



# New results on symmetric quantum cryptanalysis

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## ► To cite this version:

María Naya-Plasencia. New results on symmetric quantum cryptanalysis. Crossfyre 2018 - 8th international workshop on cryptography, robustness, and provably secure schemes for female young researchers, Sep 2018, Surrey, United Kingdom. hal-01953997

**HAL Id: hal-01953997**

**<https://inria.hal.science/hal-01953997>**

Submitted on 13 Dec 2018

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# New Results on Symmetric Quantum Cryptanalysis

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CrossFyre 2018

University of Surrey - 13 September 2018

# Outline

- ▶ Introduction  
On Quantum-Safe **Symmetric** Cryptography
- ▶ Efficient Quantum Collision Search  
joint work with **A. Chailloux** and **A. Schrottenloher**  
[Asiacrypt17]
- ▶ Efficient Quantum k-XOR search  
joint work with **L. Grassi** and **A. Schrottenloher**

# Symmetric Cryptography

# Classical Cryptography

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Enable secure communications even in the presence of malicious adversaries.

Asymmetric (e.g. RSA) (*no key exchange/computationally costly*)  
Security based on well-known hard mathematical problems (e.g. factorization).

Symmetric (e.g. AES) (*key exchange needed/efficient*)  
Ideal security defined by generic attacks ( $2^{|K|}$ ).  
Need of continuous security evaluation (cryptanalysis).

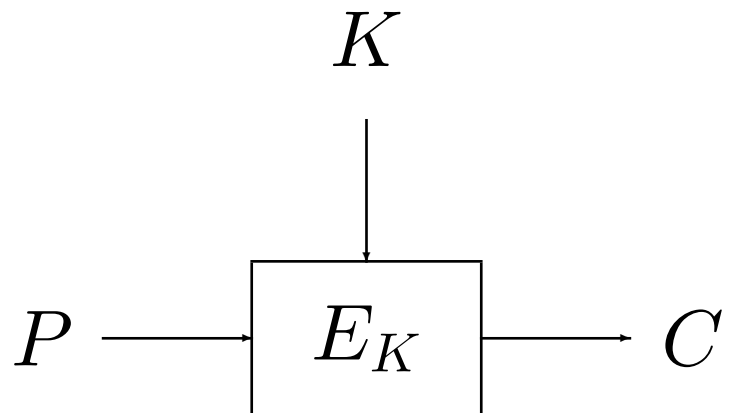
⇒ Hybrid systems! (e.g. in SSH)

# Symmetric primitives

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- ▶ Block ciphers, (stream ciphers, hash functions..)

Message decomposed into blocks, each transformed by the same function  $E_K$ .



$E_K$  is composed of a round transform repeated through several similar rounds.

# Generic Attacks on Ciphers

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- ▶ Security provided by an **ideal block cipher** defined by the best generic attack:  
exhaustive search for the key in  $2^{|K|}$ .
- ▶ Recovering the key from a secure cipher must be infeasible.  
 $\Rightarrow$  typical key sizes  $|K| = 128$  to 256 bits.

# Cryptanalysis: Foundation of Confidence

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Any attack better than the generic one is considered a “break”.

- ▶ Proofs on symmetric primitives need to make unrealistic assumptions.
- ▶ We are often left with an **empirical measure** of the security: cryptanalysis.
- ▶ Security redefinition when a new generic attack is found (e.g. accelerated key search with bicliques [BKR 12])



# Current scenario

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- ▶ Competitions (AES, SHA-3, eSTREAM, CAESAR).
- ▶ New needs: lightweight, FHE-friendly, easy-masking.  
⇒ Many good proposals/candidates.
- ▶ How to choose?
- ▶ How to be ahead of possible weaknesses?
- ▶ How to keep on trusting the chosen ones?

# Cryptanalysis: Foundation of Confidence

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When can we consider a primitive as secure?

- A primitive is secure as far as no attack on it is known.
- The more we analyze a primitive without finding any weaknesses, the more reliable it is.

**Design new attacks + improvement of existing ones:**

- ▶ essential to keep on **trusting** the primitives,
- ▶ or to stop using the insecure ones!

# On weakened versions

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If no attack is found on a given cipher, what can we say about its robustness, security margin?

The security of a cipher is not a 1-bit information:

- Round-reduced attacks.
  - Analysis of components.
- ⇒ determine and adapt the security margin.

# On high complexities

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When considering large keys, sometimes attacks breaking the ciphers might have a very high complexity far from practical e.g..  $2^{120}$  for a key of 128 bits.

Still dangerous because:

- Weak properties not expected by the designers.
  - Experience shows us that attacks only get better.
  - Other existing ciphers without the "ugly" properties.
- When determining the security margin: find the highest number of rounds reached.

# Post-Quantum Symmetric Cryptography

# Post-Quantum Cryptography

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Adversaries have access to **quantum computers**.

Asymmetric (e.g. RSA):

Shor's algorithm: Factorization in polynomial time

⇒ **current systems not secure!**

Solutions: lattice-based, code-based cryptography...

Symmetric (e.g. AES):

Grover's algorithm: Exhaustive search from  $2^{|K|}$  to  $2^{|K|/2}$ .

Double the key length for equivalent ideal security.

**We don't know much about cryptanalysis of current ciphers when having quantum computing available.**

# Post-Quantum Cryptography

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Problem for present existing long-term secrets.  
⇒ start using quantum-safe primitives NOW.

## Important tasks:

- ▶ Conceive the **cryptanalysis algorithms** for evaluating the security of symmetric primitives in the P-Q world.
- ▶ Use them to evaluate and **design** symmetric primitives for the P-Q world.

# Quantum Symmetric Cryptanalysis

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Some recent results on Q-symmetric cryptanalysis:

3-R Feistel [Kuwakado-Morii10], Even-Mansour [Kuwakado-Morii12], Mitm [Kaplan14], Related-Key [Roetteler-Steinwand15], Diff-lin [Kaplan-Leurent-Leverrier-NP16], Simon's [Kaplan-Leurent-Leverrier-NP16], FX [Leander-May17], parallel multi-preim. [Banegas-Bernstein17], Multicollision [Hosoyamada-Sasaki-Xagawa17], AEZ [Bonnetain17], Modular additions [Bonnetain-NP18]...



# Quantum Symmetric Cryptanalysis

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Two main models used:

- ▶ Q1:  
classical queries and access to a quantum computer.
- ▶ Q2:  
+superposition queries to a quantum cryptog. oracle.

Very powerful, BUT...

## Q2: Superposition Model

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Many good reasons to study security in this scenario:

- ▶ Simple
- ▶ Non-trivial: Many constructions still seem resistant: AES, SALSA20, NMAC, HMAC...
- ▶ Inclusive of all intermediate scenarios

Defined and used in: [Zhandry12], [Boneh-Zhandry13], [Damgård-Funder-Nielsen-Salvail13], [Mossayebi-Schack16], [Song-Yun17], Simon's attacks, FX, AEZ...

An attack in this model  $\Rightarrow$  might not be safe to implement the primitive in a quantum computer.

# On Quantum attacks

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- ▶ Compare to best generic attack,
- ▶ generic attack is accelerated, so
- ▶ broken classical primitive might be unbroken in a quantum setting.

# Collision Search

*w. A. Chailloux & A. Schrottenloher*

# Collision Search Problem

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Given a random function  $H : \{0, 1\}^n \rightarrow \{0, 1\}^n$ , find  $x, y \in \{0, 1\}^n$  with  $x \neq y$  such that  $H(x) = H(y)$ .

Many applications: *i.e.* generic attacks on hash functions.

(Multi-preimage search can be seen as a particular case).

# Best known algorithms

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	Time	Queries	Memory
Pollard's rho	$2^{n/2}$	$2^{n/2}$	$poly(n)$
Parallelization ( $2^s$ )	$2^{n/2-s}$	$2^{n/2}$	$2^s$

	Time	Queries	Qubits
Grover	$2^{n/2}$	$2^{n/2}$	$poly(n)$
BHT	$2^{2n/3}$ *	$2^{n/3}$	$poly(n)$ *
Ambainis	$2^{n/3}$	$2^{n/3}$	$2^{n/3}$

# Considered Model

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- ▶ The **same** one as in all the previous quantum algorithms BUT we limit the amount of **quantum memory available** to a **small** amount  $\text{poly}(n)$ .
- ▶ Available small quantum computers seems like the most plausible scenario.
- ▶ We are interested in the theoretical algorithm and we did not take into account implementation aspects.

# Starting Point: BHT Algorithm

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- ▶ Optimal number of queries,
- ▶  $\text{poly}(n)$  qbits,
- ▶ But time?



# BHT: Summarized procedure

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- ▶ Build a list  $L$  of size  $2^{n/3}$  elements (classic memory),
- ▶ Exhaustive search for finding one element that collides:  
With AA, the number of iterations is  $(\frac{2^n}{2^{n/3}})^{1/2} = 2^{n/3}$ .

Testing the membership with  $L$  for the superposition of states costs  $2^{n/3}$  with  $n$  qbits:

$$\text{Time: } 2^{n/3} + 2^{n/3}(1 + 2^{n/3}) \approx 2^{2n/3}$$

# Can we improve this?

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Lets build the list  $L$  with distinguished points

e.g.  $H(x_i) = 0^u || z$ , for  $z \in \{0, 1\}^{n-u}$ .

The cost of building the list is bigger:  $2^{n/3+u/2}$ .

The setup of AA is bigger:  $2^{u/2}$

The membership test stays the same:  $|L| = 2^{n/3}$

**BUT** The number of iterations is smaller:  $2^{n/3-u/2}$

Time:  $2^{n/3+u/2} + 2^{n/3-u/2}(2^{u/2} + 2^{n/3}) \approx 2^{2n/3-u/2} + 2^{n/3+u/2}$

## With optimal parameters

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The cost will be optimized for a certain size of  $L$ :  $2^v \neq 2^{n/3}$ .

Time:  $2^{v+u/2} + 2^{\frac{n-v-u}{2}}(2^{u/2} + 2^v)$

For  $v = n/5$ ,  $u = 2n/5$ : Time:  $\tilde{O}(2^{2n/5})$

For multiple preimage search, the algorithm is similar, but we only keep in  $L$  the distinguished points amongst the already given ones.

# Comparison

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	Time	Queries	Qubits	Classic Memory
Pollard	$2^{n/2}$	$2^{n/2}$	0	$poly(n)$
Grover	$2^{n/2}$	$2^{n/2}$	$poly(n)$	0
BHT	$2^{2n/3}$	$2^{n/3}$	$poly(n)$	$2^{n/3}$
Ambainis	$2^{n/3}$	$2^{n/3}$	$2^{n/3}$	0
New algorithm	$2^{2n/5}$	$2^{2n/5}$	$poly(n)$	$2^{n/5}$

# Parallelization

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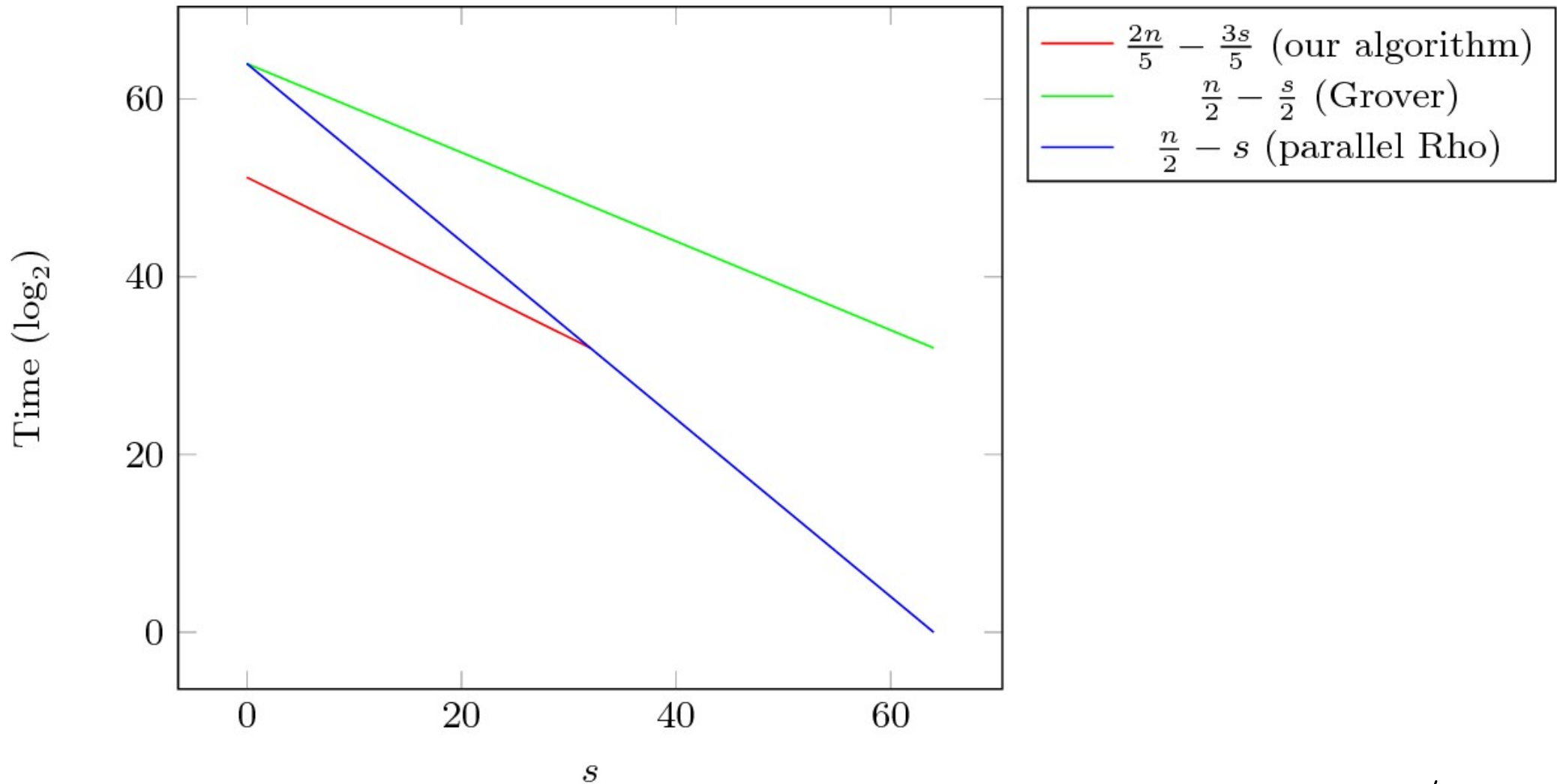
With  $2^s$   $n$ -qbit registers and "external" parallelization we can achieve:

$$\text{Time: } 2^{v+u/2-s} + 2^{\frac{n-v-u}{2}-s/2}(2^{u/2} + 2^v)$$

Our theoretical algorithm seems more efficient than classical parallelization/Beal up to  $s = n/4$

# Comparison example: $n=128$

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# Example of Applications (1)

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- ▶ **1. Hash functions:** Collision and Multi-preimages time from  $2^{n/2}$  to  $2^{2n/5}$  and  $2^{3n/7}$  (Q1).  
Ex.- time and queries for  $n = 128$ :  
 $\text{rho} = 2^{64}$ ,  $\text{ours} = 2^{51.2}$  (with less than 1GB classical)
- ▶ **2. Multi-user setting:** Recover Ctxt, from same Ptxt,  $2^t$  different keys: apply multi-preimage algorithm (Q1).  
Depending on the value of  $t$  different gain.

## Example of Applications (2)

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- ▶ 3. Operation modes: Collision attacks on CBC:  
 $2^t$  Ctxt, find one preimage  $\Rightarrow$  Ptxt. (Q2). If frequent rekeying (Q1).
- ▶ 4. Bricks for Cryptanalysis: Collision, multi-preimage search: often bricks of more technical cryptanalysis: improve the steps.



# Conclusion 1

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New efficient collision search algorithm with small quantum memory.

Many applications in symmetric cryptography.

**Open question:** is it possible to meet the optimal  $2^{n/3}$  in time with small quantum memory? (Quantum random walks, quantum learning graphs...?)

# Quantum Efficient Algorithms for the k-XOR Problem

*w. L. Grassi & A. Schrottenloher*

# k-XOR problem with random functions

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Given query access to a random function

$H : \{0, 1\}^n \rightarrow \{0, 1\}^n$ , find  $x_1, \dots, x_k$  such that  
 $H(x_1) \oplus \dots \oplus H(x_k) = 0$ .

For us, **equivalent** to the case with  $k$  different random functions.

**Many applications** (with k-SUM, similar algorithms apply),  
ex.: attacks on FSB, XLS, SWIFFT; correlation attacks.

# The 3-XOR problem

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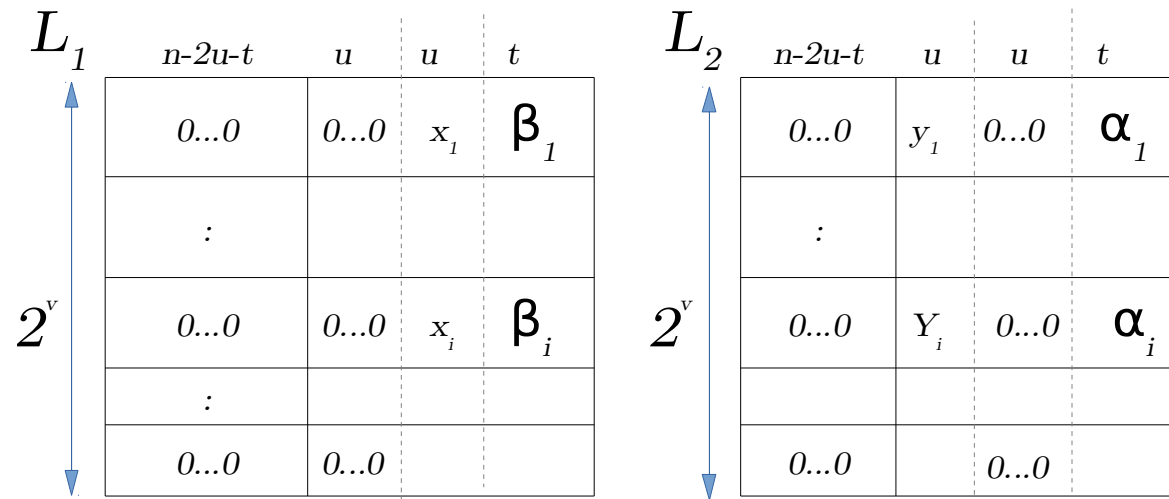
Find 3 elements that XOR to 0: not much better than collision in classical setting.

Classically, no exponential acceleration, only logarithmic factors:

Complexity of about  $2^{n/2}$  without these factors.

# 3-XOR: Low Quantum Memory Algorithm

- ▶ 1st approach, distinguished point:  $2^v = 2^{n/8}$ ,  $T = 2^{3n/8}$
- ▶ 2nd approach, techniques linked to "list merging":



Improved time =  $2^{5n/14}$ , with  $2^v = 2^{n/7}$ .

- ▶ More efficient than collision, contrary to classical!

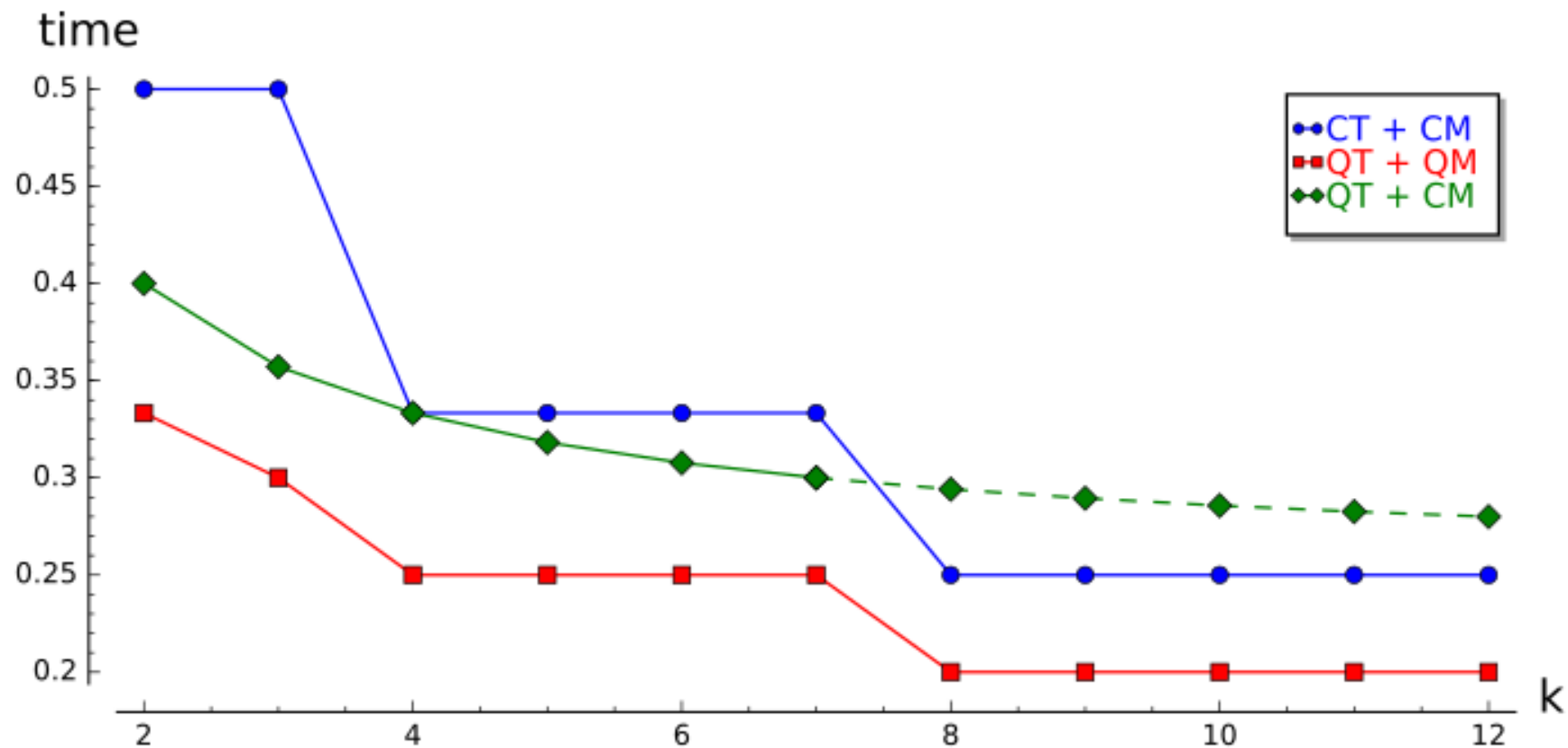
# 3-XOR: High Quantum Memory Algorithm

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- ▶ Same technique as before, but no need for the positions to '0' in both lists.
- ▶ Complexity of:  
$$2^{v+u/2} + 2^{\frac{n-2v}{2}}(2^{v-u}).$$
- ▶ This becomes optimal for  
QM =  $2^{n/5}$  and Time =  $2^{3n/10}$ .

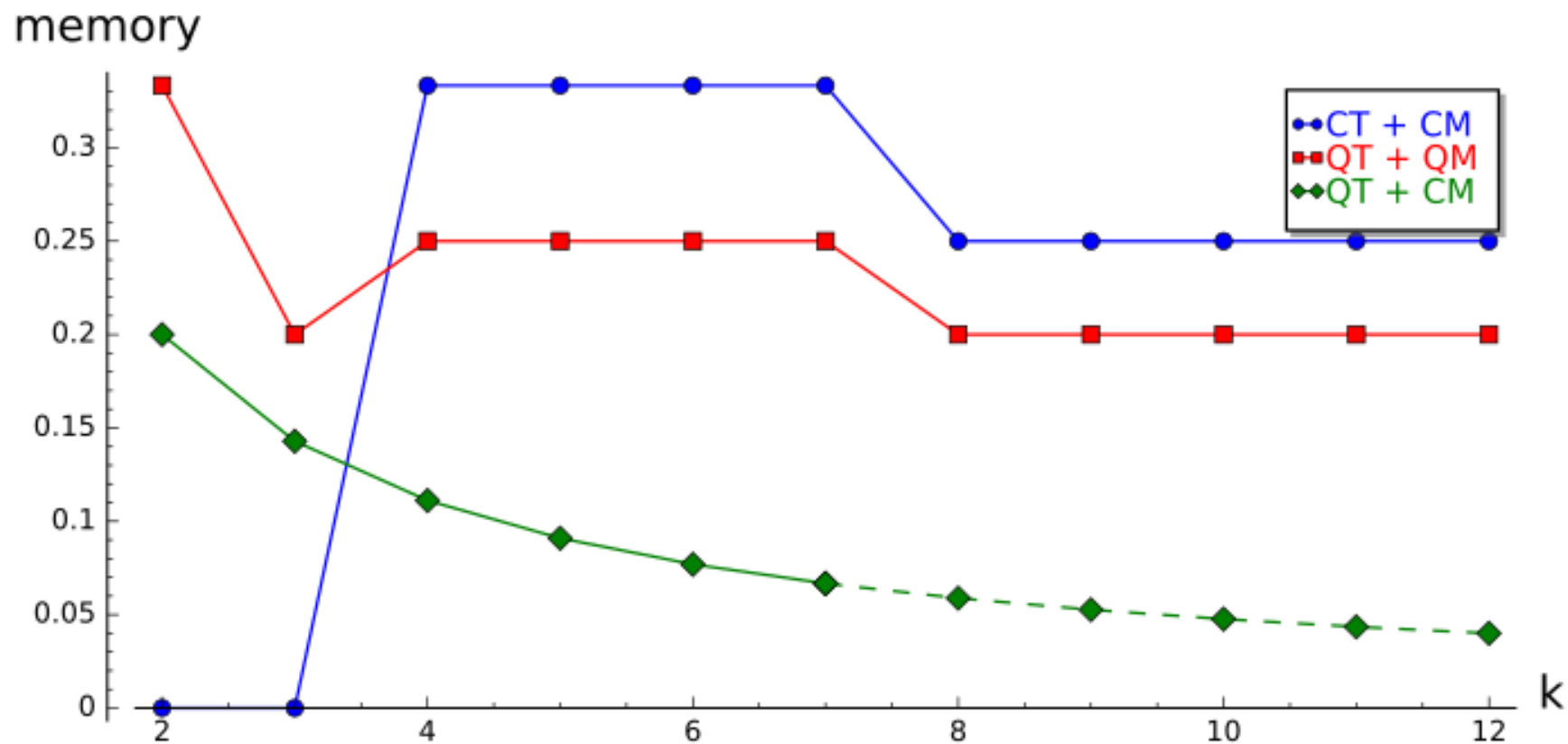
# The $k$ -XOR algorithms

Similar algorithms can be applied to other values of  $k$



# The $k$ -XOR algorithms

Similar algorithms can be applied to other values of  $k$





## Conclusion 2

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- ▶ We have shown that quantum 3-xor problem is **exponentially easier** than the quantum collision problem (in both settings), contrary to classical.
- ▶ The complexity of solving the 3-xor problem with allowed quantum memory **beats the lower bound** for quantum collision of  $2^{n/3}$
- ▶ For generic  $k$ , low quantum memory **improves Wagner** up to  $k = 8$ , and allowed quantum memory for all  $k$ .

Final Conclusion

# Open problems

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- ▶ Optimal collision time  $2^{n/3}$ ?
- ▶ Algebraic attacks
- ▶ Boomerang attacks
- ▶ FSE Stevens: Quantum cryptanalysis of SHA-2?
- ▶ AES quantum evaluation- on going work.
- ▶ Generic key-length extensions?
- ▶ What about state size? ...

# Symmetric Quantum Cryptanalysis

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Lots of things to do !