$ is composed of a round transform repeated through several similar rounds.
Generic Attacks on Ciphers

- Security provided by an ideal block cipher defined by the best generic attack: exhaustive search for the key in $2^{|K|}$.

- Recovering the key from a secure cipher must be infeasible.

  $\Rightarrow$ typical key sizes $|K| = 128$ to 256 bits.
Cryptanalysis: Foundation of Confidence

Any attack better than the generic one is considered a “break”.

- Proofs on symmetric primitives need to make unrealistic assumptions.
- We are often left with an empirical measure of the security: cryptanalysis.
- Security redefinition when a new generic attack is found (e.g. accelerated key search with bicliques [BKR 12])
Current scenario

- Competitions (AES, SHA-3, eSTREAM, CAESAR).
- New needs: lightweight, FHE-friendly, easy-masking.
  ➞ Many good proposals/candidates.

- How to choose?

- How to be ahead of possible weaknesses?

- How to keep on trusting the chosen ones?
When can we consider a primitive as secure?

- A primitive is secure as far as no attack on it is known.
- The more we analyze a primitive without finding any weaknesses, the more reliable it is.

**Design new attacks + improvement of existing ones:**
- essential to keep on trusting the primitives,
- or to stop using the insecure ones!
On weakened versions

If no attack is found on a given cipher, what can we say about its robustness, security margin?

The security of a cipher is not a 1-bit information:

- Round-reduced attacks.
- Analysis of components.

⇒ determine and adapt the security margin.
On high complexities

When considering large keys, sometimes attacks breaking the ciphers might have a very high complexity far from practical e.g. $2^{120}$ for a key of 128 bits.

Still dangerous because:

• Weak properties not expected by the designers.
• Experience shows us that attacks only get better.
• Other existing ciphers without the ”ugly” properties.

► When determining the security margin: find the highest number of rounds reached.
Post-Quantum Symmetric Cryptography
Adversaries have access to **quantum computers**.

**Asymmetric (e.g. RSA):**
- Shor’s algorithm: Factorization in polynomial time
  - \( \Rightarrow \) current systems not secure!
- Solutions: lattice-based, code-based cryptography...

**Symmetric (e.g. AES):**
- Grover’s algorithm: Exhaustive search from \(2^{|K|}\) to \(2^{|K|}/2\).
  - Double the key length for equivalent ideal security.
- We don’t know much about cryptanalysis of current ciphers when having quantum computing available.
Post-Quantum Cryptography

Problem for present existing long-term secrets.  
⇒ start using quantum-safe primitives NOW.

Important tasks:

- Conceive the cryptanalysis algorithms for evaluating the security of symmetric primitives in the P-Q world.

- Use them to evaluate and design symmetric primitives for the P-Q world.
Quantum Symmetric Cryptanalysis

Some recent results on Q-symmetric cryptanalysis:

3-R Feistel [Kuwakado-Morii10], Even-Mansour [Kuwakado-Morii12], Mitm [Kaplan14], Related-Key [Roetteler-Steinwandt15], Diff-lin [Kaplan-Leurent-Leverrier-NP16], Simon’s[Kaplan-Leurent-Leverrier-NP16], FX [Leander-May17], parallel multi-preim. [Banegas-Bernstein17], Multicollision [Hosoyamada-Sasaki-Xagawa17], AEZ [Bonnetain17]...
Collision Search

w. A. Chailloux & A. Schrottenloher
Collision Search Problem

Given a random function \( H : \{0, 1\}^n \rightarrow \{0, 1\}^n \), find \( x, y \in \{0, 1\}^n \) with \( x \neq y \) such that \( H(x) = H(y) \).

Many applications: i.e. generic attacks on hash functions.

(Multi-preimage search can be seen as a particular case).
## Best known algorithms

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<th>Time</th>
<th>Queries</th>
<th>Memory</th>
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<td>Ambainis</td>
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</tr>
</tbody>
</table>
Open Questions

Challenge 1: Find an algorithm for collision and/or element distinctness which gives a searching speedup greater than merely a square-root factor over the number of available processing qubits

Grover and Rudolph, How significant are the known collision and element distinctness quantum algorithms? 2004.
Considered Model

- The same one as in all the previous quantum algorithms BUT we limit the amount of quantum memory available to a small amount $\text{poly}(n)$.

- Available small quantum computers seems like the most plausible scenario.

- We are interested in the theoretical algorithm and we did not take into account implementation aspects.
Starting Point: BHT Algorithm

- Optimal number of queries,

- $poly(n)$ qbits,

- But time?
BHT: Summarized procedure

- Build a list \( L \) of size \( 2^{n/3} \) elements (classic memory),

- Exhaustive search for finding one element that collides:
  With AA, the number of iterations is \( (\frac{2^n}{2^{n/3}})^{1/2} = 2^{n/3} \).

Testing the membership with \( L \) for the superposition of states costs \( 2^{n/3} \) with \( n \) qbits:

\[
\text{Time: } 2^{n/3} + 2^{n/3}(1 + 2^{n/3}) \approx 2^{2n/3}
\]
Can we improve this?

Let's build the list $L$ with distinguished points
e.g. $H(x_i) = 0^u||z$, for $z \in \{0, 1\}^{n-u}$.

The cost of building the list is bigger: $2^{n/3+u/2}$.
The setup of AA is bigger: $2^{u/2}$
The membership test stays the same: $|L| = 2^{n/3}$
BUT The number of iterations is smaller: $2^{n/3-u/2}$

Time: $2^{n/3+u/2} + 2^{n/3-u/2}(2^{u/2} + 2^{n/3}) \approx 2^{2n/3-u/2} + 2^{n/3+u/2}$
With optimal parameters

The cost will be optimized for a certain size of $L$: $2^v \neq 2^{n/3}$.

Time: $2^{v+u/2} + 2^{n-v-u/2} (2^{u/2} + 2^v)$

For $v = n/5$, $u = 2n/5$: Time: $\tilde{O}(2^{2n/5})$

For multiple preimage search, the algorithm is similar, but we only keep in $L$ the distinguished points amongst the already given ones.
## Comparison

<table>
<thead>
<tr>
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<th>Classic Memory</th>
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<td>0</td>
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<tr>
<td>New algorithm</td>
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<td>$2^{2n/5}$</td>
<td>$poly(n)$</td>
<td>$2^{n/5}$</td>
</tr>
</tbody>
</table>
With $2^s$ n-qbit registers and "external" parallelization we can achieve:

Time: $2^{v+u/2-s} + 2^{n-v-u-s/2}(2^{u/2} + 2^v)$

Our theoretical algorithm seems more efficient than classical parallelization/Beal up to $s = n/4$
Comparison example:  n=128

\[
\begin{align*}
\text{Time (log}_2) & \\
0 & 20 & 40 & 60 \\
0 & 20 & 40 & 60 \\
\end{align*}
\]

- \(\frac{2n}{5} - \frac{3s}{5}\) (our algorithm)
- \(\frac{n}{2} - \frac{s}{2}\) (Grover)
- \(\frac{n}{2} - s\) (parallel Rho)
1. Hash functions: Collision and Multi-preimages time from $2^{n/2}$ to $2^{2n/5}$ and $2^{3n/7}$.

Ex.- time and queries for $n = 128$:

$\rho = 2^{64}$, ours $= 2^{51.2}$ (with less than 1GB classical)
Conclusion 1

We solved challenge 1 for Grover and Rudolph 2004: new efficient collision search algorithm with small quantum memory.

Many applications in symmetric cryptography.

Open question: is it possible to meet the optimal $2^{n/3}$ in time with small quantum memory? (Quantum random walks, quantum learning graphs...?)
On Modular Additions
with X. Bonnetain
Quantum cryptanalysis: Simon’s algorithm

Simon’s problem: Given $f : \{0, 1\}^n \to \{0, 1\}^n$ such that
\[
\exists s \mid f(x) = f(y) \iff [x = y \text{ or } x \oplus y = s], \text{ find } s.
\]

- Classical complexity: $\Omega(2^{n/2})$.
- Quantum complexity [Simon 94]: $O(n)$.  

25/40
Simon’s algorithm in Symmetric Cryptography

- Even-Mansour cipher [Even Mansour 97]: $DT > 2^n$

$$f(x) = E_K(x) \oplus P(x) \rightarrow f(x) = f(x \oplus K_1)$$

Simon’s algo. on $f \Rightarrow K_1$ in $O(n)$ [Kuwakado Morii 12] (Q2)

- Related-key attacks [Roetteler Steinwandt 15]
- 3-round Feistel [Kuwakado Morii 10]
- LWR, modes of operation for authentication (CBC-MAC, PMAC, OCB..), some CAESAR candidates [KLLN-P 16b]
Simon’s algorithm and Slide attacks

- Classical: $\mathcal{O}(2^{n/2})$ [Biryukov Wagner 99]

- Quantum: Simon $\mathcal{O}(n)$ [KLLN-P 16b]

$f : \{0, 1\} \times \{0, 1\}^n \to \{0, 1\}^n$

\[ b, x \mapsto \begin{cases} P(E_K(x)) \oplus x & \text{if } b = 0, \\ E_K(P(x)) \oplus x & \text{if } b = 1. \end{cases} \]

\[ f(x) = f(x \oplus (1||K)) \]
Some (NOT ALL) primitives secure in the classical world become completely broken in the superposition model.

This does not seem a priori to imply that these primitives are unsafe in other settings.
In [Alagic Russell 17] several proposals. Most efficient: replace xor by modular additions.

Hidden shift problem in $\mathbb{Z}/(N)$.

No algorithm in polynomial time: Kuperberg in $2^{O(\sqrt{n})}$

Up to what point do primitives resist?
Motivation and results

► 5. Dimensionate symmetric primitives

► 1. More precise evaluation of Kuperberg’s algorithm complexity improvement

► 2. Example of application with Poly1305

► 3. What about parallel modular additions?

► 4. New Quantum attacks (Feistel’s slide, FX)
Our improvement: all the bits with one iteration.

\[ O(n^2 2^{\sqrt{2 \log_2(3)n}}) \Rightarrow O(n2^{\sqrt{2 \log_2(3)n}}) \]

Our simulations give: \( 0.7 \times 2^{1.8\sqrt{n}} \) for recovering full \( s \).

Code available: ask Xavier Bonnetain if interested.

xavier.bonnetain@inria.fr
Application example with Poly1305

Poly1305 in the superposition model.

Two 128-bit keys \((r, k)\), 128-bit nonce \(n\), message \(m\) array of 128-bit blocks, output 128-bit tag.

\[
\text{Poly1305-AES}_{(r,k,n)}(m_1, \ldots, m_q) = (\sum_{i=1}^{q}(m_{q-i+1} + 2^{128})r^i \mod (2^{130} - 5)) + \text{AES}_k(n)
\]

Access to:

\[
Poly_n^2 : |m_1\rangle |m_2\rangle |0\rangle \leftrightarrow |m_1\rangle |m_2\rangle \left| \text{Poly1305-AES}_{(r,k,n)}(m_1, m_2) \right>,
\]
We denote
\[ F(x) = \text{Poly1305-AES}_{(r,k,n)}(1, x) = (f(x) \mod (2^{130} - 5)) + \text{AES}_k(n) \] and
\[ G(x) = \text{Poly1305-AES}_{(r,k,n)}(0, x) = (g(x) \mod (2^{130} - 5)) + \text{AES}_k(n), \]
which satisfy, for the same nonce, \( F(x) = G(x + r) \).

As \( f(x) = xr + r^2 + 2^{128}(r + r^2) \), \( g(x) = xr + 2^{128}(r + r^2) \) and \( f(x) = g(x + r) \).

Apply Kuperberg to find the hidden shift \( r \).
Two issues:

- One nonce, one query to both $F(x)$ and $G(x)$: we can compute $(1, x)$ and $(0, x)$ in superposition in one register and call the oracle $Poly_n^2$ on it.

- We cannot sample all group elements: consider $2^{18}$ possible intervals for $r$ of size $2^{106}$: $r \in [2^{106}c, 2^{106}(c + 1))$ for $c \in [0, 2^{18})$ and the functions $f(x)$ and $g(x + 2^{106}c)$. Bad element with pb $2^{-21}$. Apply Kuperberg to each interval: $2^{20}$. Complexity: $2^{38}$ for $r$ (thanks to our improvement!).
Algorithm for Parallel Modular Additions?

- HSP problem for groups product of cyclic groups
- Recurrent problem in symmetric cryptography
- Kuperberg not optimal
Algorithm for solving the case of $p$ modular additions of words of $w$, matching Simon’s ($w = 1$) and Kuperberg’s ($p = 1$)

- First Idea: Kuperberg’s variant - better worst-case gain

- Second Idea: $p + 1$ equations always gain $p$ zeros

- Combining both: best method depends on parameters and thresholds.
New Quantum Attacks

- Advanced slide attacks on Feistel ciphers
- Attacks on Feistel ciphers with non-invertible functions
- FX construction (quantum [Leander-May17]) with modular additions
Conclusion 2

- Improved Kuperberg’s algorithm and new algorithm for parallel modular additions.

- State size needed for a 128-bit security. at least 5200 bits (but for FX) ⇒ not very realistic.

- Might be better to just avoid vulnerable constructions, or try different patches (if we are concerned by superposition attacks).

- Superposition-Poly1305 broken implies that Poly1305 is not safe in the superposition model.
Final Conclusion
Optimal collision time $2^{n/3}$?

- $\alpha$–XOR problem.
- Algebraic attacks.
- Boomerang attacks.
- FSE Stevens: Quantum cryptanalysis of SHA-2?
- AES quantum evaluation- on going work.
- Generic key-length extensions?
- What about state size? ...
Symmetric Quantum Cryptanalysis

Lots of things to do!

\footnote{Thanks to X. Bonnetain, A. Chailloux and A. Schrottenloher for their help with the slides}