



S-Box Reverse-Engineering: Boolean Functions, American/Russian Standards, and Butterflies

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S-Box Reverse-Engineering

Boolean Functions, American/Russian Standards, and Butterflies

Léo Perrin

Based on joint works with Biryukov, Canteaut, Duval and Udovenko

June 6, 2018

CECC'18



Outline

- 1 Building Blocks for Symmetric Cryptography
- 2 Statistics and Skipjack
- 3 TU-Decomposition and Kuznyechik
- 4 The Butterfly Permutations and Functions
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Symmetric Cryptography

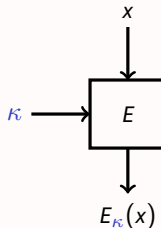
There are many **symmetric** algorithms! Hash functions, MACs...

Symmetric Cryptography

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Definition (Block Cipher)

- Input: n -bit block x
- Parameter: k -bit key κ
- Output: n -bit block $E_{\kappa}(x)$
- Symmetry: E and E^{-1} use the same κ

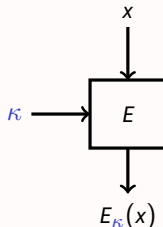


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Properties needed:

Diffusion

Confusion

No cryptanalysis!

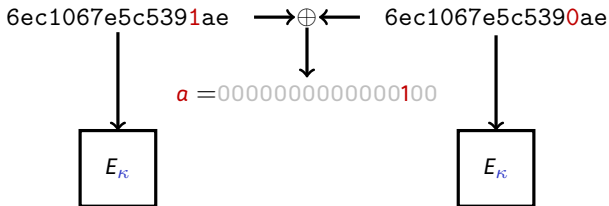
No Cryptanalysis?

Let us look at a typical cryptanalysis technique: the **differential attack**.

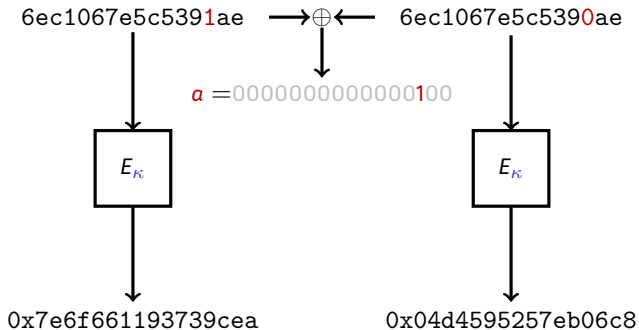
Differential Attacks

$$\begin{array}{ccc} 6ec1067e5c5391ae & \xrightarrow{\oplus} & 6ec1067e5c5390ae \\ & \downarrow & \\ a = 000000000000000100 & & \end{array}$$

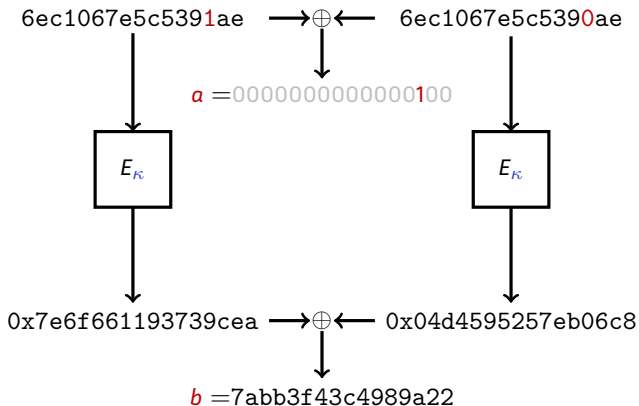
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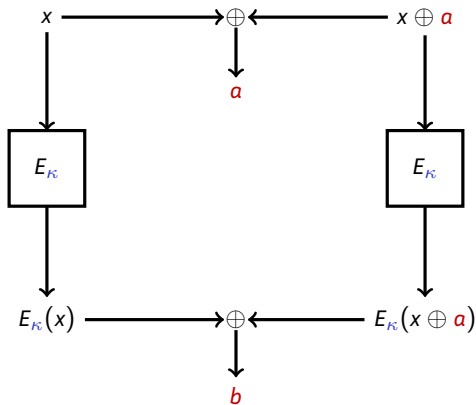
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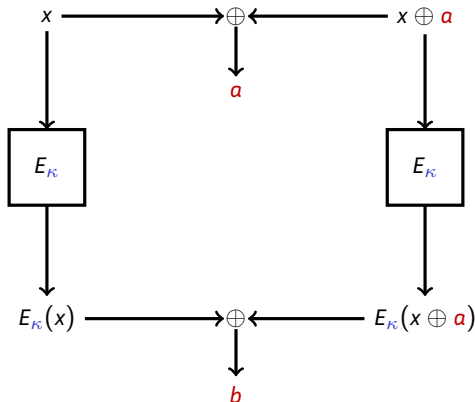
Differential Attacks



Differential Attacks



Differential Attacks



Differential Attack

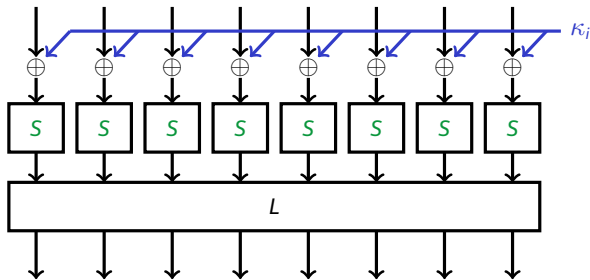
If there are many x such that $E_K(x) \oplus E_K(x \oplus a) = b$, then the cipher is **not secure**.

Basic Block Cipher Structure

How do we build block ciphers that prevent such attacks (as well as others)?

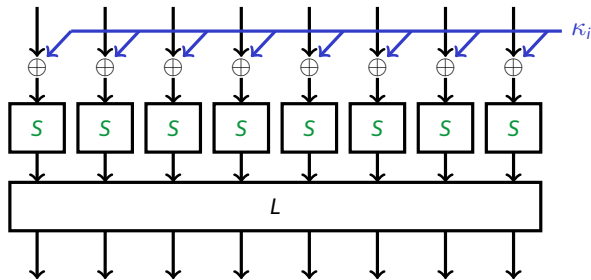
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Basic Block Cipher Structure

How do we build block ciphers that prevent such attacks (as well as others)?



Substitution-Permutation Network

Such a block cipher iterates the round function above several times. **S** is the Substitution Box (S-Box).

The S-Box (1/2)

$\pi' = (252, 238, 221, 17, 207, 110, 49, 22, 251, 196, 250, 218, 35, 197, 4, 77, 233, 119, 240, 219, 147, 46, 153, 186, 23, 54, 241, 187, 20, 205, 95, 193, 249, 24, 101, 90, 226, 92, 239, 33, 129, 28, 60, 66, 139, 1, 142, 79, 5, 132, 2, 174, 227, 106, 143, 160, 6, 11, 237, 152, 127, 212, 211, 31, 235, 52, 44, 81, 234, 200, 72, 171, 242, 42, 104, 162, 253, 58, 206, 204, 181, 112, 14, 86, 8, 12, 118, 18, 191, 114, 19, 71, 156, 183, 93, 135, 21, 161, 150, 41, 16, 123, 154, 199, 243, 145, 120, 111, 157, 158, 178, 177, 50, 117, 25, 61, 255, 53, 138, 126, 109, 84, 198, 128, 195, 189, 13, 87, 223, 245, 36, 169, 62, 168, 67, 201, 215, 121, 214, 246, 124, 34, 185, 3, 224, 15, 236, 222, 122, 148, 176, 188, 220, 232, 40, 80, 78, 51, 10, 74, 167, 151, 96, 115, 30, 0, 98, 68, 26, 184, 56, 130, 100, 159, 38, 65, 173, 69, 70, 146, 39, 94, 85, 47, 140, 163, 165, 125, 105, 213, 149, 59, 7, 88, 179, 64, 134, 172, 29, 247, 48, 55, 107, 228, 136, 217, 231, 137, 225, 27, 131, 73, 76, 63, 248, 254, 141, 83, 170, 144, 202, 216, 133, 97, 32, 113, 103, 164, 45, 43, 9, 91, 203, 155, 37, 208, 190, 229, 108, 82, 89, 166, 116, 210, 230, 244, 180, 192, 209, 102, 175, 194, 57, 75, 99, 182).$

The S-Box π of the latest Russian standards, Kuznyechik (BC) and Streebog (HF).

The S-Box (2/2)

Importance of the S-Box

If S is such that

$$S(x) \oplus S(x \oplus a) = b$$

does not have many solutions x for all (a, b) then the cipher may be proved secure against differential attacks.

The S-Box (2/2)

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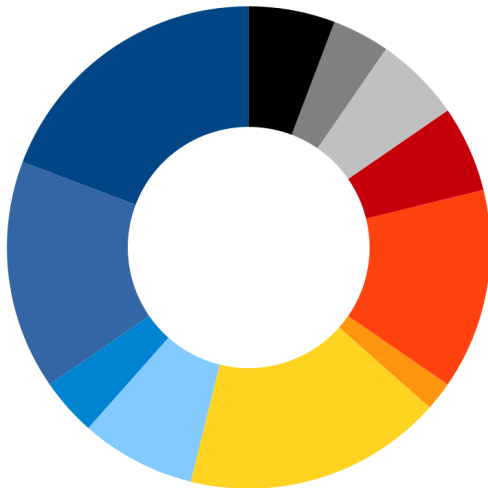
In **academic** papers presenting new block ciphers, the choice of S is carefully explained.

S-Box Design

- AES S-Box
- Inverse (other)
- Exponential
- Math (other)
- SPN
- Misty
- Feistel
- Lai-Massey
- Pseudo-random
- Hill climbing
- Unknown

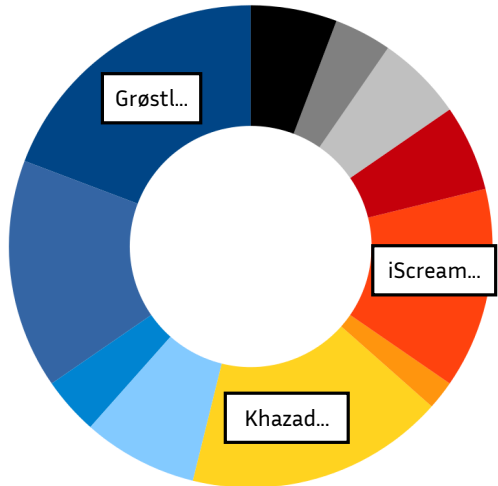
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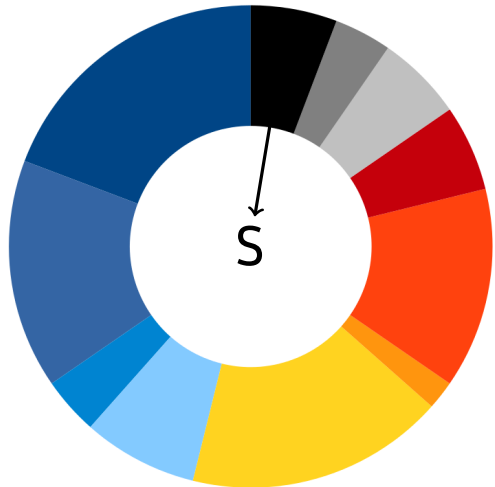
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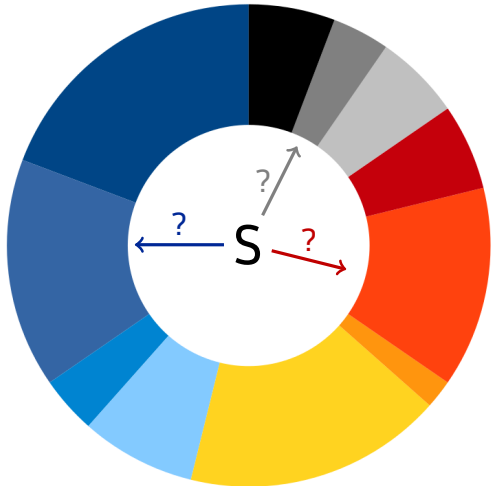
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Motivation (1/3)

A malicious designer can easily hide a structure in an S-Box.

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To keep an advantage in implementation (WB crypto)...

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A malicious designer can easily hide a structure in an S-Box.

To keep an advantage in implementation (WB crypto)...
... or an advantage in cryptanalysis (backdoor).

Motivation (2/3)

Definition (Kleptography)

The study of trapdoored cryptography is called **kleptography** (term introduced by Jung and Young).

S-Box based backdoors in the literature

- Rijmen, V., & Preneel, B. (1997). *A family of trapdoor ciphers*. FSE'97.
- Patterson, K. (1999). *Imprimitive Permutation Groups and Trapdoors in Iterated Block Ciphers*. FSE'99.
- Blondeau, C., Civino, R., & Sala, M. (2017). *Differential Attacks: Using Alternative Operations*. eprint report 2017/610.
- Bannier, A., & Filiol, E. (2017). *Partition-based trapdoor ciphers*. InTech'17.

Motivation (3/3)

Even without malicious intent, an unexpected structure can be a problem.

⇒ We need tools to *reverse-engineer* S-Boxes!

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Summary



We can recover parts of the design process of an S-Box using some statistics.

- 1 The two tables (basics of Boolean functions for cryptography)
- 2 A statistical tool based on the two tables
- 3 Application to NSA's Skipjack

The Two Tables

Let $S : \mathbb{F}_2^n \rightarrow \mathbb{F}_2^n$ be an S-Box.

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Definition (DDT)

The *Difference Distribution Table* of S is a matrix of size $2^n \times 2^n$ such that

$$\text{DDT}[a, b] = \#\{x \in \mathbb{F}_2^n \mid S(x \oplus a) \oplus S(x) = b\}.$$

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Definition (LAT)

The *Linear Approximations Table* of S is a matrix of size $2^n \times 2^n$ such that

$$\text{LAT}[a, b] = \#\{x \in \mathbb{F}_2^n \mid x \cdot a = S(x) \cdot b\} - 2^{n-1}.$$

Example

$$S = [4, 2, 1, 6, 0, 5, 7, 3]$$

The **DDT** of S .

8	0	0	0	0	0	0	0
0	0	0	0	2	2	2	2
0	0	0	0	2	2	2	2
0	0	4	4	0	0	0	0
0	0	0	0	2	2	2	2
0	4	4	0	0	0	0	0
0	4	0	4	0	0	0	0
0	0	0	0	2	2	2	2

The **LAT** of S .

4	0	0	0	0	0	0	0
0	0	2	2	0	0	2	-2
0	2	2	0	0	2	-2	0
0	2	0	2	0	-2	0	2
0	2	0	-2	0	-2	0	-2
0	-2	2	0	0	-2	-2	0
0	0	-2	2	0	0	-2	-2
0	0	0	0	-4	0	0	0

Coefficient Distribution in the DDT

If an n -bit S-Box is bijective, then its DDT coefficients behave like independent and identically distributed random variables following a Poisson distribution:

$$\Pr [\text{DDT}[a, b] = 2z] = \frac{e^{-1/2}}{2^z} .$$

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- Always even, ≥ 0
- Typically between 0 and 16.
- Lower is better.

Coefficient Distribution in the LAT

If an n -bit S-Box is bijective, then its LAT coefficients behave like independent and identically distributed random variables following this distribution:

$$\Pr [\text{LAT}[a, b] = 2^z] = \frac{\binom{2^{n-1}}{2^{n-2+z}}}{\binom{2^n}{2^{n-1}}} .$$

Coefficient Distribution in the LAT

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$$\Pr [\text{LAT}[a, b] = 2z] = \frac{\binom{2^{n-1}}{2^{n-2+z}}}{\binom{2^n}{2^{n-1}}}.$$

- Always even, signed.
- Typically between -40 and 40.
- Lower absolute value is better.

Looking Only at the Maximum

δ	$\log_2 (\Pr [\max(\text{DDT}) \leq \delta])$
14	-0.006
12	-0.094
10	-1.329
8	-16.148
6	-164.466
4	-1359.530

DDT

ℓ	$\log_2 (\Pr [\max(\text{LAT}) \leq \ell])$
38	-0.084
36	-0.302
34	-1.008
32	-3.160
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28	-25.623
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LAT

Probability that the maximum coefficient in the DDT/LAT of an 8-bit permutation is at most equal to a certain threshold.

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What is Skipjack? (1/2)

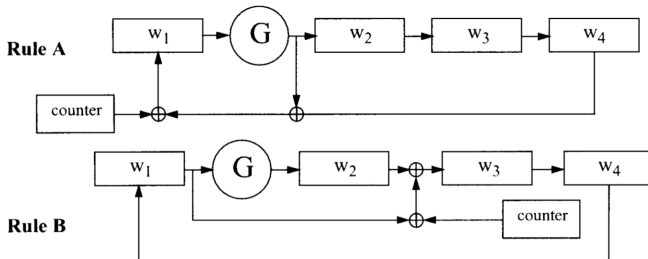
Type Block cipher

Bloc 64 bits

Key 80 bits

Authors NSA

Publication 1998



What is Skipjack? (2/2)

- Skipjack was supposed to be secret...
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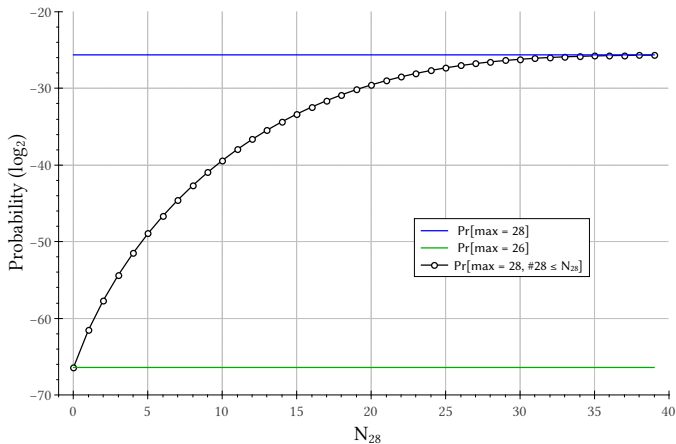
- Skipjack was supposed to be secret...
- ... but eventually published in 1998.
- Skipjack was to be used by the *Clipper Chip*,
- It uses an 8×8 S-Box (*F*) specified only by its LUT.

Reverse-Engineering F

For Skipjack's F , $\max(\text{LAT}) = 28$ and $\#28 = 3$.

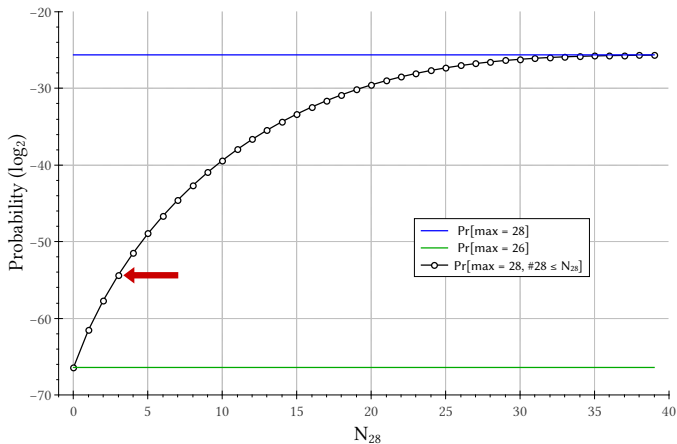
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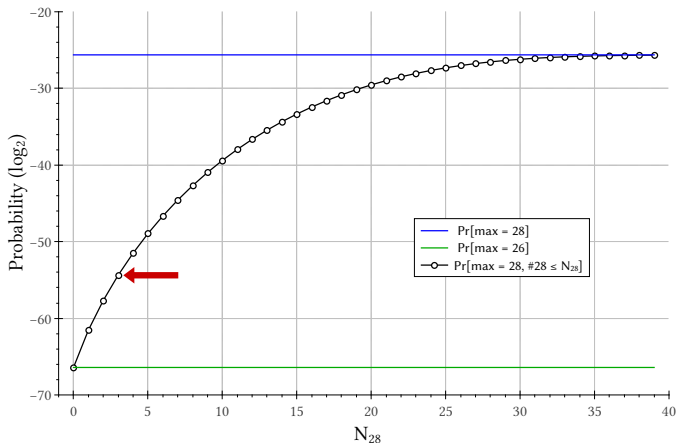
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Reverse-Engineering F

For Skipjack's F , $\max(\text{LAT}) = 28$ and $\#28 = 3$.



$$\Pr [\max(\text{LAT}) = 28 \text{ and } \#28 \leq 3] \approx 2^{-55}$$

What Can We Deduce?

- F has not been picked uniformly at random.
- F has not been picked among a feasibly large set of random S-Boxes.
- Its linear properties were optimized (though poorly).

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**The S-Box of Skipjack was built
using a dedicated algorithm.**

Timeline

Jun 98 Declassification of Skipjack

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1987 Initial design of Skipjack

Jul 93 “interim report” on Skipjack published by external cryptographers

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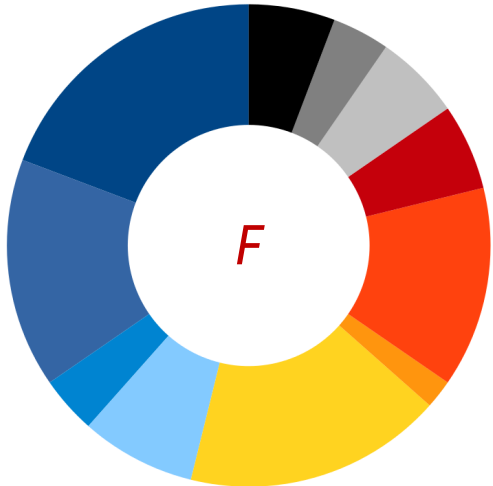
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Timeline

- 1987** Initial design of Skipjack
- Aug 90** (CRYPTO) Gilbert et al. use linear relations for key recovery (FEAL)
- Aug 91** (CRYPTO) Attack against FEAL using linear relations between key, plaintext and ciphertext
- May 92** (EUROCRYPT) Other attack against FEAL using linear relations between key, plaintext and ciphertext
- Aug 92** The S-Box ("F-table") of Skipjack is changed
- Jul 93** "interim report" on Skipjack published by external cryptographers
- Aug 95** Alleged "Skipjack" (actually not) is leaked to usenet
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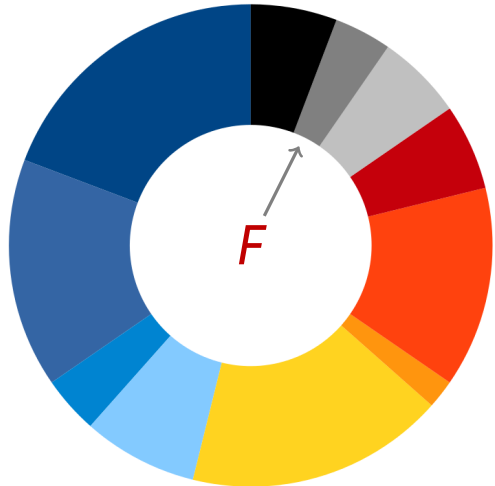
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We can recover an **actual decomposition** using patterns in the LAT.

- 1 Our target, the S-Box of Kuznyechik and Streebog
- 2 TU-decomposition: what is it and how to apply it to Kuznyechik

Kuznyechik/Streebog

Stribog

Type Hash function

Publication 2012

Kuznyechik

Type Block cipher

Publication 2015



Kuznyechik/Streebog

Stribog

Type Hash function

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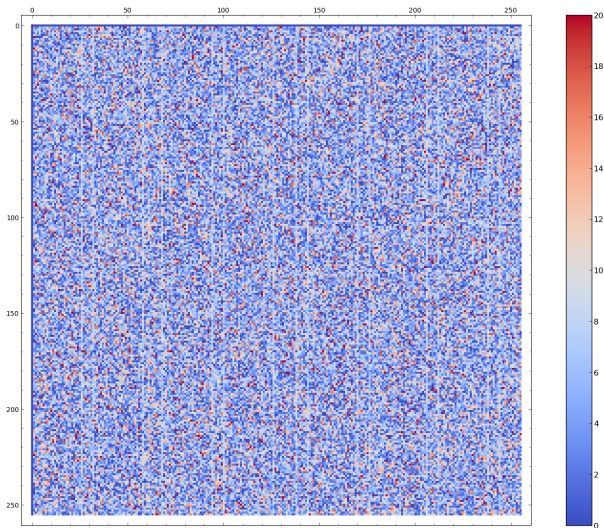
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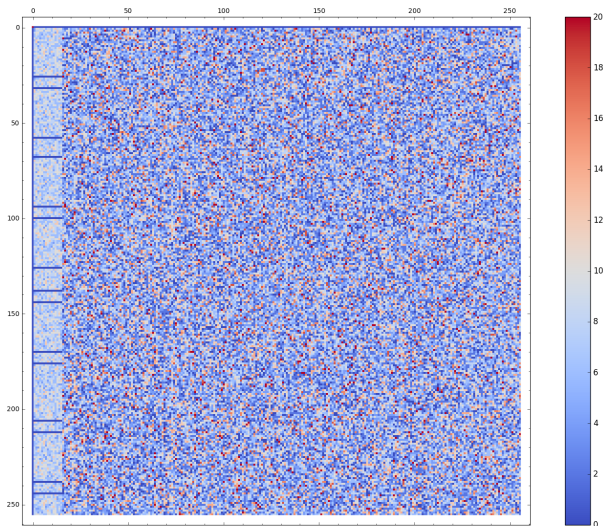
Common ground

- Both are standard symmetric primitives in Russia.
- Both were designed by the FSB (TC26).
- Both use the same 8×8 S-Box, π .

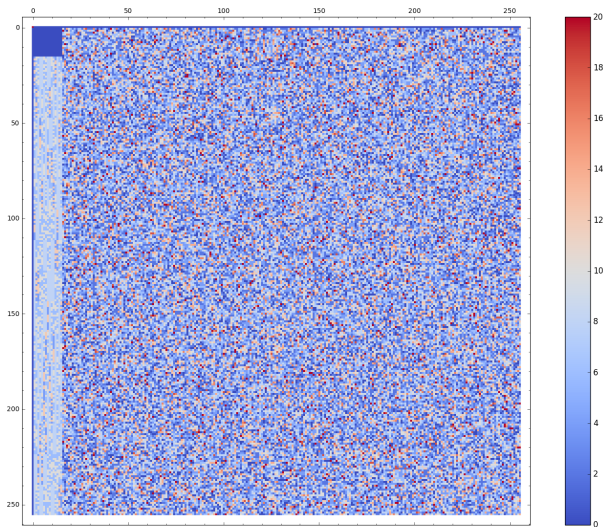
The LAT of π



The LAT of η (reordered columns)



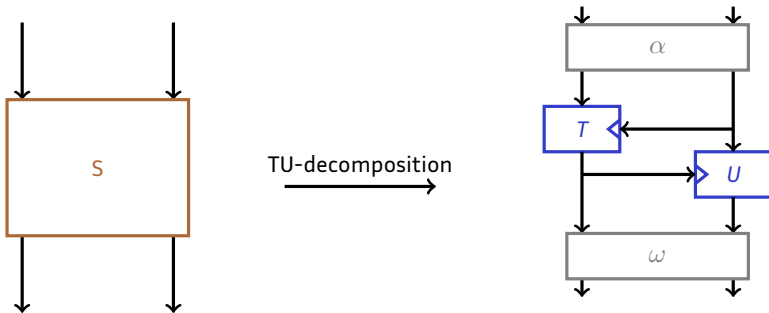
The LAT of $\eta \circ \pi \circ \mu$



The TU-Decomposition

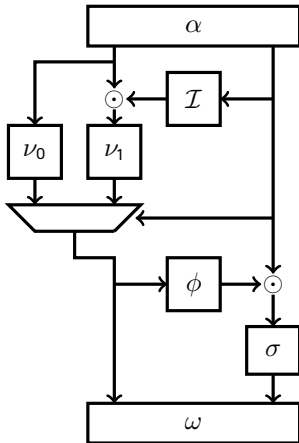
Definition

The **TU-decomposition** is a decomposition algorithm working against S-Boxes with vector spaces of zeroes in their LAT.



T and U are mini-block ciphers; μ and η are linear permutations.

Final Decomposition Number 1



\odot Multiplication in \mathbb{F}_{2^4}

α Linear permutation

\mathcal{I} Inversion in \mathbb{F}_{2^4}

ν_0, ν_1, σ 4×4 permutations

ϕ 4×4 function

ω Linear permutation

Hardware Performance

Structure	Area (μm^2)	Delay (ns)
Naive implementation	3889.6	362.52
Feistel-like	1534.7	61.53
Multiplications-first	1530.3	54.01
Feistel-like (with tweaked MUX)	1530.1	46.11

Conclusion for Kuznyechik/Streebog?

**The Russian S-Box was built like a
strange Feistel...**

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Belarussian inspiration

- The last standard of Belarus (BelT) uses an 8-bit S-box,
- somewhat similar to π ...

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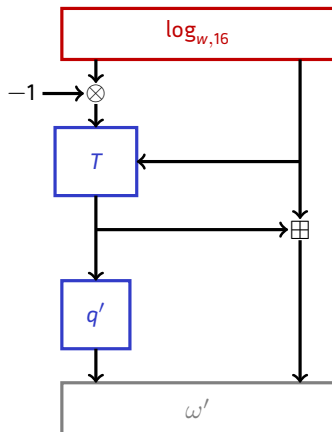
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Belarussian inspiration

- The last standard of Belarus (BelT) uses an 8-bit S-box,
- somewhat similar to π ...
- ... based on a **finite field exponential!**

Final Decomposition Number 2 (!)



	0	1	2	3	4	5	6	7	8	9	a	b	c	d	e	f
T_0	0	1	2	3	4	5	6	7	8	9	a	b	c	d	e	f
T_1	0	1	2	3	4	5	6	7	8	9	a	b	c	d	e	f
T_2	0	1	2	3	4	5	6	7	8	9	a	b	c	d	f	e
T_3	0	1	2	3	4	5	6	7	8	9	a	b	c	f	d	e
T_4	0	1	2	3	4	5	6	7	8	9	a	b	f	c	d	e
T_5	0	1	2	3	4	5	6	7	8	9	a	f	b	c	d	e
T_6	0	1	2	3	4	5	6	7	8	9	f	a	b	c	d	e
T_7	0	1	2	3	4	5	6	7	8	f	9	a	b	c	d	e
T_8	0	1	2	3	4	5	6	7	f	8	9	a	b	c	d	e
T_9	0	1	2	3	4	5	6	f	7	8	9	a	b	c	d	e
T_a	0	1	2	3	4	5	f	6	7	8	9	a	b	c	d	e
T_b	0	1	2	3	4	f	5	6	7	8	9	a	b	c	d	e
T_c	0	1	2	3	f	4	5	6	7	8	9	a	b	c	d	e
T_d	0	1	2	f	3	4	5	6	7	8	9	a	b	c	d	e
T_e	0	1	f	2	3	4	5	6	7	8	9	a	b	c	d	e
T_f	0	f	1	2	3	4	5	6	7	8	9	a	b	c	d	e

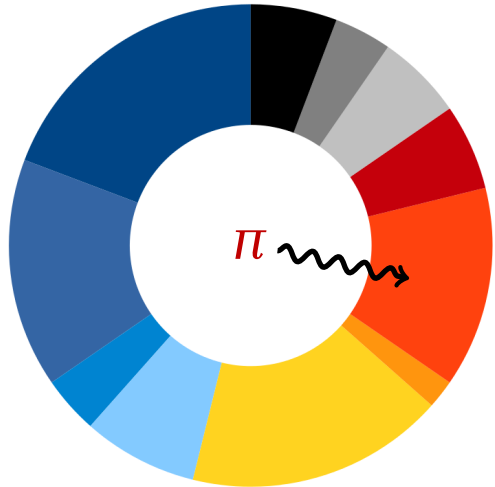
Conclusion on Kuznyechik/Streebog

- AES S-Box
- Inverse (other)
- Exponential
- Math (other)
- SPN
- Misty
- Feistel
- Lai-Massey
- Pseudo-random
- Hill climbing
- Unknown



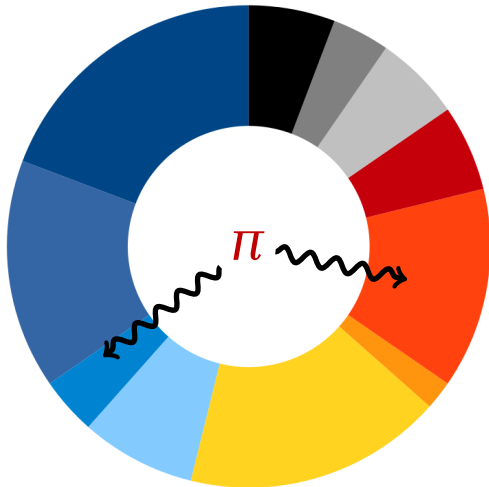
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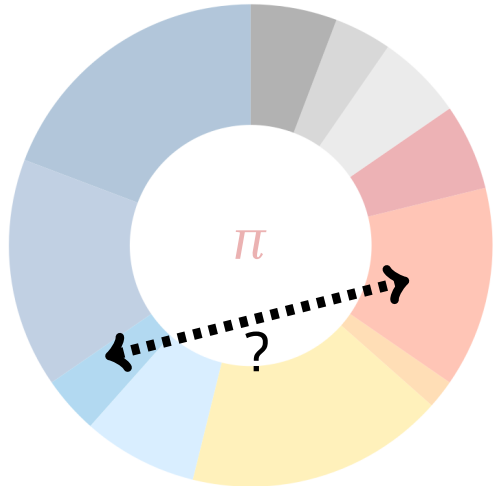
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Outline

- 1 Building Blocks for Symmetric Cryptography
- 2 Statistics and Skipjack
- 3 TU-Decomposition and Kuznyechik
- 4 The Butterfly Permutations and Functions**
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Summary



We can obtain new mathematical results using reverse-engineering techniques.

- 1 The big APN problem and its only known solution
- 2 Decomposing and generalizing this solution as butterflies

NSUCRYPTO (Olympiad in Cryptography)

Siberian Student's Olympiad in Cryptography with International participation — 2014
Second round NSUCRYPTO November 17-24



Task 2. «An APN Permutation»

“Try to find an APN permutation on 8 variables or prove that it doesn't exist.”

<https://nsucrypto.nsu.ru/>

The Big APN Problem

Definition (APN function)

A function $S : \mathbb{F}_2^n \rightarrow \mathbb{F}_2^n$ is **Almost Perfect Non-linear (APN)** if

$$S(x \oplus a) \oplus S(x) = b$$

has 0 or 2 solutions for all $a \neq 0$ and for all b .

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Big APN Problem

Are there APN permutations operating on \mathbb{F}_2^n where n is even?

Dillon et al.'s Permutation

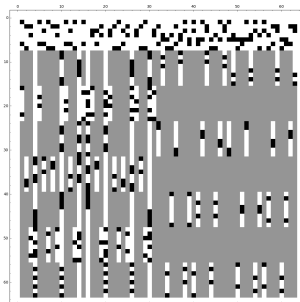
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For $n = 6$, Dillon et al. found an APN permutation.

Dillon et al.'s Permutation

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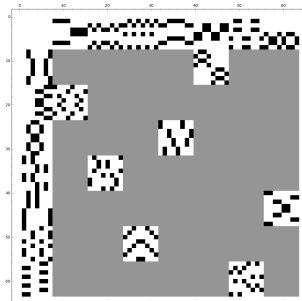
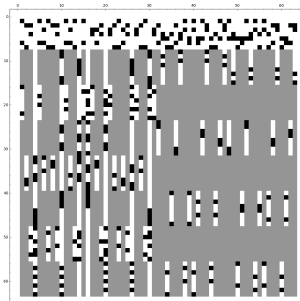
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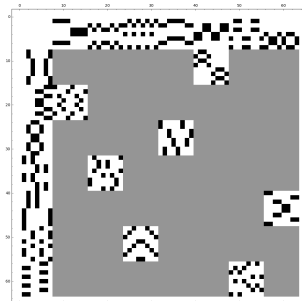
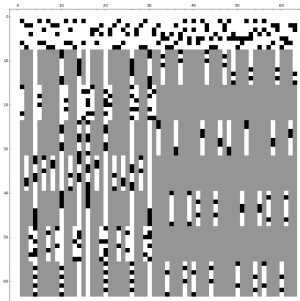
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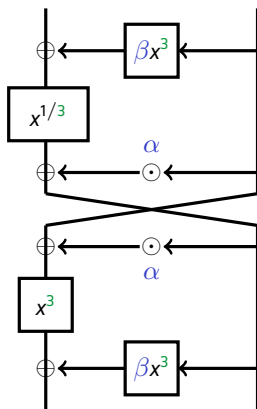
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It is possible to make a TU-decomposition!

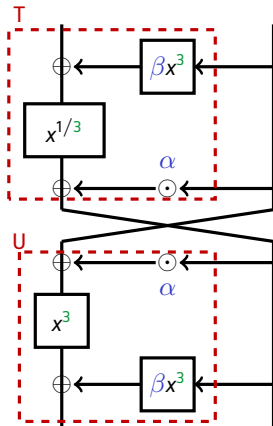
On the Butterfly Structure



Definition (Open Butterfly $H_{\alpha, \beta}^3$)

This permutation is an **open butterfly**.

On the Butterfly Structure



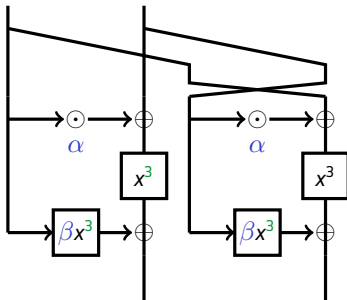
Definition (Open Butterfly $H_{\alpha, \beta}^3$)

This permutation is an **open butterfly**.

Lemma

Dillon's permutation is affine-equivalent to $H_{w, 1}^3$, where $\text{Tr}(w) = 0$.

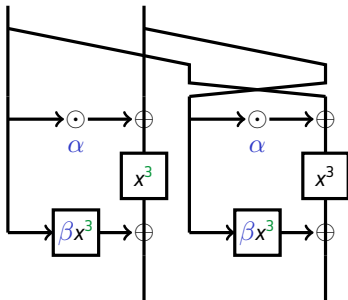
Closed Butterflies



Definition (Closed butterfly $V_{\alpha, \beta}^3$)

This quadratic function is a **closed butterfly**.

Closed Butterflies



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Lemma (Equivalence)

Open and closed butterflies with the same parameters are CCZ-equivalent.

Some Properties of Butterflies

Theorem (Properties of butterflies)

Let $V_{\alpha,\beta}^3$ and $H_{\alpha,\beta}^3$ be butterflies operating on $2n$ bits, n odd. Then:

- $\deg(V_{\alpha,\beta}^3) = 2$,
- if $n = 3$, $\text{Tr}(\alpha) = 0$ and $\beta + \alpha^3 \in \{\alpha, 1/\alpha\}$, then

$$\max(\text{DDT}) = 2, \max(\mathcal{W}) = 2^{n+1} \text{ and } \deg(H_{\alpha,\beta}^3) = n + 1,$$
- if $\beta = (1 + \alpha)^3$, then

$$\max(\text{DDT}) = 2^{n+1}, \max(\mathcal{W}) = 2^{(3n+1)/2} \text{ and } \deg(H_{\alpha,\beta}^3) = n,$$
- otherwise,

$$\max(\text{DDT}) = 4, \max(\mathcal{W}) = 2^{n+1} \text{ and } \deg(H_{\alpha,\beta}^3) \in \{n, n + 1\}$$
 and $\deg(H_{\alpha,\beta}^3) = n$ if and only if

$$1 + \alpha\beta + \alpha^4 = (\beta + \alpha + \alpha^3)^2.$$

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Open Problem

Cellular Message Encryption Algorithm

From Wikipedia, the free encyclopedia

In [cryptography](#), the **Cellular Message Encryption Algorithm** (**CMEA**) is a [block cipher](#) which was used for securing [mobile phones](#) in the [United States](#). CMEA is one of four cryptographic primitives specified in a [Telecommunications Industry Association](#) (TIA) standard, and is designed to [encrypt](#) the control channel, rather than the voice data. In 1997, a group of cryptographers published attacks on the [cipher](#) showing it had several weaknesses which give it a trivial effective strength of a 24-bit to 32-bit cipher.^[1]

CMEA

General

Designers [James A. Reeds III](#)

First published 1991

Cipher detail

Key sizes 64 bits

Block sizes 16-64 bits

Rounds 3

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A hidden structure!

CMEA uses an 8-bit (non-bijective) S-Box... With a TU-decomposition!

What is its actual structure?

Conclusion

- 1 Cryptographers use mathematics but mathematicians could also use crypto!

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Conclusion

- 1 Cryptographers use mathematics but mathematicians could also use crypto!
- 2 If you **design** a cipher, **justify** every step of your design.
- 3 If you **choose** a cipher, **demand** a full design explanation.

The Last S-Box

14	11	60	6d	e9	10	e3	2	b	90	d	17	c5	b0	9f	c5
d8	da	be	22	8	f3	4	a9	fe	f3	f5	fc	bc	30	be	26
bb	88	85	46	f4	2e	e	fd	76	fe	b0	11	4e	de	35	bb
30	4b	30	d6	dd	df	df	d4	90	7a	d8	8c	6a	89	30	39
e9	1	da	d2	85	87	d3	d4	ba	2b	d4	9f	9c	38	8c	55
d3	86	bb	db	ec	e0	46	48	bf	46	1b	1c	d7	d9	1b	e0
23	d4	d7	7f	16	3f	3	3	44	c3	59	10	2a	da	ed	e9
8e	d8	d1	db	cb	cb	c3	c7	38	22	34	3d	db	85	23	7c
24	d1	d8	2e	fc	44	8	38	c8	c7	39	4c	5f	56	2a	cf
d0	e9	d2	68	e4	e3	e9	13	e2	c	97	e4	60	29	d7	9b
d9	16	24	94	b3	e3	4c	4c	4f	39	e0	4b	bc	2c	d3	94
81	96	93	84	91	d0	2e	d6	d2	2b	78	ef	d6	9e	7b	72
ad	c4	68	92	7a	d2	5	2b	1e	d0	dc	b1	22	3f	c3	c3
88	b1	8d	b5	e3	4e	d7	81	3	15	17	25	4e	65	88	4e
e4	3b	81	81	fa	1	1d	4	22	0	6	1	27	68	27	2e
3b	83	c7	cc	25	9b	d8	d5	1c	1f	e5	59	7f	3f	3f	ef

