Some Recent Developments around the CROCO Initiative for Complex Regional to Coastal Modeling
Francis Auclair, Rachid Benshila, Laurent Debreu, Nicolas Ducousso, Franck Dumas, Patrick Marchesiello, Florian Lemarié

To cite this version:
Francis Auclair, Rachid Benshila, Laurent Debreu, Nicolas Ducousso, Franck Dumas, et al.. Some Recent Developments around the CROCO Initiative for Complex Regional to Coastal Modeling. COMOD 2018 - Workshop on Coastal Ocean Modelling, Feb 2018, Hambourg, Germany. pp.1-47. hal-01947670

HAL Id: hal-01947670
https://inria.hal.science/hal-01947670
Submitted on 7 Dec 2018

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L’archive ouverte pluridisciplinaire HAL, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d’enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.
Some Recent Developments around the CROCO Initiative for Complex Regional to Coastal Modeling

F. Auclair†, R. Benshila‡, L. Debreu*, N. Ducousso‡, F. Dumas♠, P. Marchesiello†, F. Lemarié*

* Inria (EPI AIRSEA), Laboratoire Jean Kuntzmann, Grenoble, France
† Laboratoire d’Aérologie, Toulouse, France
‡ LEGOS, Toulouse, France
§ Ifremer, Brest, France
♠ National Hydrographic Service, SHOM, Brest, France
Context – the COMODO project (2012-2016; PI: L. Debreu, Inria)
Funded by the french national research agency (9 postdocs/engineers)

- Intercomparison and evaluation of models
- Improved numerical methods
- Definition of a suite of standardized test cases
- Develop associated diagnostic tools (pyCOMODO tools)

Garnier et al., GMD, in prep.
Context – the COMODO project (2012-2016; PI: L. Debreu, Inria)

- Modeling groups involved in the project

Most modeling groups had common objectives for coastal applications (e.g. NH option, flexible horiz. and vert. grids, coupling with waves, etc)

- minimize the duplication of efforts
- promote interoperability across numerical models (via tools like Oasis, XIOS)
the Croco initiative: an outcome from COMODO

- Croco as a way to sustain the COMODO group
- Complementary to NEMO in terms of target applications

- Roms-Agrif numerical kernel
- Online nesting capability via Agrif library
- Non-hydrostatic Non-Boussinesq from S-NH
- Sediment module from Mars3d
- OAW coupling module (shared w. Nemo)
- ALE-type vertical coordinate (ongoing + shared w. Nemo)

Supported by Ifremer, Shom, Cnrs, Ird, Inria
Content: recent developments in CROCO

1. Stability analysis of the mode-splitting and control of numerical dissipation

2. A non-hydrostatic non-Boussinesq algorithm, the how and why

3. Multiresolution simulations using block structured mesh refinement

4. Future perspectives and concluding remarks
Stability analysis of the mode-splitting and control of numerical dissipation
Objectives

Spurious ”numerical mixing” not only associated with the space/time discretization of the advection operator

- **Objective**: characterize the impact of the ”inexact” mode-splitting on the stability of numerical models

**Our approach**: 

1. Stability analysis of the mode splitting technique based on an eigenvector decomposition using the exact barotropic mode
2. Quantify the amount of dissipation required to stabilize the approximative splitting
Normal mode decomposition (e.g. Gill, 1982; Kundu, 1990)

2D linearized primitive equations
\( (u_0 = w_0 = 0, f_{\text{cor}} = 0) \)

\[
\begin{align*}
\frac{\partial_t u}{\rho_0} + \frac{1}{\partial_x} \partial_x \tilde{p} &= 0 \\
\partial_z \tilde{p} &= -\tilde{\rho} g \\
\partial_x u + \partial_z w &= 0 \\
\partial_t \tilde{\rho} + w \frac{d\tilde{\rho}}{dz} &= 0
\end{align*}
\]

with boundary conditions
\[
\begin{align*}
w(z = 0) &= \partial_t \eta & \eta \ll H \\
w(z = -H) &= 0 & \text{(flat bottom)} \\
\tilde{p}(z = 0) &= \rho_0 g \eta
\end{align*}
\]
Normal mode decomposition (e.g. Gill, 1982; Kundu, 1990)

2D linearized primitive equations
\( u_0 = w_0 = 0, \ f_{\text{cor}} = 0 \)

\[
\begin{align*}
\partial_t u + \frac{1}{\rho_0} \partial_x \tilde{p} &= 0 \\
\partial_z \tilde{p} &= -\tilde{\rho} g \\
\partial_x u + \partial_z w &= 0 \\
\partial_t \tilde{\rho} + w \frac{d\tilde{\rho}}{dz} &= 0
\end{align*}
\]

with boundary conditions
\( w(z = 0) = \partial_t \eta \quad \eta \ll H \)
\( w(z = -H) = 0 \quad \text{(flat bottom)} \)
\( \tilde{p}(z = 0) = \rho_0 g \eta \)

Sturm-Liouville eigenvalue problem

\[
\Lambda = -\frac{d}{dz} \left( N^{-2} \frac{d}{dz} \right)
\]

\[
\begin{align*}
\Lambda M_q(z) &= c_q^{-2} M_q(z) \\
\frac{dM_q}{dz}(-H) &= 0 \\
\frac{dM_q}{dz}(0) &= -\frac{N^2(0)}{g} M_q(0)
\end{align*}
\]

\( \langle M_q, M_m \rangle = \delta_{q,m} \) w.r.t. the inner product
\( \langle f, g \rangle = \frac{1}{H} \int_{-H}^{0} fg dz \)
Normal mode decomposition (e.g. Gill, 1982; Kundu, 1990)

2D linearized primitive equations
($u_0 = w_0 = 0$, $f_{\text{cor}} = 0$)

\[
\begin{align*}
\partial_t u + \frac{1}{\rho_0} \partial_x \tilde{p} &= 0 \\
\partial_z \tilde{p} &= -\tilde{\rho} g \\
\partial_x u + \partial_z w &= 0 \\
\partial_t \tilde{\rho} + w \frac{d\tilde{\rho}}{dz} &= 0
\end{align*}
\]

with boundary conditions

\[
\begin{align*}
w(z = 0) &= \partial_t \eta \\
w(z = -H) &= 0 \quad \text{(flat bottom)} \\
\tilde{p}(z = 0) &= \rho_0 g \eta
\end{align*}
\]

Expand the variables $u, \tilde{p}$ in the eigenfunctions $M_q(z)$ of $\Lambda$

\[
\begin{align*}
u(x, z, t) &= \sum_{q=0}^{K-1} u_q(x, t) M_q(z) \\
\tilde{p}(x, z, t) &= g \rho_0 \sum_{q=0}^{K-1} h_q(x, t) M_q(z)
\end{align*}
\]

to obtain a set of uncoupled systems

\[
\begin{align*}
\partial_t u_q + g \partial_x h_q &= 0 \\
\partial_t h_q + \frac{c_q^2}{g} \partial_x u_q &= 0
\end{align*}
\]

In particular

- $q = 0$ → barotropic mode
- $q > 1$ → baroclinic modes
Usual derivation of the barotropic mode in oceanic models
(e.g. Blumberg & Mellor, 1987; Killworth et al., 1991)

Starting from the PEs, horizontal velocities are decomposed as

\[ u = \left( \frac{1}{H} \int_{-H}^{\eta} u \, dz \right) + u_{3d} = \bar{u} + u_{3d}, \quad \int_{-H}^{\eta} u_{3d} \, dz = 0 \]

Standard barotropic mode (associated to the 2D linearized PEs)
Integrating vertically the continuity and momentum equations we end up with

\[ \begin{cases} 
\frac{\partial}{\partial t} \eta + \frac{\partial}{\partial x} (H \bar{u}) &= 0 \\
\frac{\partial}{\partial t} \bar{u} + g \frac{\partial}{\partial x} \eta &= -\frac{1}{\rho_0} \frac{\partial}{\partial x} \left( \frac{1}{H} \int_{-H}^{0} p_h \, dz \right) 
\end{cases} \]

Remark : the red term is a "slow" term absent from the normal mode analysis.

→ In practice it is kept frozen during the barotropic integration
Interpretation in terms of normal modes

The underlying assumption is that the barotropic mode is depth-independent

\[ M_0^*(z) = 1, \quad u_0^*(x, t) = \bar{u}, \quad \int_{-H}^{0} M_q^*(z) dz = 0 \quad (q \geq 1) \]

The orthogonality condition between modes is lost

→ Exact normal mode decomposition → \( K \) independent systems
→ Depth-independent barotropic mode assumption → \( K - 1 \) independent systems (for \( q \geq 1 \)) + a system which includes contributions from all modes
Consequences of the depth-independent assumption

1. Some (fast) barotropic contributions are treated as ”slow” terms

\[
\begin{align*}
\frac{\partial_t \eta + \partial_x (H \bar{u})}{\partial_t \bar{u} + g \partial_x \eta} &= 0 \\
\bar{u} &= -\frac{1}{\rho_0} \partial_x \left( \frac{1}{H} \int^{0}_{-H} p_{h} \, dz \right)
\end{align*}
\]

\[
\frac{1}{H} \int^{0}_{-H} p_{h} \, dz = \frac{1}{H} \int^{0}_{-H} (p - \rho_0 g \eta) \, dz = \rho_0 g \sum_{q=0}^{K-1} \left\{ \frac{1}{H} \int^{0}_{-H} M_{q}(z) - M_{q}(0) \right\} h_{q}
\]

\[
\Rightarrow \text{the contribution of the barotropic mode to the red term is proportional to}
\]

For \( N^2 = \text{cste} \)

\[
\frac{1}{H} \int^{0}_{-H} M_{0}(z) - M_{0}(0) \approx \frac{\varepsilon}{3}, \quad \varepsilon = \sqrt{\frac{N^2 H}{g}}
\]

2. The consistency between the barotropic and baroclinic modes must be enforced since they are no longer independent
Time stepping the coupled baroclinic-barotropic system

→ Dissipation is necessary to stabilize the time integration procedure because some fast contributions are treated with the slow time-step.

**Objective**: determine the minimum amount of dissipation required to stabilize the coupled baroclinic-barotropic system integration
Adding dissipation within the barotropic mode

1. Averaging filters (e.g. Nadiga et al., 1997; Shchepetkin & McWilliams, 2005)

\[
\bar{u}^{n+1,f} = \sum_{m=0}^{N_{\text{filter}}} a_m \bar{u}^m \quad (N_{\text{filter}} \geq N_{\text{split}})
\]

under the constraints

\[
\sum_{m=0}^{N_{\text{filter}}} a_m = 1,
\sum_{m=0}^{N_{\text{filter}}} a_m \frac{m}{N_{\text{split}}} = 1
\]

Several options:
- Flat filter
- Cosine filter

With averaging filters
- the amount of dissipation is somewhat ad-hoc (no quantification of its impact)
- the barotropic integration must go beyond \( n + 1 \)
- the barotropic components are not continuous in time
Adding dissipation within the barotropic mode

1. Averaging filters (e.g. Nadiga et al., 1997; Shchepetkin & McWilliams, 2005)

2. Dissipative time-stepping (e.g. Hallberg, 1997)

In the case of the CROCO ocean model:

→ a generalized forward-backward scheme (AB3-like, AM4-like) is used for the
   barotropic mode (on top of an averaging filter)

\[
\eta^{n+1} = \eta^n - H \Delta t \partial_x \left( \left( \frac{3}{2} + \beta \right) u^n - \left( \frac{1}{2} + 2\beta \right) u^{n-1} + \beta u^{n-2} \right) \\
\bar{u}^{n+1} = \bar{u}^n - g \Delta t \partial_x \left( \delta \eta^{n+1} + \left( 1 - \delta - \gamma - \epsilon \right) \eta^n + \gamma \eta^{n-1} + \epsilon \eta^{n-2} \right)
\]

- **Revised choice**: no averaging filter + additional constraint for the choice of
  \((\delta, \gamma, \beta, \epsilon)\) to guarantee stability of the split-explicit formulation

  ⇒ Minimum level of dissipation to guarantee stability

**Impact on nonlinear simulations?**
Linear testcase \((N = 10^{-3} \, \text{s}^{-1}, \, H = 4000 \, \text{m})\)

Figure: Time evolution of the maximum absolute value of the free surface elevation. Comparaison of usual filters against a reference solution without splitting.
Internal tide test case

- **Testcase**: nonlinear internal tide generation by topography (Marsaleix et al., 2008)
  - 2D x-z, closed boundaries, $L = 4800\text{km}$
  - Start from rest + linear stratification
  - Barotropic seiche at a 12-h period
  - Production of internal waves over the ridge

In the nonlinear case there are other sources of instability:

- The rhs of the barotropic equations is integrated from $-H$ to $\eta$
- Aliasing errors when the advective terms are added
Internal tide test case

Barotropic mechanical energy

\[ E(t) = \int_0^L \frac{\rho_0}{2} (H + \eta) \overline{u^2} \, dx + \int_0^L \frac{\rho_0}{2} g\eta^2 \, dx \]

◮ Comparison of standard ROMS vs modified CROCO scheme (for a fixed baroclinic time-step)
Summary & perspectives on mode splitting analysis

▷ The barotropic mode is traditionally considered as depth independent which is an assumption (e.g. work of Higdon, Bennett, de Szoeke)

▷ CROCO approach:
  → Provide a general framework for the stability analysis of the mode splitting approach
  → The framework allow the design of efficient 2D/3D time stepping algorithms
  → Approach implemented in standard Croco oceanic model

▷ Perspectives:
  → Transfer toward operational centers (in progress with Mercator-Ocean)

A non-hydrostatic non-Boussinesq algorithm, the how and why
Classical formulation of a non-hydrostatic oceanic model

2D linearized Non hydrostatic (Boussinesq) equations

\[
\begin{align*}
\partial_t u &= -\partial_x p / \rho_0 \\
\partial_t w &= - (\partial_z p - \rho g) / \rho_0 \\
\partial_x u + \partial_z w &= 0 \\
\rho &= \rho_{bq}(\theta, S, -\rho_0 g z)
\end{align*}
\]

Pressure decomposition:

\[
p = p_a + p_H + q, \quad p_H = \rho_0 g \eta + g \int_0^z (\rho_{bq} - \rho_0) dz'
\]

Boundary conditions:

\[
\begin{align*}
\partial_t \eta &= w(0), \quad w(-H) = 0, \quad p(z = 0) = \rho_0 g \eta \quad \rightarrow \quad q(z = 0) = 0
\end{align*}
\]

$q$ : non-hydrostatic pressure which cancels the divergent part of velocity field
Classical formulation of a non-hydrostatic oceanic model
Interaction with barotropic mode

Homogeneous linearized equations

\[ \begin{align*}
\partial_x u + \partial_z w &= 0 \\
\partial_t u &= -g\partial_x \eta - \partial_x q / \rho_0 \\
\partial_t w &= -\partial_z q / \rho_0 \\
\partial_t \eta &= w(0) = -H\partial_x \bar{u}
\end{align*} \]

Implicit algorithm

1. Advance \( \eta \) with \( \bar{q}^* = 0 \) or \( \bar{q}^* = \bar{q}^n \)

\[
\frac{\eta^{n+1} - \eta^n}{\Delta t} - gH \Delta t \partial_x^2 \eta^{n+1} = -H \partial_x \bar{u}^n + \frac{\Delta t H}{\rho_0} \partial_x \bar{q}^*
\]

2. Compute provisional velocity field

\[
\tilde{u}^{n+1} = u^n - \Delta t g \partial_x \eta^{n+1}, \quad \tilde{w}^{n+1} = w^n
\]

3. Solve \( \Delta q = \frac{\rho_0}{\Delta t} (\partial_x \tilde{u}^{n+1} + \partial_z \tilde{w}^{n+1}) \)

4. Correct velocity field to remove divergent part

\[
u^{n+1} = \tilde{u}^{n+1} - \Delta t \partial_x q, \quad w^{n+1} = \tilde{w}^{n+1} - \Delta t \partial_z q
\]

But in the end: \( \frac{\eta^{n+1} - \eta^n}{\Delta t} \neq -H \partial_x \bar{u}^{n+1} \)

\( \rightarrow \) constancy preservation for tracers is lost + 2D/3D inconsistencies
Classical formulation of a non-hydrostatic oceanic model

Interaction with barotropic mode

Homogeneous linearized equations

\[ \begin{align*}
\partial_x u + \partial_z w &= 0 \\
\partial_t u &= -g \partial_x \eta - \partial_x q / \rho_0 \\
\partial_t w &= -\partial_z q / \rho_0 \\
\partial_t \eta &= w(0) = -H \partial_x \bar{u}
\end{align*} \]

Implicit algorithm (Casulli; 1999)

1. Compute provisional \( \eta \) with \( \bar{q}^* = 0 \) or \( \bar{q}^* = \bar{q}^n \)

\[ \frac{\eta^{n+1} - \eta^n}{\Delta t} - gH \Delta t \partial_x^2 \tilde{\eta}^{n+1} = -H \partial_x \bar{u}^n + \frac{\Delta t H}{\rho_0} \partial_x^2 \bar{q}^* \]

\[ \vdots \]

5. Correct free-surface to satisfy B.C.

\[ \frac{\eta^{n+1} - \eta^n}{\Delta t} = -H \partial_x \bar{u}^{n+1} \]

Still 2D/3D inconsistencies (may require a decrease of \( \Delta t_{3D} \))
Classical formulation of a non-hydrostatic oceanic model

Interaction with barotropic mode

Homogeneous linearized equations

\[
\begin{align*}
\partial_x u + \partial_z w &= 0 \\
\partial_t u &= -g \partial_x \eta - \partial_x q/\rho_0 \\
\partial_t w &= -\partial_z q/\rho_0 \\
\partial_t \eta &= w(0) = -H \partial_x \bar{u}
\end{align*}
\]

Split-explicit algorithm $0 \leq m \leq N_{\text{split}} - 1$

1. Advance $\eta$ and $\bar{u}$ with $\bar{q}^* = 0$ or $\bar{q}^* = \bar{q}^n$

\[
\begin{align*}
\bar{u}^{m+1} &= \bar{u}^m - g(\delta t) \partial_x \eta^m - \frac{\delta t}{\rho_0} \partial_x \bar{q}^* \\
\eta^{m+1} &= \eta^m - \delta t H \partial_x \bar{u}^{m+1},
\end{align*}
\]

2. Compute provisional fields $\tilde{u}^{n+1}$ and $\tilde{w}^{n+1}$

3. Correct $\tilde{u}^{n+1}$ to enforce $\tilde{u}^{n+1} = \bar{u}^{n+1}$

4. Solve $\Delta q = \frac{\rho_0}{\Delta t} (\partial_x \tilde{u}^{n+1} + \partial_z \tilde{w}^{n+1})$

5. Correct velocity field to remove divergent part

\[
\begin{align*}
\bar{u}^{n+1} &= \tilde{u}^{n+1} - \Delta t \partial_x q, \\
\bar{w}^{n+1} &= \tilde{w}^{n+1} - \Delta t \partial_z q
\end{align*}
\]

However: $\bar{u}^{n+1} \neq \bar{u}^{n+1}$
Classical formulation of a non-hydrostatic oceanic model
Interaction with barotropic mode

Homogeneous linearized equations

\[
\begin{align*}
\partial_x u + \partial_z w & = 0 \\
\partial_t u & = -g \partial_x \eta - \partial_x q/\rho_0 \\
\partial_t w & = -\partial_z q/\rho_0 \\
\partial_t \eta & = w(0) = -H \partial_x \bar{u}
\end{align*}
\]

Split-explicit algorithm \(0 \leq m \leq N_{\text{split}} - 1\)

1. Advance \(\eta\) and \(\bar{u}\) with \(\bar{q}^* = 0\) or \(\bar{q}^* = \bar{q}^n\)
   \[
   \begin{align*}
   \bar{u}^{m+1} & = \bar{u}^m - g(\delta t) \partial_x \eta^m - \frac{\delta t}{\rho_0} \partial_x \bar{q}^* \\
   \eta^{m+1} & = \eta^m - \delta t H \partial_x \bar{u}^{m+1},
   \end{align*}
   \]
2. Compute provisional fields \(\tilde{u}^{n+1}\) and \(\tilde{w}^{n+1}\)
3. Correct \(\tilde{u}^{n+1}\) to enforce \(\tilde{u}^{n+1} = \bar{u}^{n+1}\)
4. Solve \(\Delta q = \frac{\rho_0}{\Delta t} (\partial_x \tilde{u}^{n+1} + \partial_z \tilde{w}^{n+1})\)
5. Correct velocity field to remove divergent part
   \[
   \begin{align*}
   u^{n+1} & = \tilde{u}^{n+1} - \Delta t \partial_x q, \\
   w^{n+1} & = \tilde{w}^{n+1} - \Delta t \partial_z q
   \end{align*}
   \]

However: \(\bar{u}^{n+1} \neq u^{n+1}\)

Solution 1: change boundary condition on \(q\) to \(\partial_z q \big|_{z=0} = 0\)

\[
\Rightarrow \bar{u}^{n+1} = \tilde{u}^{n+1} = u^{n+1}
\]
Classical formulation of a non-hydrostatic oceanic model
Interaction with barotropic mode

Homogeneous linearized equations

\[
\begin{align*}
\partial_x u & + \partial_z w = 0 \\
\partial_t u & = -g \partial_x \eta - \partial_x q / \rho_0 \\
\partial_t w & = -\partial_z q / \rho_0 \\
\partial_t \eta & = w(0) = -H \partial_x \bar{u}
\end{align*}
\]

Split-explicit algorithm \( 0 \leq m \leq N_{\text{split}} - 1 \)

1. Advance \( \eta \) and \( \bar{u} \) with \( q^* = 0 \) or \( q^* = \bar{q}^n \)

\[
\begin{align*}
\bar{u}^{m+1} & = \bar{u}^m - g(\delta t) \partial_x \eta^m - \frac{\delta t}{\rho_0} \partial_x \bar{q}^* \\
\eta^{m+1} & = \eta^m - \delta t H \partial_x \bar{u}^{m+1},
\end{align*}
\]

2. Compute provisional fields \( \tilde{u}^{n+1} \) and \( \tilde{w}^{n+1} \)

3. Correct \( \tilde{u}^{n+1} \) to enforce \( \tilde{u}^{n+1} = \bar{u}^{n+1} \)

4. Solve \( \Delta q = \frac{\rho_0}{\Delta t} (\partial_x \tilde{u}^{n+1} + \partial_z \tilde{w}^{n+1}) \)

5. Correct velocity field to remove divergent part

\[
\begin{align*}
\bar{u}^{n+1} & = \tilde{u}^{n+1} - \Delta t \partial_x q, \\
\bar{w}^{n+1} & = \tilde{w}^{n+1} - \Delta t \partial_z q
\end{align*}
\]

However: \( \bar{u}^{n+1} \neq \bar{u}^{n+1} \)

Solution 2: reset \( \bar{u}^{n+1} = \bar{u}^{n+1} \) and \( \eta^{n+1} = \eta^{n+1} - \frac{\eta^{n+1} - \eta^n}{\Delta t} = \Delta t \partial_x \bar{u}^{n+1} \)

\( \Rightarrow \) loose the gain in accuracy of the split-explicit algorithm
Classical formulation of a non-hydrostatic oceanic model
Interaction with barotropic mode and Poisson pressure equation

Pressure correction / Pressure projection method

- In generalized vertical coordinates, the Poisson pressure equation is hard to solve (25 diagonals, non symmetric)

- Mode splitting: choices
  - relax the compatibility of 2D/3D fluxes (large source of instabilities)
  - relax the true boundary condition \( q(z = 0) = 0 \)
  - relax the accuracy of the 2D integration (for split-explicit algorithms)

- Likely to require to decrease the baroclinic time-step compared to hydrostatic simulations
Pseudo-compressible approach (Auclair et al., 2017)

2D linearized NH Non-Boussinesq equations

Taylor expansion of density field (with $\frac{\partial \rho}{\partial p} = c_s^{-2}$)

$$
\rho = \rho(\theta, S, p) = \rho_{bq}(\theta, S, p_{\text{ref}}) + \frac{\partial \rho}{\partial p} \delta p + \mathcal{O}(\delta p^2)
$$

$$
\begin{align*}
\partial_t u &= -\frac{\partial_x p}{\rho_0} \\
\partial_t w &= -\left(\partial_z p - \rho g\right)/\rho_0 \\
\partial_t \delta \rho &= -\rho_0 (\partial_x u + \partial_z w)
\end{align*}
$$

Pressure decomposition:

$$
p = p_a + pH + c_s^2 \delta \rho, \quad pH = \rho_0 g \eta + g \int_0^z (\rho_{bq} - \rho_0) dz'
$$

Boundary conditions:

$$
\partial_t \eta = w(0), \quad w(-H) = 0, \quad p(z = 0) = \rho_0 g \eta \quad \rightarrow \quad \delta \rho(z = 0) = 0
$$
Pseudo-compressible approach (Auclair et al., 2017)

Homogeneous linearized equations

\[
\begin{align*}
\partial_t u &= -g \partial_x \eta - c_s^2 \partial_x \delta \rho \\
\partial_t w &= -c_s^2 \partial_z \delta \rho \\
\partial_t \delta \rho &= -\rho_0 (\partial_x u + \partial_z w) \\
\partial_t \eta &= w \big|_{z=0} \\
\left. w \right|_{z=-H} &= 0 \\
\left. \delta \rho \right|_{z=0} &= 0
\end{align*}
\]

In practice:

- \( c_s \) is chosen such that: \( \epsilon = \frac{\sqrt{gH}}{c_s} \ll 1 \)

- The acoustic waves are integrated, in a split-explicit free surface approach, at the same level (i.e. with the same time step) than the barotropic mode.

Semi-implicit forward-backward

\[
\begin{align*}
\left. u \right|^{m+1} &= u^m - \delta t \left( g \partial_x \eta^m + c_s^2 \partial_x \delta \rho^m \right) \\
\left. w \right|^{m+1} &= w^m - \delta t c_s^2 \partial_z \left( \delta \rho^{m+\theta} \right) \\
\left. \delta \rho \right|^{m+1} &= \delta \rho^m - \rho_0 \delta t \left( \partial_x u^{m+1} + \partial_z w^{m+\theta} \right) \\
\left. \eta \right|^{m+1} &= \eta^m + \delta t \left( \left. w \right|_{z=0} \right)^{m+\theta}
\end{align*}
\]
Advantages of the pseudo-compressible approach (Auclair et al., 2017)

- Surface waves "feel" the NH effects
- Main limiting CFL condition: horizontal acoustic waves propagation → may require to decrease the barotropic time-step
- This approach allows internal waves with high order divergence - pressure gradient computation
- Scales well with the resolution
- Compressibility effects physically significant?
A semi-realistic application (Gibraltar strait)

Simulation characteristics

- NH-NBQ Croco model
- $\Delta x = 150$ m
- 40 vertical layers
- 5-days simulation
- Forced by tides at boundaries

$\Rightarrow$ Robust enough to be used for arbitrary realistic simulations

$\Rightarrow$ F. Auclair, L. Bordois
Multiresolution simulations using block structured mesh refinement
Mesh refinement in structured grid models
Berger & Oliger algorithm

- Time integration

- Possibility to locally adjust the physics, the numerics, the geometry, etc.

Marchesiello et al., 2012, OM
”Standard” nesting in Croco

- Full two-way coupling (i.e. at the barotropic time-step level)
  → model solution unaffected by nesting when the refinement coefficient is one.
- Local space and time refinement (unlimited number of grids)
- Fully conservative (volume and tracer via refluxing)

- Implemented via the AGRIF library [http://agrif.imag.fr/]

From nesting to truly multiresolution

What is needed for multiresolution?

- Neighboring/overlapping grids connection
- Bathymetry smoothing
- Refinement criterion
  - e.g. distance to the coast
- Load balancing on parallel computers
- Inputs / Outputs and visualization
  - Inputs: online interpolation
  - Outputs: Boxlib Format
  - Visualization: VisIt
    https://visit.llnl.gov
A realistic application (westernmost tip of Brittany)

From 2.8 km to 350 m
A realistic application (westernmost tip of Brittany)

From 800 m to 100 m
A realistic application (westernmost tip of Brittany)

4 levels of refinement

- **Grid resolutions**
  - Level 1: 2.8 km
  - Level 2: 1.4 km
  - Level 3: 700 m
  - Level 4: 350 m

- **Area covered by the different levels**
  - Level 1: 100%
  - Level 2: 43%
  - Level 3: 18%
  - Level 4: 7%

- **Number of grids per levels**
  - Level 1: 1
  - Level 2: 8
  - Level 3: 20
  - Level 4: 46
A realistic application (westernmost tip of Brittany)

- Minimum number of grid points per grids: 256 (16x16)
- Percentage of land points at the finer level: 20%
- Cost of intergrid operations: 15%
A realistic application

Sea surface temperature (multiresolution from $\Delta x = 800 \text{ m}$ to $\Delta x = 100 \text{ m}$)

Simulation characteristics
- PE Croco model
- Refinement of 2
- 4 levels of refinement
- Tides
- Wetting & drying
Future perspectives and concluding remarks
Some future challenges for Croco

- (Non-hydrostatic) - (Hydrostatic) coupling (for OBCs and nesting)
- 3D closure (LES vs MILES)
- Relative role of truncation errors vs subgrid terms
- Coupling with external components
- Perpetual revision of time-stepping and numerical schemes
Feedback from model intercomparison
Experience feedback from COMODO project

Positive aspects
- Inter-disciplinary collaborations around specific scientific questions
- Useful for code debugging and checking of numerical implementation
- Helpful to prioritize future code developments (e.g. Ilicak et al., 2011)
- Dynamic effect on the community with experience-sharing

Difficulties/limitations
- Difficult to define reference solutions
- ”Good” solutions obtained for the wrong reasons
- Focused on dynamical core, no parameterization
- Generally simple geometries
- Common format for structured and unstructured grid models

An illustration from COMODO project

▷ Computational and numerical aspects
Numerical and computational aspects
Stability range

Theoretical stability limit (e.g. Lemarié et al., 2015)

<table>
<thead>
<tr>
<th></th>
<th>NEMO</th>
<th>CROCO</th>
<th>Mars3D</th>
</tr>
</thead>
<tbody>
<tr>
<td>Internal gravity waves</td>
<td>0.46</td>
<td>0.85</td>
<td>2</td>
</tr>
<tr>
<td>External gravity waves</td>
<td>0.46</td>
<td>0.9</td>
<td>∞</td>
</tr>
<tr>
<td>Advection</td>
<td>0.46</td>
<td>0.87</td>
<td>1</td>
</tr>
</tbody>
</table>

Baroclinic jet testcase

→ time-step constrained by IGW propagation

<table>
<thead>
<tr>
<th></th>
<th>NEMO</th>
<th>CROCO</th>
<th>Mars3D</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta t_{3D}[s]$</td>
<td>200</td>
<td>340</td>
<td>320</td>
</tr>
<tr>
<td>$\Delta t_{2D}[s]$</td>
<td>3</td>
<td>6.1</td>
<td>320</td>
</tr>
</tbody>
</table>

→ Development of offline diagnostics to compute stability limits within the Pycomodo tools
Numerical and computational aspects
Sequential performance (Intel VTune profiler)

<table>
<thead>
<tr>
<th></th>
<th>NEMO</th>
<th>CROCO</th>
<th>Mars3D</th>
</tr>
</thead>
<tbody>
<tr>
<td>Memory size</td>
<td>2Gb</td>
<td>800Mb</td>
<td>1,4Gb</td>
</tr>
<tr>
<td>Number of instructions</td>
<td>5.5 bil.</td>
<td>3.3 bil.</td>
<td>13.9 bil.</td>
</tr>
<tr>
<td>Vectorization (%)</td>
<td>40</td>
<td>78</td>
<td>45</td>
</tr>
<tr>
<td>Cache bound(^1) (%)</td>
<td>14</td>
<td>14</td>
<td>71</td>
</tr>
<tr>
<td>FP Arith./Mem. Rd Instr.(^2)</td>
<td>0.56</td>
<td><strong>1.43</strong></td>
<td>0.62</td>
</tr>
<tr>
<td>Execution time (s)</td>
<td>609</td>
<td>160</td>
<td>686</td>
</tr>
</tbody>
</table>

\(^1\) percentage of execution time spent in cache memory accesses
\(^2\) floating point arithmetic instructions per Memory Read or Write

→ e.g. results used as a basis for the NEMO development strategy
• COMODO Project: http://www.comodo-ocean.fr/ (now an online shopping site !)
• Pycomodo tools: http://pycomodo.forge.imag.fr/

https://www.croco-ocean.org/