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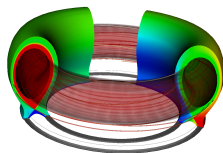
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# Tokamesh : A software for mesh generation in Tokamaks

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- High anisotropy ( $10^9$ ) in magnetized fusion plasmas
- Tokamak is an axisymmetric machine :  
$$\mathbf{B} = F(\psi)\nabla\phi + \nabla\psi \times \nabla\phi$$
 $\psi$  is the magnetic flux

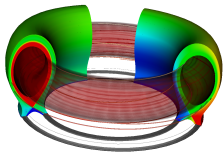
⇒

- use Fourier discretization in the toroidal direction
- use flux aligned meshes in the poloidal direction

# EoCoE WP5.1 task: Constructing flux-surface aligned mesh-grids in the poloidal plane

## Task WP5.1 : Propose unified software for

- different codes and models
  - gyrokinetic (Gysela)
  - Fluid (Jorek, Tokam3X, SOLPS, SOLEdge, Plato )
- different type of meshes
  - block structured - non structured
- different numerical methods
  - Semi-Lagrangian approaches
  - Finite difference, Finite volume
  - $C^0$  Finite Element (Lagrange, spectral elements)
  - $C^1$  Finite Element (Spline or Hermite-Bezier on quadrangles, Powell-Sabin-Clough-Tocher on triangles )



# Computing the flux surfaces : Grad-Shafranov Equilibrium solver

## The problem

Generate meshes adapted to the magnetic flux surfaces  
(anisotropy)

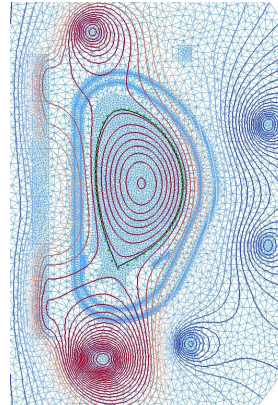
First part of the problem : Compute the magnetic flux surfaces

Magnetic field Ansatz :  $\mathbf{B} = F(\psi)\nabla\phi + \nabla\psi \times \nabla\phi$

Force balance  $\nabla p = \mathbf{J} \times \mathbf{B}$  and toroidal symmetry yield

Non-linear elliptic free boundary problem :

$$-\nabla \left( \frac{1}{\mu r} \nabla \psi \right) = \mathbf{J}_{\text{toro}} = \begin{cases} \mathbf{J}_{\text{plasma}}(x, \psi) & \text{in plasma} \\ \mathbf{J}_{\text{coil}}(\text{voltage}, \partial_t \psi) & \text{in coils} \\ \text{vanishing} & \text{elsewhere} \end{cases}$$



# Tokamesh work flow

- ① Input from equilibrium codes (EFIT, Cedres++) solving Grad-Shafranov equation
  - get flux field data
- ② Automated domain segmentation
  - Decomposition of the domain into patches homeomorphic to a logical square
- ③ Construction of meshes-grids of individual patches
- ④ Gluing of patches to obtain the global mesh

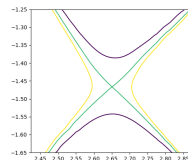
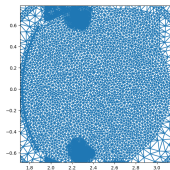
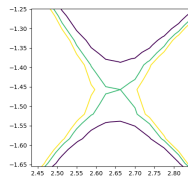
# Tokamesh work flow

- 1 Input from equilibrium code (EFIT, Cedres++) solving Grad-Shafranov equation
  - get flux field data
- 2 **Data regularization**
- 3 Automated domain segmentation
  - Decomposition of the domain into patches homeomorphic to a logical square
- 4 Construction of meshes-grids of individual patches
- 5 Gluing of patches to obtain the global mesh

# Data regularization

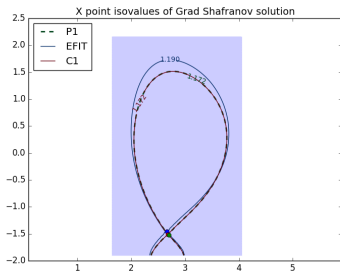
Results from standard equilibrium solvers are not smooth enough

- use of low-order discretization
- coarse resolution
- Mesh refinement around x-points +  $\mathcal{C}^1$  Splines interpolation on triangles (Clough-Tocher)





# Free boundary Grad-Shafranov Equilibrium



Free boundary Grad-Shafranov Equilibrium solver using Clough-Tocher  $C^1$  finite element method on triangular meshes

- Regularize the data coming from low-order GS solver
- Recompute an equilibrium using  $C^1$  high-order method

# Domain decomposition

## Segmentation problem :

Given a domain  $\Omega$  : Find  $n$  patches  $\Omega_n$  such that :

$$\Omega = \cup \Omega_n \quad \text{and} \quad \overset{\circ}{\Omega}_n \cap \overset{\circ}{\Omega}_m = \emptyset$$

and there exists a one to one mapping between  $\Omega_n$  and the the unit square :

$$\exists f_n : [0, 1] \times [0, 1] \leftrightarrow \Omega_n$$

In general : extremely difficult question ( $10^3$  of papers on this question)

Additional constraint : we want Flux aligned grids

The problem becomes much easier :

**use Morse theory**

Morse. M, The Calculus of Variations in the Large, American Mathematical Society Colloquium Publication 18; New York, (1934)



# Morse theory

A function  $f$  is a Morse function if all its critical points are regular.

## Critical points

Let  $C^r$  be the space of  $r$  differentiable scalar field defined on  $\Omega$ . For  $r \geq 2$ , a point  $\mathbf{p} \in \Omega$  is a critical or singular point of  $f$  if  $\nabla f = 0$ .

## Regular Critical points

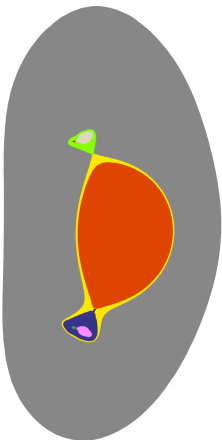
A critical point is regular (or non-degenerate) if the Hessian of  $f$  at  $\mathbf{p}$  is invertible.

## In 2-D

if we note  $\mu_1, \mu_2$  the two eigenvalues of  $H_f$  the only possible critical points of a Morse function are therefore :

Maxima	index = 2	$\mu_1 < 0$	$\mu_2 < 0$
Saddles	index = 1	$\mu_1 < 0$	$\mu_2 > 0$
Minima	index = 0	$\mu_1 > 0$	$\mu_2 > 0$

# Morse theory



**Mountainer theorem :** Let  $f$  be a Morse function defined on  $\Omega$ , a region defined by a closed iso-contour, then the number of critical points (counting a virtual extremum) verifies the relation :

$$C_M - C_S + C_m = 2$$

**Topology :** Let  $f$  be a Morse function defined on  $\Omega$ . Then the topological set of the iso-contours of  $f$  consists of finite connected components that are either

- Circle cells which are homeomorphic to open disks
- Circle bands which are homeomorphic to open annulus
- Saddle connections

**Stability :** Let  $f$  be a Morse function defined on  $\Omega$ . This field is structurally stable if and only if all saddle points are self-connected.

# Morse theory and Reeb Graph

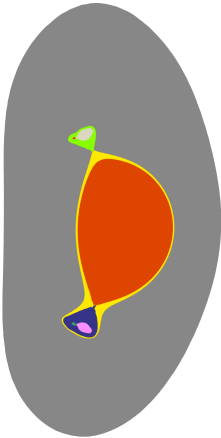


Figure: Domain decomposition

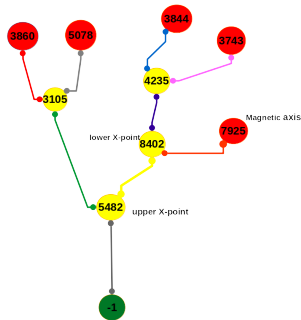
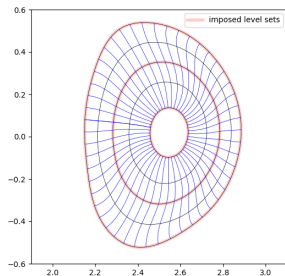


Figure: Associated Reeb graph : Each edge corresponds to a subdomain

# Algorithm for block structured mesh generation

## Meshing individual patches



Into each subdomain the level set  $f^{-1} = c$  is unique and closed

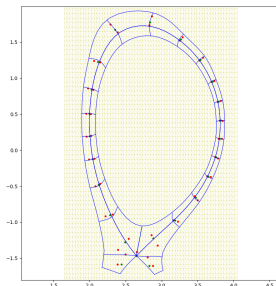
- Choose a finite number of level sets  $f^{-1}(c_i), i = 1, \dots, l$
- Approximate each level set by a cubic spline :  $f^{-1}(c_i) \sim C_i(t) = \sum_j \mathbf{P}_j^i N_j(t)$  :  $N_j$  spline basis function  $\mathbf{P}_j^i$  : control points
- Construct a 2D tensor product mapping from  $[0, 1] \times [0, 1] \rightarrow \Omega_k$   $S(s, t) = \sum_{i=1}^l \sum_{j=1}^J \mathbf{P}_{i,j} N_i(s) N_j(t)$
- such that

$$\forall i = 1, \dots, l \quad S(s_i, t) = C_i(t)$$

Note that  $\mathbf{P}_{i,j} \neq \mathbf{P}_j^i$

# Algorithm for block structured mesh generation

## $\mathcal{G}^1$ Gluing of patches



Given 2 subdomains  $\Omega_1$  and  $\Omega_2$  described by 2 mappings

$$S^1(s, t) = \sum_{i=1}^{I^1} \sum_{j=1}^J \mathbf{P}_{i,j}^1 N_i(s) N_j(t)$$

$$S^2(s, t) = \sum_{i=1}^{I^2} \sum_{j=1}^J \mathbf{P}_{i,j}^2 N_i(s) N_j(t)$$

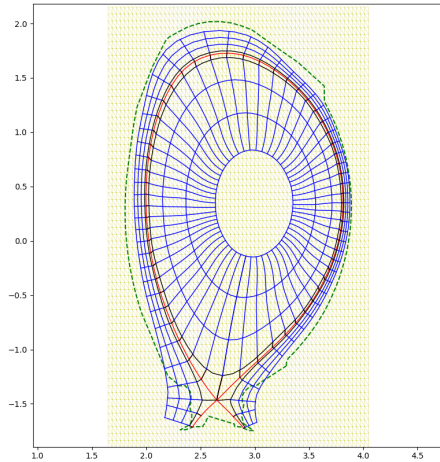
with a common boundary for instance at  $s = 0$

$$S^1(0, t) = S^2(0, t)$$

- $\mathcal{G}^0$  continuity :  $\mathbf{P}_{0,j}^1 = \mathbf{P}_{0,j}^2 \quad \forall j$
- $\mathcal{G}^1$  continuity : geometric condition on the control points  $\mathbf{P}_{1,j}^1, \mathbf{P}_{0,j}^1 = \mathbf{P}_{0,j}^2, \mathbf{P}_{1,j}^2$  have to be aligned.

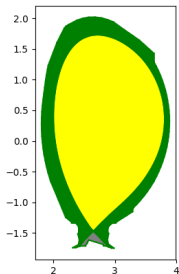


# $\mathcal{G}^1$ meshes : example



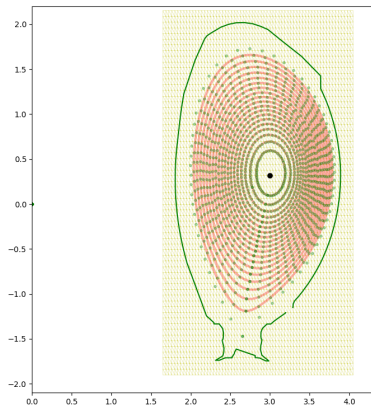


# Algorithm for non-structured triangular mesh generation



- Segmentation of the domain
- choice of a set of isolines into each subdomain (depend on user's interest)
- cubic spline approximation of these isolines
- node sampling on each isolines (no need to have the same number of nodes on each isoline) → cloud a nodes
- triangulation of the resulting set of nodes by a **constrained anisotropic** Delaunay algorithm

# Algorithm for non-structured triangular mesh generation



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# Algorithm for non-structured triangular mesh generation

## Constrained Anisotropic Delaunay algorithm

### Standard Delaunay algorithm

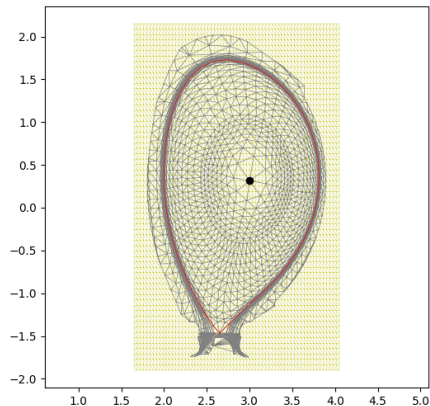
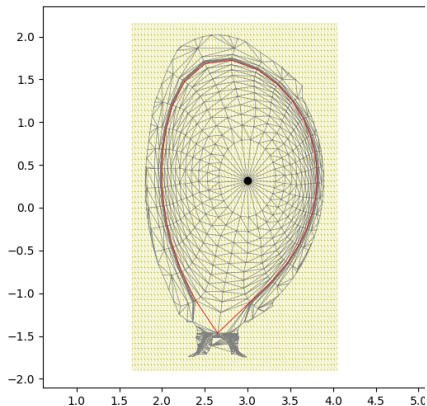
empty sphere property : no point is inside the circumcircle of any triangle

### Constrained Anisotropic Delaunay algorithm

Some edges of the triangulation are forced to exist in the triangulation  
The distance are evaluated using Riemannian metric instead of Euclidian one

$$d(\mathbf{a}, \mathbf{b}) = \sqrt{(\mathbf{b} - \mathbf{a})^t M^t \Lambda M (\mathbf{b} - \mathbf{a})}$$

# Algorithm for non-structured triangular mesh generation



# Conclusions

- Design of a mesh generation software for tokamak simulations
  - can be used by different codes
  - handle different types of meshes
  - written in python with C and FORTRAN bindings
    - using open source libraries
    - (Except feflo.a but free for academics)
- future developments
  - GUI
  - extension for unstructured P2- P3 meshes
  - extends the library to geophysical applications.

[git@gitlab.inria.fr:EuCoE\\_Mesh/tokamesh.git](mailto:git@gitlab.inria.fr:EuCoE_Mesh/tokamesh.git)