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Toward improved ocean-atmosphere coupling algorithms

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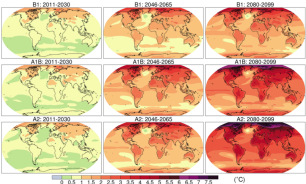
Toward improved ocean-atmosphere coupling algorithms



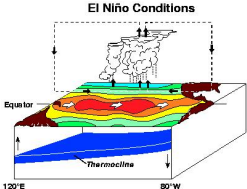
*Eric Blayo, Florian Lemarié,
Charles Pelletier and Sophie They*
Univ. Grenoble Alpes and Inria, France

Context

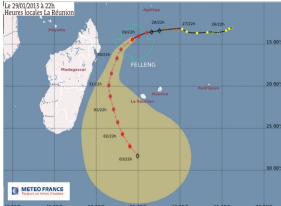
- ▶ Various applications require coupling an oceanic model and an atmospheric model.



climate modeling



seasonal forecasts



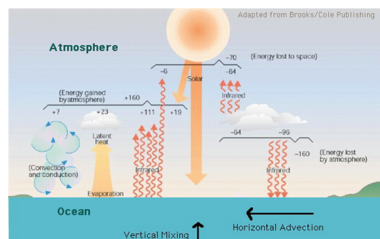
short term predictions

- ▶ **Objective:** revisit the coupling strategies presently used in such systems

Air-sea interactions

$$\begin{cases} \mathcal{L}_{\text{atm}} \mathbf{U}^a = f_{\text{atm}} & \text{in } \Omega_{\text{atm}} \times [0, T] \\ \mathcal{L}_{\text{oce}} \mathbf{U}^o = f_{\text{oce}} & \text{in } \Omega_{\text{oce}} \times [0, T] \end{cases}$$

$$\text{with } \mathbf{U}^a = \begin{pmatrix} \mathbf{u}_h^a \\ T^a \end{pmatrix} \quad \mathbf{U}^o = \begin{pmatrix} \mathbf{u}_h^o \\ T^o \end{pmatrix}$$

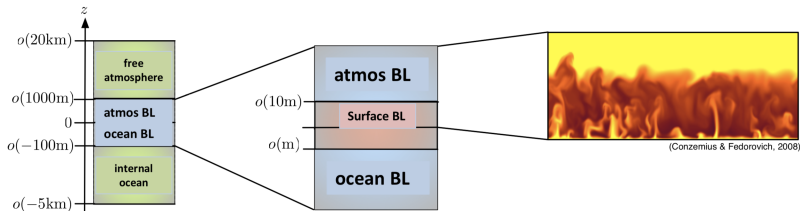


Interface conditions:

$$\text{momentum} \quad \rho^a K_m^a \frac{\partial \mathbf{u}_h^a}{\partial z} = \rho^o K_m^o \frac{\partial \mathbf{u}_h^o}{\partial z} = \boldsymbol{\tau} \quad \text{on } \Gamma \times [0, T]$$

$$\text{heat flux} \quad \rho^a c_p^a K_T^a \frac{\partial T^a}{\partial z} = \rho^o c_p^o K_T^o \frac{\partial T^o}{\partial z} = Q_S + \mathcal{R} \quad \text{on } \Gamma \times [0, T]$$

Estimation of OA fluxes



(Conzemius & Fedorovich, 2008)

$$\begin{cases} \tau = \rho^a C_D \| [\mathbf{u}]_o^a \| [\mathbf{u}]_o^a \\ Q_S = \rho^a c_p^a C_H \| [\mathbf{u}]_o^a \| [T]_o^a \end{cases}$$

$[\mathbf{u}]_o^a, [T]_o^a =$ jumps of \mathbf{u} and T

C_D, C_H : exchange coefficients given by
(complicated) bulk formulas
 $= f([\mathbf{u}]_o^a, [T]_o^a, [q]_o^a, z_{\text{atm}}, z_{\text{oce}}, \dots)$

Keywords: Monin-Obukhov theory (wall law + stratified fluid), bulk aerodynamic formulas...

Pelletier C., F. Lemarié, E. Blayo, J.-L. Redelsperger and P.-E. Brilouet, 2018: Comprehensive and coupled ocean-atmosphere turbulent parameterization schemes. *To be submitted*.

In summary

We have to solve:

$$\begin{cases} \mathcal{L}_{\text{atm}} \mathbf{U}^{\text{a}} = f_{\text{atm}} & \text{in } \Omega_{\text{atm}} \times [0, T] \\ \mathcal{L}_{\text{oce}} \mathbf{U}^{\text{o}} = f_{\text{oce}} & \text{in } \Omega_{\text{oce}} \times [0, T] \\ \mathcal{F}_{\text{atm}} \mathbf{U}^{\text{a}} = \mathcal{F}_{\text{oce}} \mathbf{U}^{\text{o}} = F_{\text{OA}}(\mathbf{U}^{\text{a}}, \mathbf{U}^{\text{o}}, \mathcal{R}) & \text{on } \Gamma \times [0, T] \end{cases}$$

In summary

We have to solve:

$$\begin{cases} \mathcal{L}_{\text{atm}} \mathbf{U}^{\text{a}} = f_{\text{atm}} & \text{in } \Omega_{\text{atm}} \times [0, T] \\ \mathcal{L}_{\text{oce}} \mathbf{U}^{\text{o}} = f_{\text{oce}} & \text{in } \Omega_{\text{oce}} \times [0, T] \\ \mathcal{F}_{\text{atm}} \mathbf{U}^{\text{a}} = \mathcal{F}_{\text{oce}} \mathbf{U}^{\text{o}} = F_{\text{OA}}(\mathbf{U}^{\text{a}}, \mathbf{U}^{\text{o}}, \mathcal{R}) & \text{on } \Gamma \times [0, T] \end{cases}$$

Two actual approaches:

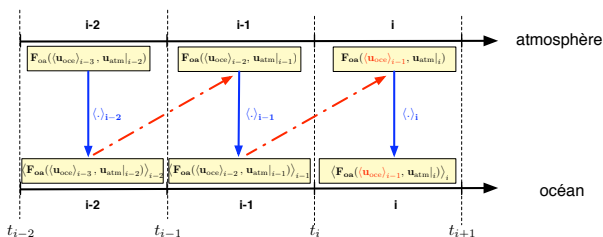
- ▶ Asynchronous coupling by time windows (averaged fluxes)
- ▶ Synchronous coupling at the time step (instantaneous fluxes)

Asynchronous coupling (time windows: $[0, T] = \cup_{i=1}^M [t_i, t_{i+1}]$)

$$\begin{cases} \mathcal{L}_{\text{atm}} \mathbf{U}^a = f_{\text{atm}} & \text{in } \Omega_{\text{atm}} \times [t_i, t_{i+1}] \\ \mathcal{F}_{\text{atm}} \mathbf{U}^a = F_{\text{OA}}(\langle \mathbf{U}^o \rangle_{i-1}, \mathbf{U}^a, \mathcal{R}) & \text{on } \Gamma \times [t_i, t_{i+1}] \end{cases}$$

then

$$\begin{cases} \mathcal{L}_{\text{oce}} \mathbf{U}^o = f_{\text{oce}} & \text{in } \Omega_{\text{oce}} \times [t_i, t_{i+1}] \\ \mathcal{F}_{\text{oce}} \mathbf{U}^o = \langle \mathcal{F}_{\text{atm}} \mathbf{U}^a \rangle_i & \text{on } \Gamma \times [t_i, t_{i+1}] \end{cases}$$



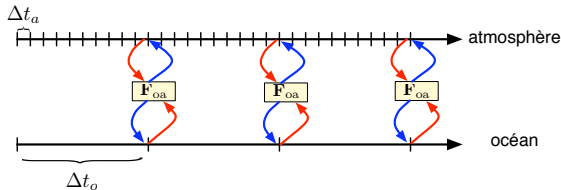
- ▶ same mean fluxes over each time window $[t_i, t_{i+1}]$
- ▶ ... but synchrony issue

Synchronous coupling at the time step

$$\begin{cases} \mathcal{L}_{\text{atm}} \mathbf{U}^a = f_{\text{atm}} & \text{in } \Omega_{\text{atm}} \times [t_i, t_i + N \Delta t_a] \\ \mathcal{F}_{\text{atm}} \mathbf{U}^a = F_{\text{OA}}(\mathbf{U}^o(t_i), \mathbf{U}^a(t), \mathcal{R}(t)) & \text{on } \Gamma \times [t_i, t_i + N \Delta t_a] \end{cases}$$

and

$$\begin{cases} \mathcal{L}_{\text{oce}} \mathbf{U}^o = f_{\text{oce}} & \text{in } \Omega_{\text{oce}} \times [t_i, t_i + \Delta t_o] \\ \mathcal{F}_{\text{oce}} \mathbf{U}^o = F_{\text{OA}}(\mathbf{U}^o(t_i), \mathbf{U}^a(t_i), \mathcal{R}(t_i)) & \text{on } \Gamma \times [t_i, t_i + \Delta t_o] \end{cases}$$



- ▶ still a (much smaller) synchrony issue
- ▶ difficult to implement efficiently
- ▶ validity issue

Stability analysis

Model problem: 1-D diffusion with variable and discontinuous coefficients

$$\begin{aligned} \frac{\partial u_{\text{atm}}}{\partial t} - \frac{\partial}{\partial z} \left(\nu_{\text{atm}}(z) \frac{\partial u_{\text{atm}}}{\partial z} \right) &= f_{\text{atm}} && \text{in } \Omega_{\text{atm}} \times [0, T] \\ \frac{\partial u_{\text{oce}}}{\partial t} - \frac{\partial}{\partial z} \left(\nu_{\text{oce}}(z) \frac{\partial u_{\text{oce}}}{\partial z} \right) &= f_{\text{oce}} && \text{in } \Omega_{\text{oce}} \times [0, T] \end{aligned}$$

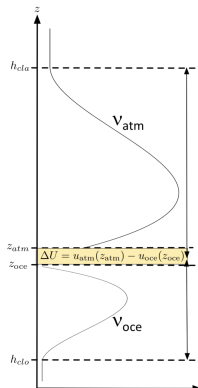
with interface conditions:

Dirichlet-Neumann:
$$\begin{cases} u_{\text{atm}}(0, t) = u_{\text{oce}}(0, t) \\ \nu_{\text{oce}}(0) \frac{\partial u_{\text{oce}}}{\partial z}(0, t) = \nu_{\text{atm}}(0) \frac{\partial u_{\text{atm}}}{\partial z}(0, t) \end{cases}$$

or

linearized bulk:

$$\nu_{\text{atm}}(0) \frac{\partial u_{\text{atm}}}{\partial z}(0, t) = \nu_{\text{oce}}(0) \frac{\partial u_{\text{oce}}}{\partial z}(0, t) = \alpha (u_{\text{atm}}(0^+, t) - u_{\text{oce}}(0^-, t))$$



Stability analysis: synchronous coupling

Consider a natural discretization scheme: Euler + implicit \rightarrow **each model is unconditionally stable.**

- ▶ Stability analysis shows that **the coupled scheme is unstable for usual configurations** of ocean-atmosphere models

$$\text{stable iff } \begin{cases} \Delta z_{\text{atm}} \ll \Delta z_{\text{oce}} & \text{for Dirichlet-Neumann} \\ \alpha \leq \min \left(\frac{\nu_{\text{oce}}(0)}{\Delta z_{\text{oce}}} \frac{\rho_{\text{oce}}}{\rho_{\text{atm}}}; \frac{\nu_{\text{atm}}(0)}{\Delta z_{\text{atm}}} \right) & \text{for linearized bulk} \end{cases}$$

Lemarié F., E. Blayo and L. Debreu, 2015: Analysis of ocean-atmosphere coupling algorithms: consistency and stability issues. *Procedia Computer Science*, **51**, 2066-2075.

- ▶ This does not mean that realistic coupled OA models systematically blow up, due to other processes (additional diffusion and viscosity).

Beljaars A., E. Dutra, G. Balsamo and F. Lemarié, 2017: On the numerical stability of surface-atmosphere coupling in weather and climate models. *Geosci. Model Dev.*, **10**, 977-989.

Conclusion on current coupling methods

- ▶ Current coupling methods are simple ad-hoc algorithms, which ensure that fluxes are balanced and are computationally cheap.
- ▶ These methods are inadequate from a mathematical point of view.

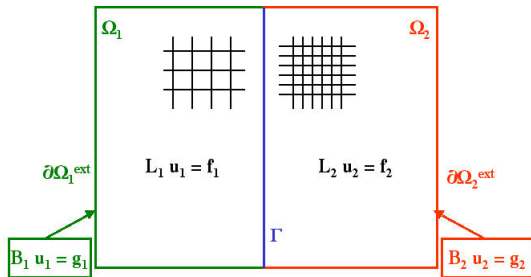
Conclusion on current coupling methods

- ▶ Current coupling methods are simple ad-hoc algorithms, which ensure that fluxes are balanced and are computationally cheap.
- ▶ These methods are inadequate from a mathematical point of view.

Issues:

- ▶ Can we improve the coupling method ?
- ▶ Does it improve the physics of the coupled solution ?
- ▶ Can this be done for a reasonable CPU cost ?

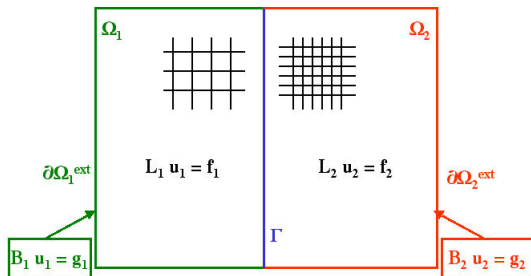
A convenient framework: Schwarz methods



$$\left\{ \begin{array}{l} L_1 u_1 = f_1 \quad \Omega_1 \times [0, T] \\ u_1 \text{ given} \quad \text{at } t = 0 \\ B_1 u_1 = g_1 \quad \partial\Omega_1^{\text{ext}} \times [0, T] \end{array} \right. \quad \left\{ \begin{array}{l} L_2 u_2 = f_2 \quad \Omega_2 \times [0, T] \\ u_2 \text{ given} \quad \text{at } t = 0 \\ B_2 u_2 = g_2 \quad \partial\Omega_2^{\text{ext}} \times [0, T] \end{array} \right.$$

+ physical constraints at the interface : $\mathcal{F}(u_1, u_2) = 0$

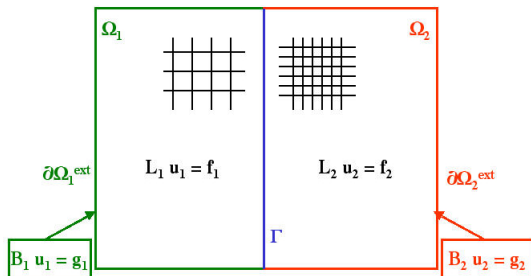
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$$\text{with } (C u_1 = C' u_2 \text{ and } D u_2 = D' u_1) \iff \mathcal{F}(u_1, u_2) = 0$$

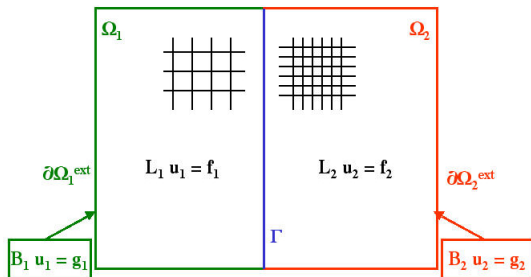
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$$\text{with } (C u_1 = C' u_2 \text{ and } D u_2 = D' u_1) \iff \mathcal{F}(u_1, u_2) = 0$$

- Present ocean-atmosphere coupling methods correspond to **one single iteration of a Schwarz-like coupling method**

Toward iterative ocean-atmosphere coupling

$$\begin{cases} \mathcal{L}_{\text{atm}} \mathbf{U}^a = f_{\text{atm}} & \text{in } \Omega_{\text{atm}} \times [t_i, t_{i+1}] \\ \mathcal{F}_{\text{atm}} \mathbf{U}^a = F_{\text{OA}}(\mathbf{U}^o, \mathbf{U}^a, \mathcal{R}) & \text{on } \Gamma \times [t_i, t_{i+1}] \end{cases}$$

$$\begin{cases} \mathcal{L}_{\text{oce}} \mathbf{U}^o = f_{\text{oce}} & \text{in } \Omega_{\text{oce}} \times [t_i, t_{i+1}] \\ \mathcal{F}_{\text{oce}} \mathbf{U}^o = \mathcal{F}_{\text{atm}} \mathbf{U}^a & \text{on } \Gamma \times [t_i, t_{i+1}] \end{cases}$$

Toward iterative ocean-atmosphere coupling

Iterate until convergence

$$\begin{cases} \mathcal{L}_{\text{atm}} \mathbf{U}_{k+1}^{\text{a}} = f_{\text{atm}} & \text{in } \Omega_{\text{atm}} \times [t_i, t_{i+1}] \\ \mathcal{F}_{\text{atm}} \mathbf{U}_{k+1}^{\text{a}} = F_{\text{OA}}(\mathbf{U}_k^{\text{o}}, \mathbf{U}_{k+1}^{\text{a}}, \mathcal{R}_{k+1}) & \text{on } \Gamma \times [t_i, t_{i+1}] \end{cases}$$

then

$$\begin{cases} \mathcal{L}_{\text{oce}} \mathbf{U}_{k+1}^{\text{o}} = f_{\text{oce}} & \text{in } \Omega_{\text{oce}} \times [t_i, t_{i+1}] \\ \mathcal{F}_{\text{oce}} \mathbf{U}_{k+1}^{\text{o}} = \mathcal{F}_{\text{atm}} \mathbf{U}_{k+1}^{\text{a}} & \text{on } \Gamma \times [t_i, t_{i+1}] \end{cases}$$

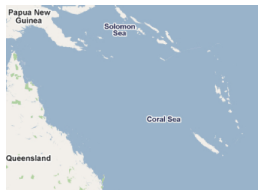
Impact on the physics

A major difficulty There is no idealized coupled ocean-atmosphere testcase with a known reference solution.

Impact on the physics

Simulation of the **tropical cyclone Erica (2003)**, by coupling

- ▶ ROMS: primitive equation ocean model (Shchepetkin-McWilliams, 2005)
- ▶ WRF: non hydrostatic atmospheric model (Skamarock-Klemp, 2007)



$$\Delta x_{\text{atm}} = 35\text{km}, \Delta t_{\text{atm}} = 180\text{s}$$

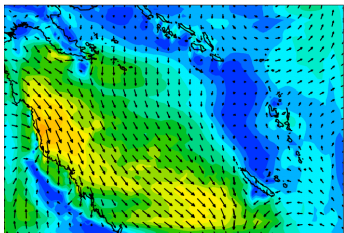
$$\Delta x_{\text{oce}} = 18\text{km}, \Delta t_{\text{oce}} = 1800\text{s}$$

15-day simulation

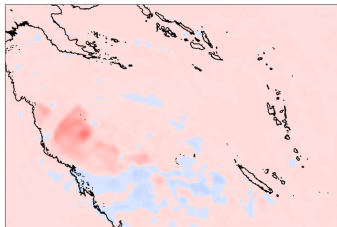
Interface conditions: vertical fluxes for momentum, heat and fresh water

Impact on the physics (cont'd)

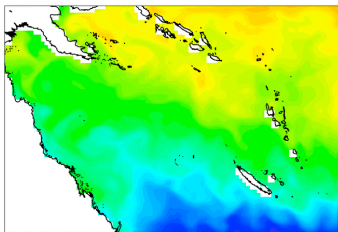
WRF - Vents (m/s) 2003-03-01_07:00:00gmt



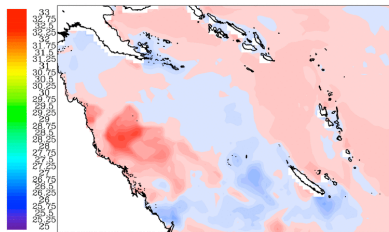
WRF Vents iter9 - iter1 (m/s) 2003-03-01_06:00:00



Sea Surface Temperature (deg C) 2003-03-01_06:00:00



SST iter9 - iter1 (deg C) 2003-03-01_06:00:00



10-meter wind (m/s) and sea surface temperature (°C).

Impact on the physics (cont'd)

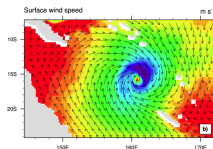
To assess the robustness of the coupled solution: **ensemble simulations**

- ▶ Initial conditions
- ▶ Length of the time windows: 6h vs 3h

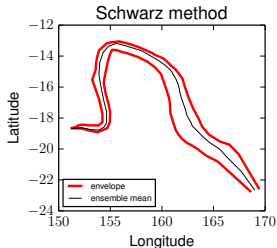
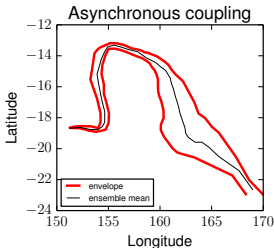
Impact on the physics (cont'd)

To assess the robustness of the coupled solution: **ensemble simulations**

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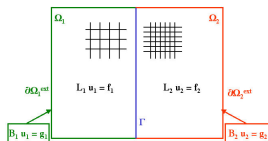


Trajectory of the cyclone



- ▶ The uncertainty on the cyclone trajectory and intensity is decreased by 30%-40%. (see Lemarié et al, 2014, for further details)

Decreasing the cost: absorbing boundary conditions

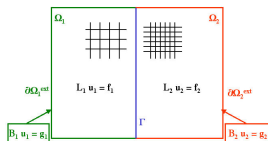


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Systems satisfied by the errors $e_i^k = u_i^k - u_i$:

$$\left\{ \begin{array}{l} L_1 e_1^{k+1} = 0 \\ e_1^{k+1} = 0 \\ B_1 e_1^{k+1} = 0 \\ C e_1^{k+1} = C' e_2^k \end{array} \right. \quad \begin{array}{l} \Omega_1 \times [0, T] \\ \text{at } t = 0 \\ \partial\Omega_1^{\text{ext}} \times [0, T] \\ \Gamma \times [0, T] \end{array} \quad \left\{ \begin{array}{l} L_2 e_2^{k+1} = 0 \\ e_2^{k+1} = 0 \\ B_2 e_2^{k+1} = 0 \\ D e_2^{k+1} = D' e_1^k \end{array} \right. \quad \begin{array}{l} \Omega_2 \times [0, T] \\ \text{at } t = 0 \\ \partial\Omega_2^{\text{ext}} \times [0, T] \\ \Gamma \times [0, T] \end{array}$$

Decreasing the cost: absorbing boundary conditions



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Systems satisfied by the errors $e_i^k = u_i^k - u_i$:

$$\left\{ \begin{array}{ll} L_1 e_1^{k+1} = 0 & \Omega_1 \times [0, T] \\ e_1^{k+1} = 0 & \text{at } t = 0 \\ B_1 e_1^{k+1} = 0 & \partial\Omega_1^{\text{ext}} \times [0, T] \\ C e_1^{k+1} = C' e_2^k & \Gamma \times [0, T] \end{array} \right. \quad \left\{ \begin{array}{ll} L_2 e_2^{k+1} = 0 & \Omega_2 \times [0, T] \\ e_2^{k+1} = 0 & \text{at } t = 0 \\ B_2 e_2^{k+1} = 0 & \partial\Omega_2^{\text{ext}} \times [0, T] \\ D e_2^{k+1} = D' e_1^k & \Gamma \times [0, T] \end{array} \right.$$

If one finds C', D' such that $C' e_2 = 0$ and/or $D' e_1 = 0$, then convergence in 2 iterations. \rightarrow **exact absorbing conditions** (Engquist & Majda, 77)

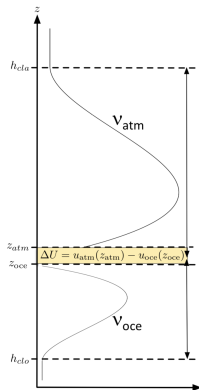
Improving the convergence speed

Model problem: coupling of two Ekman layers

$$\begin{aligned}\partial_t u_{\text{atm}} - f v_{\text{atm}} - \partial_z (\nu_{\text{atm}}(z) \partial_z u_{\text{atm}}) &= F_{\text{atm}}^x \\ \partial_t v_{\text{atm}} + f u_{\text{atm}} - \partial_z (\nu_{\text{atm}}(z) \partial_z v_{\text{atm}}) &= F_{\text{atm}}^y \quad \text{in } \Omega_{\text{atm}} \times [0, T] \\ u_{\text{atm}}(z, t = 0) &= u_0(z) \quad z \in \Omega_{\text{atm}}\end{aligned}$$

$$\begin{aligned}\partial_t u_{\text{oce}} - f v_{\text{oce}} - \partial_z (\nu_{\text{oce}}(z) \partial_z u_{\text{oce}}) &= F_{\text{oce}}^x \\ \partial_t v_{\text{oce}} + f u_{\text{oce}} - \partial_z (\nu_{\text{oce}}(z) \partial_z v_{\text{oce}}) &= F_{\text{oce}}^y \quad \text{in } \Omega_{\text{oce}} \times [0, T] \\ u_{\text{oce}}(z, t = 0) &= u_0(z) \quad z \in \Omega_{\text{oce}}\end{aligned}$$

$$\begin{pmatrix} u_{\text{atm}} \\ v_{\text{atm}} \end{pmatrix} = \begin{pmatrix} u_{\text{oce}} \\ v_{\text{oce}} \end{pmatrix} \quad \text{and} \quad \nu_{\text{atm}}(0) \partial_z \begin{pmatrix} u_{\text{atm}} \\ v_{\text{atm}} \end{pmatrix} = \nu_{\text{oce}}(0) \partial_z \begin{pmatrix} u_{\text{oce}} \\ v_{\text{oce}} \end{pmatrix} \quad \text{on } \Gamma \times [0, T]$$



Difficulties:

- ▶ coupling of u and v
- ▶ variable in space diffusivity + discontinuity at $z = 0$

Improving the convergence speed

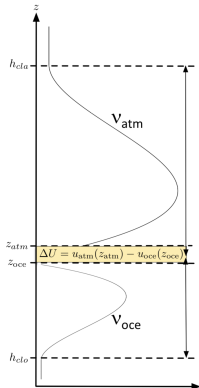
Model problem: coupling of two Ekman layers

Schwarz iteration:

$$\begin{aligned} \partial_t u_{\text{atm}}^k - f v_{\text{atm}}^k - \partial_z \left(\nu_{\text{atm}}(z) \partial_z u_{\text{atm}}^k \right) &= F_{\text{atm}}^x \\ \partial_t v_{\text{atm}}^k + f u_{\text{atm}}^k - \partial_z \left(\nu_{\text{atm}}(z) \partial_z v_{\text{atm}}^k \right) &= F_{\text{atm}}^y && \text{in } \Omega_{\text{atm}} \times \\ u_{\text{atm}}^k(z, t = 0) &= u_0(z) && z \in \Omega_{\text{atm}} \\ \mathcal{C}' u_{\text{atm}}^k(0, t) &= \mathcal{C}' u_{\text{atm}}^{k-1}(0, t) && t \in [0, T] \end{aligned}$$

$$\begin{aligned} \partial_t u_{\text{oce}}^k - f v_{\text{oce}}^k - \partial_z \left(\nu_{\text{oce}}(z) \partial_z u_{\text{oce}}^k \right) &= F_{\text{oce}}^x \\ \partial_t v_{\text{oce}}^k + f u_{\text{oce}}^k - \partial_z \left(\nu_{\text{oce}}(z) \partial_z v_{\text{oce}}^k \right) &= F_{\text{oce}}^y && \text{in } \Omega_{\text{oce}} \times [0, \\ u_{\text{oce}}^k(z, t = 0) &= u_0(z) && z \in \Omega_{\text{oce}} \\ \mathcal{D}' u_{\text{oce}}^k(0, t) &= \mathcal{D}' u_{\text{oce}}^{k-1}(0, t) && t \in [0, T] \end{aligned}$$

for a given $(u_{\text{oce}}^0, v_{\text{oce}}^0)$ on Γ .



Improving the convergence speed (cont'd)

		Constant diffusion	Space dependent diffusion
Steady state	No Coriolis effect	Dubois (2007) opt. Schwarz 2-D adv-diff eq. Gander-Zhang (2016) opt. Schwarz Helmholtz eq.	Lions (1990) convergence Schwarz diffusion eq.
	Coriolis effect		
Time dependent	No Coriolis effect	Gander-Halpern (2002) opt. Schwarz heat eq. Blayo-Rousseau-Tayachi (2017) lin. viscous SW eq. Bennequin-Gander-Gouarin-Halpern (2004) opt. Schwarz 2-D diff-reaction	Lemarié-Debreu-Blayo (2013) opt. Schwarz 1-D diffusion
	Coriolis effect	Martin (2003) opt. Schwarz 2-D SW Audusse-Dreyfuss-Merlet (2010) opt. Schwarz 3-D primitive eqs	

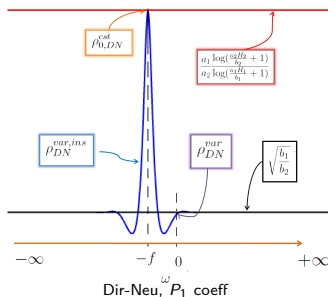
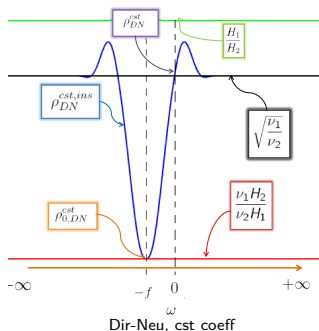
Improving the convergence speed (cont'd)

		Constant diffusion	Space dependent diffusion
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Improving the convergence speed (cont'd)

Exact analytical expression for the convergence factor:

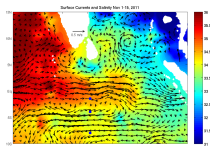
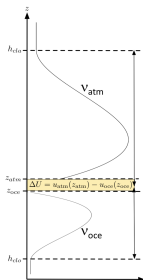
- ▶ P_0 , P_1 and P_2 diffusion profiles
- ▶ Dirichlet-Neumann and Robin-Robin interface conditions
- ▶ optimized coefficients for Robin-Robin conditions



Blayo E., F. Lemarié, C. Pelletier and S. They, 2018: Coupling two Ekman layers with a Schwarz algorithm. *In preparation.*

Current and future work

- ▶ Integrate all these results in a model of the three boundary layers (atmospheric, surface, oceanic)
- ▶ Study the coherence of the different parameterizations (well-posedness issues)



In collaboration with climate scientists:

- ▶ Build 1D coupled reference test cases (idealized and realistic)
- ▶ Include this coupling strategy in the French IPSL climate model
- ▶ Mitigate the cost (perform the iterations only for the boundary layers, model reduction...)