



## Some Recent Developments around the CROCO Initiative for Complex Regional to Coastal Modeling

Francis Auclair, Rachid Benshila, Laurent Debreu, Nicolas Ducousso, Franck Dumas, Patrick Marchesiello, Florian Lemarié

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LABORATOIRE  
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MATHÉMATIQUES APPLIQUÉES • INFORMATIQUE

COMOD 2018

COASTAL OCEAN MODELLING WORKSHOP

## Some Recent Developments around the CROCO Initiative for Complex Regional to Coastal Modeling

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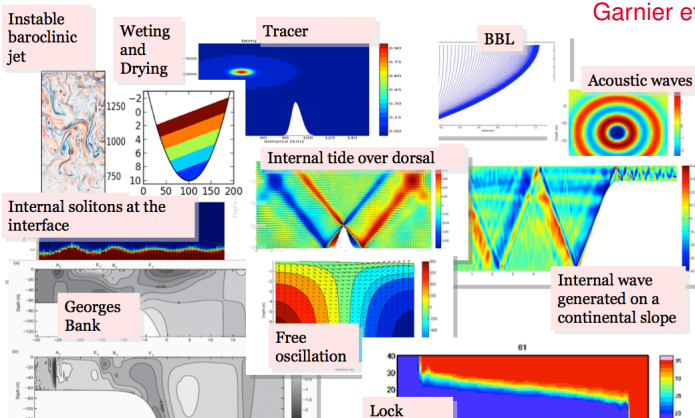
♠ National Hydrographic Service, SHOM, Brest, France

# Context – the COMODO project (2012-2016; PI: L. Debreu, Inria)

Funded by the french national research agency (9 postdocs/engineers)

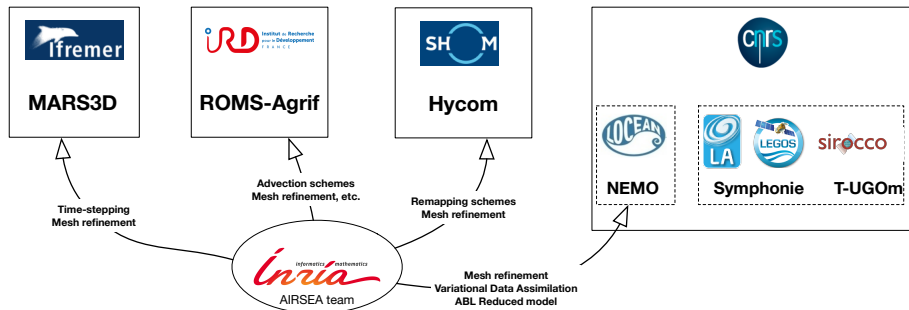
- Intercomparison and evaluation of models
- Improved numerical methods
- Definition of a suite of standardized test cases
- Develop associated diagnostic tools (pyCOMODO tools)

Garnier et al., GMD, in prep.



## Context – the COMODO project (2012-2016; PI: L. Debreu, Inria)

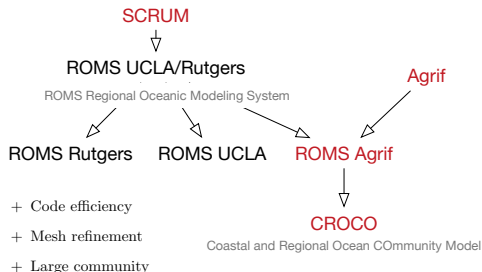
- Modeling groups involved in the project



- Most modeling groups had common objectives for coastal applications (e.g. NH option, flexible horiz. and vert. grids, coupling with waves, etc)
  - minimize the duplication of efforts
  - promote interoperability across numerical models (via tools like Oasis, XIOS)



## the Croco initiative : an outcome from COMODO



- ▷ **Roms-Agrif** numerical kernel
- ▷ Online nesting capability via **Agrif** library
- ▷ Non-hydrostatic Non-Boussinesq from **S-NH**
- ▷ Sediment module from **Mars3d**
- ▷ OAW coupling interface (shared w. **Nemo**)
- ▷ ALE-type vertical coordinate (ongoing + shared w. **Nemo**)

Supported by Ifremer, Shom, Cnrs, Ird, Inria

- Croco as a way to sustain the COMODO group
- Complementary to NEMO in terms of target applications

- ▷ Ifremer : Mars3d  $\xrightarrow{\text{transition}}$  Croco
- ▷ Shom : Hycom  $\xrightarrow{\text{transition}}$  Croco

## Content: recent developments in CROCO

1. Stability analysis of the mode-splitting and control of numerical dissipation
2. A non-hydrostatic non-Boussinesq algorithm, the how and why
3. Multiresolution simulations using block structured mesh refinement
4. Future perspectives and concluding remarks

# 1

## **Stability analysis of the mode-splitting and control of numerical dissipation**

# Objectives

- Spurious "numerical mixing" not only associated with the space/time discretization of the advection operator
- **Objective** : characterize the impact of the "inexact" mode-splitting on the stability of numerical models

## Our approach :

1. Stability analysis of the mode splitting technique based on an eigenvector decomposition using the exact barotropic mode
2. Quantify the amount of dissipation required to stabilize the approximative splitting

## Normal mode decomposition (e.g. Gill, 1982; Kundu, 1990)

### 2D linearized primitive equations

$$(u_0 = w_0 = 0, f_{\text{cor}} = 0)$$

$$\left\{ \begin{array}{lcl} \partial_t u + \frac{1}{\rho_0} \partial_x \tilde{p} & = & 0 \\ \partial_z \tilde{p} & = & -\tilde{\rho} g \\ \partial_x u + \partial_z w & = & 0 \\ \partial_t \tilde{\rho} + w \frac{d\bar{\rho}}{dz} & = & 0 \end{array} \right.$$

with boundary conditions

$$\begin{aligned} w(z=0) &= \partial_t \eta & \eta \ll H \\ w(z=-H) &= 0 & \text{(flat bottom)} \\ \tilde{p}(z=0) &= \rho_0 g \eta \end{aligned}$$

## Normal mode decomposition (e.g. Gill, 1982; Kundu, 1990)

### 2D linearized primitive equations

$$(u_0 = w_0 = 0, f_{\text{cor}} = 0)$$

$$\begin{cases} \partial_t u + \frac{1}{\rho_0} \partial_x \tilde{p} &= 0 \\ \partial_z \tilde{p} &= -\tilde{\rho} g \\ \partial_x u + \partial_z w &= 0 \\ \partial_t \tilde{\rho} + w \frac{d\bar{\rho}}{dz} &= 0 \end{cases}$$

with boundary conditions

$$\begin{aligned} w(z=0) &= \partial_t \eta & \eta &\ll H \\ w(z=-H) &= 0 & \text{(flat bottom)} \\ \tilde{p}(z=0) &= \rho_0 g \eta \end{aligned}$$

### Sturm-Liouville eigenvalue problem

$$\text{with } \Lambda = -\frac{d}{dz} \left( N^{-2} \frac{d}{dz} \right)$$

$$\begin{cases} \Lambda M_q(z) &= c_q^{-2} M_q(z) \\ \frac{dM_q}{dz}(-H) &= 0 \\ \frac{dM_q}{dz}(0) &= -\frac{N^2(0)}{g} M_q(0) \end{cases}$$

$\langle M_q, M_m \rangle = \delta_{q,m}$  w.r.t. the inner

$$\text{product } \langle f, g \rangle = \frac{1}{H} \int_{-H}^0 f g dz$$

## Normal mode decomposition (e.g. Gill, 1982; Kundu, 1990)

2D linearized primitive equations  
( $u_0 = w_0 = 0, f_{\text{cor}} = 0$ )

$$\left\{ \begin{array}{l} \partial_t u + \frac{1}{\rho_0} \partial_x \tilde{p} = 0 \\ \partial_z \tilde{p} = -\tilde{\rho} g \\ \partial_x u + \partial_z w = 0 \\ \partial_t \tilde{\rho} + w \frac{d\bar{\rho}}{dz} = 0 \end{array} \right.$$

with boundary conditions

$$\begin{aligned} w(z=0) &= \partial_t \eta & \eta \ll H \\ w(z=-H) &= 0 & \text{(flat bottom)} \\ \tilde{p}(z=0) &= \rho_0 g \eta \end{aligned}$$

Expand the variables  $u, \tilde{p}$  in the eigenfunctions  $M_q(z)$  of  $\Lambda$

$$\left\{ \begin{array}{l} u(x, z, t) = \sum_{q=0}^{K-1} u_q(x, t) M_q(z) \\ \tilde{p}(x, z, t) = g \rho_0 \sum_{q=0}^{K-1} h_q(x, t) M_q(z) \end{array} \right.$$

to obtain a set of uncoupled systems

$$\left\{ \begin{array}{l} \partial_t u_q + g \partial_x h_q = 0 \\ \partial_t h_q + \frac{c_q^2}{g} \partial_x u_q = 0 \end{array} \right.$$

In particular

- $q = 0 \rightarrow$  barotropic mode
- $q > 1 \rightarrow$  baroclinic modes

## Usual derivation of the barotropic mode in oceanic models (e.g. Blumberg & Mellor, 1987; Killworth et al., 1991)

Starting from the PEs, horizontal velocities are decomposed as

$$u = \left( \frac{1}{H} \int_{-H}^{\eta} u \, dz \right) + u_{3d} = \bar{u} + u_{3d}, \quad \int_{-H}^{\eta} u_{3d} \, dz = 0$$

Standard barotropic mode (associated to the 2D linearized PEs)

Integrating vertically the continuity and momentum equations we end up with

$$\begin{cases} \partial_t \eta + \partial_x (H \bar{u}) &= 0 \\ \partial_t \bar{u} + g \partial_x \eta &= -\frac{1}{\rho_0} \partial_x \left( \frac{1}{H} \int_{-H}^0 p_h \, dz \right) \end{cases}$$

**Remark :** the red term is a "slow" term absent from the normal mode analysis.

→ In practice it is kept frozen during the barotropic integration



## Interpretation in terms of normal modes

The underlying assumption is that the barotropic mode is depth-independent

$$M_0^*(z) = 1, \quad u_0^*(x, t) = \bar{u}, \quad \int_{-H}^0 M_q^*(z) dz = 0 \quad (q \geq 1)$$

The orthogonality condition between modes is lost

- Exact normal mode decomposition →  $K$  independent systems
- Depth-independent barotropic mode assumption →  $K - 1$  independent systems (for  $q \geq 1$ ) + a system which includes contributions from all modes

## Consequences of the depth-independent assumption

### 1. Some (fast) barotropic contributions are treated as "slow" terms

$$\begin{cases} \partial_t \eta + \partial_x (H \bar{u}) &= 0 \\ \partial_t \bar{u} + g \partial_x \eta &= -\frac{1}{\rho_0} \partial_x \left( \frac{1}{H} \int_{-H}^0 p_h \, dz \right) \end{cases}$$

$$\frac{1}{H} \int_{-H}^0 p_h \, dz = \frac{1}{H} \int_{-H}^0 (p - \rho_0 g \eta) \, dz = \rho_0 g \sum_{q=0}^{K-1} \left\{ \frac{1}{H} \int_{-H}^0 M_q(z) - M_q(0) \right\} h_q$$

$\Rightarrow$  the contribution of the barotropic mode to the **red term** is proportional to

$$\text{For } N^2 = \text{cste} \quad \frac{1}{H} \int_{-H}^0 M_0(z) - M_0(0) \approx \frac{\varepsilon}{3}, \quad \varepsilon = \sqrt{\frac{N^2 H}{g}}$$

### 2. The consistency between the barotropic and baroclinic modes must be enforced since they are no longer independent

## Time stepping the coupled baroclinic-barotropic system

- Dissipation is necessary to stabilize the time integration procedure because some fast contributions are treated with the slow time-step.

**Objective :** determine the minimum amount of dissipation required to stabilize the coupled baroclinic-barotropic system integration

# Adding dissipation within the barotropic mode

## 1. Averaging filters (e.g. Nadiga et al., 1997; Shchepetkin & McWilliams, 2005)

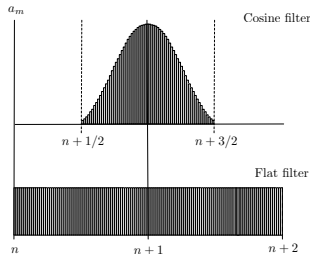
$$\bar{u}^{n+1,f} = \sum_{m=0}^{N_{\text{filter}}} a_m \bar{u}^m \quad (N_{\text{filter}} \geq N_{\text{split}})$$

under the constraints

$$\sum_{m=0}^{N_{\text{filter}}} a_m = 1, \quad \sum_{m=0}^{N_{\text{filter}}} a_m \frac{m}{N_{\text{split}}} = 1$$

Several options:

- Flat filter
- Cosine filter



### With averaging filters

- the amount of dissipation is somewhat ad-hoc (no quantification of its impact)
- the barotropic integration must go beyond  $n + 1$
- the barotropic components are not continuous in time

## Adding dissipation within the barotropic mode

1. **Averaging filters** (e.g. Nadiga et al., 1997; Shchepetkin & McWilliams, 2005)
2. **Dissipative time-stepping** (e.g. Hallberg, 1997)

In the case of the CROCO ocean model :

→ a *generalized forward-backward* scheme (*AB3-like*, *AM4-like*) is used for the barotropic mode (on top of an averaging filter)

$$\begin{aligned}\eta^{n+1} &= \eta^n - H\Delta t\partial_x \left( (3/2 + \beta)u^n - (1/2 + 2\beta)u^{n-1} + \beta u^{n-2} \right) \\ \bar{u}^{n+1} &= \bar{u}^n - g\Delta t\partial_x \left( \delta\eta^{n+1} + (1 - \delta - \gamma - \epsilon)\eta^n + \gamma\eta^{n-1} + \epsilon\eta^{n-2} \right)\end{aligned}$$

- **Revised choice** : no averaging filter + additional constraint for the choice of  $(\delta, \gamma, \beta, \epsilon)$  to guarantee stability of the split-explicit formulation  
⇒ Minimum level of dissipation to guarantee stability

**Impact on nonlinear simulations ?**

## Linear testcase ( $N = 10^{-3} \text{ s}^{-1}$ , $H = 4000 \text{ m}$ )

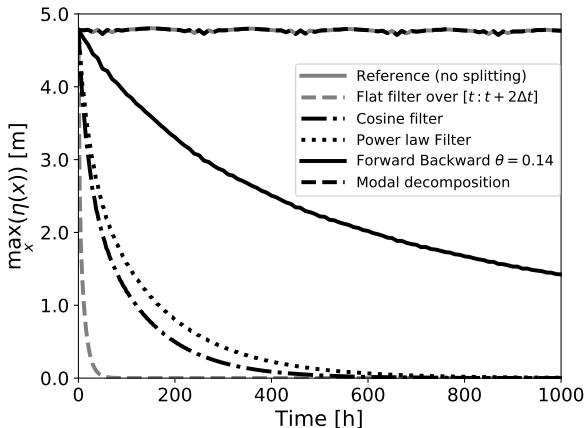
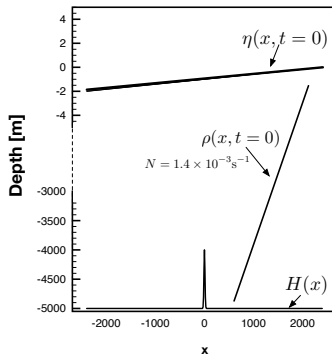


Figure : Time evolution of the maximum absolute value of the free surface elevation. Comparaison of usual filters against a reference solution without splitting.

## Internal tide test case

- **Testcase** : nonlinear internal tide generation by topography (Marsaleix et al., 2008)
  - 2D x-z, closed boundaries,  $L = 4800\text{km}$
  - Start from rest + linear stratification
- ▷ Barotropic seiche at a 12-h period
- ▷ Production of internal waves over the ridge



In the nonlinear case there are other sources of instability :

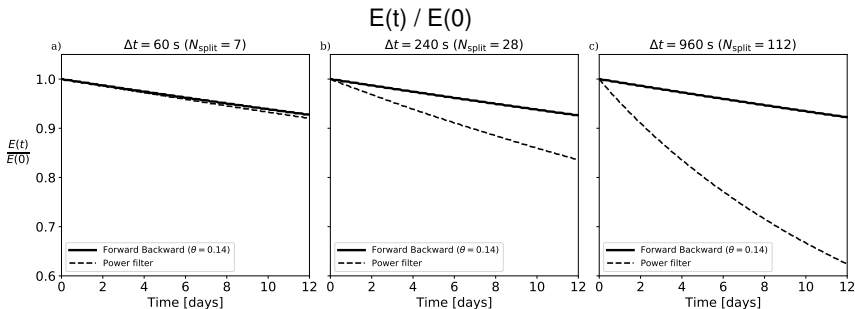
- The rhs of the barotropic equations is integrated from  $-H$  to  $\eta$
- Aliasing errors when the advective terms are added

## Internal tide test case

### Barotropic mechanical energy

$$E(t) = \int_0^L \frac{\rho_0}{2} (H + \eta) \bar{u}^2 \, dx + \int_0^L \frac{\rho_0}{2} g \eta^2 \, dx$$

- Comparison of standard ROMS vs modified CROCO scheme (for a fixed baroclinic time-step)





## Summary & perspectives on mode splitting analysis

- ▷ The barotropic mode is traditionally considered as depth independent which is an assumption (e.g. work of [Higdon](#), [Bennett](#), [de Szoeke](#))
- ▷ **CROCO approach :**
  - Provide a general framework for the stability analysis of the mode splitting approach
  - The framework allow the design of efficient 2D/3D time stepping algorithms
  - Approach implemented in standard Croco oceanic model
- ▷ **Perspectives :**
  - Transfer toward operational centers (in progress with Mercator-Ocean)

Demange J., L. Debreu, F. Lemarié, P. Marchesiello, E. Blayo: *Stability analysis of split-explicit free surface ocean models: implication of the depth-independent barotropic mode approximation*, submitted to J. Comp. Phys.

# 2

**A non-hydrostatic non-Boussinesq algorithm, the how and why**

# Classical formulation of a non-hydrostatic oceanic model

## 2D linearized Non hydrostatic (Boussinesq) equations

$$\begin{aligned}\partial_t u &= -\partial_x p / \rho_0 \\ \partial_t w &= -(\partial_z p - \rho g) / \rho_0 \\ \partial_x u + \partial_z w &= 0 \\ \rho &= \rho_{bq}(\theta, S, -\rho_0 g z)\end{aligned}$$

Pressure decomposition :

$$p = p_a + p_H + q, \quad p_H = \rho_0 g \eta + g \int_z^0 (\rho_{bq} - \rho_0) dz'$$

Boundary conditions :

$$\partial_t \eta = w(0), \quad w(-H) = 0, \quad p(z=0) = \rho_0 g \eta \rightarrow q(z=0) = 0$$

$q$  : non-hydrostatic pressure which cancels the divergent part of velocity field

# Classical formulation of a non-hydrostatic oceanic model

Interaction with barotropic mode

## Homogeneous linearized equations

$$\partial_x u + \partial_z w = 0$$

$$\partial_t u = -g\partial_x \eta - \partial_x q / \rho_0$$

$$\partial_t w = -\partial_z q / \rho_0$$

$$\partial_t \eta = w(0) = -H\partial_x \bar{u}$$

## Implicit algorithm

1. Advance  $\eta$  with  $\bar{q}^* = 0$  or  $\bar{q}^* = \bar{q}^n$

$$\frac{\eta^{n+1} - \eta^n}{\Delta t} - gH\Delta t\partial_x^2 \eta^{n+1} = -H\partial_x \bar{u}^n + \frac{\Delta t H}{\rho_0} \partial_x^2 \bar{q}^*$$

2. Compute provisional velocity field

$$\tilde{u}^{n+1} = u^n - \Delta t g \partial_x \eta^{n+1}, \quad \tilde{w}^{n+1} = w^n$$

3. Solve  $\Delta q = \frac{\rho_0}{\Delta t} (\partial_x \tilde{u}^{n+1} + \partial_z \tilde{w}^{n+1})$

4. Correct velocity field to remove divergent part

$$u^{n+1} = \tilde{u}^{n+1} - \Delta t \partial_x q, \quad w^{n+1} = \tilde{w}^{n+1} - \Delta t \partial_z q$$

But in the end :  $\frac{\eta^{n+1} - \eta^n}{\Delta t} \neq -H\partial_x \bar{u}^{n+1}$

→ constancy preservation for tracers is lost + 2D/3D inconsistencies

# Classical formulation of a non-hydrostatic oceanic model

## Interaction with barotropic mode

### Homogeneous linearized equations

$$\partial_x u + \partial_z w = 0$$

$$\partial_t u = -g \partial_x \eta - \partial_x q / \rho_0$$

$$\partial_t w = -\partial_z q / \rho_0$$

$$\partial_t \eta = w(0) = -H \partial_x \bar{u}$$

### Implicit algorithm (Casulli; 1999)

1. Compute provisional  $\eta$  with  $\bar{q}^* = 0$  or  $\bar{q}^* = \bar{q}^n$

$$\frac{\tilde{\eta}^{n+1} - \eta^n}{\Delta t} - gH \Delta t \partial_x^2 \tilde{\eta}^{n+1} = -H \partial_x \bar{u}^n + \frac{\Delta t H}{\rho_0} \partial_x^2 \bar{q}^*$$

...

5. Correct free-surface to satisfy B.C.

$$\frac{\eta^{n+1} - \eta^n}{\Delta t} = -H \partial_x \bar{u}^{n+1}$$

Still 2D/3D inconsistencies (may require a decrease of  $\Delta t_{3D}$ )

# Classical formulation of a non-hydrostatic oceanic model

Interaction with barotropic mode

## Homogeneous linearized equations

$$\partial_x u + \partial_z w = 0$$

$$\partial_t u = -g \partial_x \eta - \partial_x q / \rho_0$$

$$\partial_t w = -\partial_z q / \rho_0$$

$$\partial_t \eta = w(0) = -H \partial_x \bar{u}$$

Split-explicit algorithm  $0 \leq m \leq N_{\text{split}} - 1$

1. Advance  $\eta$  and  $\bar{u}$  with  $\bar{q}^* = 0$  or  $\bar{q}^* = \bar{q}^n$

$$\begin{cases} \bar{u}^{m+1} &= \bar{u}^m - g(\delta t) \partial_x \eta^m - \frac{\delta t}{\rho_0} \partial_x \bar{q}^* \\ \eta^{m+1} &= \eta^m - \delta t H \partial_x \bar{u}^{m+1}, \end{cases}$$

2. Compute provisional fields  $\tilde{u}^{n+1}$  and  $\tilde{w}^{n+1}$

3. Correct  $\tilde{u}^{n+1}$  to enforce  $\overline{\tilde{u}^{n+1}} = \bar{u}^{n+1}$

4. Solve  $\Delta q = \frac{\rho_0}{\Delta t} (\partial_x \tilde{u}^{n+1} + \partial_z \tilde{w}^{n+1})$

5. Correct velocity field to remove divergent part

$$u^{n+1} = \tilde{u}^{n+1} - \Delta t \partial_x q, \quad w^{n+1} = \tilde{w}^{n+1} - \Delta t \partial_z q$$

However :  $\bar{u}^{n+1} \neq \overline{u^{n+1}}$

# Classical formulation of a non-hydrostatic oceanic model

Interaction with barotropic mode

## Homogeneous linearized equations

$$\partial_x u + \partial_z w = 0$$

$$\partial_t u = -g \partial_x \eta - \partial_x q / \rho_0$$

$$\partial_t w = -\partial_z q / \rho_0$$

$$\partial_t \eta = w(0) = -H \partial_x \bar{u}$$

Split-explicit algorithm  $0 \leq m \leq N_{\text{split}} - 1$

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3. Correct  $\tilde{u}^{n+1}$  to enforce  $\overline{\tilde{u}^{n+1}} = \bar{u}^{n+1}$

4. Solve  $\Delta q = \frac{\rho_0}{\Delta t} (\partial_x \tilde{u}^{n+1} + \partial_z \tilde{w}^{n+1})$

5. Correct velocity field to remove divergent part

$$u^{n+1} = \tilde{u}^{n+1} - \Delta t \partial_x q, \quad w^{n+1} = \tilde{w}^{n+1} - \Delta t \partial_z q$$

However :  $\bar{u}^{n+1} \neq \overline{u^{n+1}}$

Solution 1 : change boundary condition on  $q$  to  $\partial_z q|_{z=0} = 0$

$$\Rightarrow \bar{u}^{n+1} = \overline{\tilde{u}^{n+1}} = \overline{u^{n+1}}$$

# Classical formulation of a non-hydrostatic oceanic model

Interaction with barotropic mode

## Homogeneous linearized equations

$$\partial_x u + \partial_z w = 0$$

$$\partial_t u = -g\partial_x \eta - \partial_x q / \rho_0$$

$$\partial_t w = -\partial_z q / \rho_0$$

$$\partial_t \eta = w(0) = -H\partial_x \bar{u}$$

Split-explicit algorithm  $0 \leq m \leq N_{\text{split}} - 1$

1. Advance  $\eta$  and  $\bar{u}$  with  $\bar{q}^* = 0$  or  $\bar{q}^* = \bar{q}^n$

$$\begin{cases} \bar{u}^{m+1} &= \bar{u}^m - g(\delta t)\partial_x \eta^m - \frac{\delta t}{\rho_0}\partial_x \bar{q}^* \\ \eta^{m+1} &= \eta^m - \delta t H \partial_x \bar{u}^{m+1}, \end{cases}$$

2. Compute provisional fields  $\tilde{u}^{n+1}$  and  $\tilde{w}^{n+1}$

3. Correct  $\tilde{u}^{n+1}$  to enforce  $\overline{\tilde{u}^{n+1}} = \bar{u}^{n+1}$

4. Solve  $\Delta q = \frac{\rho_0}{\Delta t} (\partial_x \tilde{u}^{n+1} + \partial_z \tilde{w}^{n+1})$

5. Correct velocity field to remove divergent part

$$u^{n+1} = \tilde{u}^{n+1} - \Delta t \partial_x q, \quad w^{n+1} = \tilde{w}^{n+1} - \Delta t \partial_z q$$

However :  $\bar{u}^{n+1} \neq \overline{u^{n+1}}$

Solution 2 : reset  $\bar{u}^{n+1} = \overline{u^{n+1}}$  and  $\frac{\eta^{n+1} - \eta^n}{\Delta t} = -H\partial_x \overline{u^{n+1}}$

$\Rightarrow$  loose the gain in accuracy of the split-explicit algorithm



# Classical formulation of a non-hydrostatic oceanic model

Interaction with barotropic mode and Poisson pressure equation

## Pressure correction / Pressure projection method

- In generalized vertical coordinates, the Poisson pressure equation is hard to solve (25 diagonals, non symmetric)
- Mode splitting: choices
  - ~~relax the compatibility of 2D/3D fluxes~~ (large source of instabilities)
  - relax the true boundary condition  $q(z = 0) = 0$
  - relax the accuracy of the 2D integration (for split-explicit algorithms)
- Likely to require to decrease the baroclinic time-step compared to hydrostatic simulations

# Pseudo-compressible approach (Auclair et al., 2017)

## 2D linearized NH Non-Boussinesq equations

Taylor expansion of density field (with  $\frac{\partial \rho}{\partial p} = c_s^{-2}$ )

$$\rho = \rho(\theta, S, p) = \rho_{\text{bq}}(\theta, S, p_{\text{ref}}) + \underbrace{\frac{\partial \rho}{\partial p} \delta p}_{\delta \rho} + \mathcal{O}(\delta p^2)$$

$$\partial_t u = -\partial_x p / \rho_0$$

$$\partial_t w = -(\partial_z p - \rho g) / \rho_0$$

$$\partial_t \delta \rho = -\rho_0 (\partial_x u + \partial_z w)$$

Pressure decomposition :

$$p = p_a + p_H + c_s^2 \delta \rho, \quad p_H = \rho_0 g \eta + g \int_z^0 (\rho_{\text{bq}} - \rho_0) dz'$$

Boundary conditions :

$$\partial_t \eta = w(0), \quad w(-H) = 0, \quad p(z=0) = \rho_0 g \eta \rightarrow \delta \rho(z=0) = 0$$

## Pseudo-compressible approach (Auclair et al., 2017)

### Homogeneous linearized equations

$$\begin{aligned}\partial_t u &= -g\partial_x \eta - c_s^2 \partial_x \delta\rho \\ \partial_t w &= -c_s^2 \partial_z \delta\rho \\ \partial_t \delta\rho &= -\rho_0(\partial_x u + \partial_z w)\end{aligned}$$

$$\begin{aligned}\partial_t \eta &= w|_{z=0} \\ w|_{z=-H} &= 0 \\ \delta\rho|_{z=0} &= 0\end{aligned}$$

### Semi-implicit forward-backward

$$\begin{aligned}u^{m+1} &= u^m - \delta t (g\partial_x \eta^m + c_s^2 \partial_x \delta\rho^m) \\ w^{m+1} &= w^m - \delta t c_s^2 \partial_z (\delta\rho^{m+\theta}) \\ \delta\rho^{m+1} &= \delta\rho^m - \rho_0 \delta t (\partial_x u^{m+1} + \partial_z w^{m+\theta}) \\ \eta^{m+1} &= \eta^m + \delta t (w|_{z=0})^{m+\theta}\end{aligned}$$

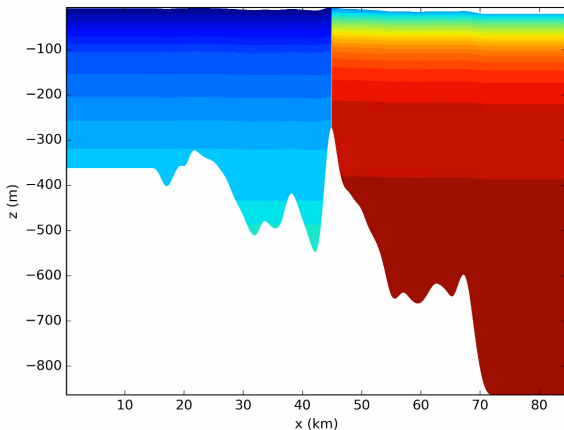
In practice :

- $c_s$  is chosen such that:  $\varepsilon = \frac{\sqrt{gH}}{c_s} \ll 1$
- The acoustic waves are integrated, in a split-explicit free surface approach, at the same level (i.e. with the same time step) than the barotropic mode.

## Advantages of the pseudo-compressible approach (Auclair et al., 2017)

- Surface waves "feel" the NH effects
- Main limiting CFL condition : horizontal acoustic waves propagation  
→ may require to decrease the barotropic time-step
- This approach allows internal waves with high order divergence - pressure gradient computation
- Scales well with the resolution
- Compressibility effects physically significant ?

## A semi-realistic application (Gibraltar strait)



### Simulation characteristics

- NH-NBQ Croco model
- $\Delta x = 150$  m
- 40 vertical layers
- 5-days simulation
- Forced by tides at boundaries

→ F. Auclair, L. Bordoïs

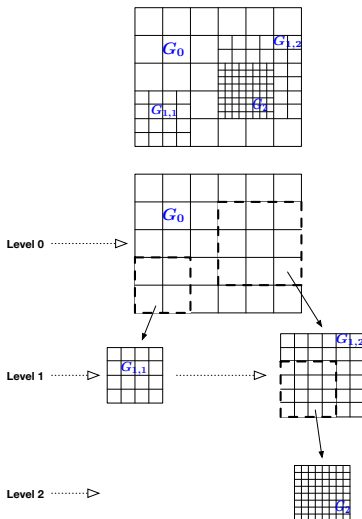
⇒ Robust enough to be used for arbitrary realistic simulations

# 3

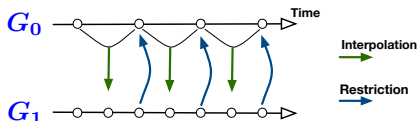
## **Multiresolution simulations using block structured mesh refinement**

# Mesh refinement in structured grid models

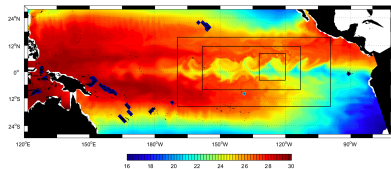
Berger & Olinger algorithm



- Time integration



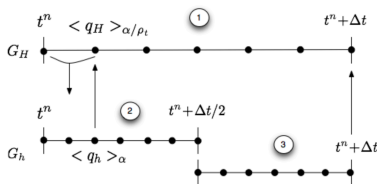
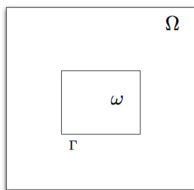
- Possibility to locally adjust the physics, the numerics, the geometry, etc.



Marchesiello et al., 2012, OM

## "Standard" nesting in Croco

- ▷ Full two-way coupling (i.e. at the barotropic time-step level)
  - model solution unaffected by nesting when the refinement coefficient is one.
- ▷ Local space and time refinement (unlimited number of grids)
- ▷ Fully conservative (volume and tracer via refluxing)
- ▷ Implemented via the AGRIF library <http://agrif.imag.fr/>



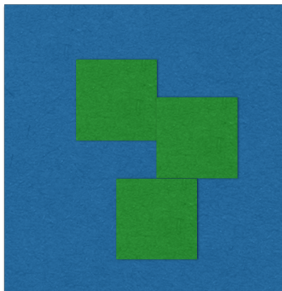
Debreu et al., *Two-way nesting in split-explicit ocean models: Algorithms, implementation and validation*, Ocean Model. (2012)



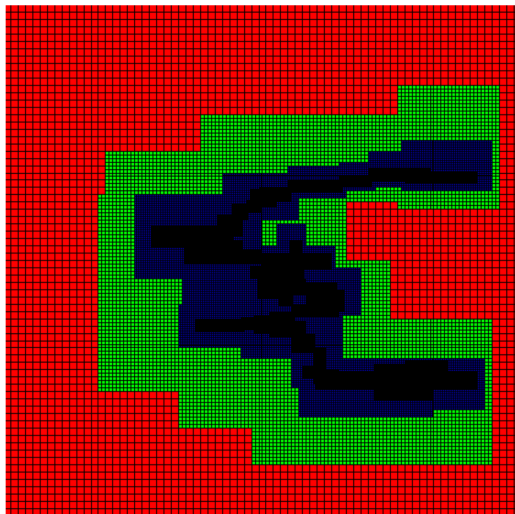
# From nesting to truly multiresolution

## What is needed for multiresolution ?

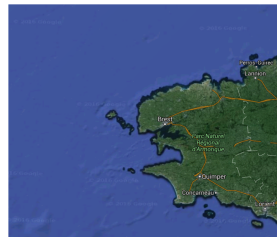
- Neighboring/overlapping grids connection
- Bathymetry smoothing
- Refinement criterion
  - e.g. distance to the coast
- Load balancing on parallel computers
- Inputs / Outputs and visualization
  - Inputs : online interpolation
  - Outputs: Boxlib Format
  - Visualization : VisIt  
<https://visit.llnl.gov>



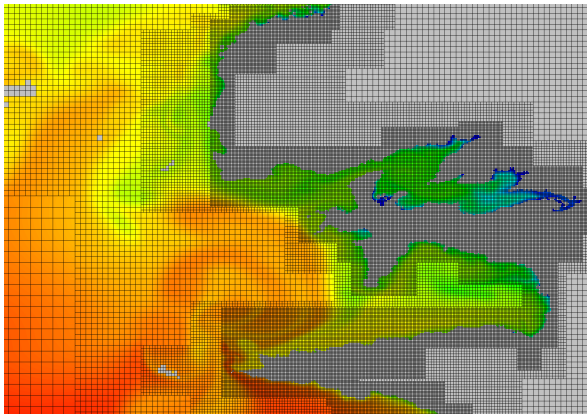
## A realistic application (westernmost tip of Brittany)



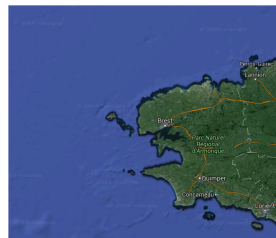
From 2.8 km to 350 m



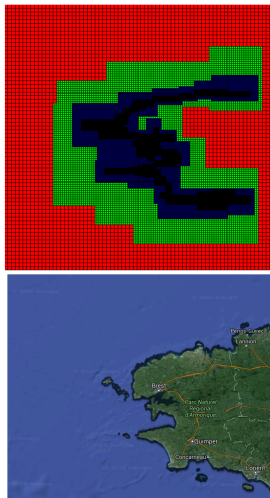
## A realistic application (westernmost tip of Brittany)



From 800 m to 100 m



## A realistic application (westernmost tip of Brittany)



### 4 levels of refinement

- **Grid resolutions**

- Level 1 : 2.8 km
- Level 2 : 1.4 km
- Level 3 : 700 m
- Level 4 : 350 m

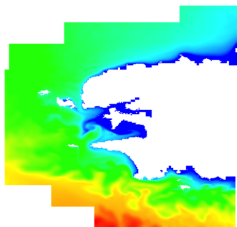
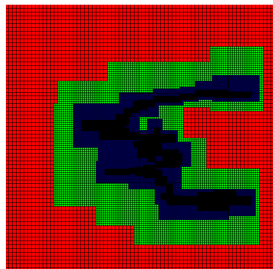
- **Area covered by the different levels**

- Level 1 : 100%
- Level 2 : 43%
- Level 3 : 18%
- Level 4 : 7%

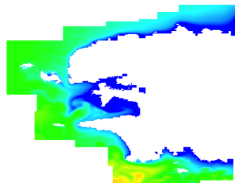
- **Number of grids per levels**

- Level 1 : 1
- Level 2 : 8
- Level 3 : 20
- Level 4 : 46

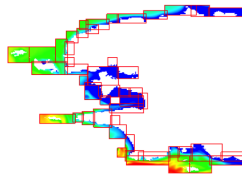
## A realistic application (westernmost tip of Brittany)



Level 2



Level 3

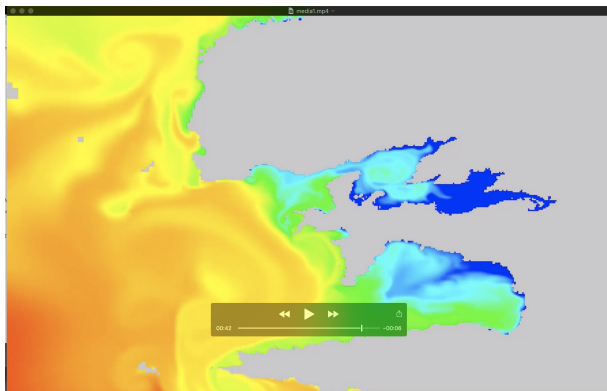


Level 4

- Minimum number of grid points per grids : 256 (16x16)
- Percentage of land points at the finer level : 20%
- Cost of intergrid operations : 15%

## A realistic application

Sea surface temperature (multiresolution from  $\Delta x = 800$  m to  $\Delta x = 100$  m)



### Simulation characteristics

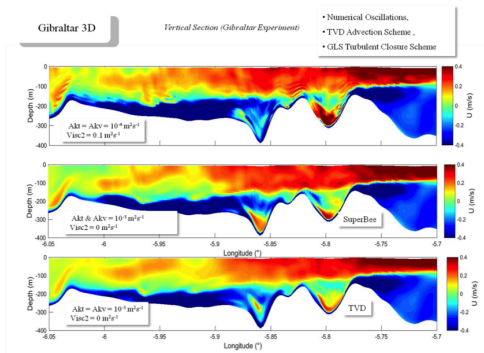
- PE Croco model
- Refinement of 2
- 4 levels of refinement
- Tides
- Wetting & drying

# 4

## Future perspectives and concluding remarks

## Some future challenges for Croco

- (Non-hydrostatic) - (Hydrostatic) coupling (for OBCs and nesting)
- 3D closure (LES vs MILES)
- Relative role of truncation errors vs subgrid terms
- Coupling with external components
- Perpetual revision of time-stepping and numerical schemes





# Feedback from model intercomparison

## Experience feedback from COMODO project

### Positive aspects

- Inter-disciplinary collaborations around specific scientific questions
- Useful for code debugging and checking of numerical implementation
- Helpful to prioritize future code developments (e.g. Ilicak et al., 2011)
- Dynamic effect on the community with experience-sharing

### Difficulties/limitations

- Difficult to define reference solutions
- "Good" solutions obtained for the wrong reasons
- Focused on dynamical core, no parameterization
- Generally simple geometries
- Common format for structured and unstructured grid models

## An illustration from COMODO project

- ▷ Computational and numerical aspects

# Numerical and computational aspects

## Stability range

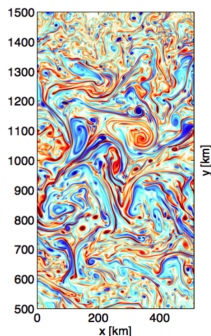
Theoretical stability limit (e.g. [Lemarié et al., 2015](#))

	NEMO	CROCO	Mars3D
Internal gravity waves	0.46	0.85	2
External gravity waves	0.46	0.9	$\infty$
Advection	0.46	0.87	1

## Baroclinic jet testcase

→ time-step constrained by IGW propagation

	NEMO	CROCO	Mars3D
$\Delta t_{3D}$ [s]	200	340	<b>320</b>
$\Delta t_{2D}$ [s]	3	6.1	320



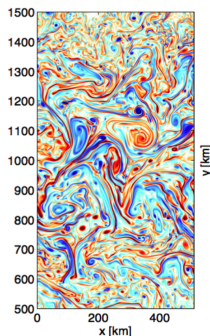
Baroclinic jet testcase  
([Soufflet et al., 2016](#))

→ Development of offline diagnostics to compute stability limits within the Pycomodo tools

# Numerical and computational aspects

Sequential performance (Intel VTune profiler)

	NEMO	CROCO	Mars3D
Memory size	2Gb	800Mb	1,4Gb
Number of instructions	5.5 bil.	3.3 bil.	<b>13.9 bil.</b>
Vectorization (%)	40	<b>78</b>	45
Cache bound <sup>1</sup> (%)	14	14	<b>71</b>
FP Arith./Mem. Rd Instr. <sup>2</sup>	0.56	<b>1.43</b>	0.62
Execution time (s)	<b>609</b>	<b>160</b>	<b>686</b>



<sup>1</sup> percentage of execution time spent in cache memory accesses

<sup>2</sup> floating point arithmetic instructions per Memory Read or Write

→ e.g. results used as a basis for the NEMO development strategy

- COMODO Project: <http://www.comodo-ocean.fr/>  
(now an online shopping site !)
- Pycomodo tools: <http://pycomodo.forge.imag.fr/>



## CROCO

Coastal and Regional Ocean COmmunity model

<https://www.croco-ocean.org/>

